Fracture mechanics, Damage and Fatigue Overview

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Before fracture mechanics

- Design with stresses lower than
  - Elastic limit ($\sigma^0_p$) or
  - Tensile strength ($\sigma_{TS}$)

- ~1860, Wöhler
  - Technologist in the German railroad system
  - Studied the failure of railcar axles
    - Failure occurred
      - After various times in service
      - At loads considerably lower than expected

- Failure due to cyclic loading/unloading
  - « Total life » approach
  - **Empirical** approach of fatigue
Before fracture mechanics

- **Empirical approach: Total life**
  - Life of a structure depends on

  - Minimal & maximal stresses: \( \sigma_{\text{min}} \) & \( \sigma_{\text{max}} \)
  - Mean stress: \( \sigma_m = (\sigma_{\text{max}} + \sigma_{\text{min}})/2 \)
  - Amplitude: \( \sigma_a = \Delta\sigma/2 = (\sigma_{\text{max}} - \sigma_{\text{min}})/2 \)
  - Load Ratio: \( R = \sigma_{\text{min}} / \sigma_{\text{max}} \)
  - Under particular environmental conditions (humidity, high temperature, …):
    - Frequency of cycles
    - Shape of cycles (sine, step, …)
Before fracture mechanics

- **Total life approach**
  - Stress life approach
    - For structures experiencing (essentially) elastic deformations
      - For \( \sigma_m = 0 \) & \( N_f \) identical cycles before failure
        - \( \sigma_e \): endurance limit (life\(>10^7 \) cycles)
          - \( \sigma_e \sim [0.35; 0.5] \sigma_{TS} \)
        - 1910, Basquin law
          \[
          \frac{\Delta \sigma}{2} = \sigma_a = \sigma'_f (2N_f)^b
          \]
          - \( \sigma'_f \): fatigue coefficient (mild steel \( T_{amb} \): \( \sim [1; 3] \) GPa)
          - \( b \): fatigue exponent (mild steel \( T_{amb} \): \( \sim [-0.1; -0.06] \))
    - Strain life approach
      - For structures experiencing (essentially) large plastic deformations
      - For \( N_f \) identical cycles before failure
        - 1954, Manson-Coffin
          \[
          \frac{\Delta \bar{\varepsilon}_p}{2} = \varepsilon'_f (2N_f)^c
          \]
        - \( \varepsilon'_f \): fatigue ductility coefficient \( \sim \) true fracture ductility (metals)
        - \( c \): fatigue ductility coefficient exponent \( \sim [-0.7, -0.5] \) (metals)
        - \( \Delta \bar{\varepsilon}_p \): plastic strain increment during the loading cycles
Design using total life approach

• 1952, De Havilland 106 Comet 1, UK (1)
  – First jetliner, 36 passengers, pressurized cabin (0.58 atm)
  – Wrong aerodynamics at high angle of attack (takeoff)
    • 1953, 2 crashes: lift loss due to swept wing and air intakes inefficient

  – The fuselage was designed using total life approach
    • 1952, a fuselage was tested against fatigue
      – Static loading at 1.12 atm, followed by
      – 10 000 cycles at 0.7 atm (> cabin pressurization at 0.58 atm)

  – Design issue
    • 1953, India, crash during storm
      – « Structural failure » of the stabilizer
      – The pilot does not “feel” the forces due to the fully powered controls (hydraulically assisted)
      – Fatigue due to overstress ?
• 1952, De Havilland 106 Comet 1, UK (2)
  – More design issues
    • 1954, January, flight BOAC 781 Rome-Heathrow
      – Plane G-ALYP disintegrated above the sea
      – After 1300 flights
      – Autopsies of passengers’ lungs revealed explosive decompression
      – Bomb? Turbine failure?
        → turbine rings with armor plates
    • 1954, April, flight SAA 201 Rome-Cairo
      – Plane G-ALYY disintegrated
    • 1954, April, reconstruction of plane ALYP from the recovered wreckages
      – Proof of fracture, but origin unknown
    • 1954, April, test of fuselage ALYU in water tank
      – Pressurization cycles of the cabin simulated
      – Rupture at port window after only 3057 pressurization cycles
  – Total life approach failed
    • Fuselages failed well before the design limit of 10000 cycles
• **1952, De Havilland 106 Comet 1, UK (3)**
  – 1954, August, ALYP roof retrieved from sea
    • Origin of failure at the communication window
    • Use of square riveted windows
    • Punched riveting instead of drill riveting
      ➞ Existence of initial defects

• **The total life approach**
  – Accounts for crack initiation in smooth specimen
  – Does not account for inherent defects
    • Metal around initial defects could have hardened during the initial static test load of the fatigue tested fuselage
    • Production planes without this static test load …

• **Life time can be improved by**
  – “Shoot peening”: surface bombarded by small spherical media
    • Compression residual stresses in the surface layer
    • Prevents crack initiation
  – Surface polishing (to remove cracks)

• **1958, Comet 3 et 4**
  – Round windows glued
  – Fuselage thicker
What is fracture mechanics?

