University of Liège
Aerospace & Mechanical Engineering

Fracture Mechanics, Damage and Fatigue
Ductile Materials & Safe Life

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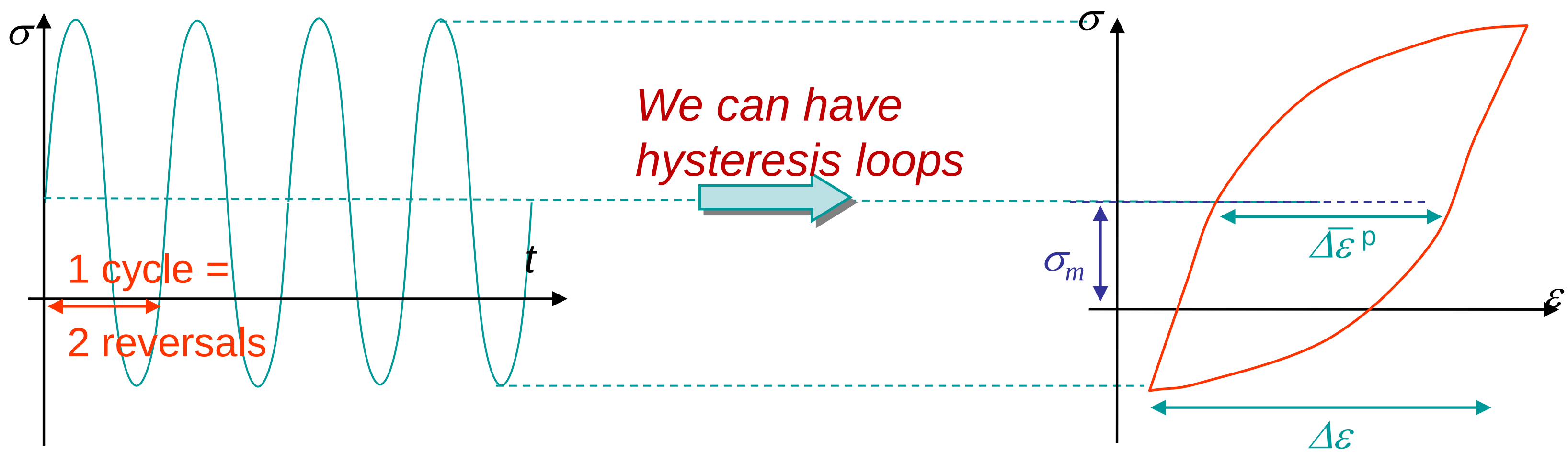
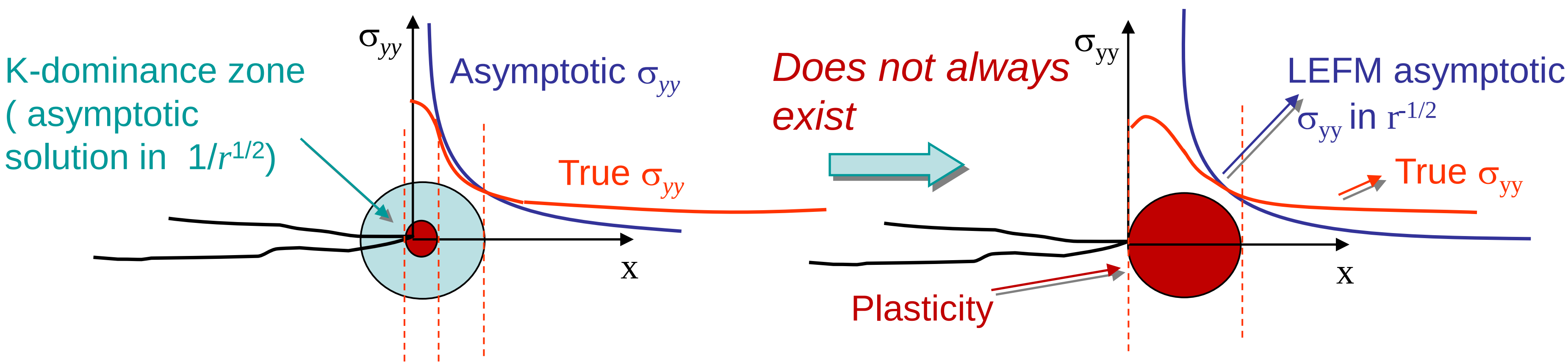
- Limit of linear elasticity
  - Safe life design

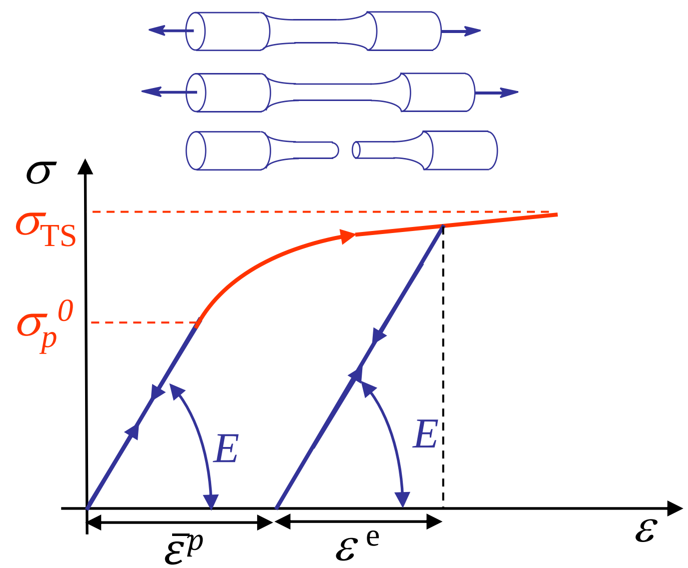


  - LEFM: we have assumed the existence of a K-dominance zone



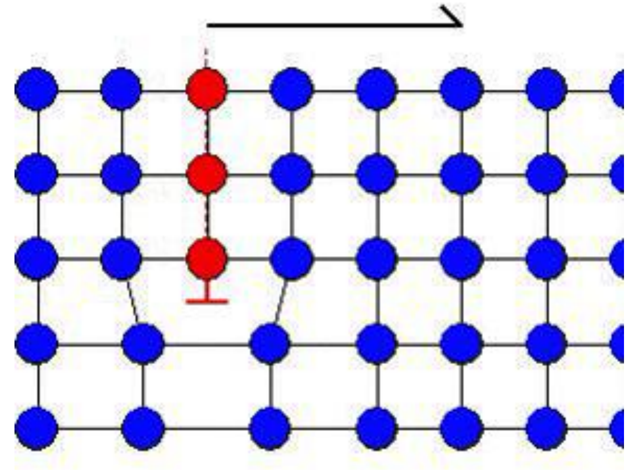
- Elasto-plasticity (small deformations)
  - Beyond a threshold the material experiences irreversible deformations
  - Typical behavior at low/room temperature
    - Curves σ-ε independent of time
    - At higher temperature: creep …
  - Yield surface

$$f(\boldsymbol{\sigma}) \leq 0 \quad \begin{cases} f < 0\text{: elastic region} \\ f = 0\text{: plasticity} \end{cases}$$



