University of Liège
Aerospace & Mechanical Engineering

Fracture mechanics, Damage and Fatigue
Overview

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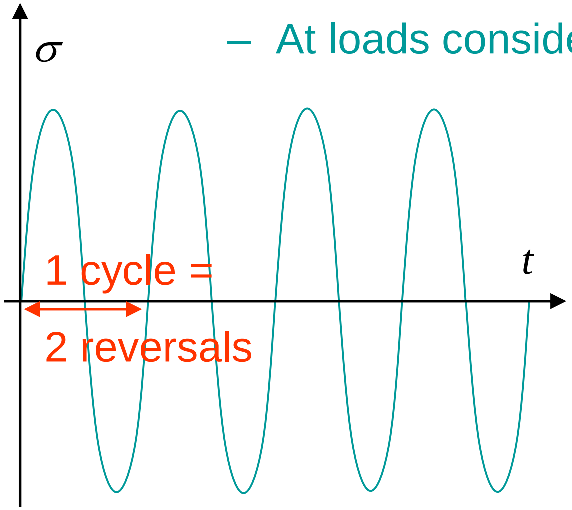
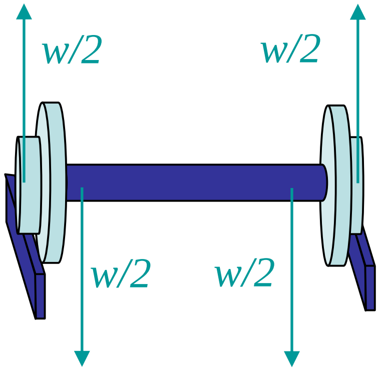
Before fracture mechanics

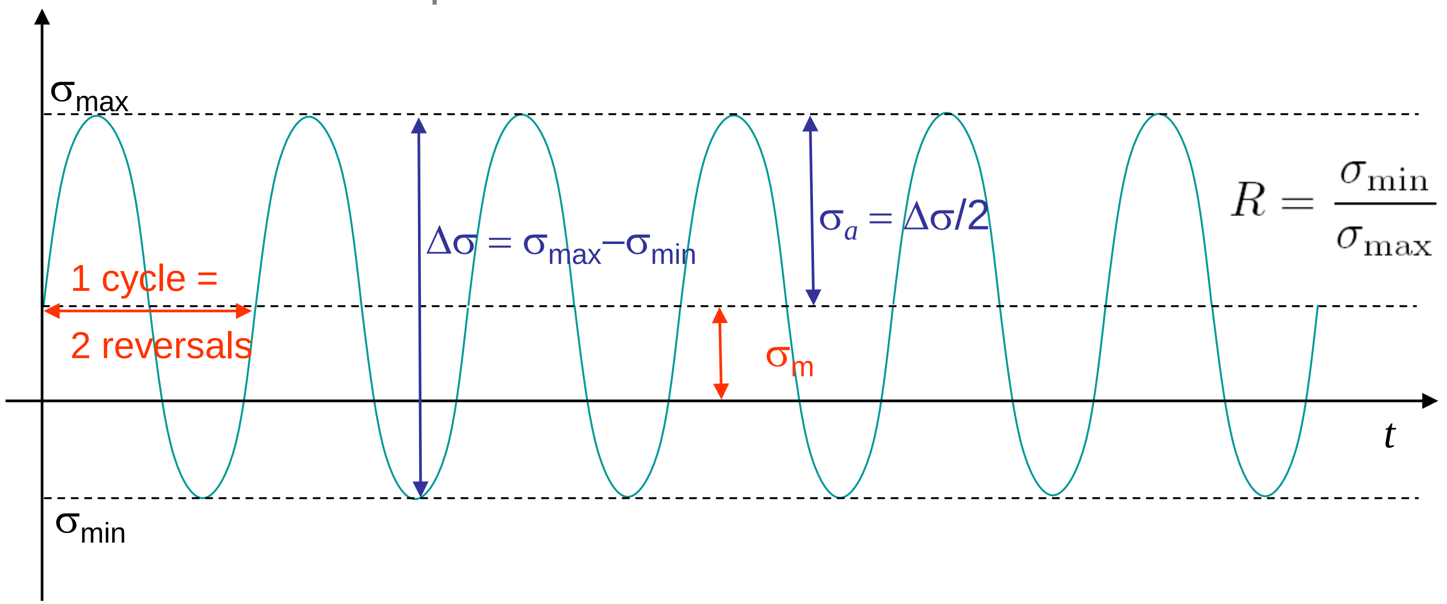
- Design with stresses lower than
  - Elastic limit (σ_p^0) or
  - Tensile strength (σ_{TS})
- ~1860, Wöhler
  - Technologist in the German railroad system
  - Studied the failure of railcar axles
    - Failure occurred
      - After various times in service
      - At loads considerably lower than expected





    - Failure due to cyclic loading/unloading
  - « Total life » approach
    - **Empirical** approach of fatigue

- Empirical approach: Total life
  - Life of a structure depends on

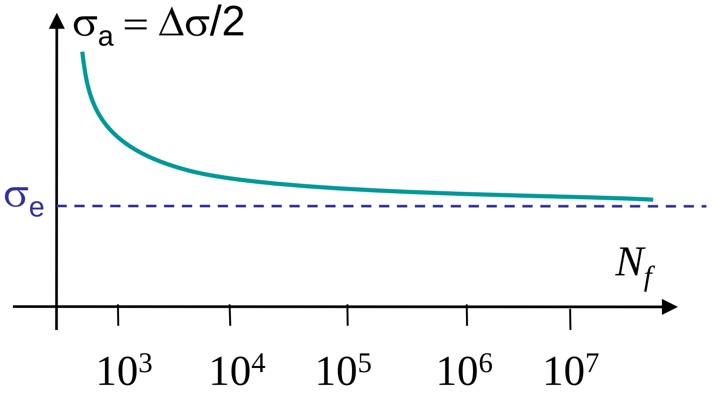


- Minimal & maximal stresses: σ_{min} & σ_{max}
- Mean stress: $\sigma_m=(\sigma_{max} + \sigma_{min})/2$
- Amplitude: $\sigma_a = \Delta\sigma/2 = (\sigma_{max} - \sigma_{min})/2$
- Load Ratio: $R = \sigma_{min} / \sigma_{max}$
- Under particular environmental conditions (humidity, high temperature, …):
  - Frequency of cycles
  - Shape of cycles (sine, step, …)

- Total life approach
  - Stress life approach
    - For (essentially) elastic deformations
    - For $\sigma_m = 0$ & N_f identical cycles before failure
      - σ_e: endurance limit (if any, >10^7 cycles)
      - 1910, Basquin law

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f \left(2N_f\right)^b$$

      - Parameters from experimental tests



  - Strain life approach
    - For (essentially) plastic deformations
    - For N_f identical cycles before failure
      - 1954, Manson-Coffin $\frac{\Delta\bar{\varepsilon}^p}{2} = \varepsilon'_f \left(2N_f\right)^c$
      - $\Delta\bar{\varepsilon}^p$ plastic strain increment during the loading cycles

- 1952, De Havilland 106 Comet 1, UK (1)
  - First jetliner, 36 passengers
    - Pressurized cabin (0.58 atm)
  - The fuselage designed using total life approach
    - 1952, a fuselage was tested against fatigue
      - Static loading at 1.12 atm, followed by
      - 10 000 cycles at 0.7 atm (> cabin pressurization at 0.58 atm)
  - 1953-1954: several explosive decompressions
    - 1954, April, test of fuselage ALYU in water tank
      - Pressurization cycles of the cabin simulated
      - Rupture at port window after only 3057 pressurization cycles

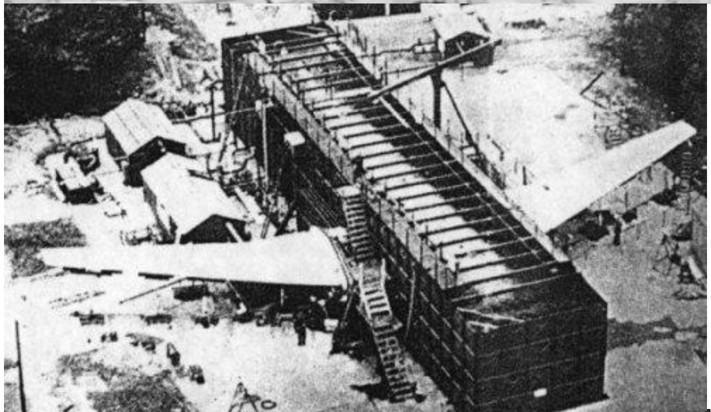
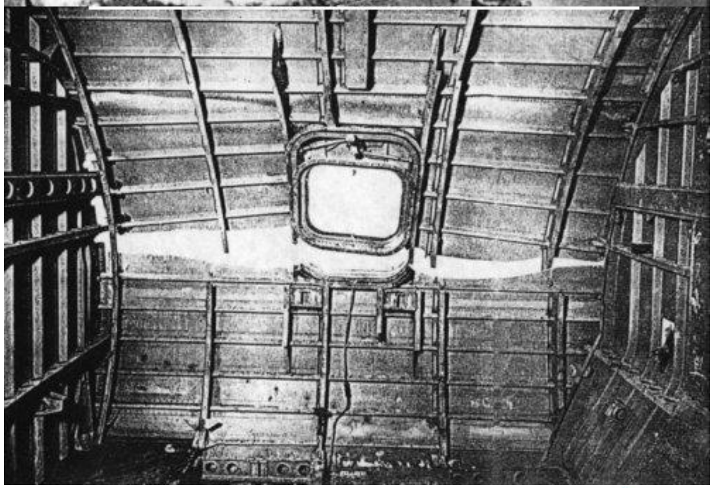


FIG. 6. AERIAL VIEW OF COMET G-ALYU IN TESTING TANK.



Design using total life approach

- 1952, De Havilland 106 Comet 1, UK (2)
  - 1954, August, ALYP roof retrieved from sea
    - Origin of failure at the communication window
    - Use of square riveted windows
    - Punched riveting instead of drill riveting
    - ⟹ Existence of initial defects
- The total life approach
  - Accounts for crack initiation in smooth specimen
  - Does not account for inherent defects
    - Metal around initial defects could have hardened during the initial static test load of the fatigue tested fuselage
    - Production planes without this static test load …
- 1958, Comet 3 et 4
  - Round windows glued
  - Fuselage thicker

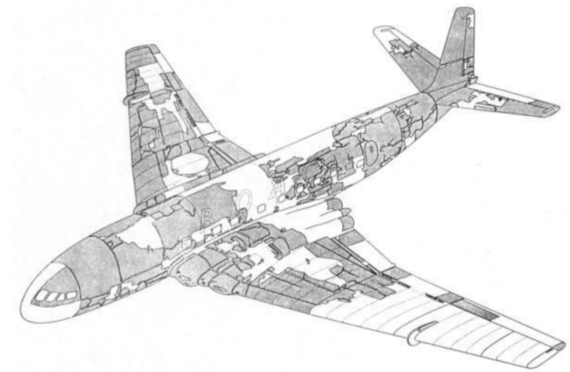


FIG. 2. DIAGRAM SHOWING AMOUNT OF WRECKAGE RECOVERED—G-ALYP.

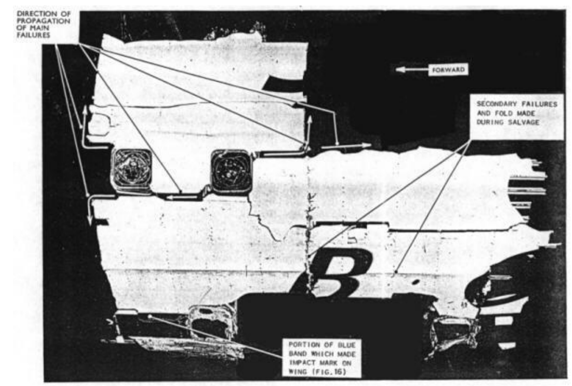


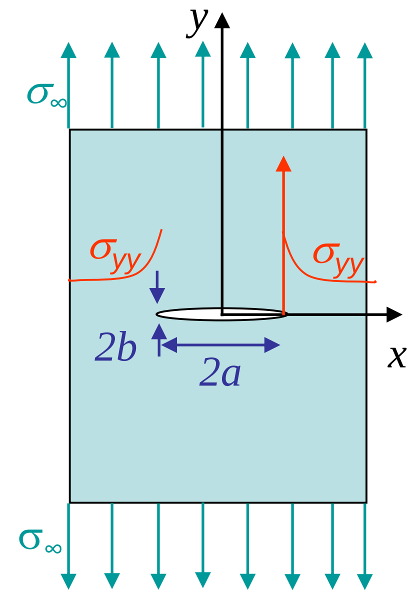
FIG. 12. PHOTOGRAPH OF WRECKAGE AROUND ADF AERIAL WINDOWS—G-ALYP.

Why fracture mechanics ?

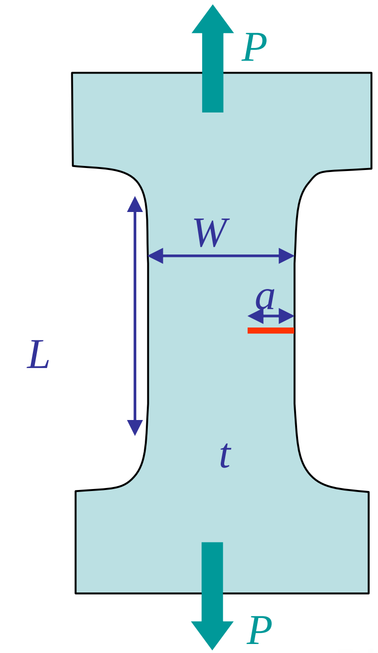
- Limits of the total life approach
  - Does not account for inherent defects
  - What is happening when a defect is present ?
    - Example: Comet
    - Theoretical stress concentration
      - Infinite plane with an ellipsoidal void (1913, Inglis)

$$\sigma_{\max} = \sigma_{yy}(a, 0) = \sigma_{\infty}\left(1 + \frac{2a}{b}\right)$$

      - b→ 0 $\Longrightarrow$ $\sigma_{max} \rightarrow \infty$ $\Longrightarrow$ breaks for $\sigma_\infty \rightarrow 0$

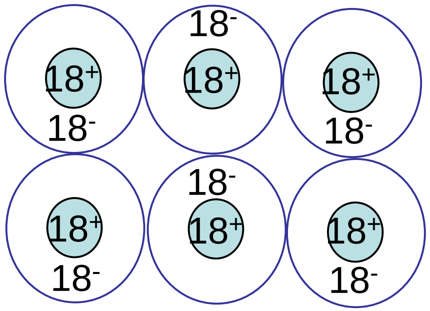


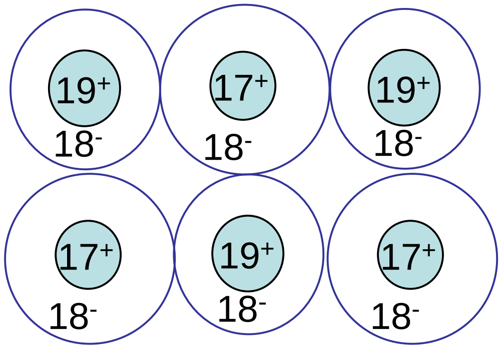
    - In contradiction with Griffith and Irwin experiments
      - Tensile tests on scratched glass samples
      - Tensile strength depends on
        - » The crack size a and on
        - » The surface energy γ_s
      - i.e. $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\, 2\gamma_s}$

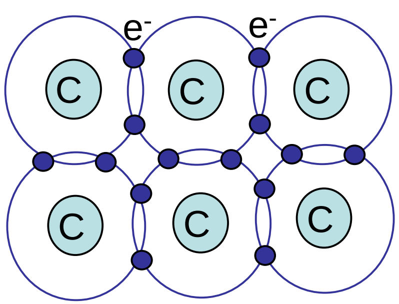


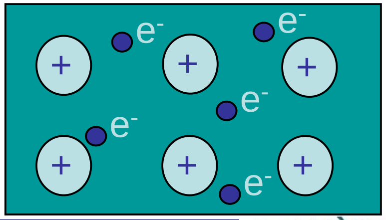
What is fracture mechanics ?

- Back to physics
  - Bonding in Crystals (attractive forces)
    - Molecular or van den Waals:
      - Interaction between dipoles
      - Ar, polymers, C_6H_6, graphite
      - Bonds easily break
    - Ionic
      - Interaction between ions
      - NaCl, …
      - Coulombic non directional forces
      - Hard & brittle crystals
    - Covalent
      - Bonds between atoms
      - CH4, Diamond
      - Directional and strong forces
      - Hard & brittle crystals
    - Metallic
      - Sea of donated valence electrons (conductors)
      - Metals & alloys
      - Non-directional forces

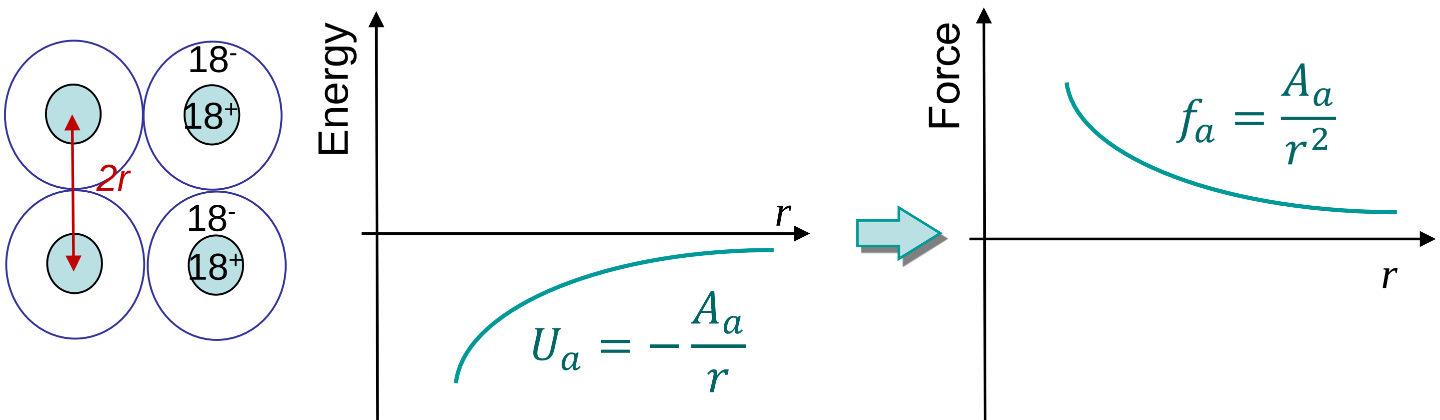








- Back to physics (2)
  - Free electrons model of a crystal structure
    - Bonding in Crystals
      - Differ from the bonding kinds, but same shape

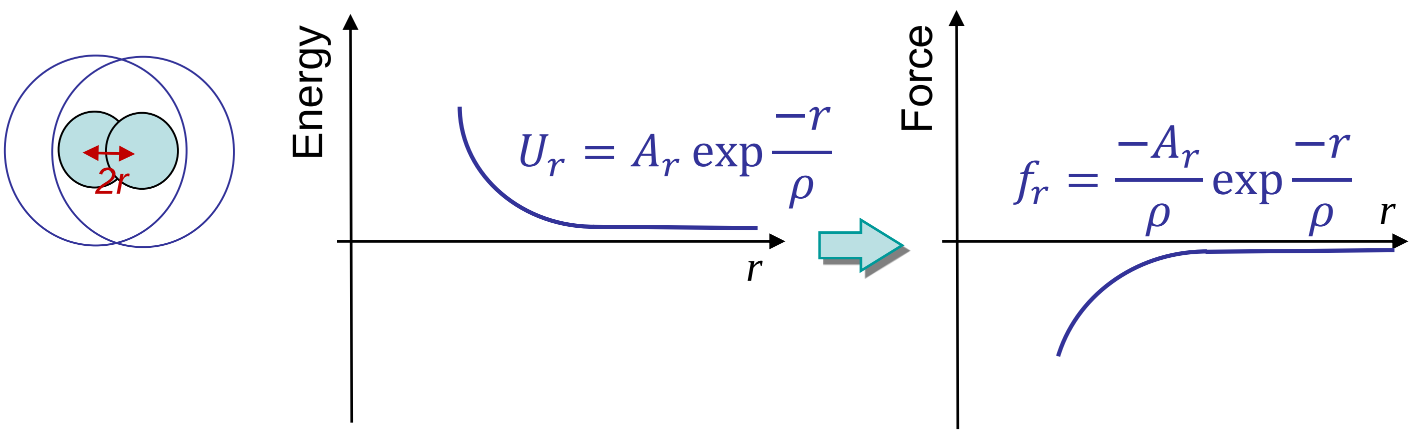


E..g.: metallic ions and electronic clouds

$$U_a = -M\frac{z_1 z_2 q^2}{4\pi\varepsilon_0 r}$$

z_i: valences,
ε_0=8.85 pF/m: permittivity,
q=1.602 10^{-19} C: electronic charge

- Repulsive energy due to the interaction of electronic clouds & nuclei



$$U_r = b\lambda\exp\frac{-r}{\rho}$$

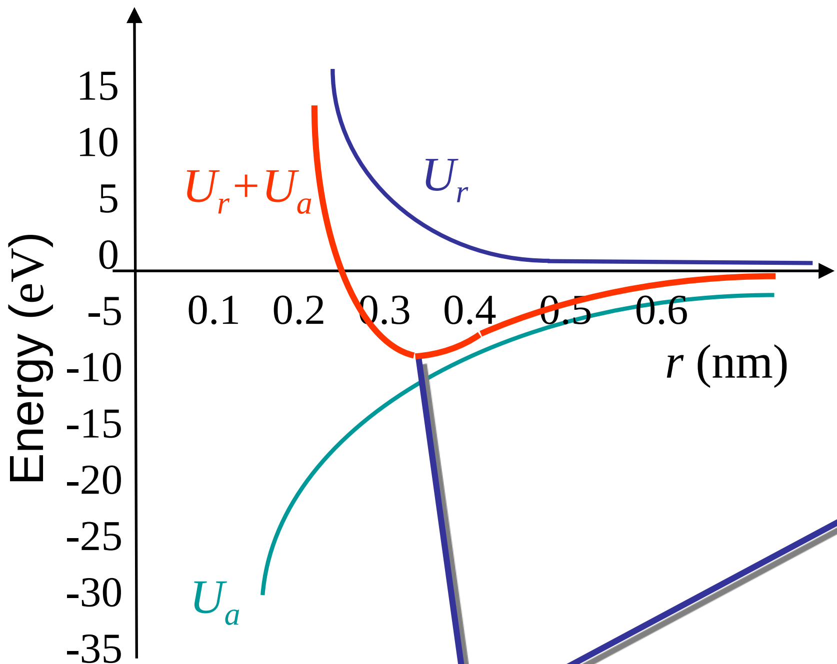
b: number of adjacent ions,
λ (eV),
ρ (nm): repulsive parameters