- **Back to physics**
  - Bonding in Crystals (attractive forces)
    - Molecular or van den Waals:
      - Interaction between dipoles
      - Ar, polymers, C₆H₆, graphite
      - Bonds easily break
    - Ionic
      - Interaction between ions
      - NaCl, …
      - Coulombic non directional forces
      - Hard & brittle crystals
    - Covalent
      - Bonds between atoms
      - CH₄, Diamond
      - Directional and strong forces
      - Hard & brittle crystals
    - Metallic
      - Sea of donated valence electrons (conductors)
      - Metals & alloys
      - Non-directional forces
What is fracture mechanics?

- **Back to physics (2)**
  - Free electrons model of a crystal structure
    - Atomic radius $r$
    - Bonding in Crystals
      - Attractive potentials differ from the bonding kinds, but same shape
      - Example metallic ions and electronic clouds

$$U_a = -M \frac{z_1 z_2 q^2}{4 \pi \varepsilon_0 r} = -\frac{A_a}{r} \quad \Rightarrow \quad f_a = \frac{A_a}{r^2}$$

$z_i$: valences, $\varepsilon_0=8.85$ pF/m: permittivity, $q=1.602 \times 10^{-19}$ C: electronic charge;

$M$: Madelung constant depends on the geometric arrangement in the crystal

- Other: Lennard-Jones (van den Waals), Morse (diatomic molecule), …

- Repulsive energy due to the interaction of electronic clouds & nuclei

$$U_r = b \lambda \exp \left( \frac{-r}{\rho} \right) = A_r \exp \left( \frac{-r}{\rho} \right) \quad \Rightarrow \quad f_r = -\frac{A_r}{\rho} \exp \left( \frac{-r}{\rho} \right)$$

$b$: number of adjacent ions, $\lambda$ (eV), $\rho$ (nm): repulsive parameters
What is fracture mechanics?

- Back to physics (3)
  - Free electrons model of a crystal structure (2)
    - Equilibrium & maximal force (NaCl)

1 eV = 1.60217646 × 10⁻¹⁹ J

Equilibrium at \( r = r_0, U_0 \)

\[
U_0 = -\frac{A_a}{r_0} \left[ 1 - \frac{\rho}{r_0} \right]
\]

Maximal force at \( r = r_*, f_{\text{max}} \)

\[
f_{\text{max}} = \frac{A_a}{r_*^2} \left[ 1 - \frac{\rho}{r_*} \right]
\]

\[
r_*^3 \exp \frac{-r_*}{\rho} = \frac{2A_a \rho^2}{A_r}
\]
What is fracture mechanics?

- From free electrons model to macroscopic behavior
  - Depends on the crystal lattice
  - Metallic crystal tends to be packed \( a_0^u \): size of the unit cell

- Examples:
  - Body-Centered-Cubic crystal
  - Face-Centered-Cubic crystal

Fe at low \( T^\circ \), Ferrite (low C-steel)
Fe at high \( T^\circ \), Austenite (low C-steel at high \( T^\circ \)), Al, Cu
What is fracture mechanics?

- From free electrons model to macroscopic behavior (2)
  - Stress

\[ \sigma = \frac{1}{S_{\text{ref}}(\text{plane}, r_0)} \sum_i f_i \]

- Young modulus

\[ E = \lim_{dr \to 0} \frac{\sigma(r_0 + dr) - \sigma(r_0)}{dr} r_0 \]

- Surface energy

\[ 2\gamma_s = \frac{N_{\text{rupture}} U_0}{S_{\text{ref}}} \] (\( \gamma_s \): energy to create a unit surface, cleavage: 2 surfaces \( S_{\text{ref}} \) are created)

- Depend on the lattice & on the plane orientation
  - Example: plane \( S(1,1,0) \) in BCC
    - \( S_{\text{ref}} = 2^{1/2} a_0^2 \)
    - \( N_{\text{rupture}} = 4 \times 1/8 \) (corner) + 2 x 1/8 (center)

- Theoretical tensile strength \( \sigma_{\text{Th}} \)
  - Planes of atoms separate at once
  - Brittle materials

Low representation: atoms are actually “in contact”
What is fracture mechanics?

• Theoretical tensile strength

- Sinus approximation

\[ \sigma = \sigma_{Th} \sin \left( \pi \frac{a^u - a_0^u}{\delta} \right) \]

\[ \frac{E}{a_0^u} \frac{d\sigma}{da^u} \bigg|_{a_0^u} = \sigma_{Th} \frac{\pi}{\delta} \]

\[ 2\gamma_s = \int_{a_0^u}^{a_0^u+\delta} \sigma \, da^u = 2\sigma_{Th} \frac{\delta}{\pi} \]

• Theoretical tensile strength of brittle materials

\[ \sigma_{Th} = \sqrt{\frac{E\gamma_s}{a_0^u}} \]
What is fracture mechanics?