  - Plastic flow
    - Assumption: deformations can be added $d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{\mathrm{e}} + d\boldsymbol{\varepsilon}^{\mathrm{p}} \Longrightarrow d\boldsymbol{\sigma} = \mathcal{H} : d\boldsymbol{\varepsilon}^{\mathrm{e}}$
    - Normal plastic flow $d\boldsymbol{\varepsilon}^{\mathrm{p}} = d\lambda \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
  - Path dependency (incremental equations in d)

- Dislocation motion (see previous lecture)
  - Metallic bonds
    - FCC or BCC above DBTT
  - A dislocation is characterized by
    - The Burger vector
    - Dislocation line
      (line along which the distortion is the largest)
    - Slip plane
      (plane where the dislocation motion occurs)



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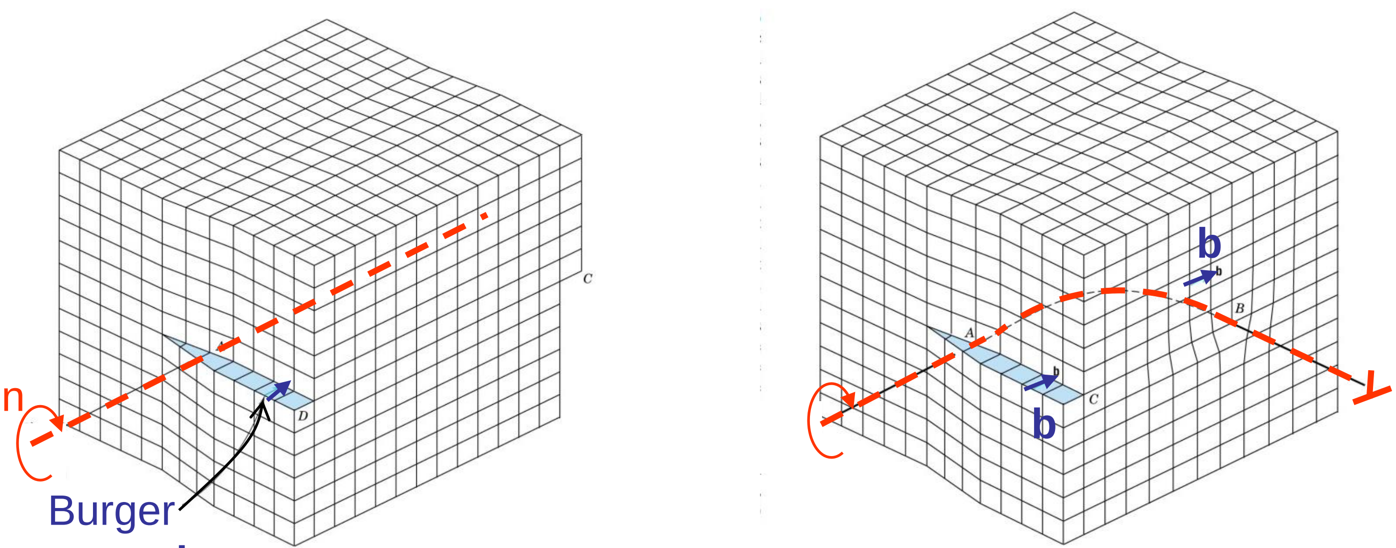
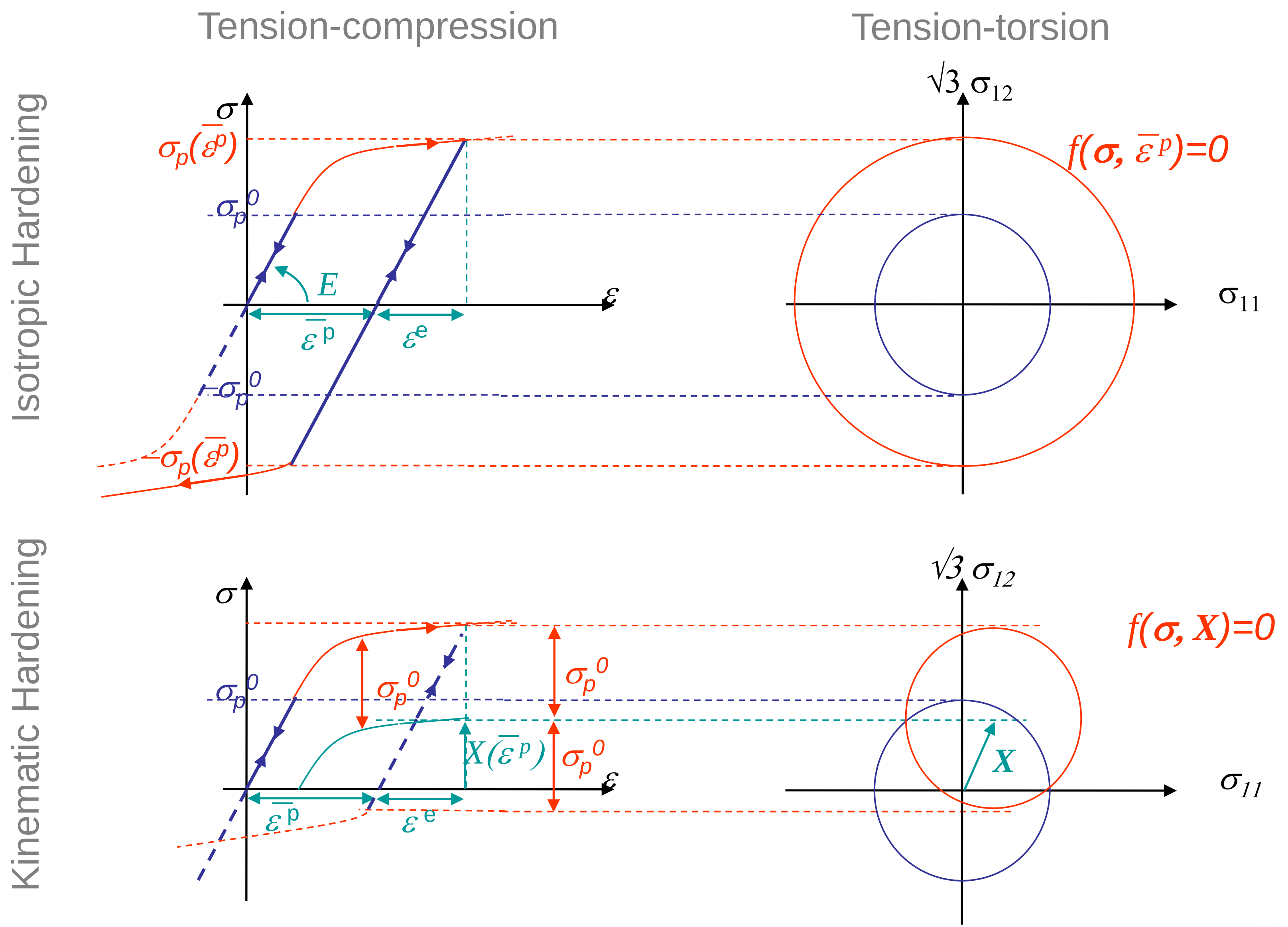
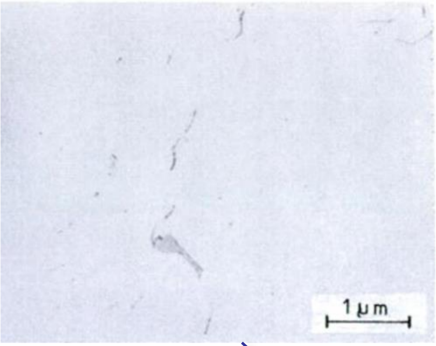
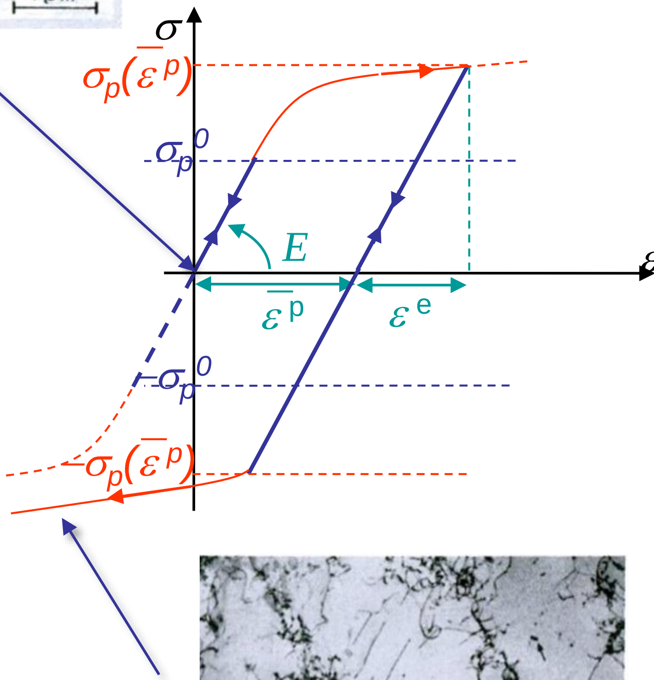