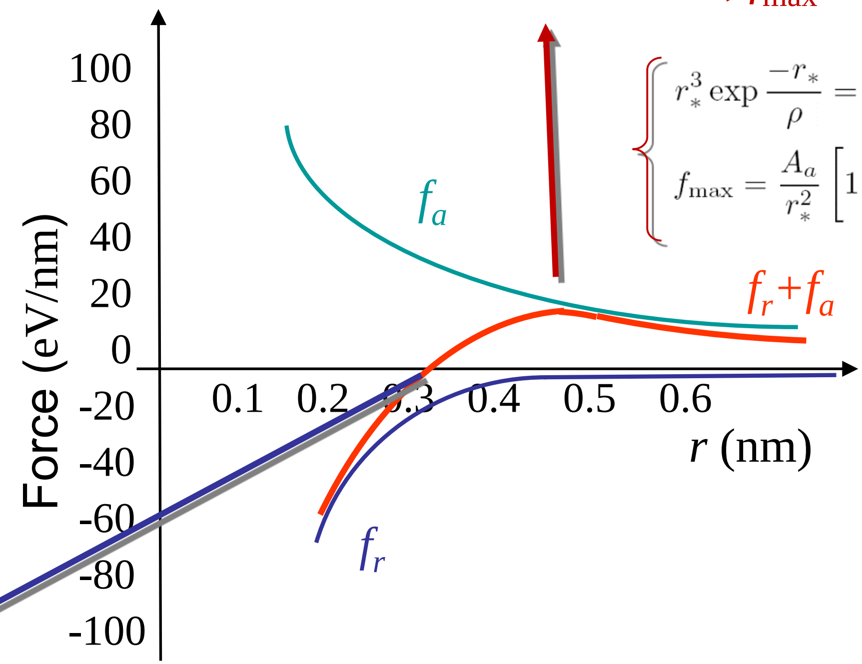
- Back to physics (3)
  - Free electrons model of a crystal structure (2)
    - Equilibrium & maximal force (NaCl)

Maximal force at $r=r^*$, $f_{\max}$

$$\begin{cases} r_*^3 \exp\dfrac{-r_*}{\rho} = \dfrac{2A_a\rho^2}{A_r} \\ f_{\max} = \dfrac{A_a}{r_*^2}\left[1-\dfrac{\rho}{r_*}\right] \end{cases}$$

1eV = 1.60217646 × 10^{-19} J



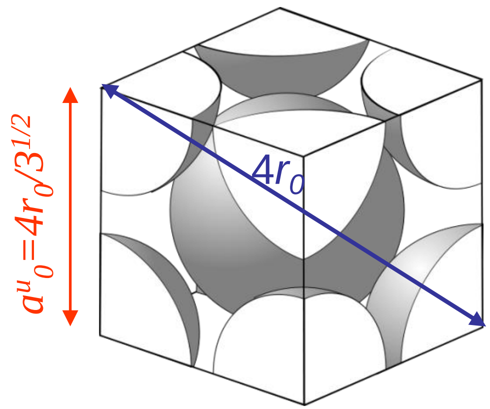


Equilibrium at $r=r_0$, U_0

$$\begin{cases} r_0^2 \exp\dfrac{-r_0}{\rho} = \dfrac{A_a\rho}{A_r} \\ U_0 = -\dfrac{A_a}{r_0}\left[1-\dfrac{\rho}{r_0}\right] \end{cases}$$

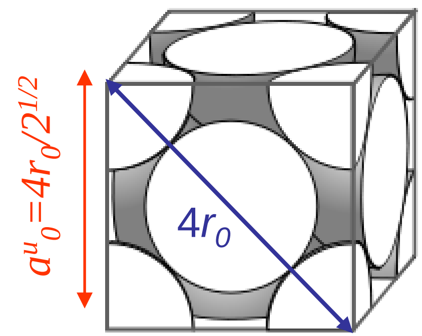
- From free electrons model to macroscopic behavior
  - Depends on the crystal lattice
  - Metallic crystal tends to be packed (a_0^u: size of the unit cell)
    - Examples:

Body-Centered-Cubic crystal



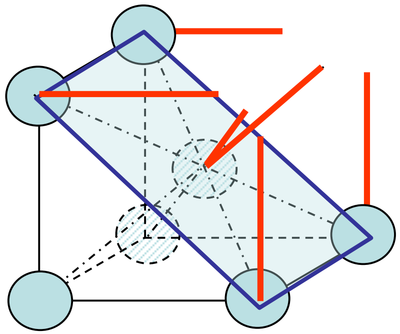
Fe at low T°, Ferrite (low C-steel)

Face-Centered-Cubic crystal

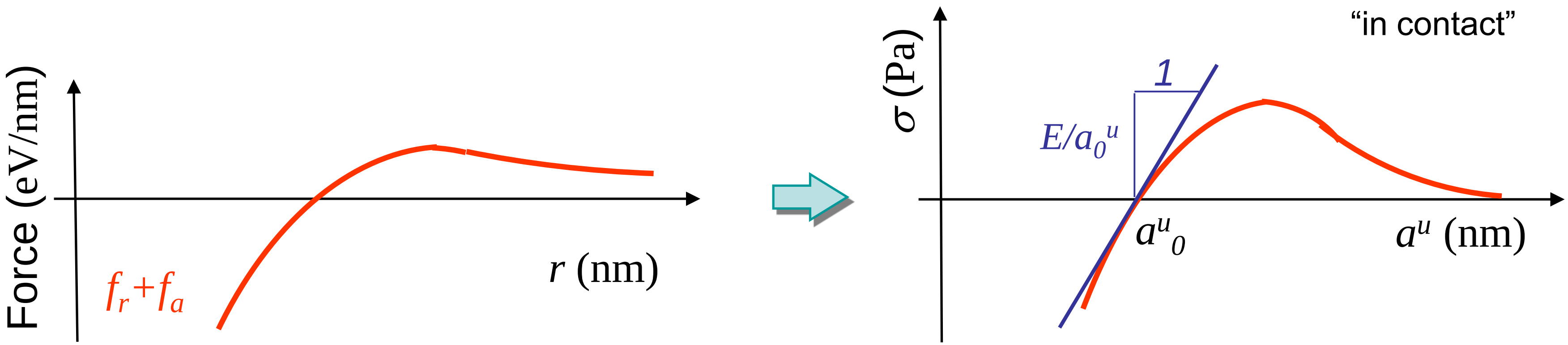


Fe at high T°, Austenite (low C-steel at high T°), Al, Cu

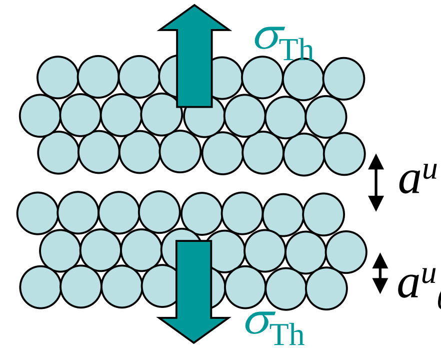
- From free electrons model to macroscopic behavior (2)
  - Elasticity
    - Stress definition $$\sigma = \frac{1}{S_{\text{ref}}\left(\text{plane},\, r_0\right)} \sum_i f_i$$
    - Young's modulus $$E = \lim_{dr \to 0} \frac{\sigma\left(r_0 + dr\right) - \sigma\left(r_0\right)}{dr} r_0$$

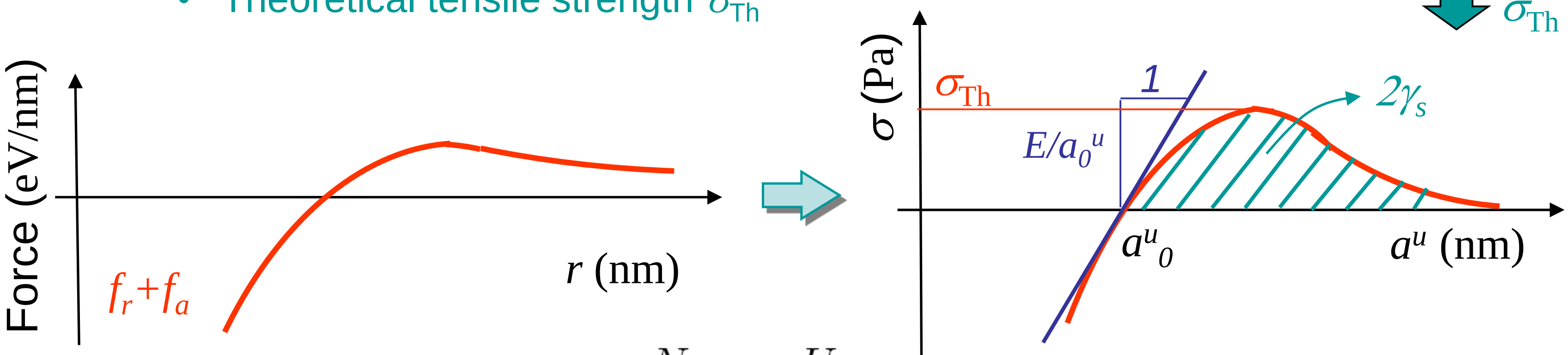


Low representation: atoms are actually "in contact"

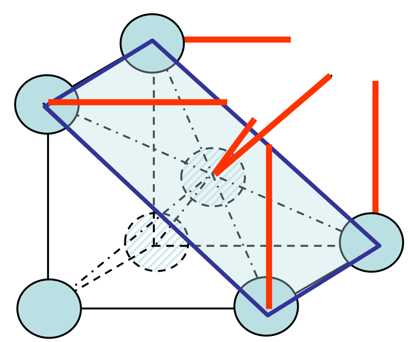


- From free electrons model to macroscopic behavior (3)
  - Cleavage
    - Planes of atoms separate at once
    - Brittle materials
    - Theoretical tensile strength σ_{Th}



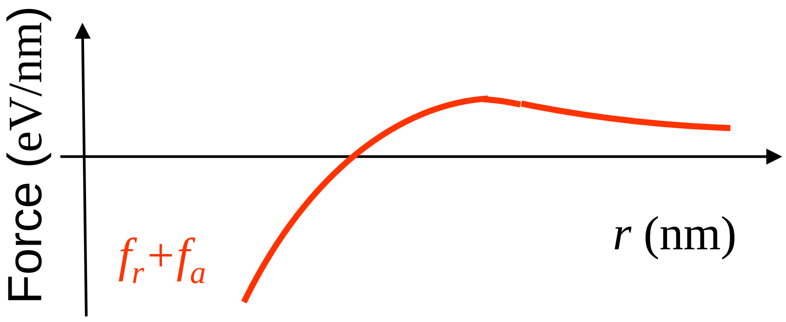


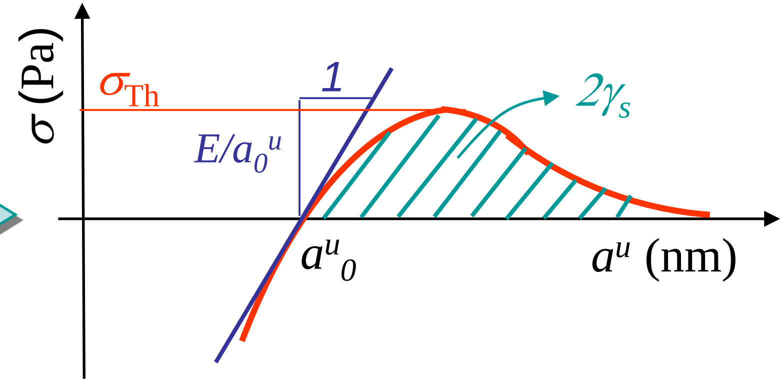
  - Surface energy γ_s: $2\gamma_s = \frac{N_{\text{rupture}} U_0}{S_{\text{ref}}}$
    - Energy to create a unit surface
    - Cleavage: 2 surfaces S_{ref} are created
    - Depend on the lattice & on the plane
    - E.g.: plane S(1,1,0) in BCC
      - $S_{ref} = 2^{1/2}\, a_0^{u\,2}$
      - $N_{rupture}$ = 4 x 1/8 (corner) + 2 x 1/8 (center)



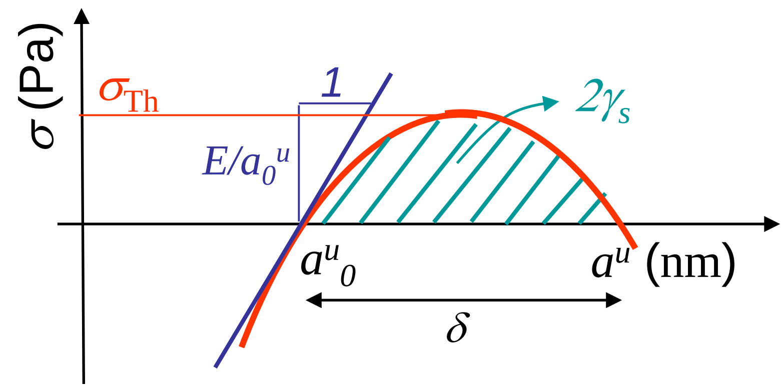
Low representation: atoms are actually “in contact”

- Theoretical tensile strength





  - Sinus approximation



$$\sigma = \sigma_{\mathrm{Th}} \sin\left(\pi \frac{a^u - a_0^u}{\delta}\right)$$

$$\left\{ \begin{array}{l} \dfrac{E}{a_0^u} = \left.\dfrac{d\sigma}{da^u}\right|_{a_0^u} = \sigma_{\mathrm{Th}} \dfrac{\pi}{\delta} \\ 2\gamma_s = \displaystyle\int_{a_0^u}^{a_0^u+\delta} \sigma da^u = 2\sigma_{\mathrm{Th}} \frac{\delta}{\pi} \end{array} \right.$$

- Theoretical tensile strength of brittle materials $\sigma_{\mathrm{Th}} = \sqrt{\dfrac{E\gamma_s}{a_0^u}}$

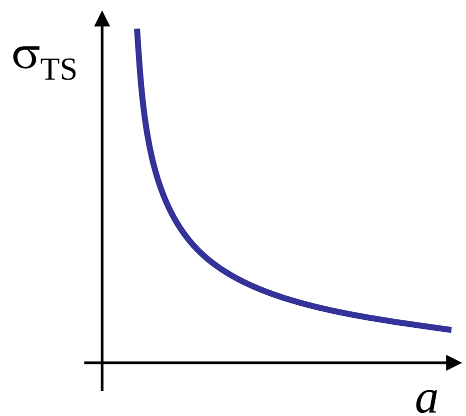
- Inherent defects play a major role in fracture mechanics
  - Theoretical tensile strength σ_{Th} vs Tensile strength σ_{TS}

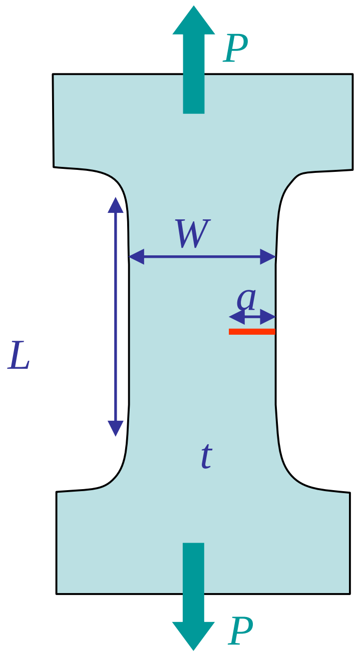
	$a^u{}_0$ [nm]	E [GPA]	γ_s [J m^{-2}]	σ_{Th}
Glass	0.3	60	21	64 GPa !
Steel (low T)	0.3	210	3400	1500 GPa !

    - Theoretical values are much higher than observed ones
  - 1920, Griffith studied the influence of a crack on σ_{TS}
    - Tensile tests on scratched glass samples
    - Experimentally:

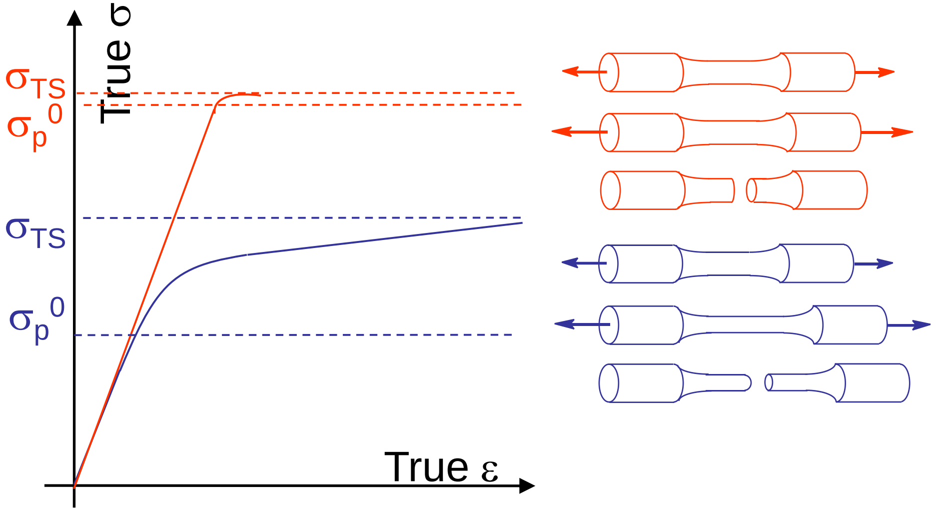
$$\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$$

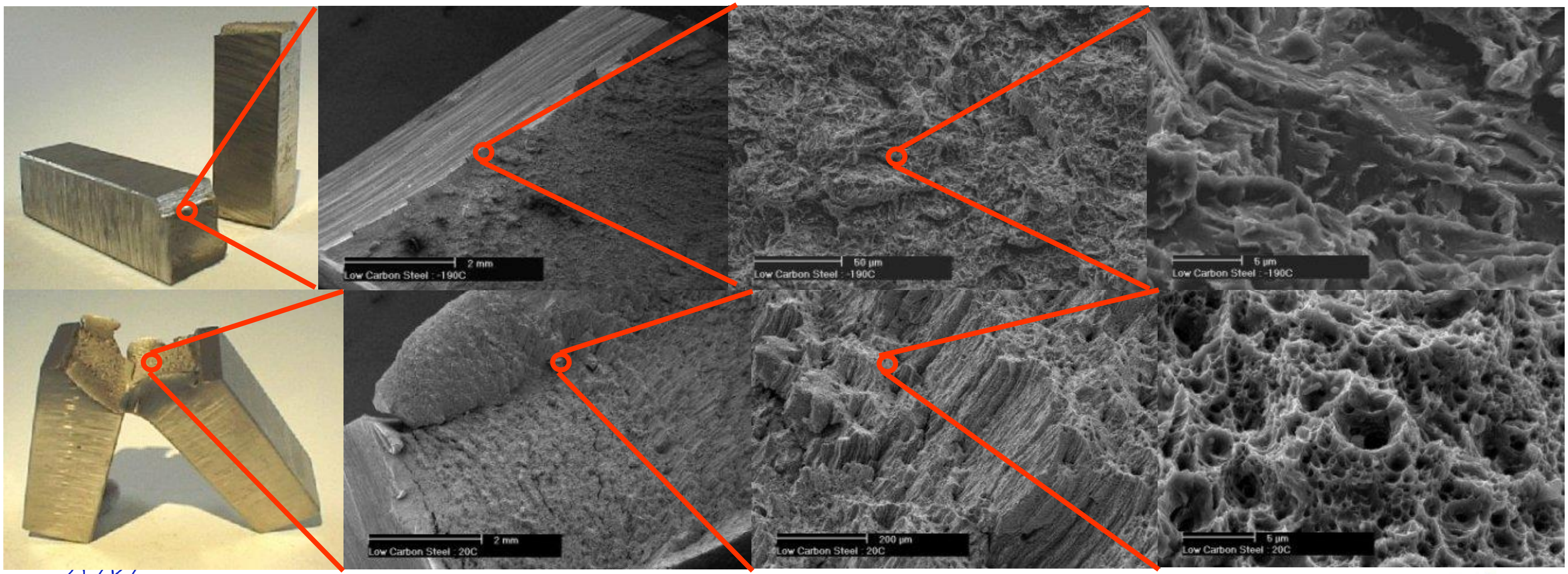
    - Brittle materials only



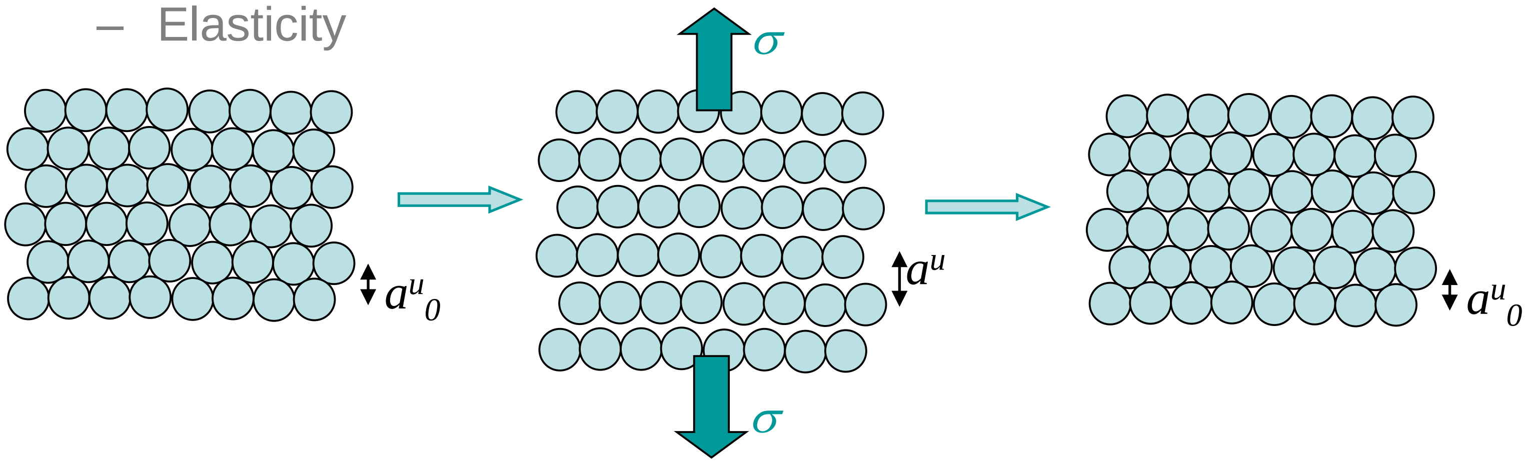


- 2 kinds of behavior
  - Brittle:
    - (Almost) no plastic deformations prior to (macroscopic) failure
  - Ductile:
    - Plastic deformations prior to (macroscopic) failure
  - Different microscopic behaviors