- Inherent defects play a major role in fracture mechanics
  - Theoretical tensile strength $\sigma_{\text{Th}}$ vs Tensile strength $\sigma_{\text{TS}}$

<table>
<thead>
<tr>
<th></th>
<th>$a_u^0$ [nm]</th>
<th>$E$ [GPA]</th>
<th>$\gamma_s$ [J m$^{-2}$]</th>
<th>$\sigma_{\text{Th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>0.3</td>
<td>60</td>
<td>21</td>
<td>64 GPa !</td>
</tr>
<tr>
<td>Steel (low T)</td>
<td>0.3</td>
<td>210</td>
<td>3400</td>
<td>1500 GPa !</td>
</tr>
</tbody>
</table>

- Theoretical values are much higher than observed ones
- 1920, Griffith studied the influence of a crack of size $2a$ on $\sigma_{\text{TS}}$
  - Brittle materials only
  - Analytically (see later on): $\sigma_{\text{TS}}\sqrt{a} \div \sqrt{E \cdot 2\gamma_s}$
  - Verified experimentally on glass fibers
    - Initially scratched to create a crack $2a$
    - Thermal treatment to remove the residual stresses
  - Important breakthrough
    - The strength depends on the defects size
    - Manufacturers improved the surface finishing (polishing, …)
    - Fracture mechanics was pioneered
Brittle / ductile fracture

• 2 kinds of behavior
  – Brittle:
    • (Almost) no plastic deformations prior to (macroscopic) failure
  – Ductile:
    • Plastic deformations prior to (macroscopic) failure
  – Different microscopic behaviors
Brittle / ductile fracture

• Deformation mechanisms
  – Elasticity

  - Results from bonds stretching
  - Reversible
  - Linear (metals, …) or non-linear (polymers, …)
Brittle / ductile fracture

- **Deformation mechanisms (2)**
  - **Plasticity**
    - The specimen experiences permanent deformations
    - Irreversible
    - Combination of
      - Bonds stretching (reversible part)
      - Plane shearing (irreversible part)

- What is the plane shearing mechanisms?
**Brittle / ductile fracture**

- **Deformation mechanisms (3)**
  - What is the plane shearing mechanisms?
    - Slip of a whole crystal lattice?

![Diagram of crystal lattice shearing](image)

- Back to sine model
  - Because of symmetry: $\tau = 0$ at $\delta = 0$, $r_0$, $2r_0$ → $\tau = \tau_{\text{max}} \sin \frac{\pi \delta}{r_0}$
  - Shear modulus: $G = \frac{d\tau}{d\delta/a_0^u}$ $|_{\delta=0} = \tau_{\text{max}} \frac{\pi a_0^u}{r_0}$
  - BCC: $\tau_{\text{max}} = \frac{G \sqrt{3}}{4\pi}$
  - Fe: $G = 82$ GPa → $\tau_{\text{max}} = 11$ GPa !!!
• Deformation mechanisms (4)
  – What is the plane shearing mechanisms (2) ?
    • Propagation of dislocations
      – A dislocation is characterized by
        » The Burger vector (difference between the distorted lattice around the dislocation and the perfect lattice)
        » Dislocation line (line along which the distortion is the largest)
        » Slip plane (plane where the dislocation motion occurs)
  – Edge dislocation
    • Dislocation line $\perp$ to Burger vector
Brittle / ductile fracture

- **Deformation mechanisms** (5)
  - Screw dislocation & Mixed dislocation

- **Example:**
  - Plastically deformed zinc single crystal: \( \tau = \sigma \cos \lambda \cos \phi \)
Brittle / ductile fracture

- Dislocations and brittle/ductile materials
  - Cristal with ionic bonding (NaCl, …)
    - Motion of dislocations difficult
      - A + would be in front of another +
      - Cleavage before \( \rightarrow \) brittle
  - Covalent bonding (Si, diamond, …)
    - Motion of dislocations difficult
      - Strong & directional bonding
      - Cleavage before \( \rightarrow \) brittle
  - Metallic bonding
    - Motion of dislocations possible
      - Non directional bonding
    - Motion along packed slip directions
      - Brittle or ductile?

BCC:
- 6 slip planes
- \( \times 2 \) directions=
- 12 slip systems

FCC:
- 4 slip planes
- \( \times 3 \) directions=
- 12 slip systems
Brittle / ductile fracture

- Are metallic bondings leading to ductile behavior?
  - FCC (Al, Cu, …)
    - Many close-packed planes
    - Dislocations motion possible
    - Always ductile
  - BCC (Ferrite with Low C)
    - No close-packed plane
    - Only directions are close-packed
    - Low $T^\circ$: atoms do not have enough energy to move
    - High $T^\circ$: atoms have enough energy for dislocations motion (Peierls stress)
    - There is a ductile/brittle transition temperature (DBTT)
  - Dislocations motion can be reduced by obstacles → harder & less ductile
    - Substitutional solutes (Sn in Cu, …, duralumin)
    -Interstitial solutes (Cr or V in Fe, …)
    - Grain boundaries (small crystal size after cold rolling: Hall-Petch effect)
    - Precipitate particles (Martensite Body-Centered-Tetragonal in ferrite after quench)
    - Other dislocations (created after cold work)
Brittle / ductile fracture

- **Mechanism of brittle failure**
  - (Almost) no plastic deformations prior to the (macroscopic) failure
  - Cleavage: separation of crystallographic planes
    - In general inside the grains
    - Preferred directions: low bonding
    - Between the grains: corrosion, H$_2$, ...
  - Rupture criterion
    - 1920, Griffith: \( \sigma_{TS} \sqrt{a} \div \sqrt{E \cdot 2\gamma_s} \)
Brittle / ductile fracture