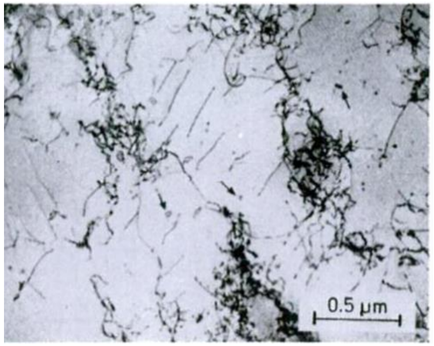
- Hardening laws



- Phenomenological explanations
  - Isotropic hardening
    - Dislocations creation due to plastic deformations
      - Irregularities at the grain boundaries (poly-crystalline material)
      - Irregular crystal surface (mono-crystalline material)
    - Increase of the dislocation density
    - Obstruct dislocations motion
      - Long range elastic force between dislocations of same sign
    - Macroscopic yield stress increases
      - In both tension and
      - Compression

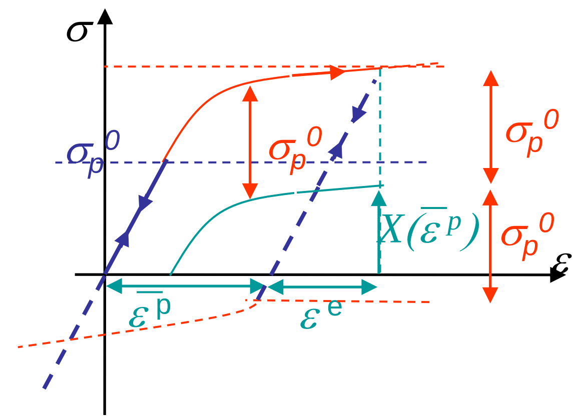


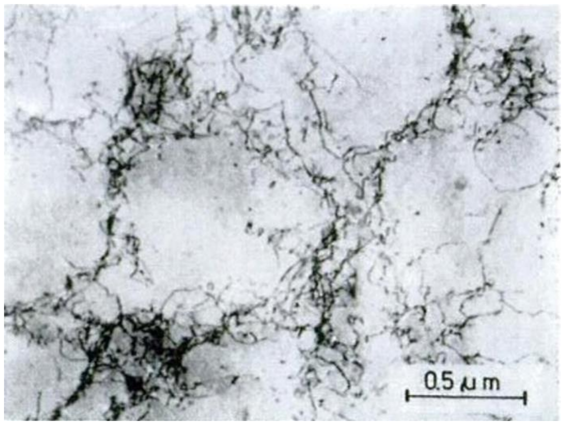




200 cycles

- Phenomenological explanations (2)
  - Kinematic hardening
    - Polycrystalline metals
    - 2 possible sources
      - Dislocations accumulate (pile-up) at barriers (grain boundaries, precipitations)
        - They can move easily in backward direction
        - Yield is reduced in reverse direction
      - When strains are reversed, dislocation sources produce dislocations of opposite sign
        - Annihilation of dislocations
        - Reduce the strength (as yield is proportional to dislocation density)





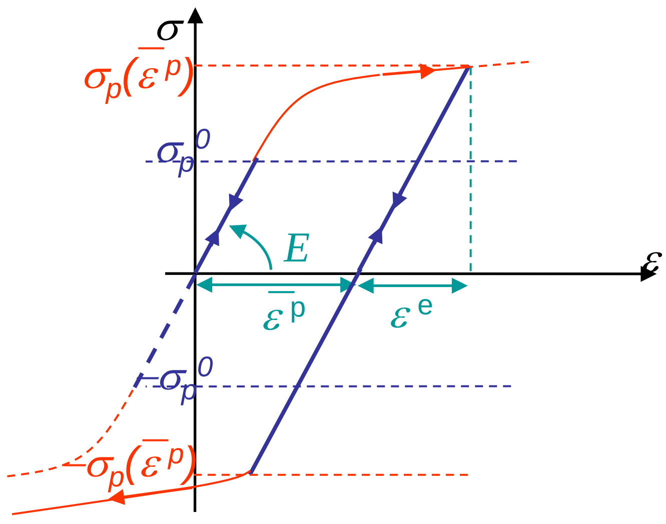
- von Mises isotropic hardening (J2-plasticity)
  - Yield surface

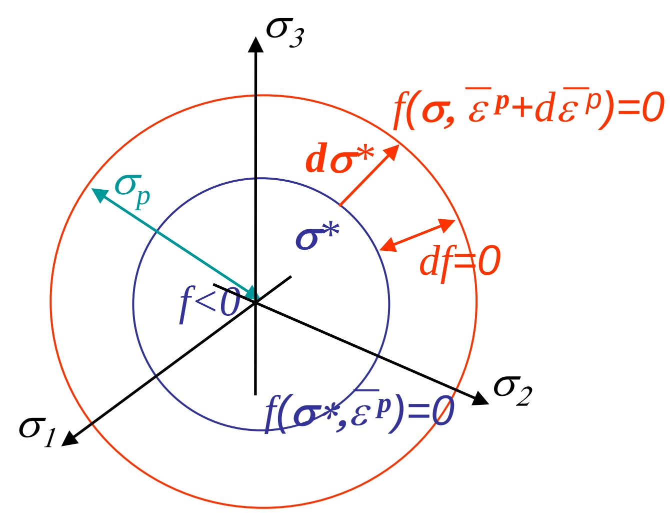
$$f = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}} - \sigma_p\left(\bar{\epsilon}^{\mathrm{p}}\right) \leq 0$$

$$\begin{cases} f < 0\text{: elastic region} \\ f = 0\text{: plasticity} \end{cases}$$