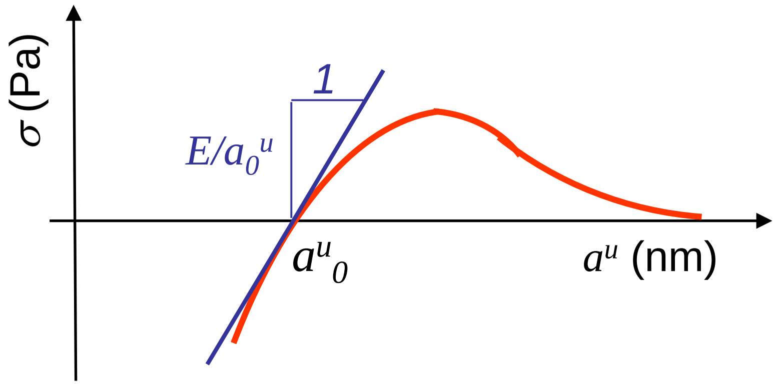


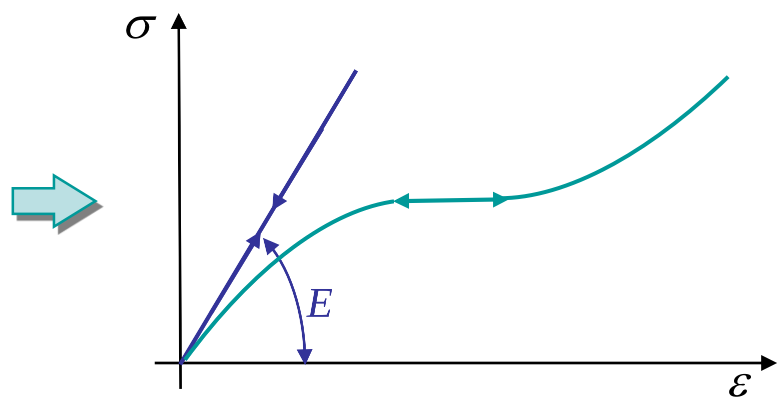


- Deformation mechanisms
  - Elasticity

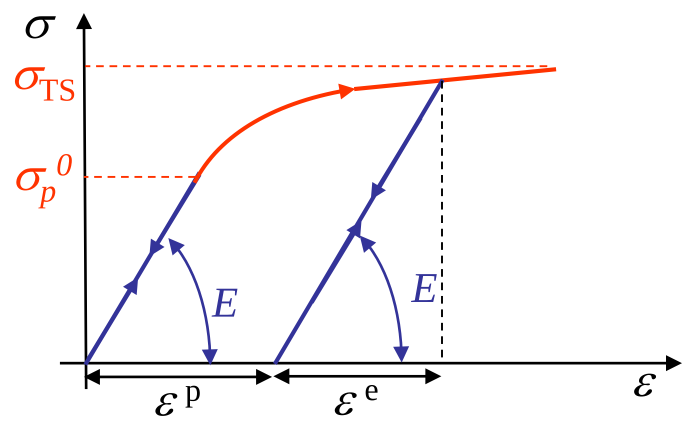


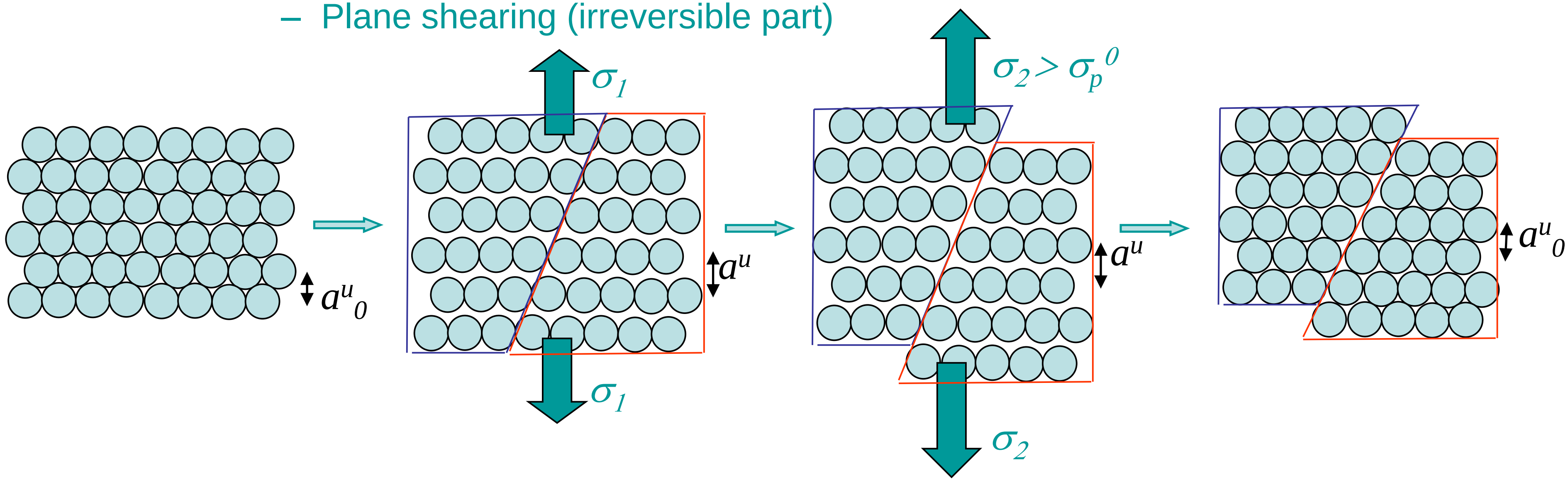
- Results from bonds stretching
- Reversible
- Linear (metals, …) or non-linear (polymers, …)





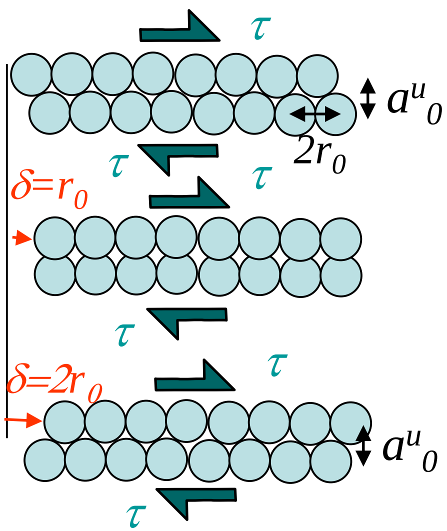
- Deformation mechanisms (2)
  - Plasticity
    - The specimen experiences permanent deformations
    - Irreversible
    - Combination of
      - Bonds stretching (reversible part)
      - Plane shearing (irreversible part)

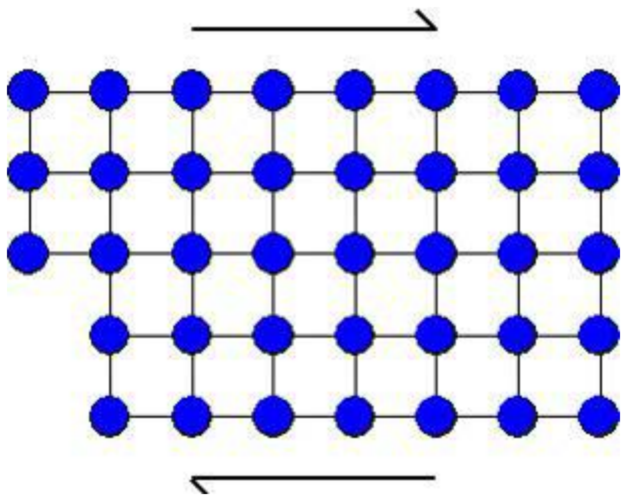




  - What is the plane shearing mechanisms?

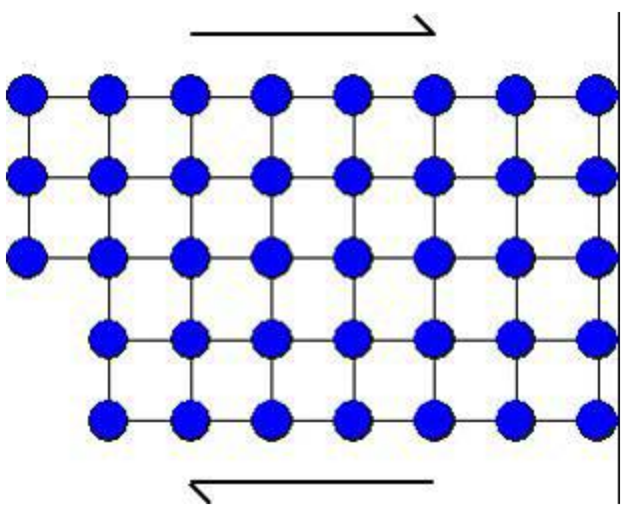
- Deformation mechanisms (3)
  - What is the plane shearing mechanisms ?
    - Slip of a whole crystal lattice?

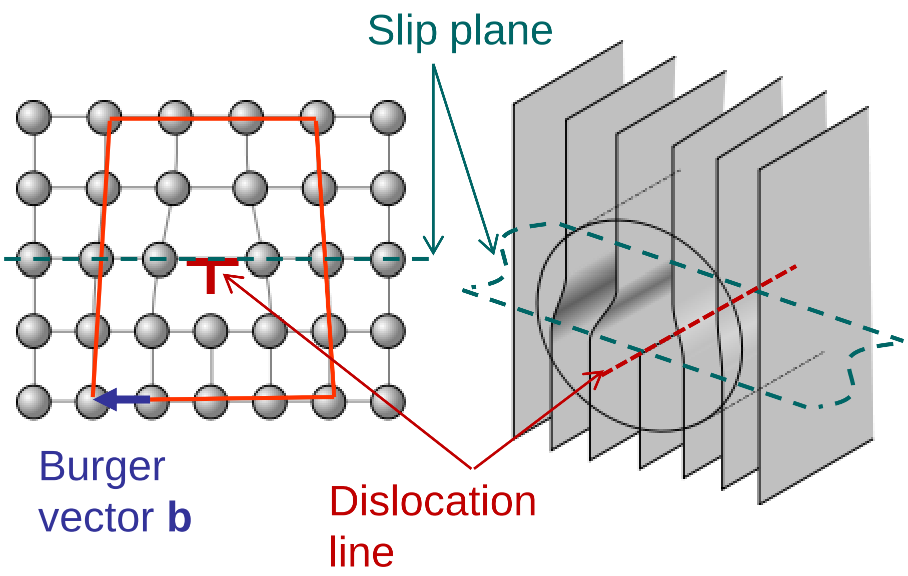




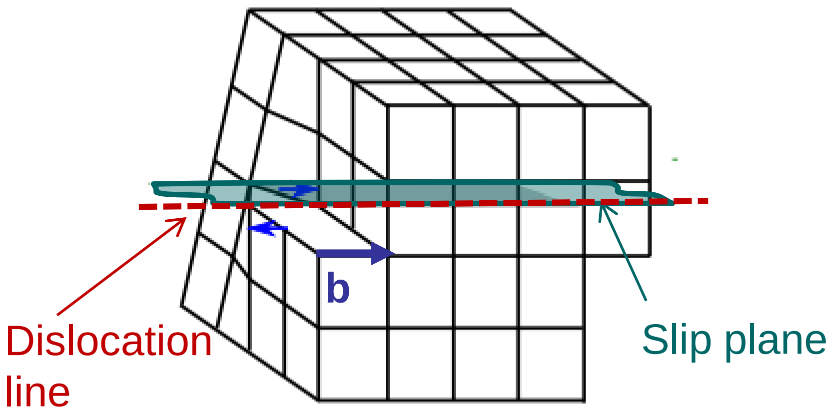
- Back to sine model
  - Because of symmetry: $\tau = 0$ at $\delta = 0, r_0, 2r_0$ $\Longrightarrow$ $\tau = \tau_{\max} \sin \frac{\pi\delta}{r_0}$
  - Shear modulus: $G = \left.\frac{d\tau}{d\delta/a^u_0}\right|_{\delta=0} = \tau_{\max}\frac{\pi a^u_0}{r_0}$
  - BCC: $\tau_{\max} = \frac{G\sqrt{3}}{4\pi}$
- Fe : G = 82 GPa $\Longrightarrow$ τ_{max} = 11 GPa !!!

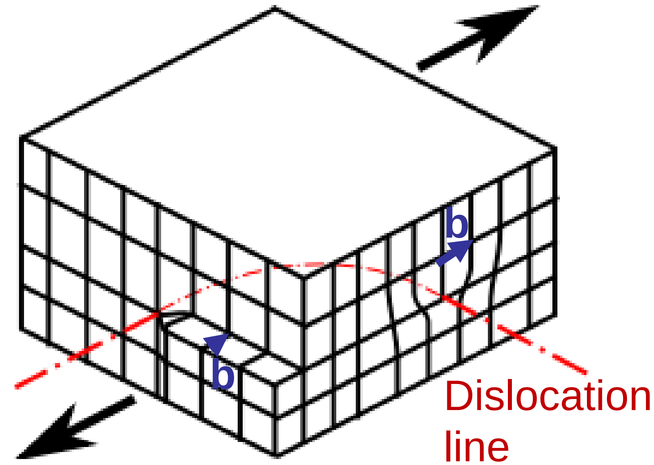
- Deformation mechanisms (4)
  - What is the plane shearing mechanisms (2) ?
    - A dislocation is characterized by
      - The Burger vector (difference between the distorted lattice around the dislocation and the perfect lattice)
      - Dislocation line (line along which the distortion is the largest)
      - Slip plane (plane where the dislocation motion occurs)
  - Edge dislocation
    - Dislocation line _|_ to Burger vector



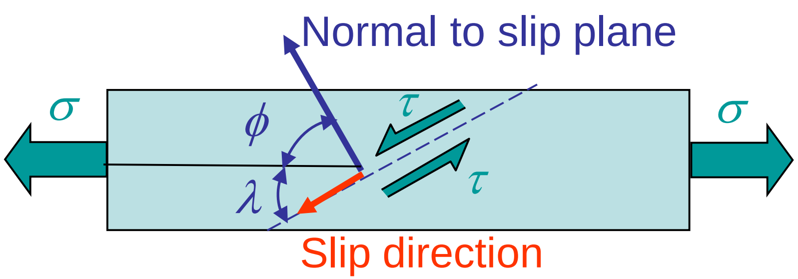


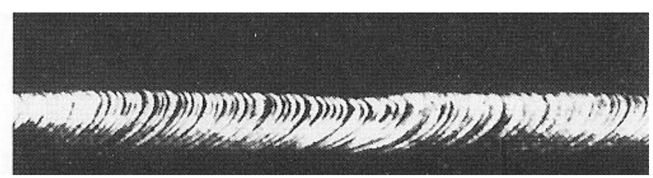
- Deformation mechanisms (5)
  - Screw dislocation & Mixed dislocation



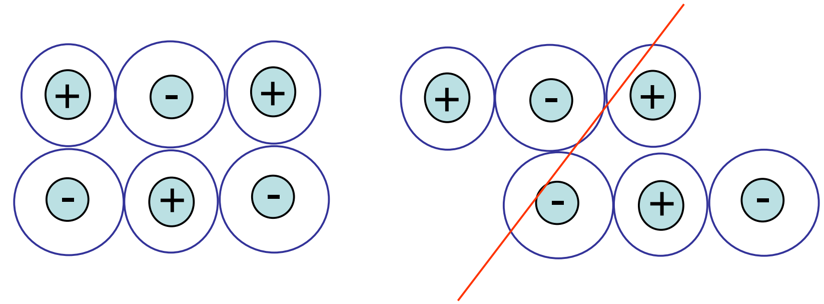


  - Example:
    - Plastically deformed zinc single crystal : $\tau = \sigma \cos\lambda \cos\phi$

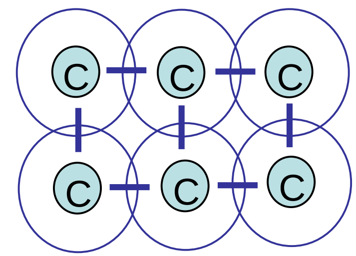




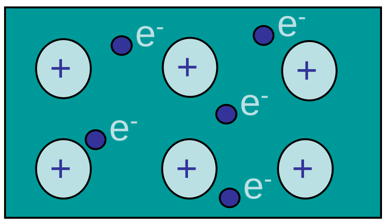
- Dislocations and brittle/ductile materials
  - Cristal with ionic bonding (NaCl, …)
    - Motion of dislocations difficult
      - A + would be in front of another +
      - Cleavage before ⟹ brittle



  - Covalent bonding (Si, diamond, …)
    - Motion of dislocations difficult
      - Strong & directional bonding
      - Cleavage before ⟹ brittle



  - Metallic bonding
    - Motion of dislocations possible
      - Non directional bonding
    - Motion along packed slip directions
      - Brittle or ductile?



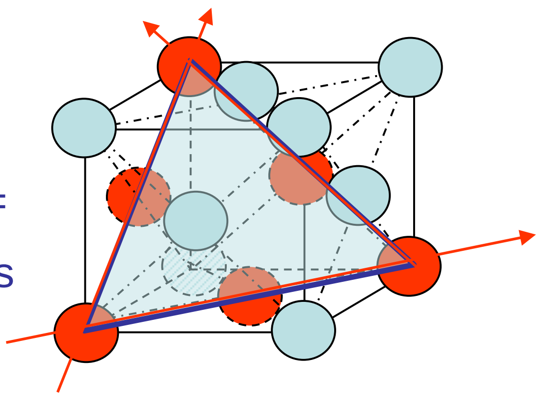
- Is metallic bonding leading to ductile behavior ?
  - FCC (Al, Cu, Austenite)
    - Many close-packed planes

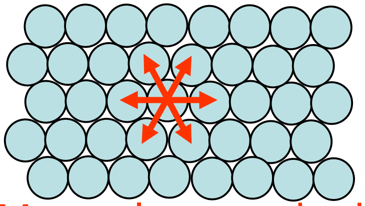
FCC:

4 slip planes

X 3 directions=

12 slip systems





Many close-packed directions in a close packed plane

- Dislocations motion always possible → always ductile

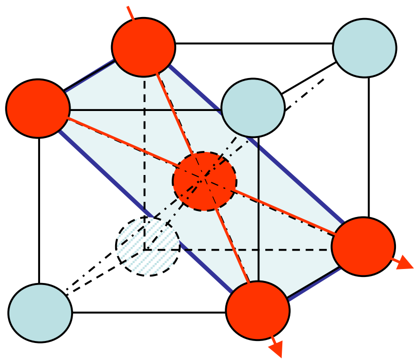
- Is metallic bonding leading to ductile behavior ?
  - BCC (Ferrite with Low C)
    - No close-packed plane

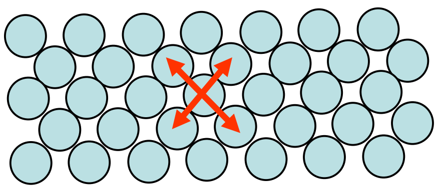
BCC:

6 slip planes

X 2 directions=

12 slip systems





The slip plane is
not close-packed

- Only directions are close-packed
- Low T°: atoms do not have enough energy to move

  → brittle

- High T°: atoms have enough energy for dislocations motion (Peierls stress)

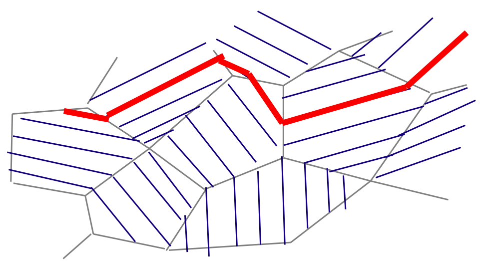
  → ductile

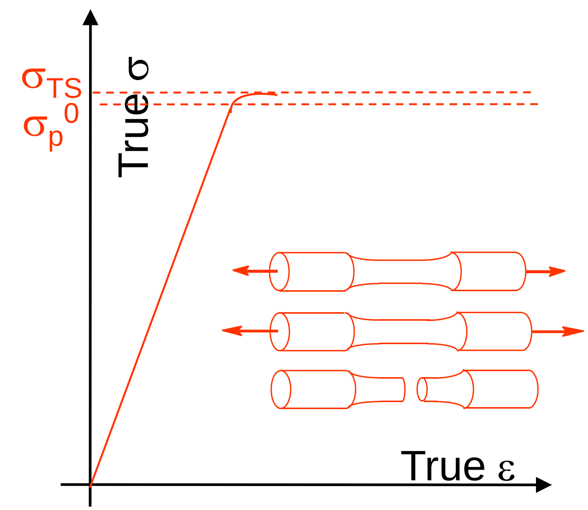
- There is a ductile/brittle transition temperature (DBTT)