- **Mechanism of ductile failure**
  - Plastic deformations prior to (macroscopic) failure of the specimen
    - Dislocations motion $\rightarrow$ void nucleation around inclusions $\rightarrow$ micro cavity coalescence $\rightarrow$ crack growth
    - Is Griffith criterion $\sigma_{TS} \sqrt{a} \div \sqrt{E \left(2\gamma_s + W_{pl}\right)}$ still correct?
    - 1950, Irwin, the plastic work at the crack tip should be added to the surface energy:

$$\sigma_{TS} \sqrt{a} \div \sqrt{E \left(2\gamma_s + W_{pl}\right)}$$
Brittle / ductile fracture: Liberty ships

• **WWII**
  
  – Steel at low $T^\circ$: brittle
    
    $\sigma_{TS} \sqrt{a} \div \sqrt{E \cdot 2\gamma_s}$ with $\gamma_s \sim 3400 \text{ J m}^{-2}$
  
  – Steel at room $T^\circ$: ductile
    
    $\sigma_{TS} \sqrt{a} \div \sqrt{E \cdot (2\gamma_s + W_{pl})}$ with $2\gamma_s + W_{pl} \sim 200 \text{ kJ m}^{-2}$
  
  – Use of low-grade steel
    
    • In cold weather:
      
      DBTT $\sim$ water temperature
    
    • When put in water existing cracks lead to failure
    
    • 30% of the liberty ships suffered from fracture
Brittle / ductile fracture

• Failure mode of polymers: rupture & disentangling of molecules
  – Behavior depends on the covalent chain structure
    • Aligned cross linked or networked (epoxy):
      – Always amorphous but directional
      – Brittle response
Brittle / ductile fracture

- Failure mode of polymers: rupture & disentangling of molecules (2)
  - Behavior depends on the covalent chain structure (2)
    - Semi crystalline (Plexiglas, PVC):
      - Linear polymers can locally crystallize into lamellar thin plates (chains fold back and forth)
      - Behavior strongly depends on temperature

![Diagram showing crystalline and amorphous regions](image)

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Brittle / ductile fracture

• Failure mode of polymers: rupture & disentangling of molecules (3)
  – Behavior depends on the covalent chain structure (3)
    • Elastomer (Rubber):
      – Amorphous chains are kinked and heavily cross linked
      – Reversible behavior

Initial: amorphous chains are kinked, heavily cross-linked.

Final: chains are straight, still cross-linked.
• Composites
  – Fibers in a matrix
    • Fibers: polymers, metals or ceramics
    • Matrix: polymers, metals or ceramics
    • Fibers orientation: unidirectional, woven, random
  – Complex failure modes
    • Transverse matrix fracture
    • Longitudinal matrix fracture
    • Fiber rupture
    • Fiber debonding
    • Delamination
    • Macroscopically: no plastic deformation
Why fracture mechanics?

- **Limits of the total life approach**
  - Does not account for inherent defect
    - What is happening when a defect is present?
    - Theoretical stress concentration?
      - Infinite plane with an ellipsoidal void (1913, Inglis)
        \[ \sigma_{\text{max}} = \sigma_{yy}(a,0) = \sigma_\infty \left( 1 + \frac{2a}{b} \right) \]
        - As \( b \to 0 \Rightarrow \sigma_{\text{max}} \to \infty \Rightarrow \text{breaks for } \sigma_\infty \to 0 \)
      - In contradiction with Griffith and Irwin experiments
        \[ \sigma_{TS} \sqrt{a} \div \sqrt{E \left( 2\gamma_s + W_p \right)} \]
  - Development of the fracture mechanics field
    - How can we predict failure when a crack is there?
    - Microscopic observations for cycling loading
      - Crack initiated at stress concentrations (nucleation)
      - Crack growth
      - Failure of the structure when the crack reaches a critical size
      - How can we model this?
Linear Elastic Fracture Mechanics (LEFM)

- Singularity at crack tip for linear and elastic materials
  - 1957, Irwin, 3 fracture modes

  **Mode I** (opening)
  \[
  \begin{align*}
  \sigma_{zz} &= 0 \quad \text{or} \quad \varepsilon_{zz} = 0 \\
  \sigma_{yy}(\theta = \pm \pi) &= 0 \\
  \sigma_{xy}(\theta = \pm \pi) &= 0 \\
  u_x(\theta > 0) &= u_x(\theta < 0) \\
  u_y(\theta > 0) &= -u_y(\theta < 0)
  \end{align*}
  \]

  **Mode II** (sliding)
  \[
  \begin{align*}
  \sigma_{zz} &= 0 \quad \text{or} \quad \varepsilon_{zz} = 0 \\
  \sigma_{yy}(\theta = \pm \pi) &= 0 \\
  \sigma_{xy}(\theta = \pm \pi) &= 0 \\
  u_x(\theta > 0) &= -u_x(\theta < 0) \\
  u_y(\theta > 0) &= u_y(\theta < 0)
  \end{align*}
  \]