- Deviatoric stress

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{3}\mathbf{I}$$

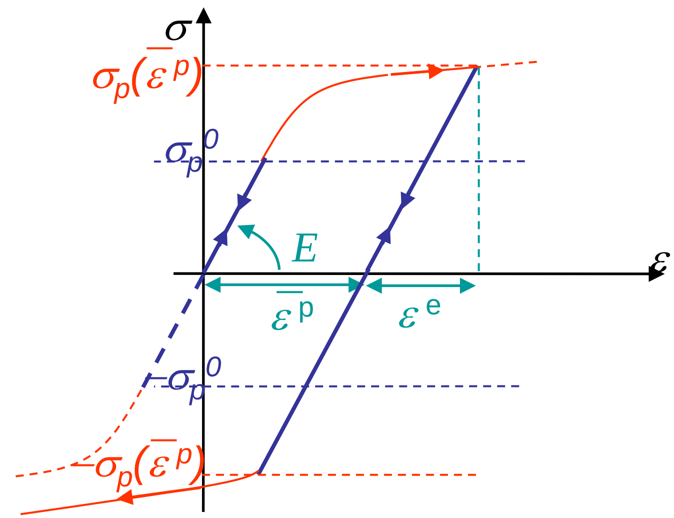




- von Mises isotropic hardening (2)
  - Yield surface

$$f = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}} - \sigma_p\left(\bar{\epsilon}^{\mathrm{p}}\right) \leq 0$$

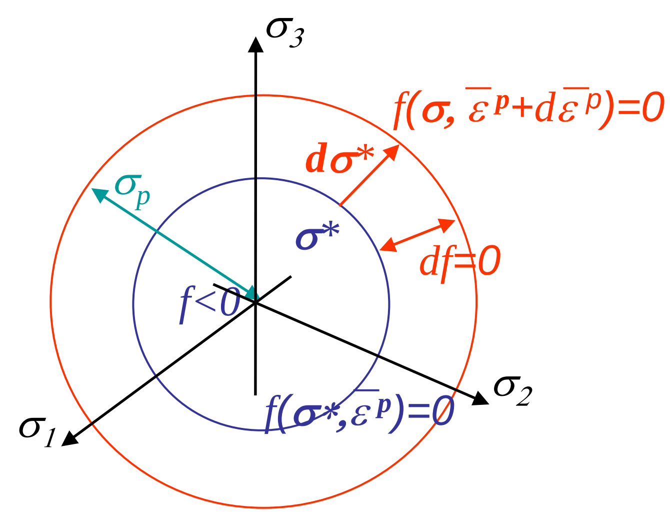
    - Deviatoric stress $\mathbf{s} = \boldsymbol{\sigma} - \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{3}\mathbf{I}$
  - Plastic flow
    - Normal to yield surface $d\boldsymbol{\varepsilon}^{\mathrm{p}} = d\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}$



$$\begin{cases} \dfrac{\partial f}{\partial \mathbf{s}} = \sqrt{\dfrac{3}{2}}\dfrac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}} = \dfrac{3}{2}\dfrac{\mathbf{s}}{\sqrt{\dfrac{3}{2}\mathbf{s}:\mathbf{s}}} \\ \dfrac{\partial \mathbf{s}}{\partial \boldsymbol{\sigma}} = \mathbb{I} - \dfrac{1}{3}\mathbf{I} \otimes \mathbf{I} \end{cases}$$

$$\Rightarrow \frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{\mathbf{s}}{\sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}}} \quad \Rightarrow \frac{\partial f}{\partial \boldsymbol{\sigma}} : \frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{3}{2}$$

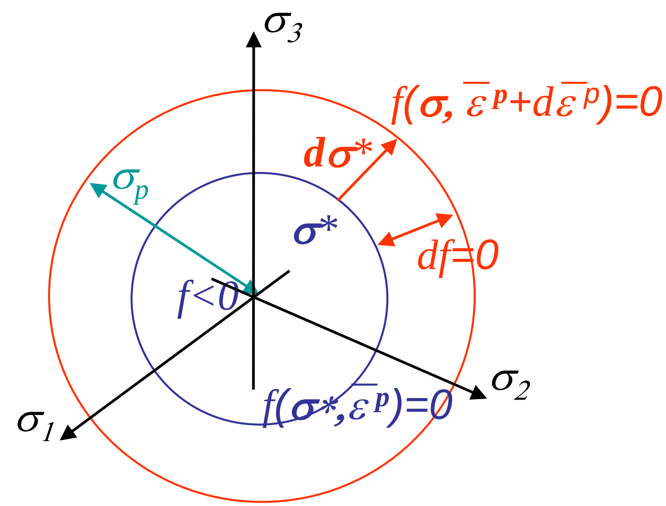
$$\Rightarrow d\bar{\varepsilon}^{\mathrm{p}} = \sqrt{\frac{2}{3}d\boldsymbol{\varepsilon}^{p}:d\boldsymbol{\varepsilon}^{p}} = d\lambda$$

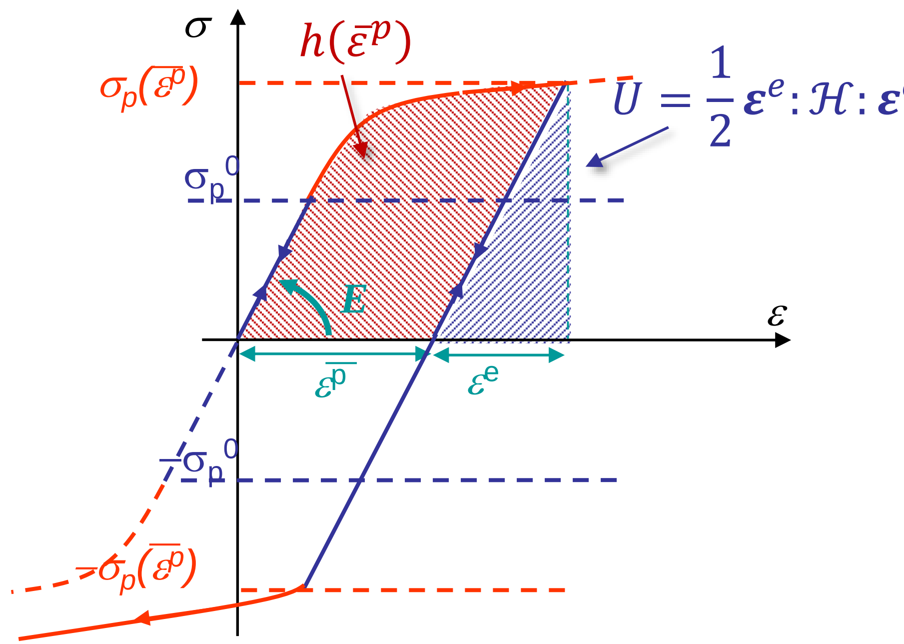


- von Mises isotropic hardening (2)
  - Yield surface

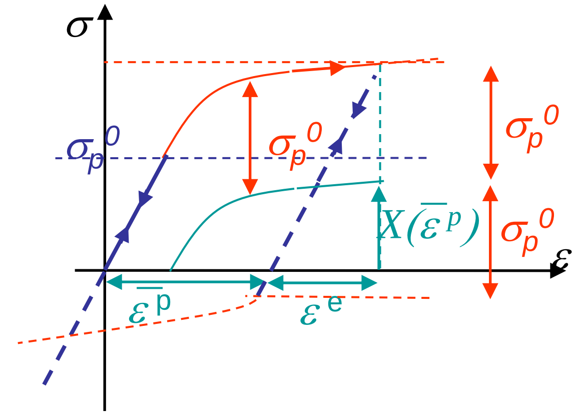
$$f = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}} - \sigma_p\left(\bar{\epsilon}^{\mathrm{p}}\right) \leq 0 \quad \begin{cases} f < 0\text{: elastic region} \\ f = 0\text{: plasticity} \end{cases}$$