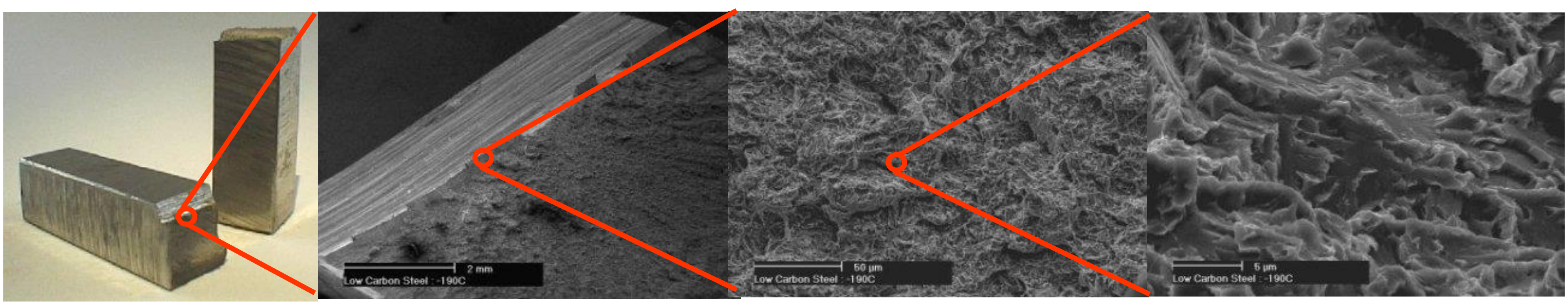
- Is metallic bonding leading to ductile behavior ?
  - Dislocations motion can be reduced by obstacles → harder & less ductile
    - Substitutional solutes (Sn in Cu, …, duralumin)
    - Interstitial solutes (Cr or V in Fe, …)
    - Grain boundaries (small crystal size after cold rolling: Hall-Petch effect)
    - Precipitate particles (Martensite Body-Centered-Tetragonal in ferrite after quench)
    - Other dislocations (created after cold work)

Brittle / ductile fracture

- Mechanism of brittle failure
  - (Almost) no plastic deformations prior to the (macroscopic) failure
  - Cleavage: separation of crystallographic planes
    - In general inside the grains
    - Preferred directions: low bonding
    - Between the grains: corrosion, H_2, …
  - Rupture criterion
    - 1920, Griffith: $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$



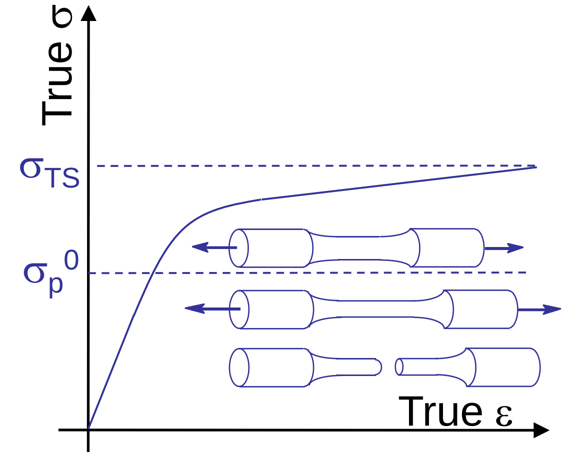


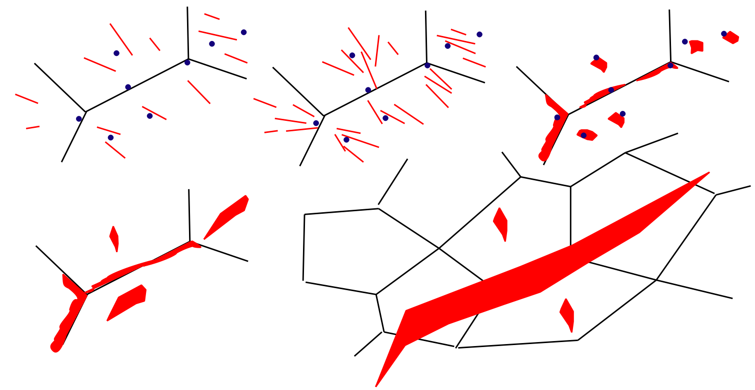


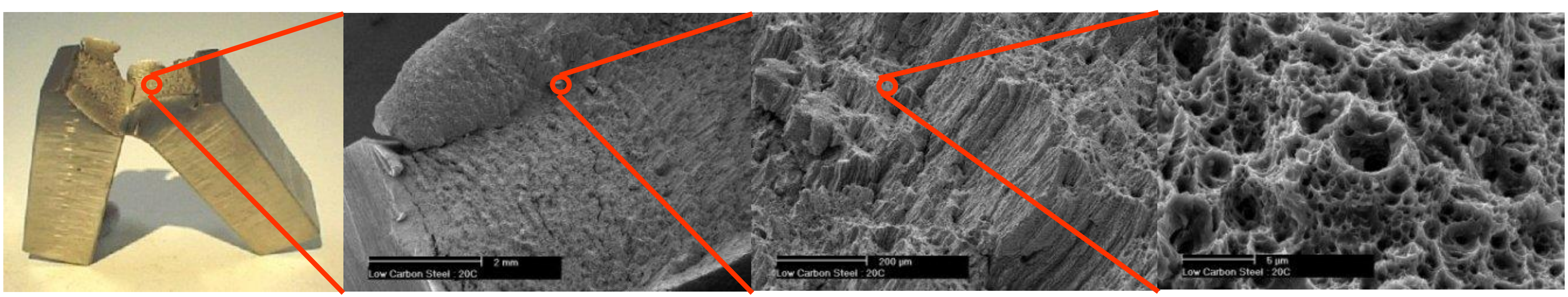
Brittle / ductile fracture

- Mechanism of ductile failure
  - Plastic deformations prior to (macroscopic) failure of the specimen
    - Dislocations motion ⟹ void nucleation around inclusions ⟹ micro cavity coalescence ⟹ crack growth
    - Is Griffith criterion $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$ still correct?
    - 1950, Irwin, the plastic work at the crack tip should be added to the surface energy:

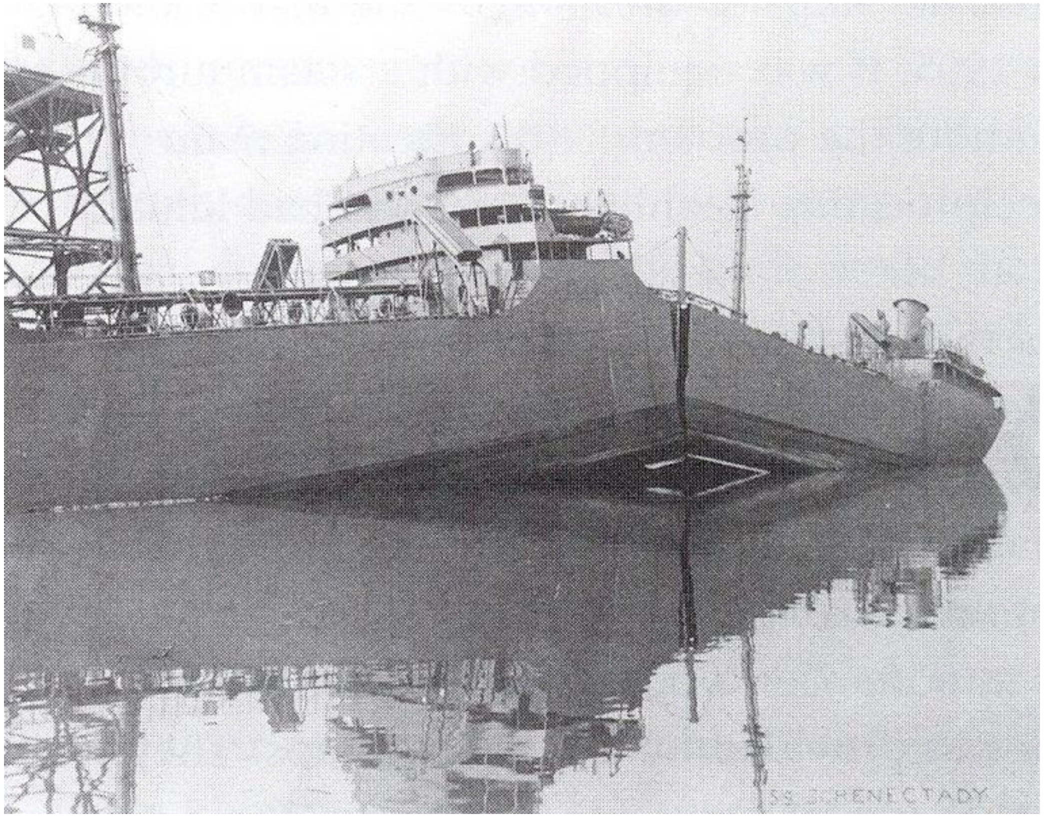
$$\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,(2\gamma_s + W_{\mathrm{pl}})}$$



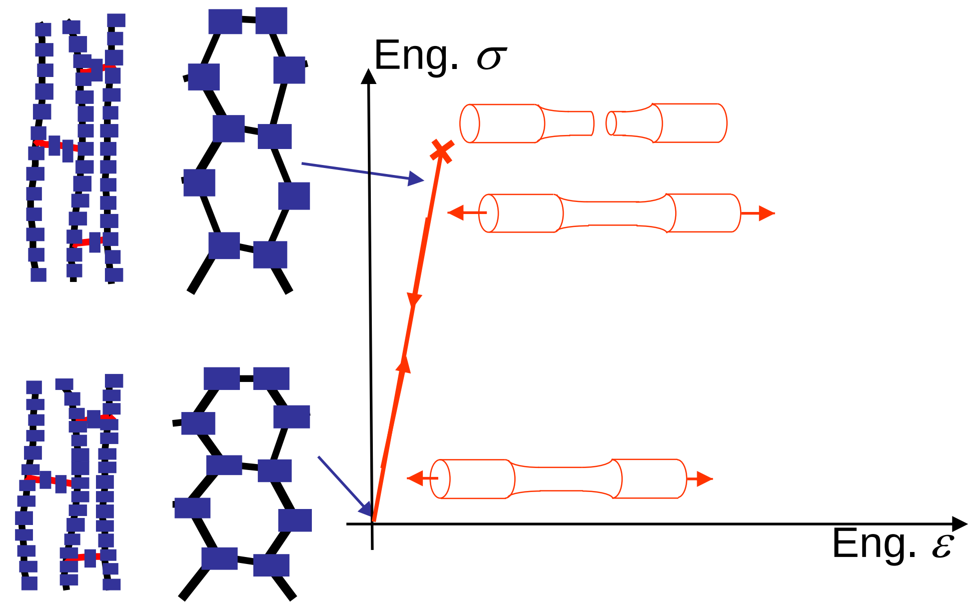




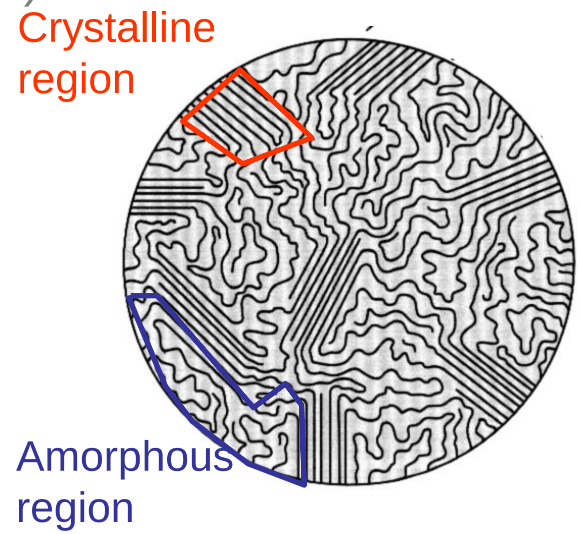
- WWII
  - Steel at low T°: brittle
    - $\sigma_{\text{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$ with γ_s ~ 3400 J m^{-2}
  - Steel at room T°: ductile
    - $\sigma_{\text{TS}}\sqrt{a} \div \sqrt{E\,(2\gamma_s + W_{\text{pl}})}$ with $2\gamma_s + W_{pl}$ ~ 200 kJ m^{-2}
  - Use of low-grade steel
    - In cold weather:
      DBTT ~ water temperature
    - When put in water existing cracks lead to failure
    - 30% of the liberty ships suffered from fracture

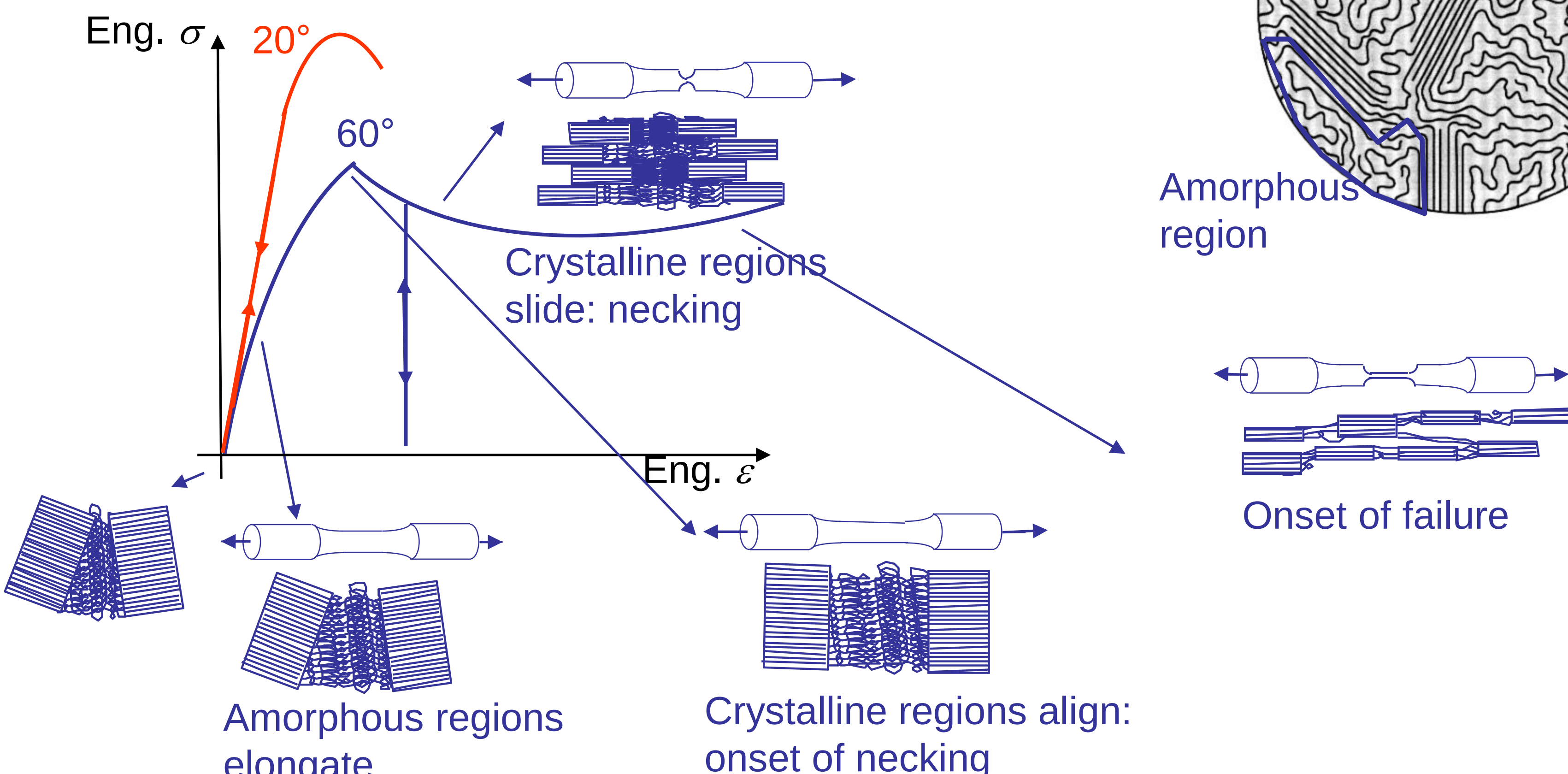


- Failure mode of polymers: rupture & disentangling of molecules
  - Behavior depends on the covalent chain structure
    - Aligned cross linked or networked (epoxy):
      - Always amorphous but directional
      - Brittle response

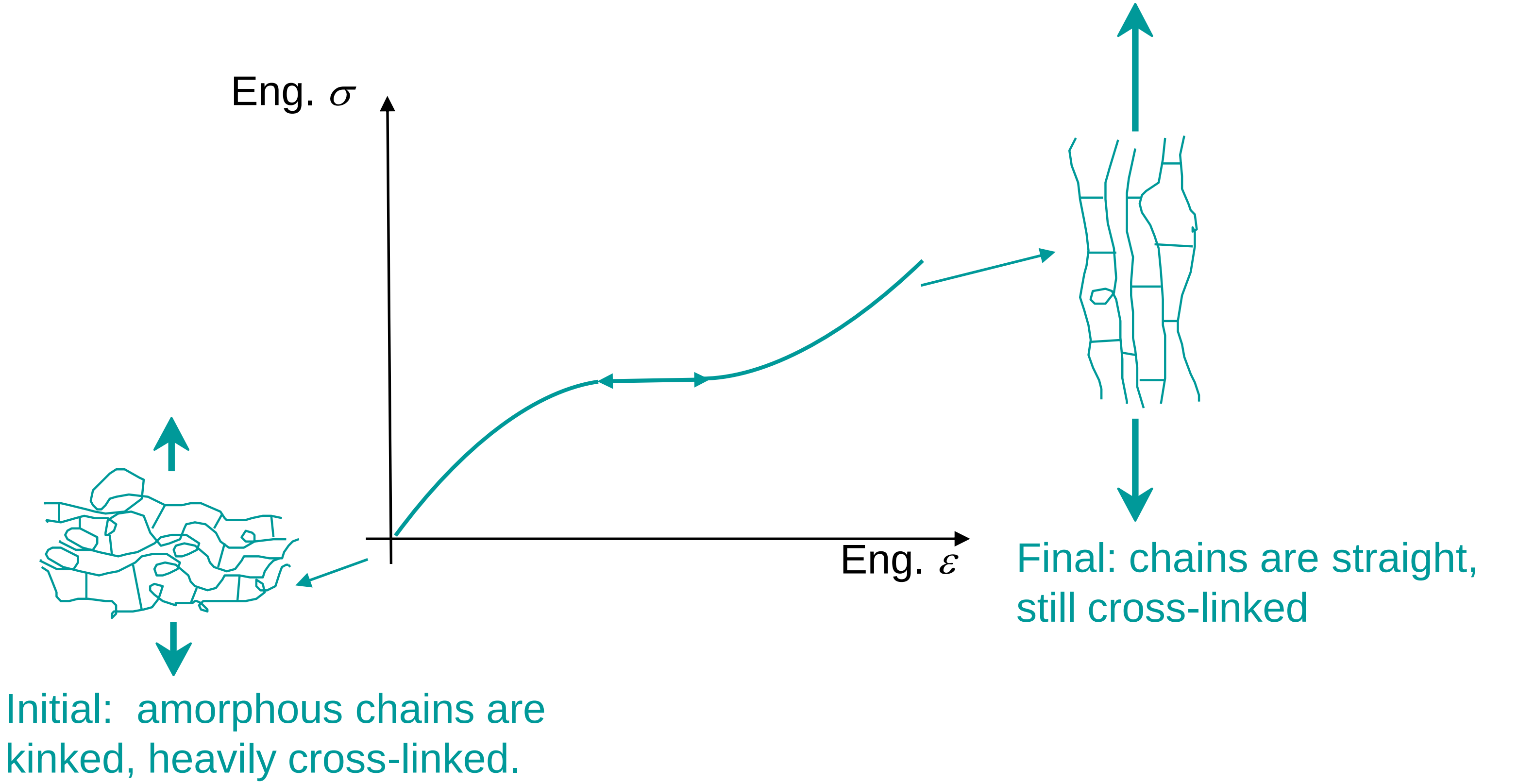


- Failure mode of polymers: rupture & disentangling of molecules (2)
  - Behavior depends on the covalent chain structure (2)
    - Semi crystalline (Plexiglas, PVC):
      - Linear polymers can locally crystallize into lamellar thin plates (chains fold back and forth)
      - Behavior strongly depends on temperature

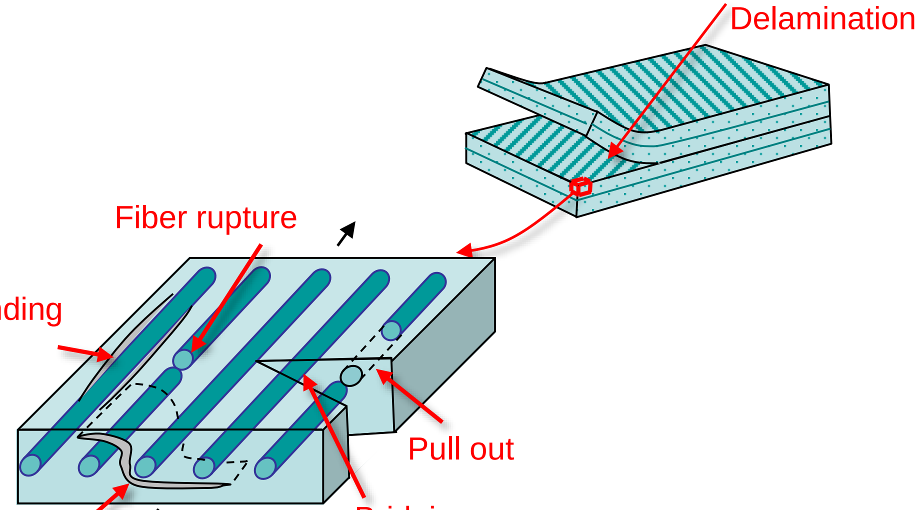




- Failure mode of polymers: rupture & disentangling of molecules (3)
  - Behavior depends on the covalent chain structure (3)
    - Elastomer (Rubber):
      - Amorphous chains are kinked and heavily cross linked
      - Reversible behavior