  **Mode III** (shearing)
  \[
  \begin{align*}
  \sigma_{xx} = \sigma_{xy} = \sigma_{yy} = \sigma_{zz} &= 0 \\
  \sigma_{yy}(\theta = \pm \pi) &= 0 \\
  \sigma_{xy}(\theta = \pm \pi) &= 0 \\
  u_y &= u_x = 0 \\
  u_z(\theta > 0) &= -u_z(\theta < 0)
  \end{align*}
  \]

- Boundary conditions
Singularity at crack tip for linear and elastic materials (2)

- Asymptotic solutions (Airy functions, see next week)

\[ \sigma_{xx} = \frac{C}{\sqrt{r}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{3\theta}{2} \sin \frac{\theta}{2} \right] + \mathcal{O} \left( r^0 \right) \]
\[ \sigma_{yy} = \frac{C}{\sqrt{r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{3\theta}{2} \sin \frac{\theta}{2} \right] + \mathcal{O} \left( r^0 \right) \]
\[ \sigma_{xy} = \frac{C}{\sqrt{r}} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \sin \frac{\theta}{2} + \mathcal{O} \left( r^0 \right) \]

\[ u_x = \frac{C (1 + \nu)}{E} \sqrt{r} \cos \left( \frac{\theta}{2} \right) \left[ \kappa - 1 + 2 \sin^2 \left( \frac{\theta}{2} \right) \right] \]
\[ u_y = \frac{C (1 + \nu)}{E} \sqrt{r} \sin \left( \frac{\theta}{2} \right) \left[ \kappa - 1 - 2 \cos^2 \left( \frac{\theta}{2} \right) \right] \]

with for plane \( \sigma \quad \kappa = \frac{3 - \nu}{1 + \nu} \)

& plane \( \varepsilon \quad \kappa = 3 - 4\nu \)

- Introduction of the Stress Intensity Factors - SIF (Pa m^{1/2})

\[ K_I = \lim_{r \to 0} \left( \sqrt{2\pi r} \sigma_{yy}^{\text{mode I}} \bigg|_{\theta=0} \right) = C \sqrt{2\pi} \]
\[ K_{II} = \lim_{r \to 0} \left( \sqrt{2\pi r} \sigma_{xy}^{\text{mode II}} \bigg|_{\theta=0} \right) = C \sqrt{2\pi} \]
\[ K_{III} = \lim_{r \to 0} \left( \sqrt{2\pi r} \sigma_{yz}^{\text{mode III}} \bigg|_{\theta=0} \right) = C \sqrt{2\pi} \]

- \( K_i \) are dependant on both loading and geometry

\[ \sigma_{\text{mode } i} = \frac{K_i}{\sqrt{2\pi r}} f^{\text{mode } i} (\theta) \]
\[ u_{\text{mode } i} = K_i \sqrt{\frac{r}{2\pi}} g^{\text{mode } i} (\theta) \]
• New failure criterion
  – 1957, Irwin, crack propagation
    • $\sigma_{\text{max}} \rightarrow \infty \quad \Rightarrow \quad \sigma$ is irrelevant
    • If $K_i = K_{ic} \quad \Rightarrow$ crack growth
  – Toughness (ténacité) $K_{ic}$
    • Steel, Al, … : see figures
    • Concrete: 0.2 - 1.4 MPa m$^{1/2}$ (brittle failure)

Diagram:
- Linear Elastic Fracture Mechanics (LEFM)
- Fracture Mechanics - Overview
- Yield $\sigma^0_y$ [MPa]
- Toughness $K_{ic}$ [MPa$\sqrt{m}$]
- Temperature $T$ [°C]
- Brittle
- Ductile
- Brittle/brittle transition regime
• **Stress Intensity Factor (SIF)**
  – Computation of the SIFs $K_i$
    • Analytical (crack $2a$ in an infinite plane)
      \[
      \begin{align*}
      K_I &= \sigma_\infty \sqrt{\pi a} \\
      K_{II} &= \tau_\infty \sqrt{\pi a} \\
      K_{III} &= \tau_\infty \sqrt{\pi a}
      \end{align*}
      \]
    • For other geometries or loadings
      \[
      \begin{align*}
      K_I &= \beta_I \sigma_\infty \sqrt{\pi a} \\
      K_{II} &= \beta_{II} \tau_\infty \sqrt{\pi a} \\
      K_{III} &= \beta_{III} \tau_\infty \sqrt{\pi a}
      \end{align*}
      \]
    • $\beta_i$ obtained by
      – Superposition
      – FEM
      – Energy approach
        » Related to Griffith’s work
        \[
        \sigma_{TS} \sqrt{a} \div \sqrt{E(2\gamma_s + W_{pl})}
        \]
        » See next slides
  – For 2 loadings $a \& b$: $K_I = K_I^a + K_I^b$
  – **BUT** for 2 modes $K \neq K_I + K_{II}$
• Measuring $K_{lc}$
  - Done by strictly following the ASTM E399 procedure
  • A possible specimen is the Single Edge Notch Bend (SENB)
    - Plane strain constraint (thick enough specimen) conservative (see later)
    - Specimen machined with a V-notch in order to start a sharp crack
  • Cyclic loading to initiate a fatigue crack
  • Toughness test performed with
    - A calibrated $P$ - $\delta$ recording equipment
    - The Crack Mouth Opening Displacement (CMOD=$v$) is measured with a clipped gauge
    - $P_c$ is obtained on $P$-$v$ curves
      » either the 95% offset value or
      » the maximal value reached before
    - $K_{lc}$ is deduced from $P_c$ using
      \[
      K_I = \frac{PL}{tW^{\frac{3}{2}}} f \left( \frac{a}{W} \right)
      \]
      » $f(a/W)$ depends on the test (SENB, …)
**Linear Elastic Fracture Mechanics (LEFM)**