    - Deviatoric stress $\mathbf{s} = \boldsymbol{\sigma} - \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{3}\mathbf{I}$
  - Free energy
    - $\Psi\left(\boldsymbol{\varepsilon}^e, \bar{\varepsilon}^p\right) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \mathcal{H} : \boldsymbol{\varepsilon}^e + h\left(\bar{\varepsilon}^p\right)$
    - $\begin{cases} \boldsymbol{\sigma} = \dfrac{\partial \Psi}{\partial \boldsymbol{\varepsilon}^e} \\ \sigma_p = \dfrac{\partial \Psi}{\partial \bar{\varepsilon}^p} = h' \end{cases}$
  - Hardening law
    - E.g.: $\sigma_p = \sigma_p^0 + Q\left(1 - e^{-b\bar{\varepsilon}^p}\right)$





- von Mises kinematic hardening
  - Yield surface



$$f(\boldsymbol{\sigma}, \mathbf{X}) \leq 0 \begin{cases} f < 0\text{: elastic region} \\ f = 0\text{: plasticity} \end{cases}$$

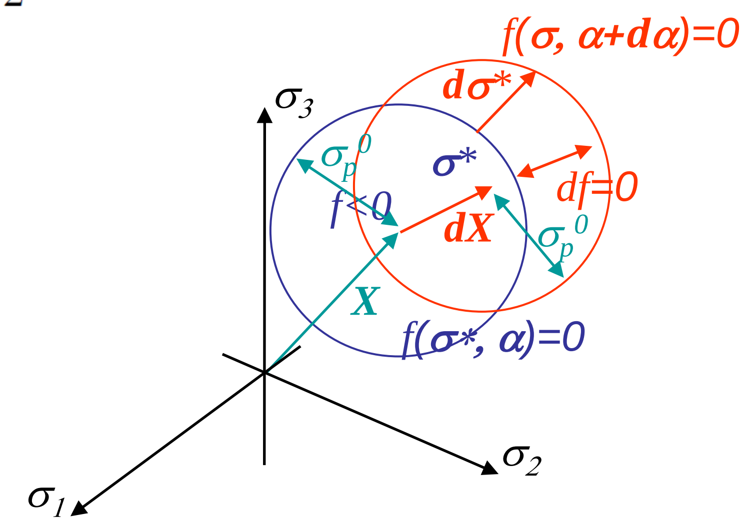
- Deviatoric part of the stress tensor

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{\mathrm{tr}(\boldsymbol{\sigma})}{3}\mathbf{I}$$

- $\boldsymbol{X}$ is deviatoric by nature
- A new yield surface is defined $f = \sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{X}):(\mathbf{s}-\mathbf{X})} - \sigma_p^0 \leq 0$

  - Plastic flow

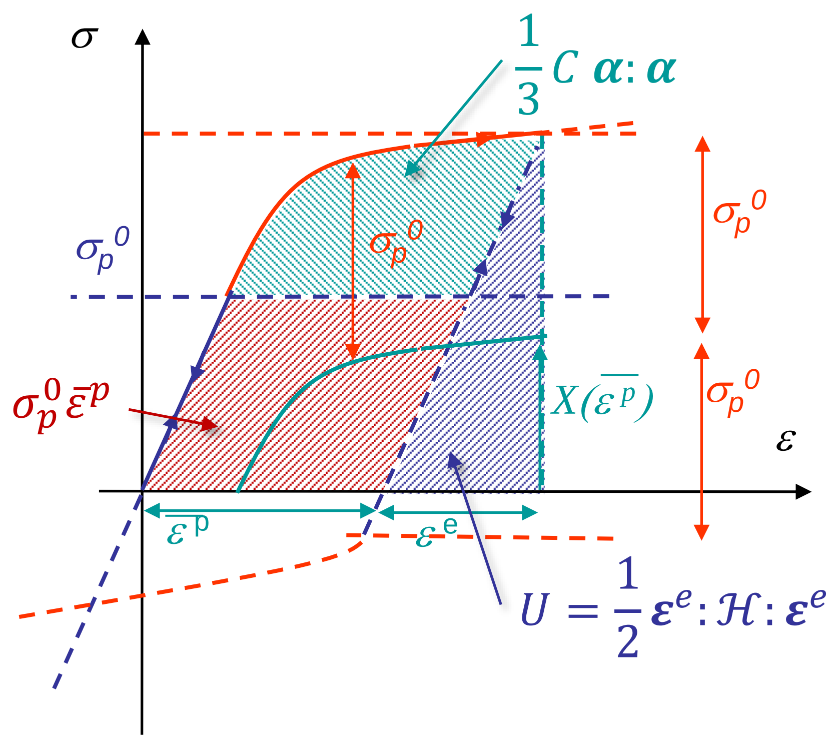
- $$\begin{cases} d\boldsymbol{\varepsilon}^{\mathrm{p}} = d\lambda \dfrac{\partial f}{\partial \boldsymbol{\sigma}} \\ \dfrac{\partial f}{\partial \boldsymbol{\sigma}} = -\dfrac{\partial f}{\partial \mathbf{X}} = \dfrac{3}{2}\dfrac{\mathbf{s}-\mathbf{X}}{\sigma_p^0} \\ d\lambda = d\bar{\varepsilon}^{\mathrm{p}} = \sqrt{\dfrac{2}{3} d\boldsymbol{\varepsilon}^{\mathrm{p}} : d\boldsymbol{\varepsilon}^{\mathrm{p}}} \end{cases}$$



- Definition of strain like variable $\boldsymbol{X} = \frac{2}{3}C\boldsymbol{\alpha}$