- Composites
  - Fibers in a matrix
    - Fibers: polymers, metals or ceramics
    - Matrix: polymers, metals or ceramics
    - Fibers orientation: unidirectional, woven, random
  - Complex failure modes
    - Transverse matrix fracture
    - Longitudinal matrix fracture
    - Fiber rupture
    - Fiber debonding
    - Delamination
    - Macroscopically: no plastic deformation

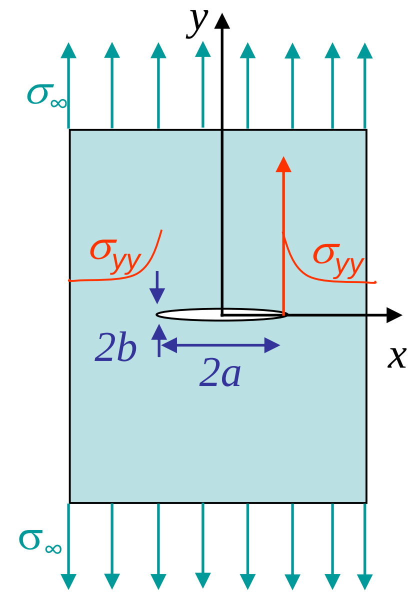


- Limits of the total life approach
  - Does not account for inherent defect
    - What is happening when a defect is present ?
    - Theoretical stress concentration ?
      - Infinite plane with an ellipsoidal void (1913, Inglis)

$$\sigma_{\max} = \sigma_{yy}(a, 0) = \sigma_\infty \left(1 + \frac{2a}{b}\right)$$

      - $b \rightarrow 0 \Longrightarrow \sigma_{max} \rightarrow \infty \Longrightarrow$ breaks for $\sigma_\infty \rightarrow 0$
      - In contradiction with Griffith and Irwin experiments

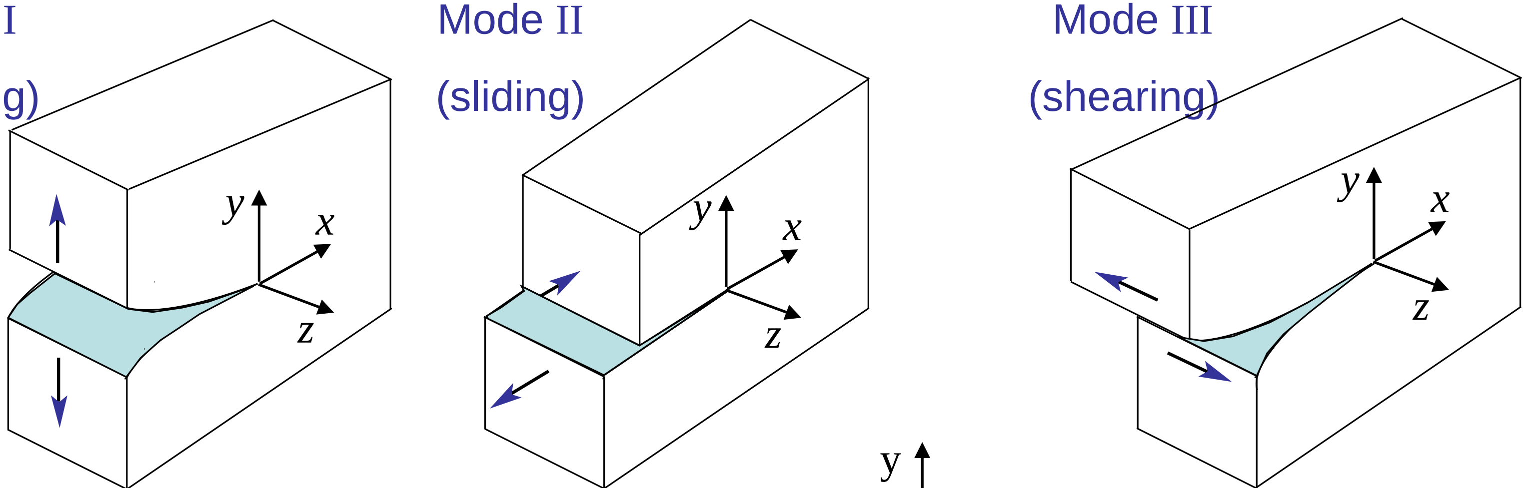
$$\sigma_{\mathrm{TS}} \sqrt{a} \div \sqrt{E\left(2\gamma_s + W_{\mathrm{pl}}\right)}$$



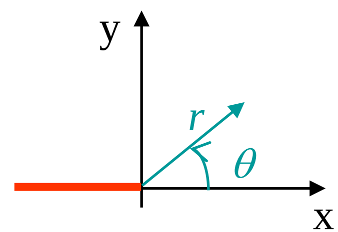
- Development of the fracture mechanics field
  - How can we predict failure when a crack is there ?
  - Microscopic observations for cycling loading
    - Crack initiated at stress concentrations (nucleation)
    - Crack growth
    - Failure of the structure when the crack reaches a critical size
    - How can we model this?

- Singularity at crack tip for linear and elastic materials
  - 1957, Irwin, 3 fracture modes

Mode I (opening) | Mode II (sliding) | Mode III (shearing)



- Boundary conditions



Mode I

$$\begin{cases} \boldsymbol{\sigma}_{zz} = 0 \quad \text{or} \quad \boldsymbol{\varepsilon}_{zz} = 0 \\ \boldsymbol{\sigma}_{yy}(\theta = \pm\pi) = 0 \\ \boldsymbol{\sigma}_{xy}(\theta = \pm\pi) = 0 \\ \boldsymbol{u}_x(\theta > 0) = \boldsymbol{u}_x(\theta < 0) \\ \boldsymbol{u}_y(\theta > 0) = -\boldsymbol{u}_y(\theta < 0) \end{cases}$$

Mode II

$$\begin{cases} \boldsymbol{\sigma}_{zz} = 0 \quad \text{or} \quad \boldsymbol{\varepsilon}_{zz} = 0 \\ \boldsymbol{\sigma}_{yy}(\theta = \pm\pi) = 0 \\ \boldsymbol{\sigma}_{xy}(\theta = \pm\pi) = 0 \\ \boldsymbol{u}_x(\theta > 0) = -\boldsymbol{u}_x(\theta < 0) \\ \boldsymbol{u}_y(\theta > 0) = \boldsymbol{u}_y(\theta < 0) \end{cases}$$

Mode III

$$\begin{cases} \boldsymbol{\sigma}_{xx} = \boldsymbol{\sigma}_{xy} = \boldsymbol{\sigma}_{yy} = \boldsymbol{\sigma}_{zz} = 0 \\ \boldsymbol{u}_y = \boldsymbol{u}_x = 0 \\ \boldsymbol{u}_z(\theta > 0) = -\boldsymbol{u}_z(\theta < 0) \end{cases}$$

Linear Elastic Fracture Mechanics (LEFM)

- Singularity at crack tip for linear and elastic materials (2)
  - Asymptotic solutions (Airy functions)

Mode I

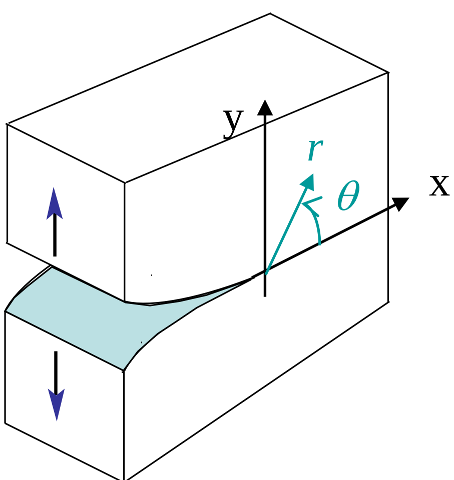
$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xx} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1-\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{yy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1+\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{\sigma}_{xy} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\sin\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
\boldsymbol{u}_{x} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\cos\left(\frac{\theta}{2}\right)\left[\kappa-1+2\sin^2\left(\frac{\theta}{2}\right)\right]\\
\boldsymbol{u}_{y} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)\left[\kappa+1-2\cos^2\left(\frac{\theta}{2}\right)\right]
\end{aligned}\right.$$

Mode II

$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xx} &= -\frac{C}{\sqrt{r}}\sin\frac{\theta}{2}\left[2+\cos\frac{3\theta}{2}\cos\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}\\
\boldsymbol{\sigma}_{yy} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{xy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1-\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{u}_{x} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)\left[\kappa+1+2\cos^2\left(\frac{\theta}{2}\right)\right]\\
\boldsymbol{u}_{y} &= -\frac{C\left(1+\nu\right)}{E}\sqrt{r}\cos\left(\frac{\theta}{2}\right)\left[\kappa-1-2\sin^2\left(\frac{\theta}{2}\right)\right]
\end{aligned}\right.$$

Mode III

$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xz} &= -\frac{C}{\sqrt{r}}\sin\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{yz} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{u}_{z} &= \frac{4C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)
\end{aligned}\right.$$



for plane σ $\kappa = \dfrac{3-\nu}{1+\nu}$ &

for plane ε $\kappa = 3-4\nu$

Linear Elastic Fracture Mechanics (LEFM)

- Singularity at crack tip for linear and elastic materials (3)
  - Asymptotic solutions (Airy functions)

Mode I

$$\boxed{\boldsymbol{\sigma}_{yy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1+\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right] + \cancel{\mathcal{O}(r^0)}}$$

Mode II

$$\boxed{\boldsymbol{\sigma}_{xy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1-\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right] + \cancel{\mathcal{O}(r^0)}}$$

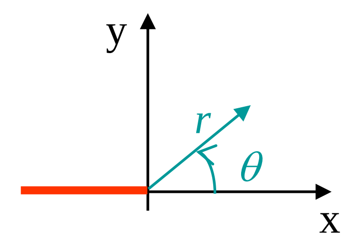
Mode III

$$\boxed{\boldsymbol{\sigma}_{yz} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2} + \cancel{\mathcal{O}(r^0)}}$$

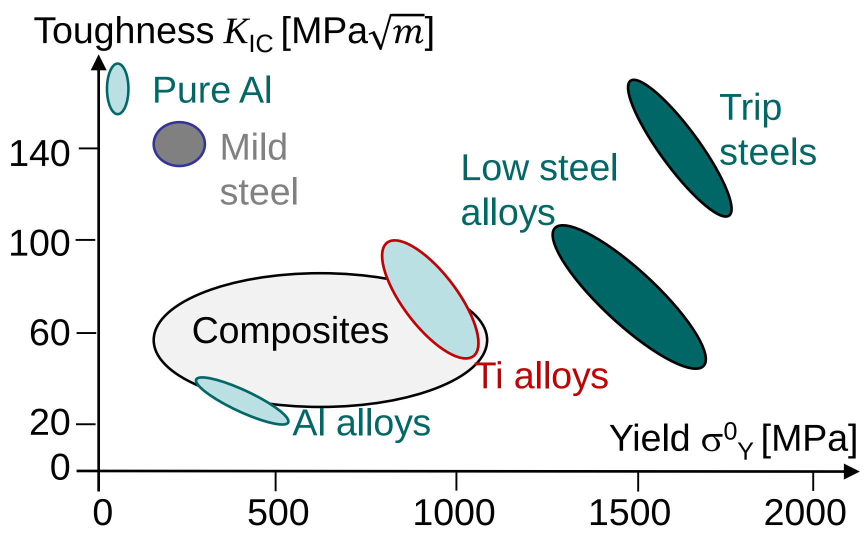
  - Introduction of the Stress Intensity Factors - SIF (Pa m$^{1/2}$)

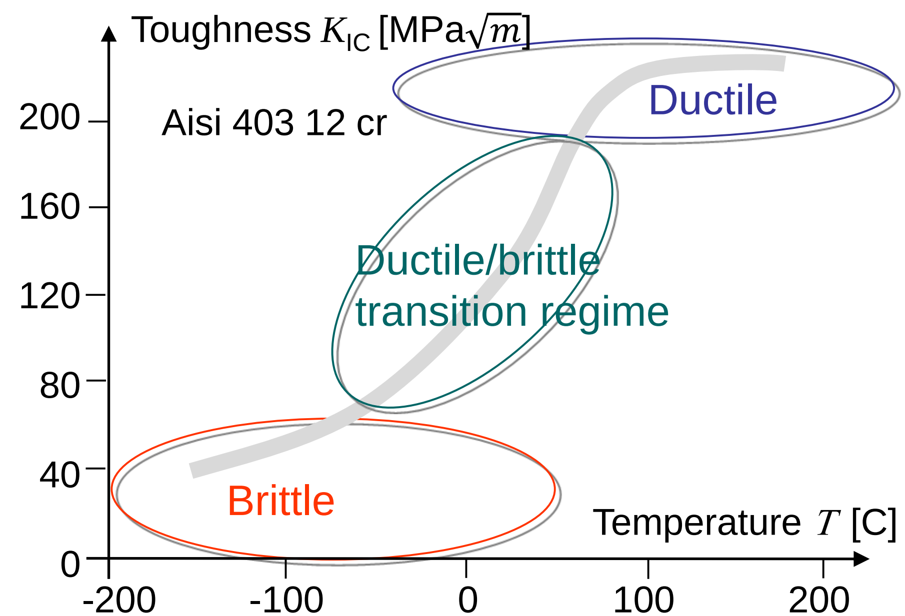
$$\left\{\begin{array}{l} K_I = \lim\limits_{r\to 0}\left(\sqrt{2\pi r}\,\boldsymbol{\sigma}_{yy}^{\text{mode I}}\,\big|_{\theta=0}\right) = C\sqrt{2\pi} \\ K_{II} = \lim\limits_{r\to 0}\left(\sqrt{2\pi r}\,\boldsymbol{\sigma}_{xy}^{\text{mode II}}\,\big|_{\theta=0}\right) = C\sqrt{2\pi} \\ K_{III} = \lim\limits_{r\to 0}\left(\sqrt{2\pi r}\,\boldsymbol{\sigma}_{yz}^{\text{mode III}}\,\big|_{\theta=0}\right) = C\sqrt{2\pi} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} \boxed{\boldsymbol{\sigma}^{\text{mode i}} = \frac{K_i}{\sqrt{2\pi r}}\mathbf{f}^{\text{mode i}}(\theta)} \\ \boxed{\boldsymbol{u}^{\text{mode i}} = K_i\sqrt{\frac{r}{2\pi}}\boldsymbol{g}^{\text{mode i}}(\theta)} \end{array}\right.$$

  - **K_i are dependent on both**
    - **Loading &**
    - **Geometry**



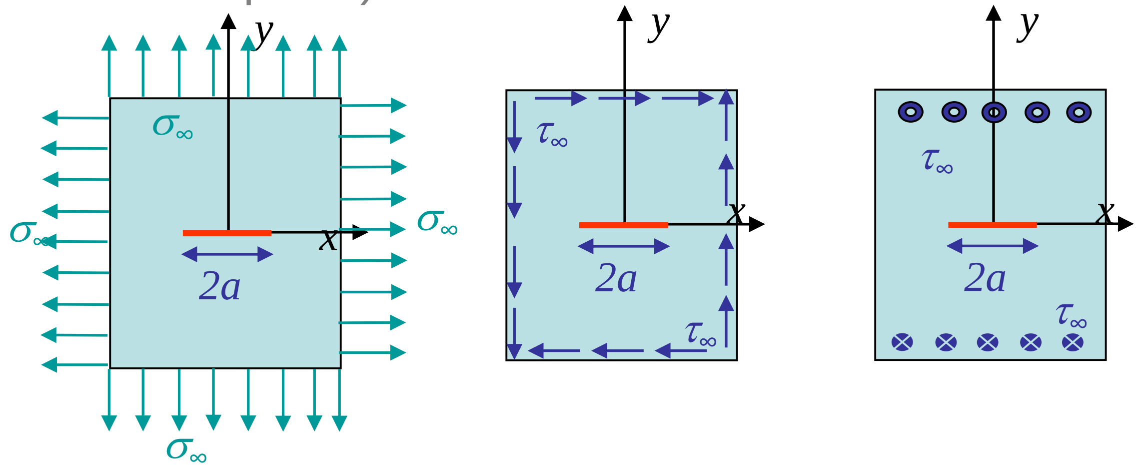
- 1957, Irwin, new failure criterion
  - $\sigma_{max} \rightarrow \infty$ ⟹ σ is irrelevant
  - Compare the SIFs (dependent on loading and geometry) to a new material property: the toughness
    - If $K_i = K_{iC}$ ⟹ crack growth
    - Toughness (ténacité) K_{Ic}
      - Steel, Al, … : see figures
      - Concrete: 0.2 - 1.4 $\mathrm{MPa\ m}^{1/2}$





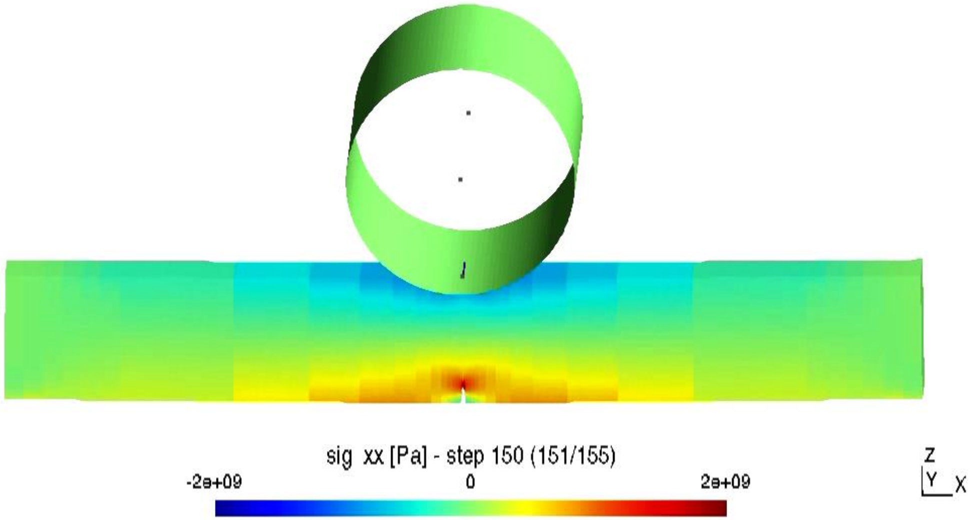
- Evaluation of the stress Intensity Factor (SIF)
  - Analytical (crack $2a$ in an infinite plane)

$$\left\{\begin{array}{l} K_I = \sigma_\infty \sqrt{\pi a} \\ K_{II} = \tau_\infty \sqrt{\pi a} \\ K_{III} = \tau_\infty \sqrt{\pi a} \end{array}\right.$$



  - Numerical

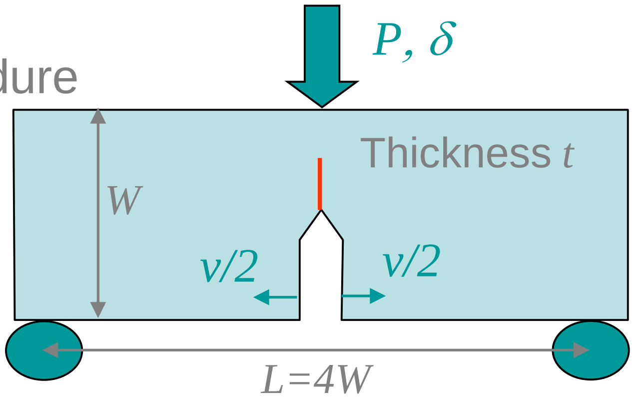
$$\left\{\begin{array}{l} K_I = \beta_I \sigma_\infty \sqrt{\pi a} \\ K_{II} = \beta_{II} \tau_\infty \sqrt{\pi a} \\ K_{III} = \beta_{III} \tau_\infty \sqrt{\pi a} \end{array}\right.$$

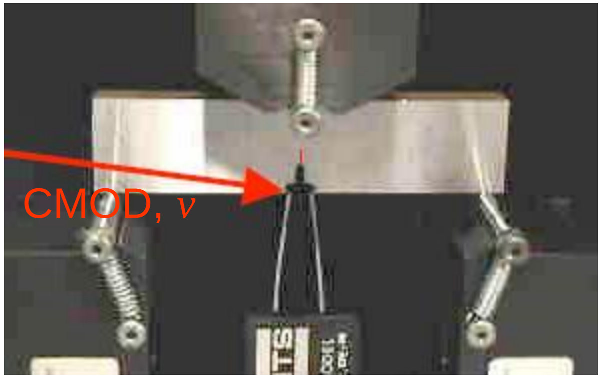


- β_i depends on
  - Geometry
  - Crack length

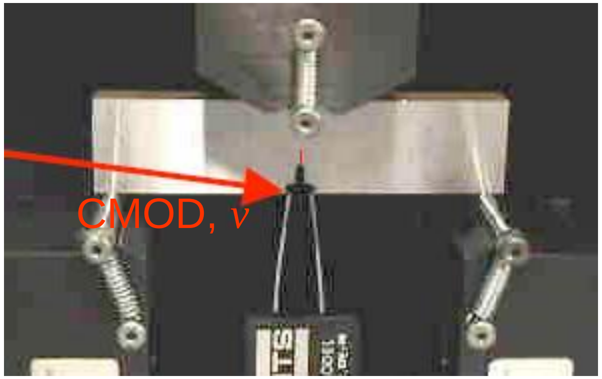
- Measuring K_{Ic}
  - Done by strictly following the ASTM E399 procedure
  - Specimen
    - Normalized, e.g. Single Notched Bend (SENB)
    - Plane strain constraint (thick enough

      ⇒ conservative (see next slide)
    - Specimen machined with a V-notch
  - Crack initiation
    - Cyclic loading to initiate a fatigue crack
    - Crack length from compliance
      - Crack Mouth Opening Displacement ($CMOD=v$) measured with a clipped gauge
      - Calibrated using FEM