- **Energy approach**
  - Mode I
    - Initial crack $2a$
      \[
      \begin{align*}
      \sigma_{yy}^0 (\theta = 0, \ r = x - a) &= \frac{\sigma_\infty \sqrt{a}}{\sqrt{2} (x - a)} \\
      u_y^0 (\theta = 0, \ r = x - a) &= 0
      \end{align*}
      \]
    - Crack grows to $2(a + \Delta a)$
      \[
      u_y^1 (\theta = \pm \pi, \ r = a + \Delta a - x) = \pm \frac{\sigma_\infty (1 + \nu) (\kappa + 1)}{E \sqrt{2}} \sqrt{a + \Delta a} \sqrt{a + \Delta a - x}
      \]
    - Energy is needed for crack to grow by $2\Delta a$ as there is a work done by $\sigma_{yy}$
      \[
      \Delta E_{\text{int}} = -4dz \int_a^{a+\Delta a} \int_{u_y^0}^{u_y^1} \sigma_{yy} du_y dx \quad (x4 \text{ as it is for } x>0, x<0 \text{ & for } y<0, y>0)
      \]
Linear Elastic Fracture Mechanics (LEFM)

- Energy approach (2)
  - Mode I
    - Energy needed for crack to grow by $\Delta a$
      $$\Delta E_{\text{int}} = -4dz \int_a^{a+\Delta a} \int_{u_y^0}^{u_y^1} \sigma_{yy} du_y dx$$
    - Assumption: $\sigma_{yy}$ linear in terms of $u_y$
      $$\Delta E_{\text{int}} = -2dz \int_a^{a+\Delta a} \sigma_{yy}^0 u_y^1 dx$$
    - Change of variable
      $$x = a + \Delta a \cos^2 \theta$$
      $$\Delta E_{\text{int}} = -\frac{dz \sigma_{yy}^2 \sqrt{a (a + \Delta a) (1 + \nu) (1 + \kappa)}}{2E} \pi \Delta a$$
    - $G$ : energy release rate for a straight ahead growth
      $$G = -\frac{dE_{\text{int}}}{dA} = -\lim_{\Delta a \to 0} \frac{\Delta E_{\text{int}}}{2\Delta a dz} = \frac{\pi a \sigma_{yy}^2}{4E} \frac{(1 + \nu) (\kappa + 1)}{E'} = \frac{K_I^2}{E'}$$
    - The crack has been assumed lying in an infinite plane. But
      $$G = \frac{K_I^2}{E'}$$ still holds for other expressions of $K_I$ (see next lectures)
Linear Elastic Fracture Mechanics (LEFM)

- Energy approach (3)
  - 1920, Griffith, energy conservation: 
    \[ E = E_{\text{int}} + \Gamma \]
    - Total energy \( E \) is the sum of the internal (elastic) energy \( E_{\text{int}} \) with the energy \( \Gamma \) needed to create surfaces \( A \)
    \[ \frac{\partial E}{\partial A} = \frac{\partial E_{\text{int}}}{\partial A} + \frac{\partial \Gamma}{\partial A} = 0 \]
    - If \( \gamma_s \) is the surface energy (material property of brittle material) 
      \[ G = G_c = -\frac{dE_{\text{int}}}{dA} = \frac{d\Gamma}{dA} = 2\gamma_s \]
      A crack creates 2 surfaces \( A \)
  - Mode I, infinite plane
    - Strength 
      \[ G = \frac{\pi a \sigma_{\infty}^2}{E'} = \frac{K_I^2}{E'} \]
      \[ \sigma_{TS} = \sqrt{\frac{2\gamma_s E}{\pi a}} \]
      Depend on the crack size 
      \[ \sigma_{TS} \sqrt{a} = 195 \text{kPa} \ m^\frac{1}{2} \]
      \[ \sigma_{TS} \sqrt{a} = 115 \text{MPa} \ m^\frac{1}{2} \]
      - Glass: \( G_c = 2 \gamma_s \sim 2 \text{Jm}^{-2}, E = 60 \text{ GPa} \)
      - Steel: \( G_c = \text{plast. dissipation} \sim 200 \text{ kJm}^{-2}, E = 210 \text{ GPa} \)
  - Straight ahead propagation for general loading
    - Proceeding as for mode I: 
      \[ G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{(1 + \nu) K_{III}^2}{E} \]
      crack growth if 
      \[ G = G_c \]
Linear Elastic Fracture Mechanics (LEFM)