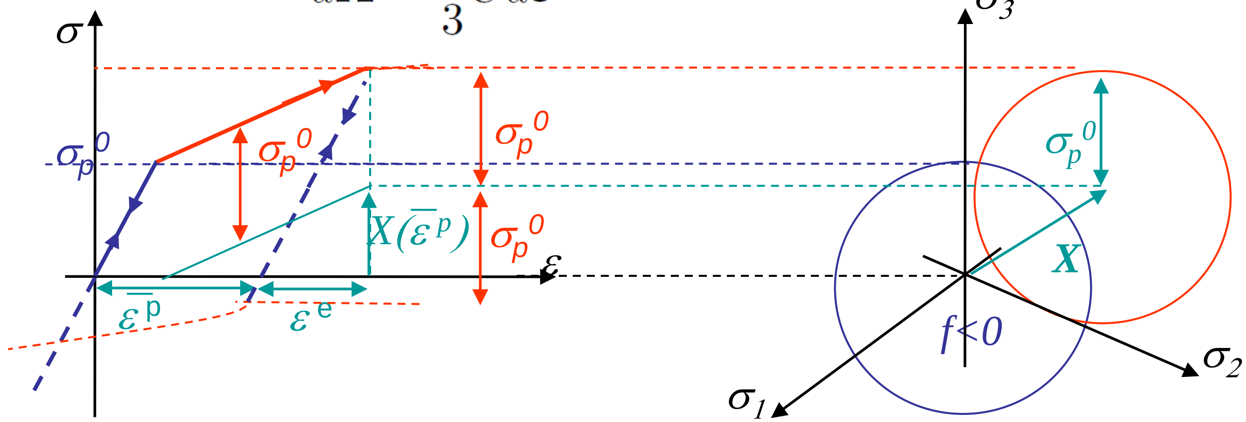
- von Mises kinematic hardening (2)
  - Yield surface
    - $f = \sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{X}):(\mathbf{s}-\mathbf{X})} - \sigma_p^0 \leq 0$
  - Free energy
    - $\psi(\boldsymbol{\varepsilon}^e, \boldsymbol{\alpha}, \bar{\varepsilon}^{\mathrm{p}}) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \mathcal{H} : \boldsymbol{\varepsilon}^e + \frac{1}{3} C \boldsymbol{\alpha} : \boldsymbol{\alpha} + \sigma_p^0 \bar{\varepsilon}^{\mathrm{p}}$
    - $\begin{cases} \boldsymbol{\sigma} = \dfrac{\partial \Psi}{\partial \boldsymbol{\varepsilon}^e} \\ \sigma_p = \dfrac{\partial \Psi}{\partial \bar{\varepsilon}^p} = \sigma_p^0 \\ \boldsymbol{X} = \dfrac{\partial \Psi}{\partial \boldsymbol{\alpha}} = \dfrac{2}{3} C \boldsymbol{\alpha} \end{cases}$

$\Rightarrow \quad d\mathbf{X} = \frac{2}{3} C d\boldsymbol{\alpha}$

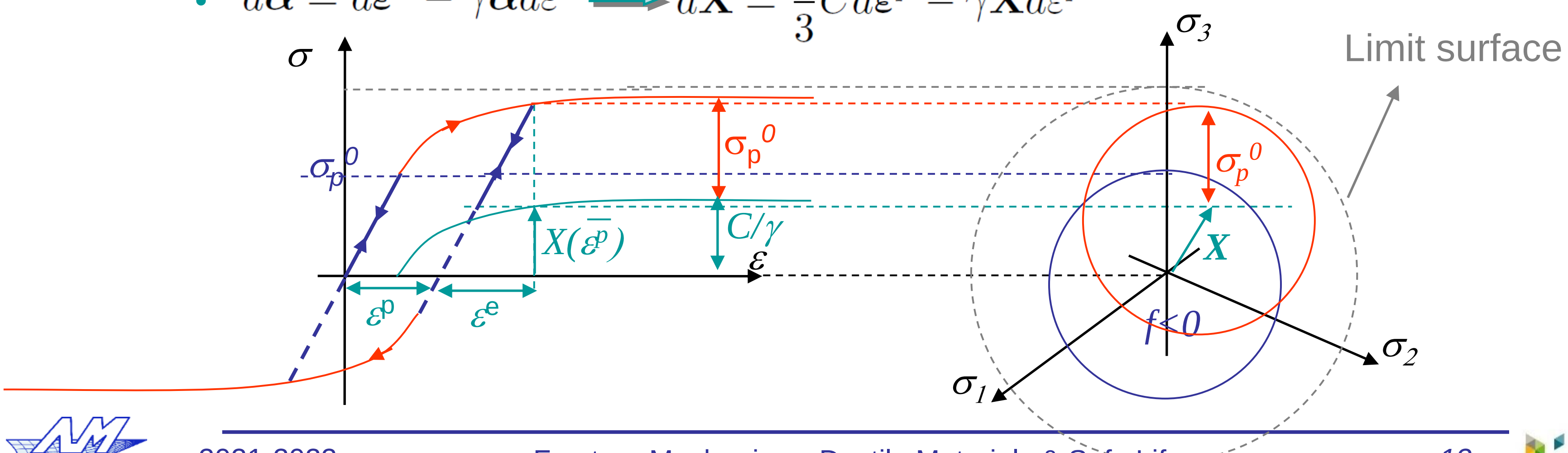


  - What is missing: hardening law
    - The law governing the evolution of $\boldsymbol{X}$

- von Mises kinematic hardening (3)
  - Prager linear hardening flow
    - $d\boldsymbol{\alpha} = d\boldsymbol{\varepsilon}^p \Longrightarrow d\mathbf{X} = \frac{2}{3} C d\boldsymbol{\varepsilon}^p$

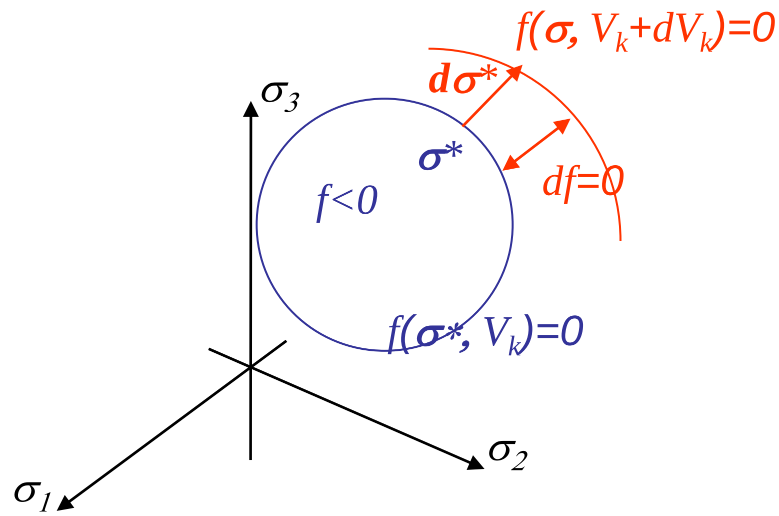


  - Armstrong-Frederick non linear hardening law
    - $d\boldsymbol{\alpha} = d\boldsymbol{\varepsilon}^p - \gamma \boldsymbol{\alpha} d\bar{\varepsilon}^p \Longrightarrow d\mathbf{X} = \frac{2}{3} C d\boldsymbol{\varepsilon}^p - \gamma \mathbf{X} d\bar{\varepsilon}^p$



- General formulation
  - Recourse to N internal variables V_k
  - Yield surface

$$f(\boldsymbol{\sigma}, V_k) \leq 0 \quad \begin{cases} f < 0\text{: elasticity} \\ f = 0\text{: plastic flow} \end{cases}$$