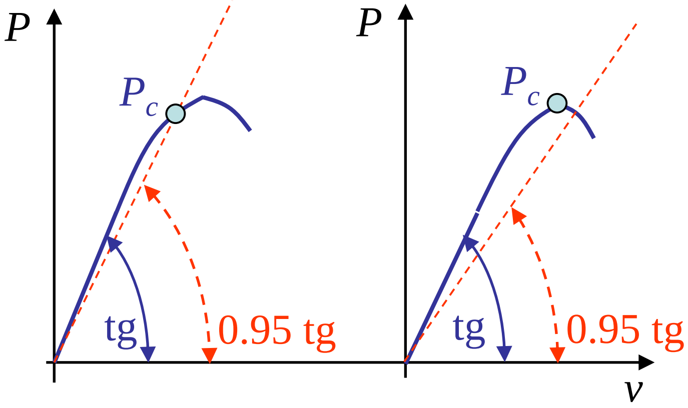




- Measuring K_{ic} (2)
  - Done by strictly following the ASTM E399 procedure
  - Toughness test
    - Calibrated P, δ recording equipment
    - Crack Mouth Opening Displacement (CMOD=v) measured with a clipped gauge



    - P_c is obtained on $P - v$ curves
      - Either the 95% offset value or
      - The maximal value reached before

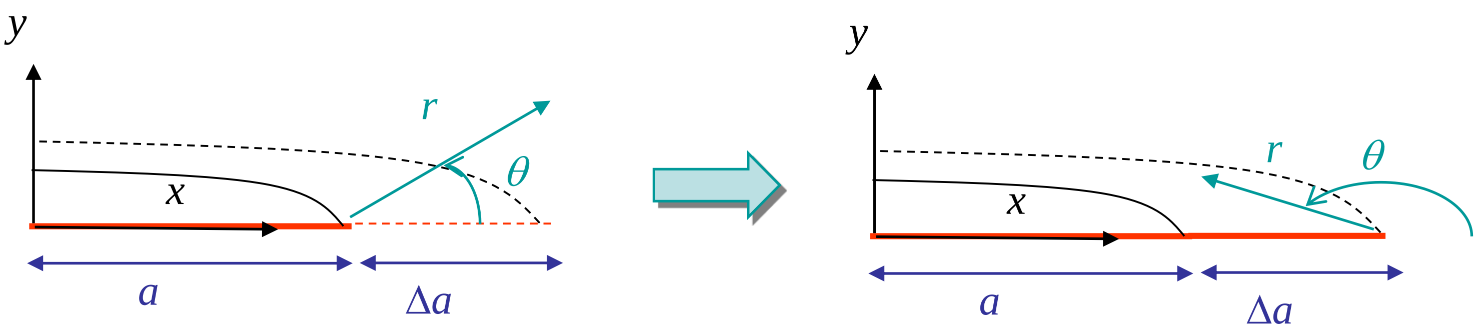


    - K_{Ic} is deduced from P_c using

$$K_I = \frac{PL}{tW^{\frac{3}{2}}} f\left(\frac{a}{W}\right)$$

      - $f(a/W)$ depends on the test (SENB, …)
      - $f(a/W)$ calibrated using FEM etc, in the norm
  - Check the constraint once you have K_{Ic}
    - Plane strain constraint (thick enough)

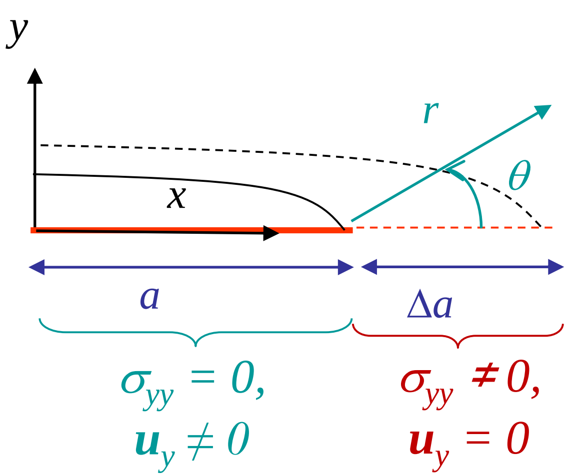
- Energy approach
  - We can now assess the crack loading
    - SIF $\boxed{\boldsymbol{\sigma}^{\text{mode i}} = \frac{K_i}{\sqrt{2\pi r}} \mathbf{f}^{\text{mode i}}(\theta)}$
    - Crack propagates if $K_i \geq K_{iC}$
  - Back to Griffith
    - $\sigma_{\text{TS}}\sqrt{a} \div \sqrt{E\, 2\gamma_s}$
    - Relates apparent strength to surface energy
  - How to make the link: Consider a virtual crack propagation
    - What happens if the crack grows?
    - How is the strain energy evolving



- Energy approach (2)
  - Mode I

$$\begin{cases} \boldsymbol{\sigma}_{yy} = \dfrac{C}{\sqrt{r}} \cos\dfrac{\theta}{2}\left[1+\sin\dfrac{3\theta}{2}\sin\dfrac{\theta}{2}\right] + \mathcal{O}\left(r^0\right) \\ \boldsymbol{u}_y = \dfrac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\dfrac{\theta}{2}\right)\left[\kappa+1-2\cos^2\left(\dfrac{\theta}{2}\right)\right] \end{cases}$$

  - Initial crack $2a$

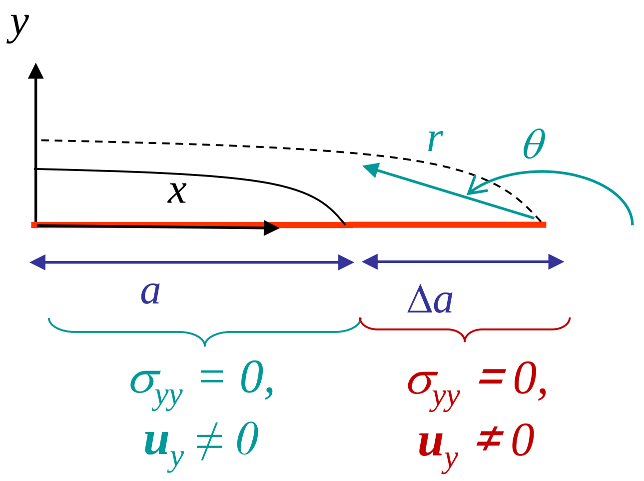


$$\begin{cases} \boldsymbol{\sigma}^0_{yy}\left(\theta = 0,\ r = x-a\right) = \dfrac{\sigma_\infty\sqrt{a}}{\sqrt{2\left(x-a\right)}} \\ \boldsymbol{u}^0_y\left(\theta=0,\ r=x-a\right) = 0 \end{cases}$$

- Energy approach (3)
  - Mode I

$$\begin{cases} \boldsymbol{\sigma}_{yy} = \frac{C}{\sqrt{r}} \cos\frac{\theta}{2}\left[1+\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right] + \mathcal{O}\left(r^0\right) \\ \boldsymbol{u}_y = \frac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)\left[\kappa+1-2\cos^2\left(\frac{\theta}{2}\right)\right] \end{cases}$$

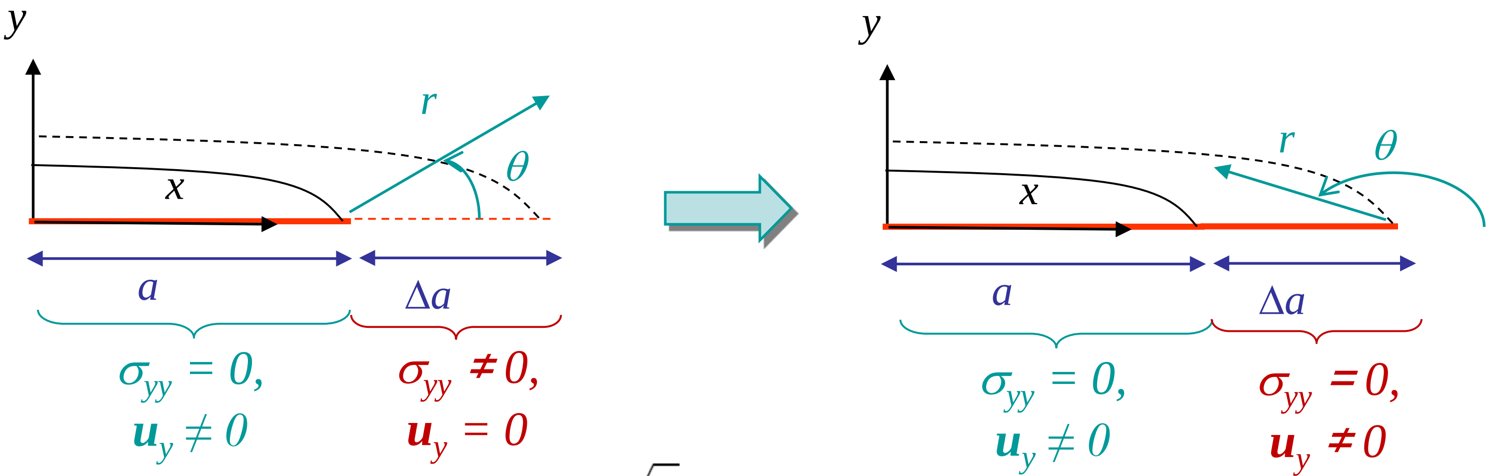
  - Virtual crack *2(a+Δa)*



$$\begin{cases} \boldsymbol{\sigma}^1_{yy}\left(\theta=\pi,\, r=a+\Delta a - x\right) = 0 \\ \boldsymbol{u}^1_y\left(\theta=\pm\pi,\, r=a+\Delta a - x\right) = \\ \quad \pm\frac{\sigma_\infty\left(1+\nu\right)\left(\kappa+1\right)}{E\sqrt{2}} \\ \quad \sqrt{a+\Delta a}\sqrt{a+\Delta a - x} \end{cases}$$

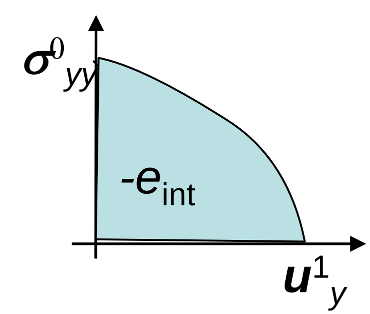
Linear Elastic Fracture Mechanics (LEFM)

- Energy approach (4)
  - Virtual crack propagation



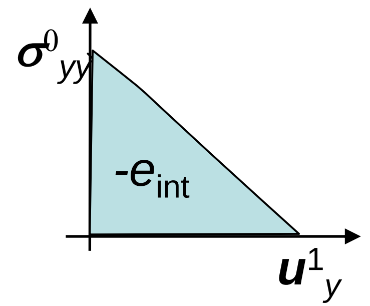
$$\begin{cases} \boldsymbol{\sigma}^0_{yy}\left(\theta = 0,\ r = x - a\right) = \dfrac{\sigma_\infty \sqrt{a}}{\sqrt{2\left(x-a\right)}} \\ \boldsymbol{u}^1_y\left(\theta = \pm\pi,\ r = a + \Delta a - x\right) = \ \pm \dfrac{\sigma_\infty\left(1+\nu\right)\left(\kappa+1\right)}{E\sqrt{2}}\sqrt{a+\Delta a}\sqrt{a+\Delta a - x} \end{cases}$$

  - Strain energy changes if crack grows by $2\Delta a$
    - $\Delta E_{\text{int}} = -4dz \int_a^{a+\Delta a} \int_{\boldsymbol{u}^0_y}^{\boldsymbol{u}^1_y} \boldsymbol{\sigma}_{yy} d\boldsymbol{u}_y dx$
    - x4 as it is for $x > 0$, $x < 0$ & for $y<0$, $y>0$



- Energy approach (5)
  - Variation of strain energy when crack grows by Δa

$$\Delta E_{\text{int}} = -4dz \int_a^{a+\Delta a} \int_{\boldsymbol{u}_y^0}^{\boldsymbol{u}_y^1} \boldsymbol{\sigma}_{yy} d\boldsymbol{u}_y dx$$



    - Linear elasticity: $\boldsymbol{\sigma}_{yy}$ linear in terms of $\boldsymbol{u}_y$

$$\Delta E_{\text{int}} = -2dz \int_a^{a+\Delta a} \boldsymbol{\sigma}_{yy}^0 \boldsymbol{u}_y^1 dx$$

    - Change of variable $x = a + \Delta a \cos^2 \theta$

$$\Rightarrow \Delta E_{\text{int}} = -\frac{dz\sigma_\infty^2 \sqrt{a\,(a+\Delta a)}\,(1+\nu)\,(1+\kappa)}{2E}\pi \Delta a \quad <0$$

  - **New concept:** G is the energy release rate **for a straight ahead growth**

$$\boxed{G = -\frac{dE_{\text{int}}}{dA} = -\lim_{\Delta a \to 0} \frac{\Delta E_{\text{int}}}{2\Delta a dz} = \frac{\pi a \sigma_\infty^2}{4E}\,(1+\nu)\,(\kappa+1) = \frac{\pi a \sigma_\infty^2}{E'} = \frac{K_I^2}{E'}}$$

  - Expression $G = \frac{K_I^2}{E'}$ with $E' = \frac{4E}{(1+\nu)\,(\kappa+1)} = \begin{cases} E & \text{P}\sigma \\ \frac{E}{1-\nu^2} & \text{P}\varepsilon \end{cases}$
    - Holds not only for infinite plate
    - In linear elasticity and crack growing straight ahead

- Energy approach (6)
  - 1920, Griffith
    - Total energy $E = E_{\text{int}} + \Gamma$ is the sum of
    - Internal (strain) energy E_{int} of the structure
    - The atomistic bond energy Γ where a crack possibly propagates
  - Energy conservation:
    - Assuming a crack do propagate by a surface ΔA

A crack creates 2 surfaces A

$$\frac{\partial E}{\partial A} = \frac{\partial E_{\text{int}}}{\partial A} + \frac{\partial \Gamma}{\partial A} = 0 \Longrightarrow G = -\frac{dE_{\text{int}}}{dA} = \frac{d\Gamma}{dA} = 2\gamma_s = G_c$$

  - We have a new crack growth criterion

$$\boxed{G \geq G_c = 2\gamma_s}$$

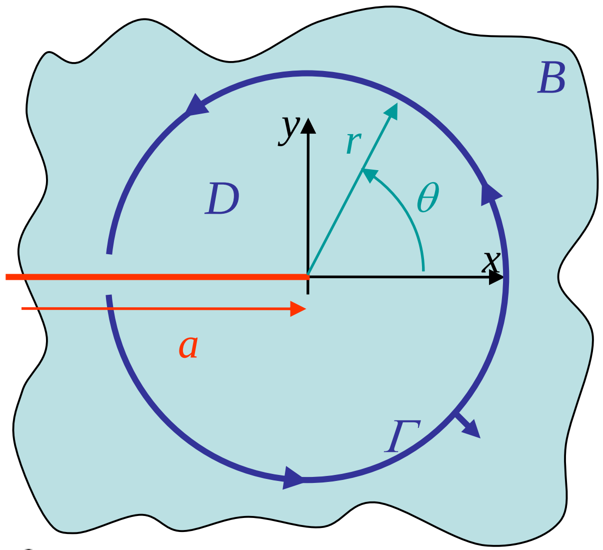
G is the energy release rate (depends on geometry and loading)

γ_s is the surface energy (material property)

  - Mode I, infinite plane
    - Strength $G = \frac{\pi a {\sigma_\infty}^2}{E'} = \frac{K_I^2}{E'} \Longrightarrow \sigma_{\text{TS}} = \sqrt{\frac{2\gamma_s E}{\pi a}}$

Depends on the crack size

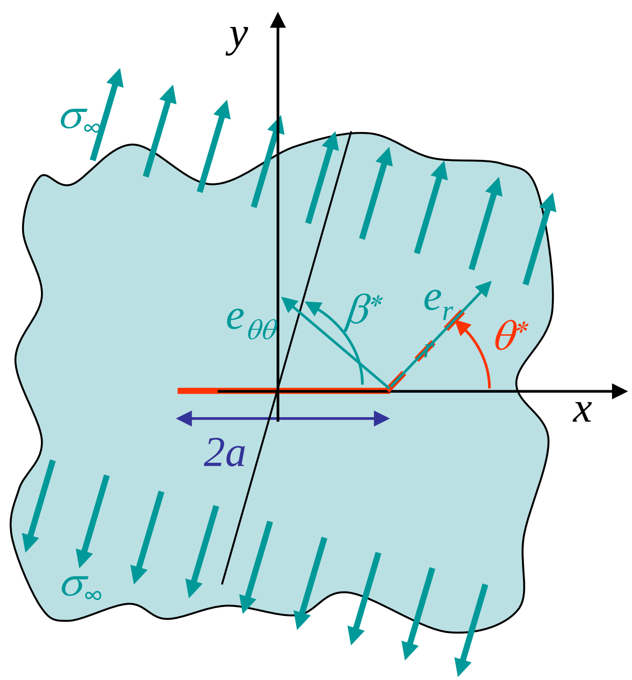
- Energy approach: J-integral
  - Energy release rate
    - Straight ahead propagation for linear elasticity $G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{(1+\nu)\, K_{III}^2}{E}$
  - Should be related to the energy flowing toward the crack tip
    - J-integral $J = \int_\Gamma \left[U(\boldsymbol{\varepsilon})\, \boldsymbol{n}_x - \boldsymbol{u}_{,x} \cdot \boldsymbol{T} \right] dl$

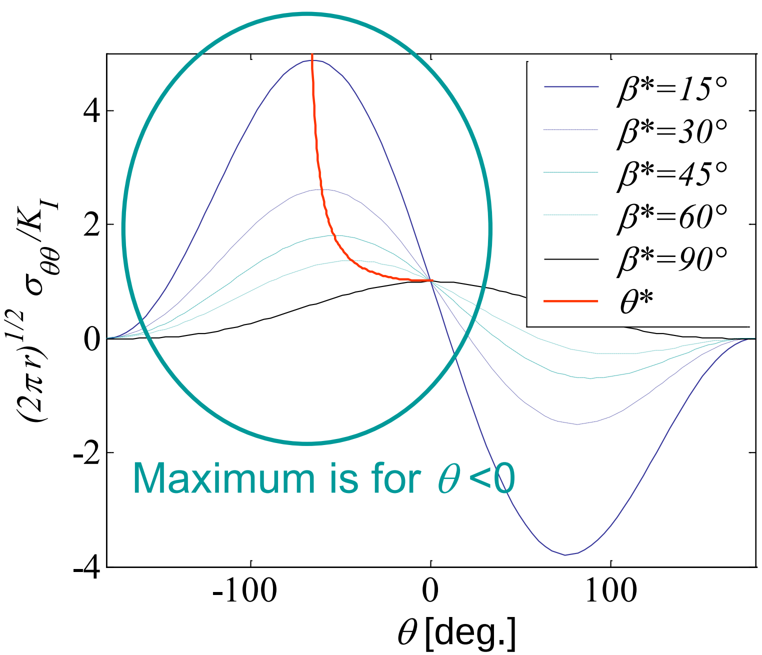


      - Defined even for non-linear materials
      - Is path independent if the contour Γ embeds a straight crack tip
      - BUT no assumption on subsequent growth direction
      - If crack grows straight ahead: $\Longrightarrow$ $G=J$
      - If linear elasticity: $\Longrightarrow$ $J = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$
      - Can be extended to plasticity if no unloading (see later)
    - Advantages
      - Efficient numerical computation of the SIFs
      - Useful for non perfectly brittle materials

- Direction of crack grow
  - Assumptions: the crack will grow in the direction where the SIF related to mode I in the new frame is maximal
    - Crack growth if $\left(\sqrt{2\pi r}\boldsymbol{\sigma}_{\theta\theta}\left(r,\ \theta*\right)\right) \geq K_C$ with $\partial_\theta \boldsymbol{\sigma}_{\theta\theta}|_{\theta*} = 0$
  - From direction of loading, one can compute the propagation direction

$$\cot\beta^* = \frac{K_{II}}{K_I} \Longrightarrow \boldsymbol{\sigma}_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\cos^3\frac{\theta}{2} - \frac{3\cot\beta^*}{2}\sin\theta\cos\frac{\theta}{2}\right]$$





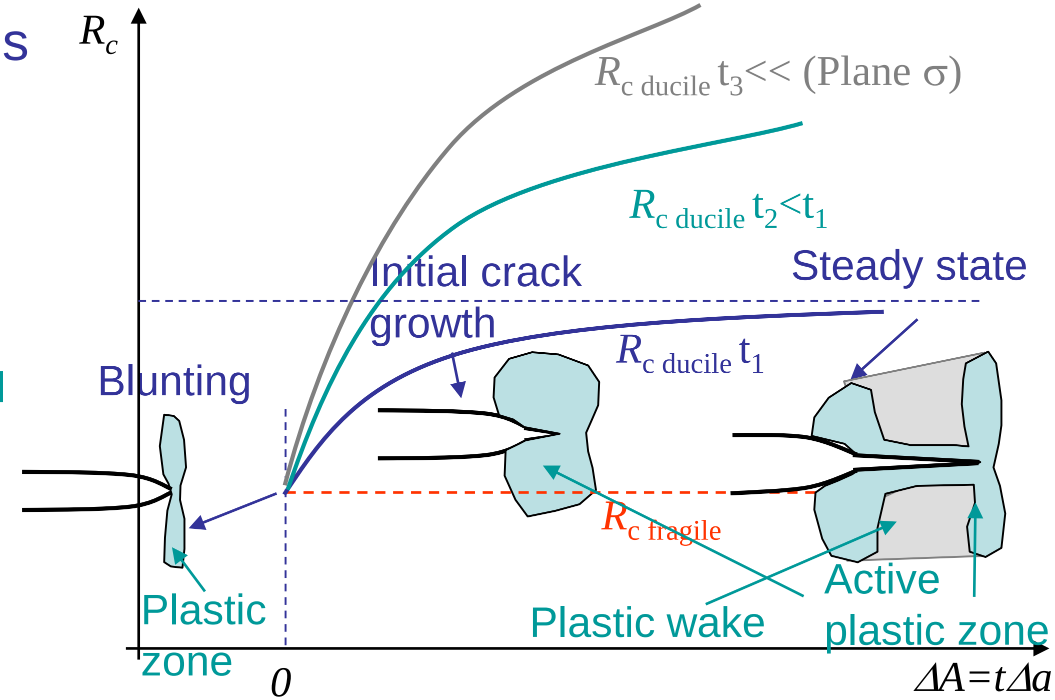
Linear Elastic Fracture Mechanics (LEFM)