- **Energy approach: $J$-integral**
  - Energy release rate
    - Straight ahead propagation for linear elasticity
    - Should be related to the energy flowing toward the crack tip
  - $J$-integral
    \[ J = \int_{\Gamma} \left[ U(\varepsilon) n_x - u_x \cdot T \right] dl \]
    - Defined even for non-linear materials
    - Is path independent if the contour $\Gamma$ embeds a straight crack tip
    - BUT no assumption on subsequent growth direction
    - If crack grows straight ahead: $G=J$
    - If linear elasticity: $J = K_I^2 E' + K_{II}^2 E' + \frac{(1+\nu)K_{III}^2}{2\mu}$
    - Can be extended to plasticity if no unloading (see later)

- **Advantages**
  - Efficient numerical computation of the SIFs
  - Useful for non perfectly brittle materials
Linear Elastic Fracture Mechanics (LEFM)

- **Direction of crack grow**
  - Assumptions: the crack will grow in the direction where the SIF related to mode I in the new frame is maximal
  - Crack growth if \( \left( \sqrt{2\pi r} \sigma_{\theta\theta} (r, \theta^*) \right) \geq K_C \) with \( \partial_{\theta} \sigma_{\theta\theta} |_{\theta^*} = 0 \)
  - From direction of loading, one can compute the propagation direction

\[
\cot \beta^* = \frac{K_{II}}{K_I} \quad \rightarrow \quad \sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left[ \cos^3 \frac{\theta}{2} - \frac{3 \cot \beta^*}{2} \sin \theta \cos \frac{\theta}{2} \right]
\]
Linear Elastic Fracture Mechanics (LEFM)

- **Non-perfectly brittle materials**
  - Plastic zone
    - Limit $G_c$ becomes $R_c(a)$
    - $R_c(a)$ depends on the loading mode and no longer only on the material
  - Stability of the crack
    - Stable
      \[
      \begin{align*}
      G &= R_c(a) \\
      \frac{dG}{da} &\leq \frac{dR_c}{da}
      \end{align*}
      \]
    - Instable: $a > a^*$
      \[
      \begin{align*}
      \frac{dG}{da} &> \frac{dR_c}{da}
      \end{align*}
      \]
Limits of the LEFM

- The stress still tends toward infinity
  - There are non-linearities (plasticity at crack tips)
  - Far away from the crack the approximation does not hold (structural response)

- Small Scale Yielding assumption
  - Holds if crack front plastic zone size is small compared to the crack length
  - Cohesive zone $r_p$ at crack tip
    \[ a \to a_{\text{eff}} = a + \frac{r_p}{3} \]
  - 1960, Dugdale-Barenblatt
    - Mode I
      \[ r_p = \frac{\pi K_I^2}{8 \sigma_p^2} (a_{\text{eff}}) \]
      Iterative method

- If applied stress larger than half the yield stress
  - The assumption does not hold $\rightarrow$ non linear fracture mechanics (NLFM)
Reactor Pressure Vessels (RPV)

- **Pressurized Water Reactor (PWR)**
  - Typical RPV operating conditions: 15 MPa, 300° C

Image from US. Nuclear Regulatory Commission,
Quadrennial Technology Review 2015, U.S. Department of Energy

Image from US. Nuclear Regulatory Commission,
http://www.eia.doe.gov/cneaf/nuclear/page/nuc_reactors/pwr.html
Reactor Pressure Vessels (RPV)

• RPV material
  – Base material:
    • Manganese–nickel–molybdenum low-alloy steel
    • E.g. SA508 Cl. 3
  – Cladding:
    • Stainless steel

\[\bar{t}_{\text{base}} \approx 0.2 \text{ m} \quad \bar{t}_{\text{clad}} \approx 0.01 \text{ m}\]

\[\phi \approx 3-5 \text{ m}\]

Reactor Pressure Vessels (RPV)

- RPV material: exhibits a DBTT
  - Definition of $T_{100}$: Reference temperature for which 50% of the normalized samples break at a toughness of 100 [MPa√m]

**Reactor Pressure Vessels (RPV)**

- **RPV operations**
  - Pressurized Thermal Shock (PTS)
    - Loss of coolant accident (LOCA)
    - Primary side injection of cold water
    - Sudden decrease of wall temperature
    - Decrease of the toughness
    - N.B. decrease of pressure, but thermal strains

Christopher Boyd, Interactions of Thermal-Hydraulics with Fuel Behavior, Structural Mechanics, and Computational Fluid Dynamics
Office of Nuclear Regulatory Research Nuclear Regulatory Commission
• **Irradiation embrittlement**
  - Irradiation during the RPV operation
  - Embrittlement increasing with
    - The fluence (number of neutrons per unit surface)
    - Neutron energy (>1MeV)

Shift in the DBTT with the operating time

---

### Fracture Mechanics - Overview

<table>
<thead>
<tr>
<th>Temperature $T$ [°C]</th>
<th>Toughness $K_{IC}$ [MPa√m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

Ductile/brittle transition regime

Fictitious material

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Reactor Pressure Vessels (RPV)