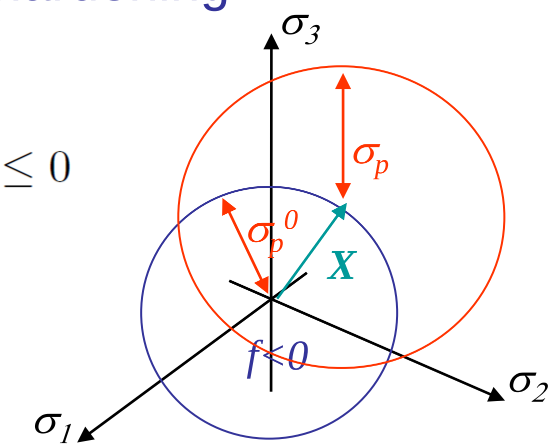
  - Plastic flow
    - The representative stress state remains on yield surface

$$\Longrightarrow \quad df(\boldsymbol{\sigma}^*) = \frac{\partial f}{\partial \boldsymbol{\sigma}} : d\boldsymbol{\sigma}^* + \frac{\partial f}{\partial V_k} dV_k = 0$$

  - Particular cases

	Internal variable	Associated stress
Isotropic hardening	ε^p	σ_p
Kinematic hardening	$\boldsymbol{\alpha}$	$\boldsymbol{X}$

- Example: Combination of isotropic and kinematic hardening



  – Yield surface

$$f\left(\boldsymbol{\sigma},\,\boldsymbol{\alpha},\,\bar{\varepsilon}^p\right)=\sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\left(\boldsymbol{\alpha}\right)\right):\left(\mathbf{s}-\mathbf{X}\left(\boldsymbol{\alpha}\right)\right)}-\sigma_p\left(\bar{\varepsilon}^p\right)\leq 0$$

  – Free energy function

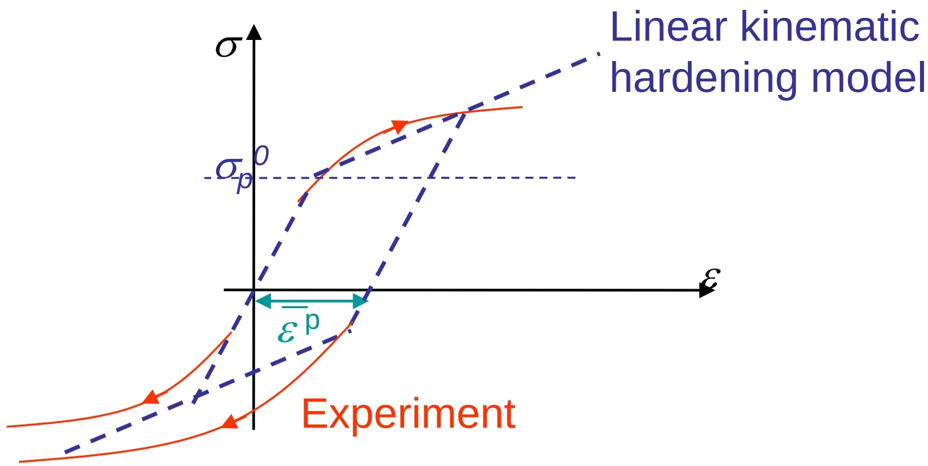
    - $\Psi\left(\boldsymbol{\varepsilon}^e,\,\boldsymbol{\alpha},\,\bar{\varepsilon}^p\right)=\frac{1}{2}\boldsymbol{\varepsilon}^e:\mathcal{H}:\boldsymbol{\varepsilon}^e+\frac{1}{3}C\boldsymbol{\alpha}:\boldsymbol{\alpha}+h\left(\bar{\varepsilon}^p\right)$

$$\Rightarrow\begin{cases}\boldsymbol{\sigma}=\dfrac{\partial\Psi}{\partial\boldsymbol{\varepsilon}^e}\\[2ex]\mathbf{X}=\dfrac{\partial\Psi}{\partial\boldsymbol{\alpha}}=\dfrac{2}{3}C\boldsymbol{\alpha}\\[2ex]\sigma_p=\dfrac{\partial\Psi}{\partial\bar{\varepsilon}^p}=h'\end{cases}$$

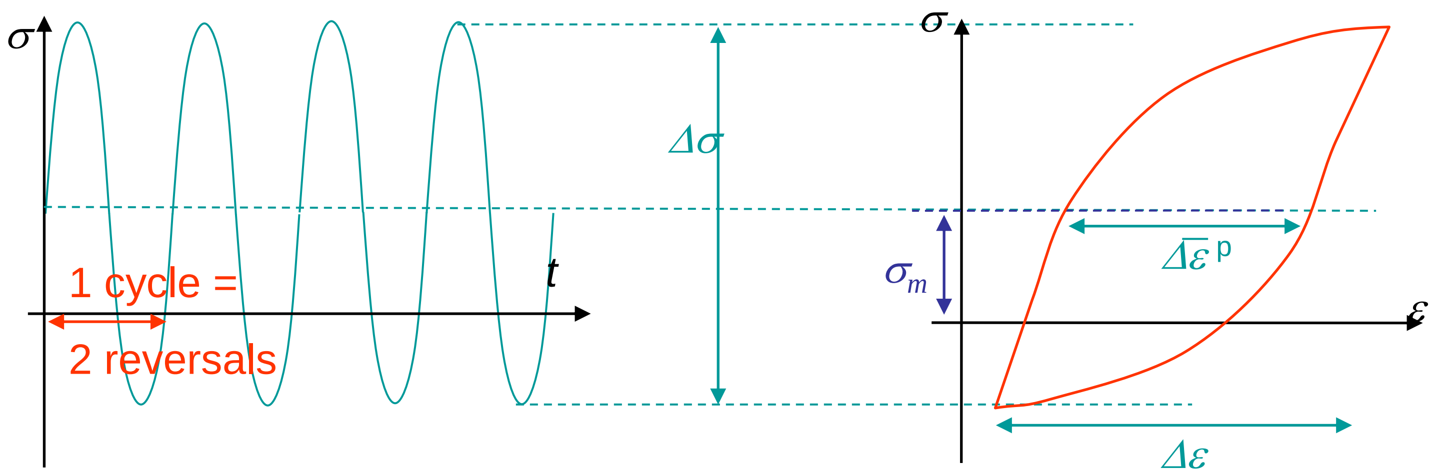
  – Plastic flow

    - $\begin{cases}d\boldsymbol{\varepsilon}^{\mathrm{p}}=d\lambda\dfrac{\partial f}{\partial\boldsymbol{\sigma}}\\[2ex]\dfrac{\partial f}{\partial\boldsymbol{\sigma}}=-\dfrac{\partial f}{\partial\mathbf{X}}=\dfrac{3}{2}\dfrac{\mathbf{s}-\mathbf{X}}{\sigma_p\left(\bar{\varepsilon}^p\right)}\end{cases}$

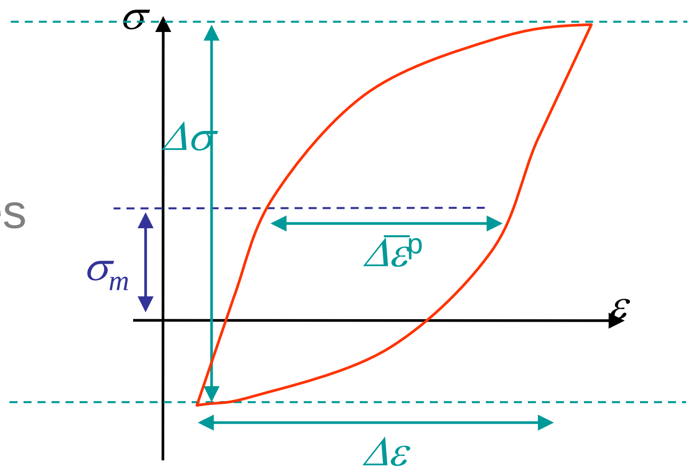
- Bauschinger effect
  - Due to kinematic hardening