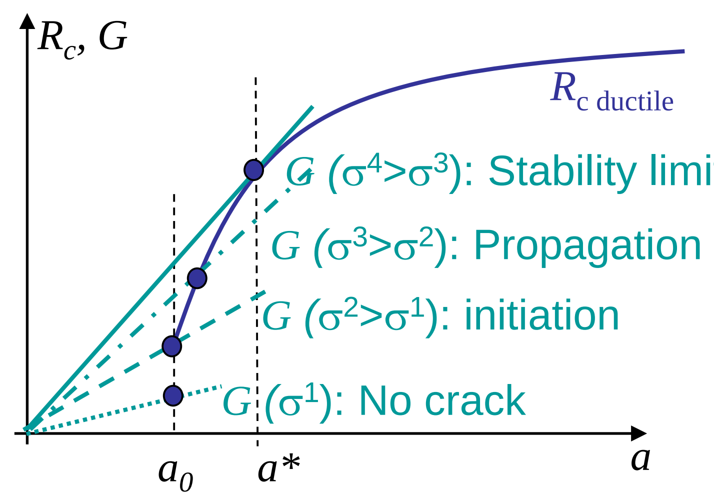
- Non-perfectly brittle materials
  - Plastic zone
    - Limit G_c becomes $R_c(a)$
    - $R_c(a)$ depends on the loading mode and no longer only on the material
  - Stability of the crack
    - Stable

$$\begin{cases} G = R_c(a) \\ \dfrac{dG}{da} \leq \dfrac{dR_c}{da} \end{cases}$$

    - Instable: $a > a^*$

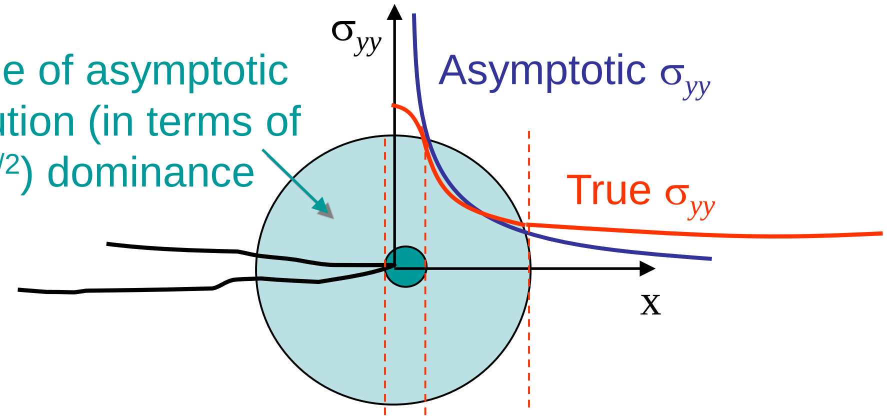
$$\frac{dG}{da} > \frac{dR_c}{da}$$





Limits of the LEFM

- The stress still tends toward infinity
  - There are non-linearities (plasticity at crack tips)
  - Far away from the crack the approximation does not hold (structural response)



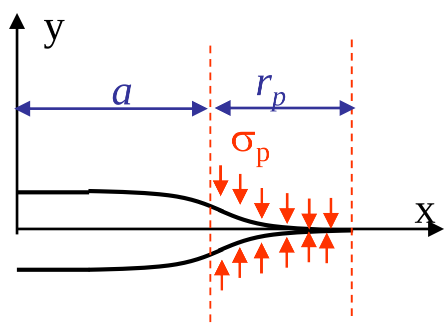
- Small Scale Yielding assumption
  - Holds if crack front plastic zone size is small compared to the crack length
  - Cohesive zone r_p at crack tip

$$a \rightarrow a_{\text{eff}} = a + \frac{r_p}{3}$$

  - 1960, Dugdale-Barenblatt
    - Mode I

$$r_p = \frac{\pi K_I^2 \left(a_{\text{eff}}\right)}{8\sigma_p^2}$$

Iterative method



- If applied stress larger than half the yield stress
  - The assumption does not hold ⇒ non linear fracture mechanics (NLFM)

- Pressurized Water Reactor (PWR)
  - Typical RPV operating conditions: 15 MPa, 300º C

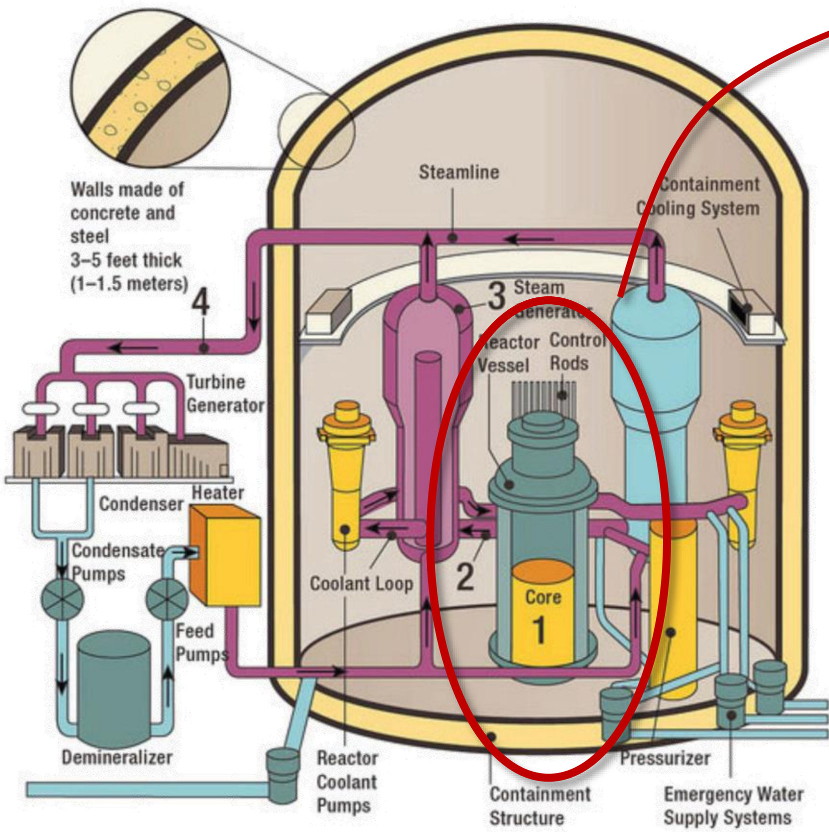


Image from US. Nuclear Regulatory Commision,
Quadrennial Technology Review 2015, U.S. Department of Energy

Ø ~ 3-5 m

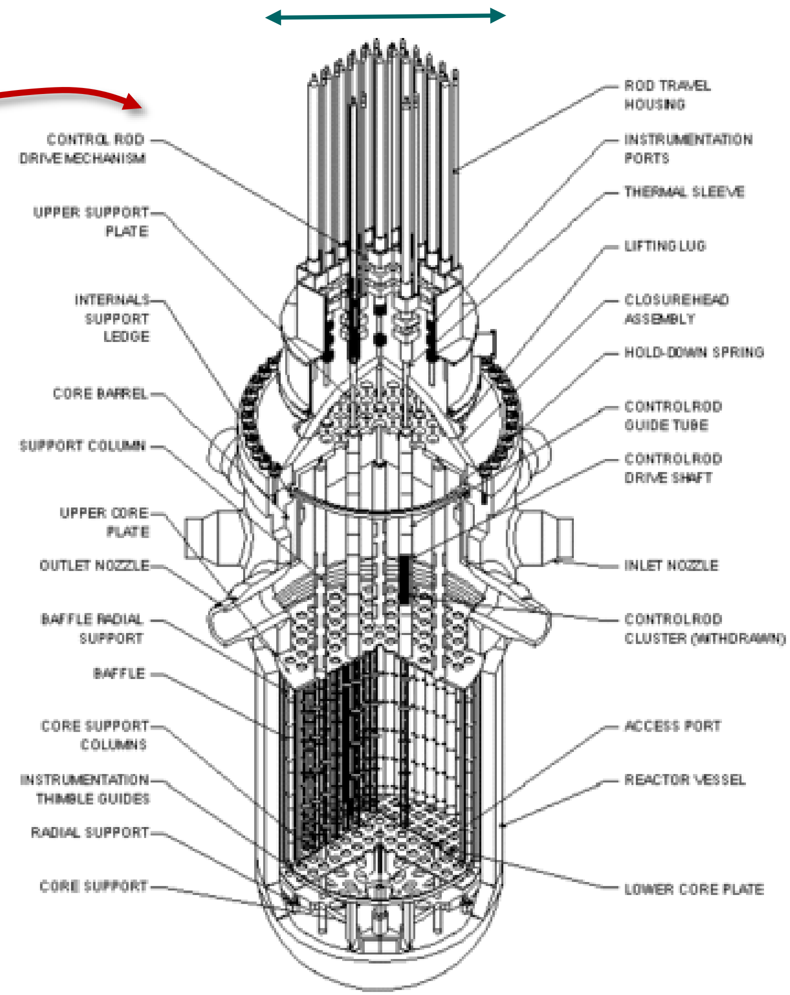
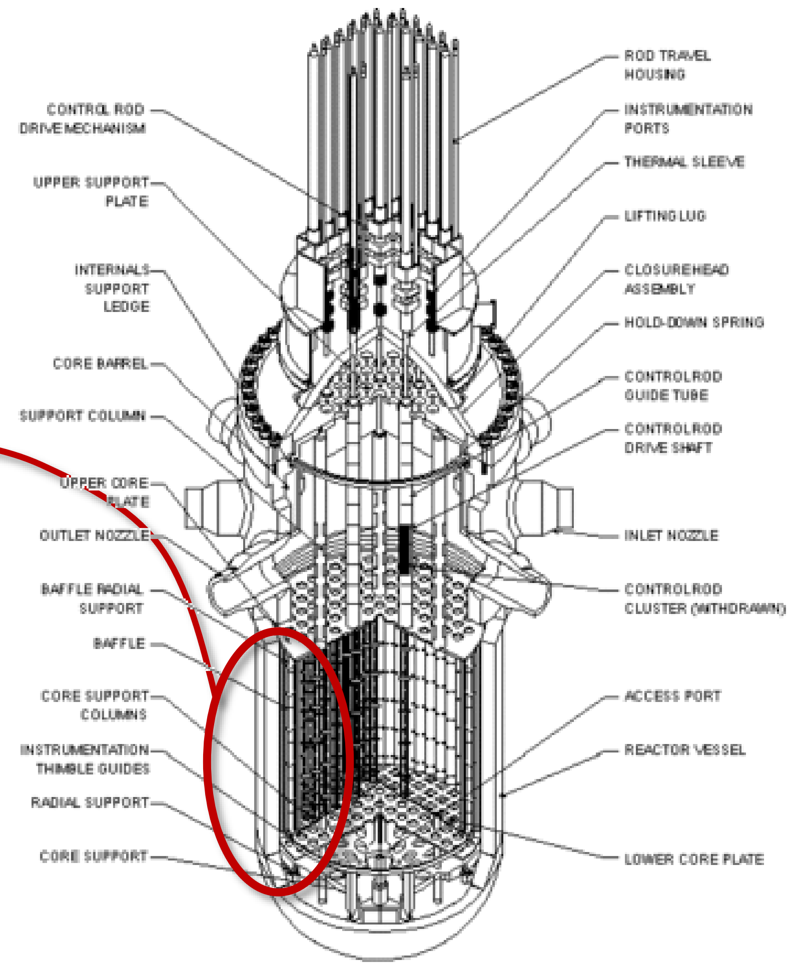
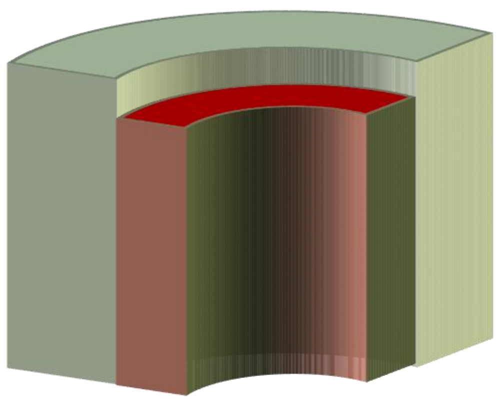


Image from US. Nuclear Regulatory Commision,
http://www.eia.doe.gov/cneaf/nuclear/page/nuc_reactors/pwr.html

- RPV material
  - Base material:
    - Manganese–nickel–molybdenum low-alloy steel
    - E.g. SA508 Cl. 3
  - Cladding:
    - Stainless steel

Ø ~ 3-5 m





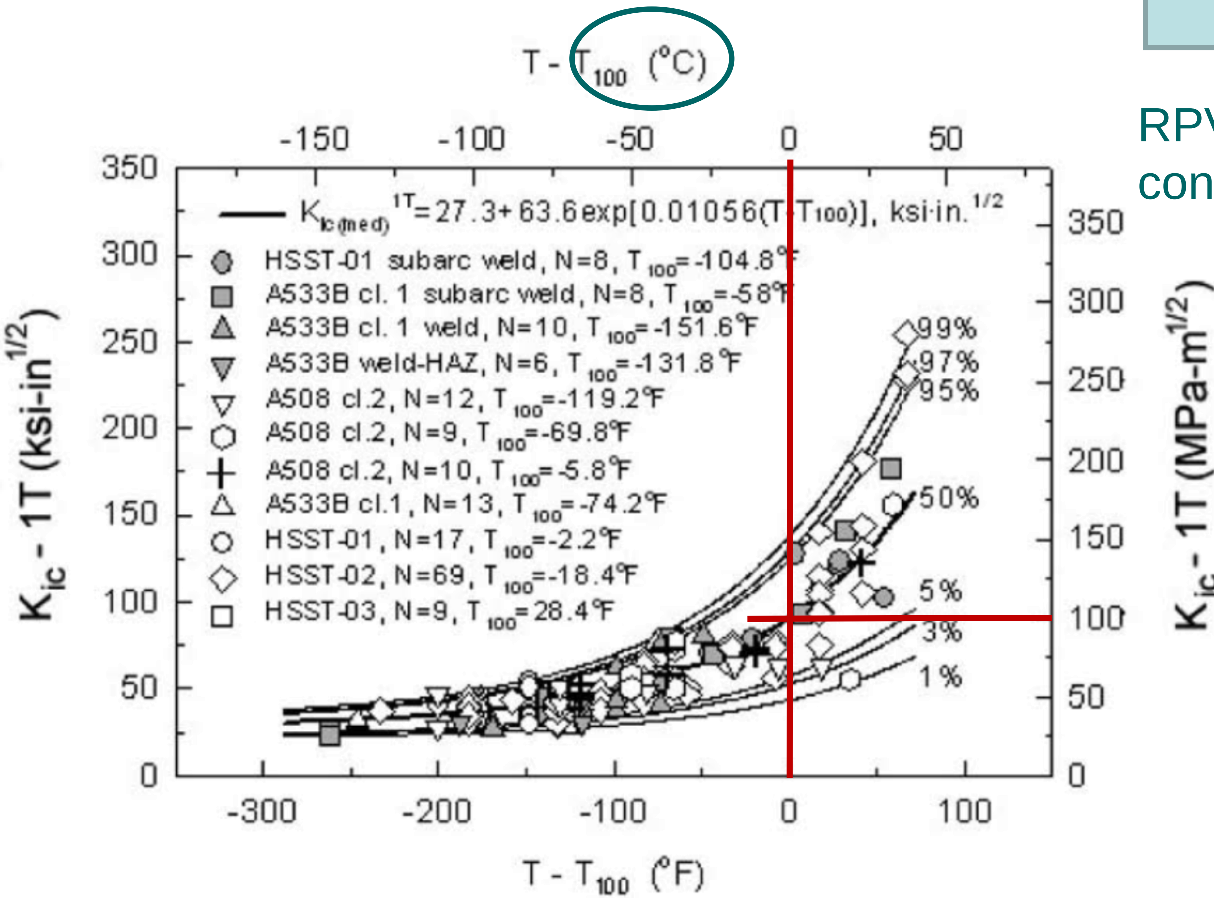
t_{base} ~ 0.2 m t_{clad} ~ 0.01 m

Image from US. Nuclear Regulatory Commision, http://www.eia.doe.gov/cneaf/nuclear/page/nuc_reactors/pwr.html

- RPV material: exhibits a DBTT
  - Definition of T_{100} :Reference temperature for which 50% of the normalized samples break at a toughness of 100 [MPa$\sqrt{m}$]

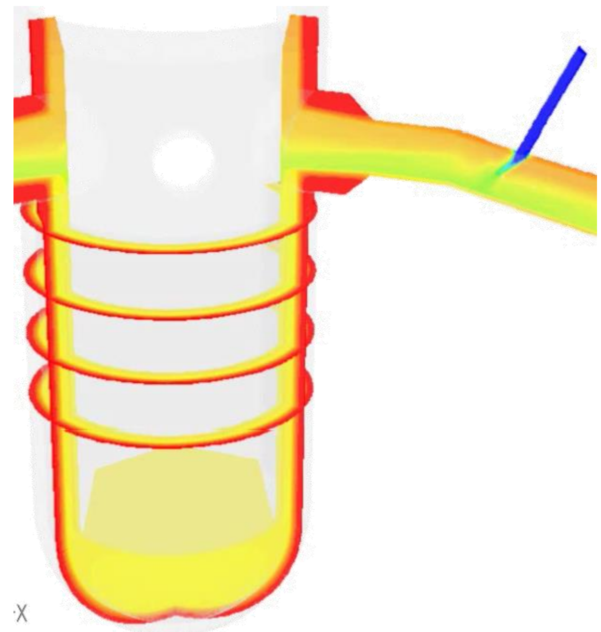
Requires NLFM

RPV operating conditions

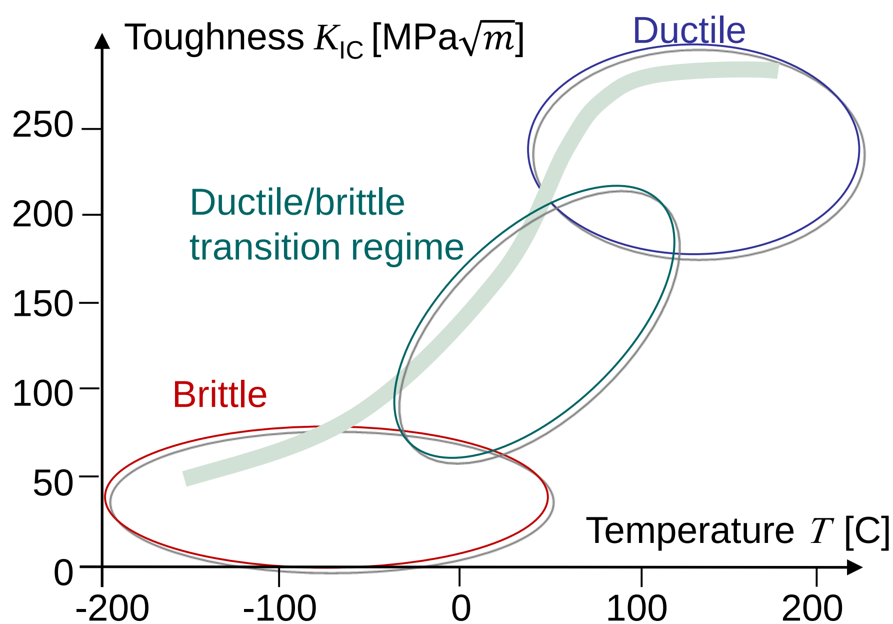


Integrity of reactor pressure vessels in nuclear power plants : assessment of irradiation embrittlement effects in reactor pressure vessel steels. International Atomic Energy Agency, 2009. (IAEA nuclear energy series), ISSN 1995-7807, ISBN 978-92-0-101709-3

- RPV operations
  - Pressurized Thermal Shock (PTS)
    - Loss of coolant accident (LOCA)
    - Primary side injection of cold water
    - Sudden decrease of wall temperature
    - Decrease of the toughness
    - N.B. decrease of pressure, but thermal strains



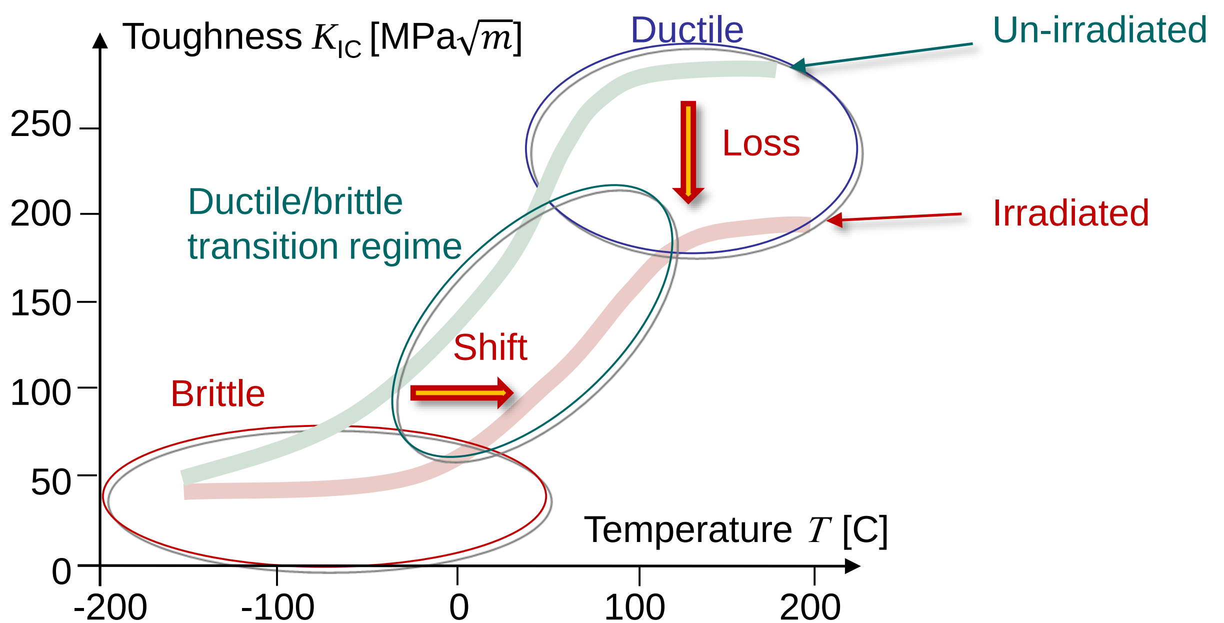
Christopher Boyd, Interactions of Thermal-Hydraulics with Fuel Behavior, Structural Mechanics, and Computational Fluid Dynamics
Office of Nuclear Regulatory Research Nuclear Regulatory Commission