Irradiation embrittlement: Shift in DBTT

- Defined by a shift in $T_{100}$
  - $T_0 = T_{100} + \Delta T_{100}$
- Computed in terms of
  - Chemical composition
  - Power of fluence ($\Phi$)
- Measured
  - Surveillance specimen capsules
  - Near the inside vessel wall

Safety assessment

- **Methodology**

**Defects characterization**
- Postulated flaws (following norms)
- Non Destructive in-service inspections (Ultrasonic …)

**Loading conditions**
- Critical loading conditions
  - Evaluation of the temperature, stress … distributions

**Material properties**
- Toughness of base material (experiments)
  - Environmental conditions (end-of-life fluence, temperature)

- Evaluation of SIFs
  - Analytical methods
  - Numerical methods

- In-service toughness
  - Models
  - Experiments
Cyclic loading

- Fatigue failure
  - Tests performed with different $\Delta P = P_{\text{max}} - P_{\text{min}}$
  - Nucleation: cracks initiated for $K < K_c$
    - Surface: deformations result from dislocations motion along slip planes
    - Can also happen around a bulk defect

Persistent slip band (PSB)
Cyclic loading

- **Fatigue failure (2)**
  - Stage I fatigue crack growth:
    - Along a slip plane
  - Stage II fatigue crack growth:
    - Across several grains
      - Along a slip plane in each grain,
      - Straight ahead macroscopically
    - Striation of the failure surface: corresponds to the cycles
• Fatigue failure (3)
  – SSY assumption
    • Tests: conditioning parameters
      – $\Delta P$ &
      – $P_{\text{min}} / P_{\text{max}}$
    • Therefore fatigue failure can be described by
      – $\Delta K = K_{\text{max}} - K_{\text{min}}$ &
      – $R = K_{\text{min}} / K_{\text{max}}$

\[ \frac{da}{dN_f} = f(\Delta K, R) \]

• There is $\Delta K_{\text{th}}$ such that if $\Delta K \sim \Delta K_{\text{th}}$:
  – The crack has a growth rate lower than one atomic spacing per cycle (statistical value)
  – Dormant crack
Cyclic loading

- **Crack growth rate**
  - Zone I
    - Stage I fatigue crack growth
    - $\Delta K_{th}$ depends on $R$
  - Zone II
    - Stage II fatigue crack growth: striation
    - 1963, Paris-Erdogan
      \[
      \frac{da}{dN_f} = C \Delta K^{-m}
      \]
      - Depends on the geometry, the loading, the frequency
    - Steel: $\Delta K_{th} \sim 2-5$ MPa $m^{1/2}$, $C \sim 0.07-0.11 \times 10^{-11}$ [m (MPa $m^{1/2}$)$^{-m}$], $m \sim 4$
    - Steel in sea water: $\Delta K_{th} \sim 1-1.5$ MPa $m^{1/2}$, $C \sim 1.6 \times 10^{-11}$ [idem], $m \sim 3.3$
  - Be careful: $K$ depends on $a$ integration required to get $a(N_f)$
    - Mode I: $K_I = \sigma_\infty \sqrt{\pi a}$ $\Rightarrow \Delta K = (\sigma_{\infty, \text{max}} - \sigma_{\infty, \text{min}}) \sqrt{\pi a}$
  - Zone III
    - Rapid crack growth until failure
    - Static behavior (cleavage) due to the effect of $K_{\text{max}}(a)$
    - There is failure once $a_f$ is reached, with $a_f$ such that $K_{\text{max}}(a_f) = K_c$
Fatigue design

• « Infinite life design »
  – $\sigma_a < \sigma_e$: « infinite » life
  – Economically deficient

• « Safe life design »
  – No crack before a determined number of cycles
    • At the end of the expected life the component is changed even if no failure has occurred
    • Emphasis on prevention of crack initiation
    • Approach theoretical in nature
      – Assumes initial crack free structures
  – Use of $\sigma_a - N_f$ curves (stress life)
    • Add factor of safety
  – Components of rotating structures vibrating with the flow cycles (blades)
    • Once cracks form, the remaining life is very short due to the high frequency of loading
Fatigue design

• « Fail safe design »
  – Even if an individual member of a component fails, there should be sufficient structural integrity to operate safely
  – Load paths and crack arresters
  – Mandate periodic inspection
  – Accent on crack growth rather than crack initiation
  – Example: 1988, B737, Aloha Airlines 243
    • 2 fuselage plates not glued
    • Sea water $\rightarrow$ rust and volume increased
    • Fatigue of the rivets
    • The crack followed a predefined path allowing a safe operation

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Fatigue design

« Damage tolerant design »

– Assume cracks are present from the beginning of service
– Characterize the significance of fatigue cracks on structural performance
  • Control initial crack sizes through manufacturing processes and (non-destructive) inspections
  • Estimate crack growth rates during service (Paris-Erdogan) & plan conservative inspection intervals (e.g. every so many years, number of flights)
  • Verify crack growth during these inspections
  • Predict end of life ($a_f$)
  • Remove old structures from service before predicted end-of-life (fracture) or implement repair-rehabilitation strategy

– Non-destructive inspections
  • Optical
  • X-rays
  • Ultrasonic (reflection on crack surface)
References

• Lecture notes

• Book

• RPV
  – Documents