Fictitious material

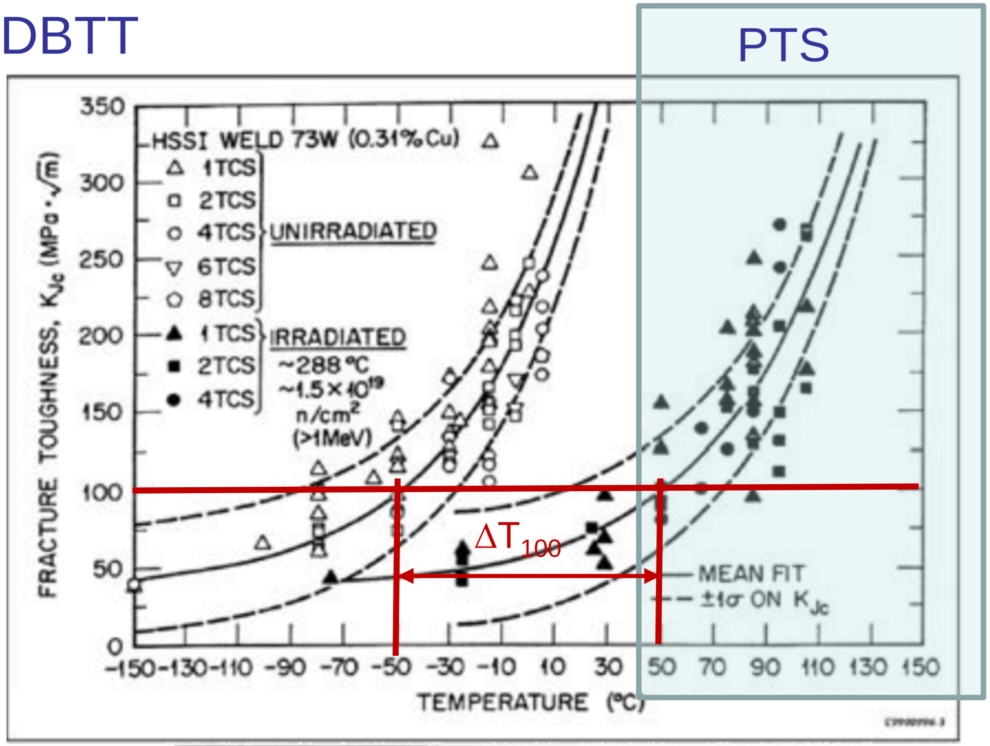
- Irradiation embrittlement
  - Irradiation during the RPV operation
  - Embrittlement increasing with
    - The fluence (number of neutrons per unit surface)
    - Neutron energy (>1MeV)

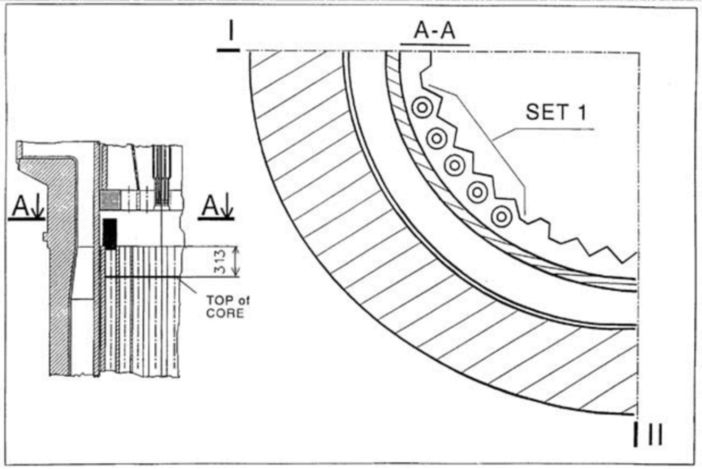
Shift in the DBTT with the operating time



Fictitious material

- Irradiation embrittlement: Shift in DBTT
  - Defined by a shift in T_{100}
    - $T_0 = T_{100} + \Delta T_{100}$
  - Computed in terms of
    - Chemical composition
    - Power of fluence (Φ^n)
  - Measured
    - Surveillance specimen capsules
    - Near the inside vessel wall





Integrity of reactor pressure vessels in nuclear power plants : assessment of irradiation embrittlement effects in reactor pressure vessel steels. International Atomic Energy Agency, 2009. (IAEA nuclear energy series), ISSN 1995-7807, ISBN 978-92-0-101709-3

- Methodology

Defects characterization

- Postulated flaws (following norms)
- Non Destructive in-service inspections (Ultrasonic …)

Loading conditions

- Critical loading conditions
- Evaluation of the temperature, stress … distributions

Material properties

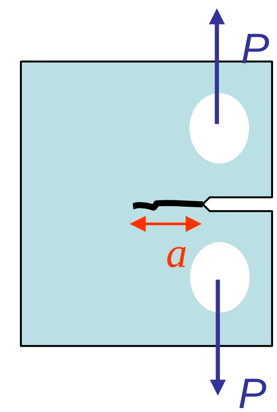
- Toughness of base material (experiments)
- Environmental conditions (end-of-life fluence, temperature)

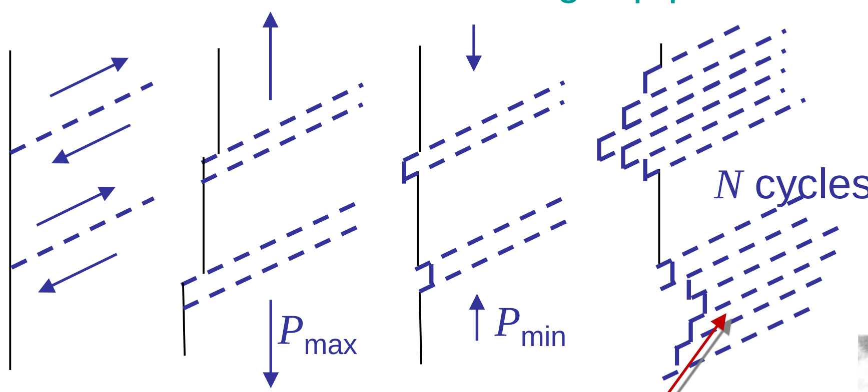
- Evaluation of SIFs
  - Analytical methods
  - Numerical methods

- In-service toughness
  - Models
  - Experiments

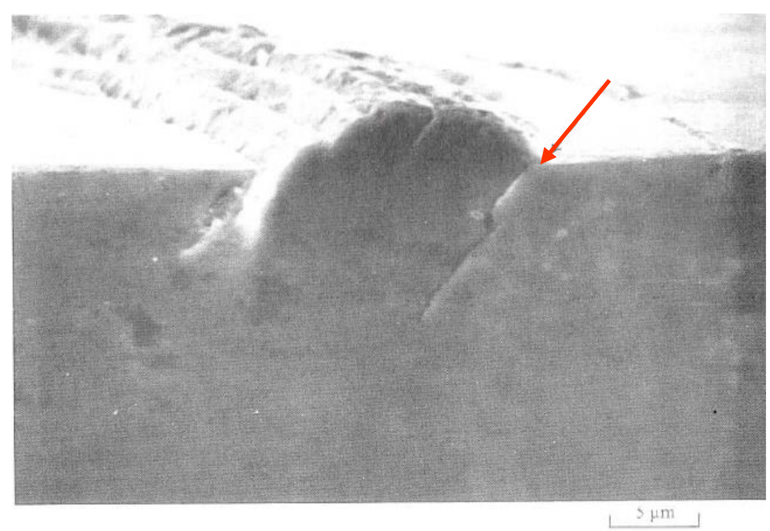
Cyclic loading

- Fatigue failure
  - Tests performed with different $\Delta P = P_{max} - P_{min}$
  - Nucleation: cracks initiated for $K < K_c$
    - Surface: deformations result from dislocations motion along slip planes



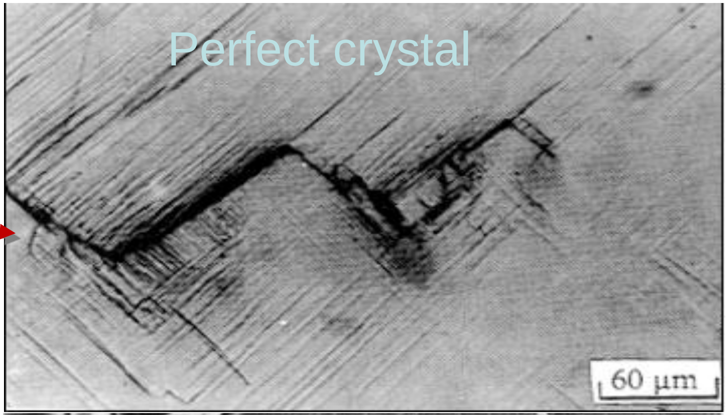


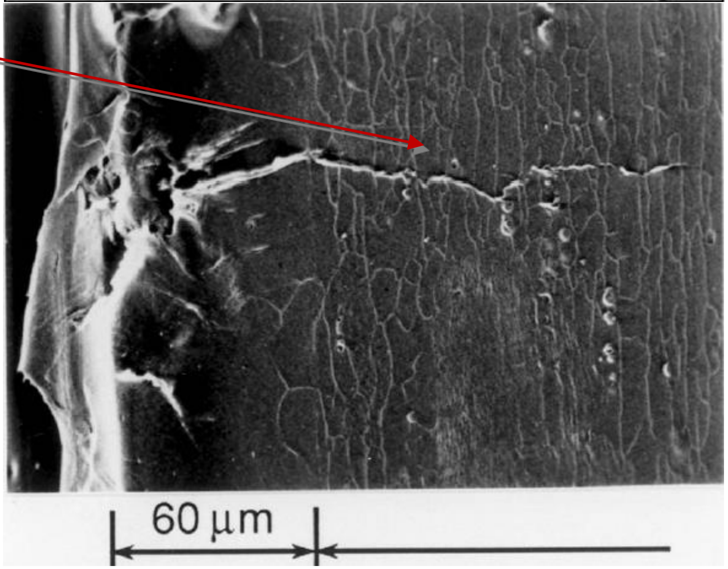
    - Can also happen around a bulk defect

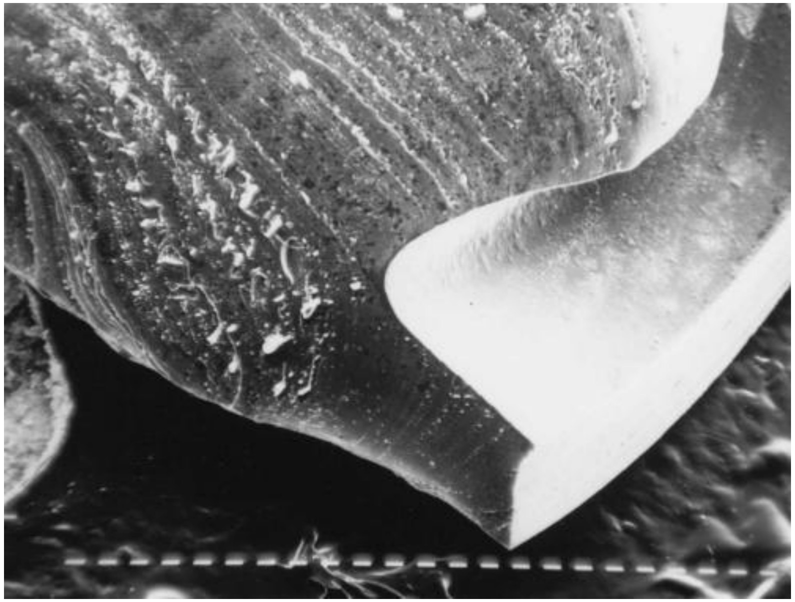


Persistent slip band (PSB)

- Fatigue failure (2)
  - Stage I fatigue crack growth:
    - Along a slip plane
  - Stage II fatigue crack growth:
    - Across several grains
      - Along a slip plane in each grain,
      - Straight ahead macroscopically
    - Striation of the failure surface: corresponds to the cycles



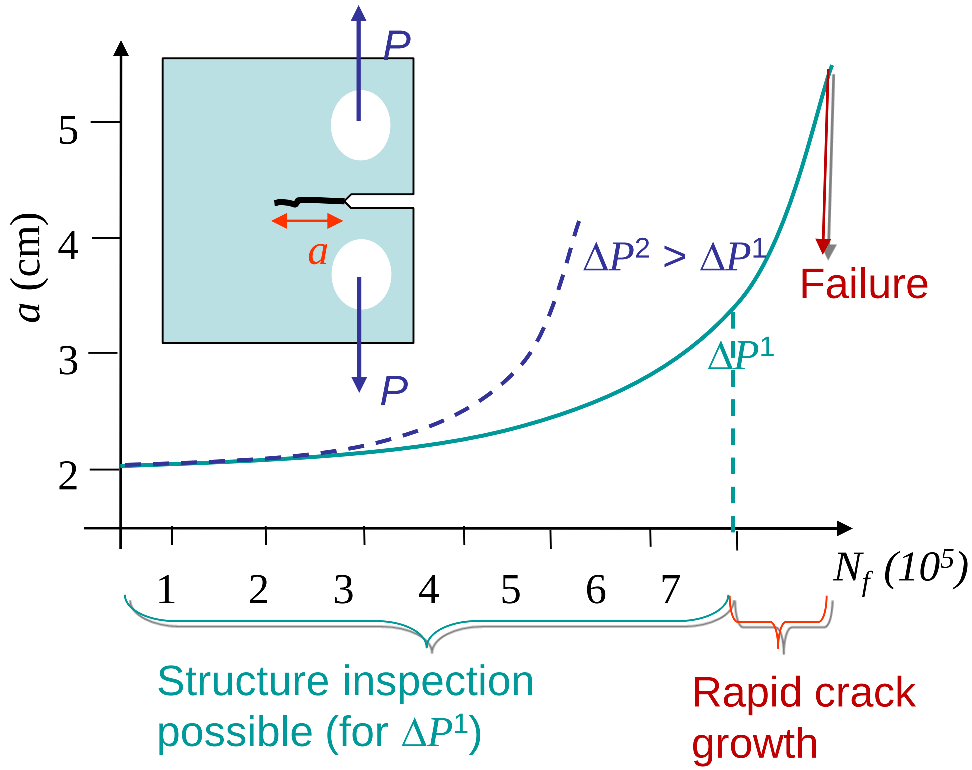




- Fatigue failure (3)
  - SSY assumption
    - Tests: conditioning parameters
      - ΔP &
      - P_{min} / P_{max}
    - Therefore fatigue failure can be described by
      - $\Delta K = K_{\text{max}} - K_{\text{min}}$ &
      - $R = K_{\text{min}}/K_{\text{max}}$

$$\boxed{\frac{da}{dN_f} = f\left(\Delta K, R\right)}$$

Interval of striations

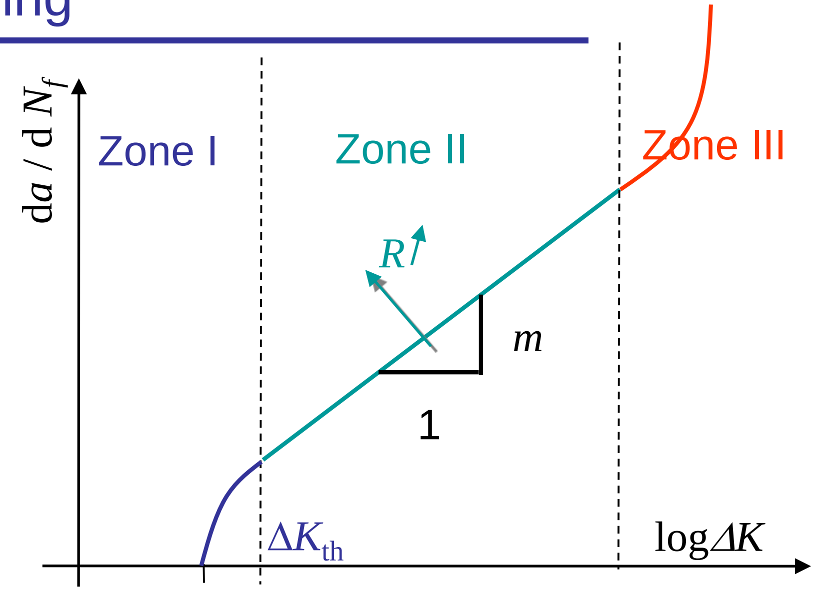


- There is ΔK_{th} such that if $\Delta K \sim \Delta K_{\text{th}}$:
  - The crack has a growth rate lower than one atomic spacing per cycle (statistical value)
  - Dormant crack

- Crack growth rate
  - Zone I
    - Stage I fatigue crack growth
    - ΔK_{th} depends on R
  - Zone II
    - Stage II fatigue crack growth: striation
    - 1963, Paris-Erdogan

$$\frac{da}{dN_f} = C \Delta K^m$$

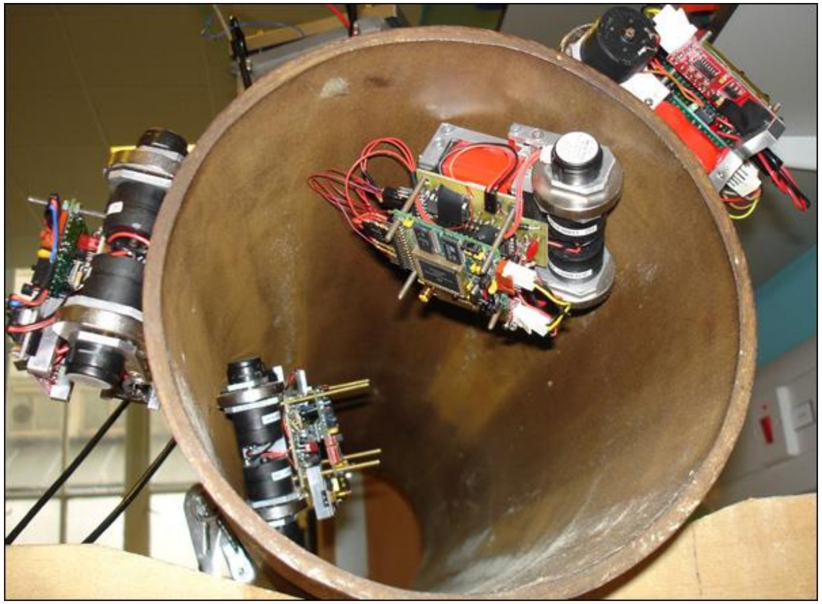
      - Depends on the geometry, the loading, the frequency
      - Steel: ΔK_{th} ~ 2-5 MPa $m^{1/2}$, C ~ 0.07-0.11 10^{-11} [m (MPa m$^{1/2}$)$^{-m}$], m ~ 4
      - Steel in sea water: ΔK_{th} ~ 1-1.5 MPa m$^{1/2}$, C ~ 1.6 10^{-11} [idem], m ~ 3.3
    - Be careful: K depends on a $\Longrightarrow$ integration required to get $a(N_f)$
      - Mode I : $K_I = \sigma_\infty \sqrt{\pi a}$ $\Longrightarrow$ $\Delta K = (\sigma_{\infty,\max} - \sigma_{\infty,\min}) \sqrt{\pi a}$
  - Zone III
    - Rapid crack growth until failure
    - Static behavior (cleavage) due to the effect of $K_{\max}(a)$
    - There is failure once a_f is reached, with a_f such that $K_{\max}(a_f) = K_c$



- « Infinite life design »
  - $\sigma_a < \sigma_e$: « infinite » life
  - Economically deficient
- « Safe life design »
  - No crack before a determined number of cycles
    - At the end of the expected life the component is changed even if no failure has occurred
    - Emphasis on prevention of crack initiation
    - Approach theoretical in nature
      - Assumes initial crack free structures
  - Use of $\sigma_a - N_f$ curves (stress life)
    - Add factor of safety
  - Components of rotating structures vibrating with the flow cycles (blades)
    - Once cracks form, the remaining life is very short due to the high frequency of loading

- « Fail safe design »
  - Even if an individual member of a component fails, there should be sufficient structural integrity to operate safely
  - Load paths and crack arresters
  - Mandate periodic inspection
  - Accent on crack growth rather than crack initiation
  - Example: 1988, B737, Aloha Airlines 243
    - 2 fuselage plates not glued
    - Sea water ⟹ rust and volume increased
    - Fatigue of the rivets
    - The crack followed a predefined path allowing a safe operation

- « Damage tolerant design »
  - Assume cracks are present from the beginning of service
  - Characterize the significance of fatigue cracks on structural performance
    - Control initial crack sizes through manufacturing processes and (non-destructive) inspections
    - Estimate crack growth rates during service (Paris-Erdogan)
    - Schedule conservative inspection intervals (e.g. every so many cycles)
    - Verify crack growth during these inspections
    - Predict end of life (a_f)
    - Remove old structures from service before predicted end-of-life (fracture) or implement repair-rehabilitation strategy



  - Non-destructive inspections
    - Optical
    - X-rays
    - Ultrasonic (reflection on crack surface)

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