  - Cyclic loading



- Fatigue hardening/softening
  - The Hysteretic loop is not always directly stabilized
  - Because of changes in material structures
    - Intensity of changes decreases with cycles number
  - The material can exhibit
    - Fatigue hardening
    - Fatigue softening

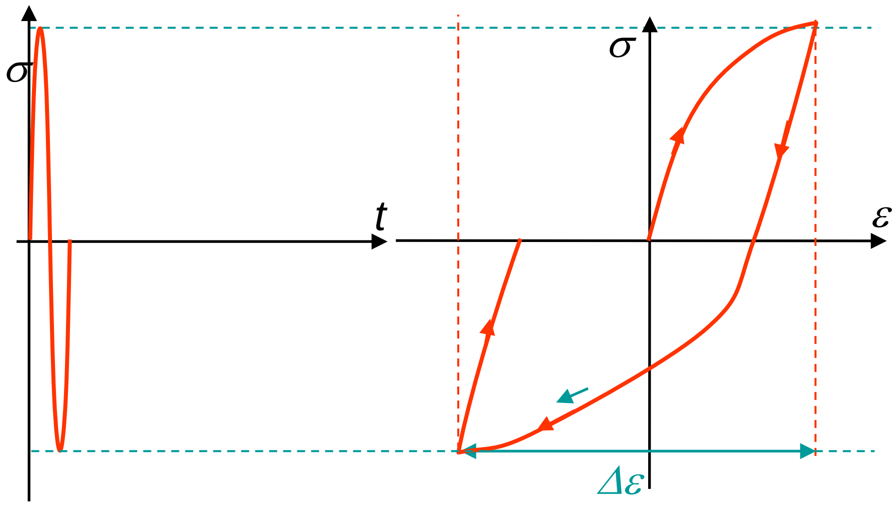


- Fatigue hardening/softening with accommodation

Cyclic softening

Cyclic hardening

Controlled $\Delta\varepsilon$ = cst



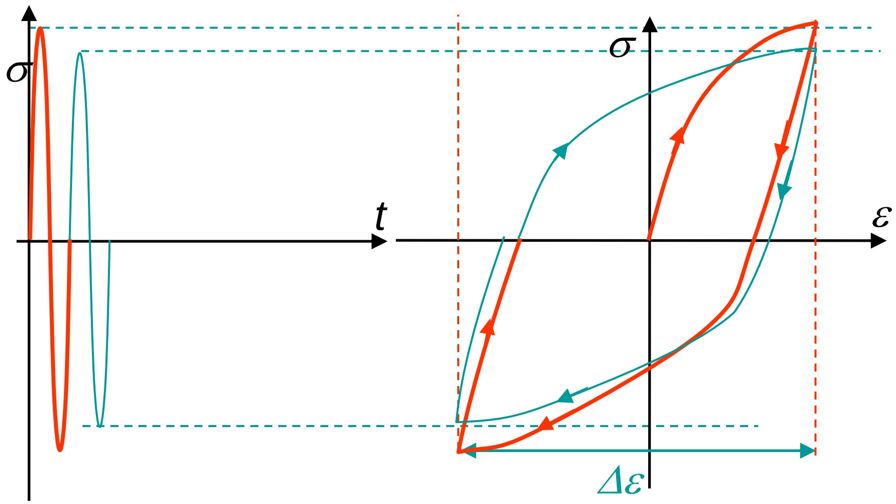
Controlled $\Delta\sigma$ = cst

- Fatigue hardening/softening with accommodation

Cyclic softening

Cyclic hardening

Controlled $\Delta\varepsilon$ = cst



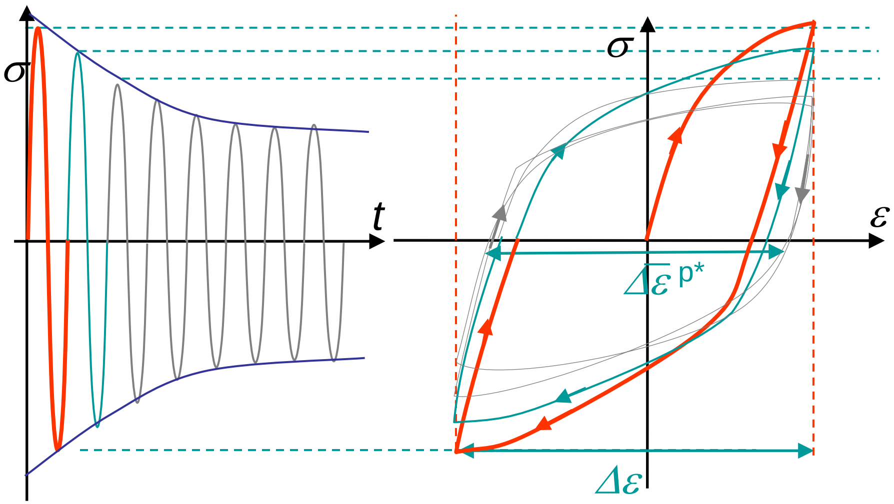
Controlled $\Delta\sigma$ = cst

- Fatigue hardening/softening with accommodation

Cyclic softening

Cyclic hardening

Controlled $\Delta\varepsilon$ = cst

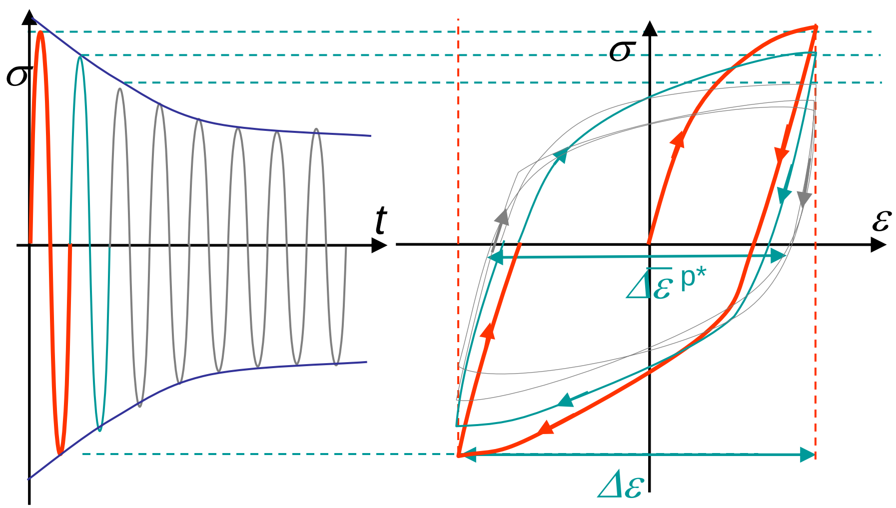


Controlled $\Delta\sigma$ = cst

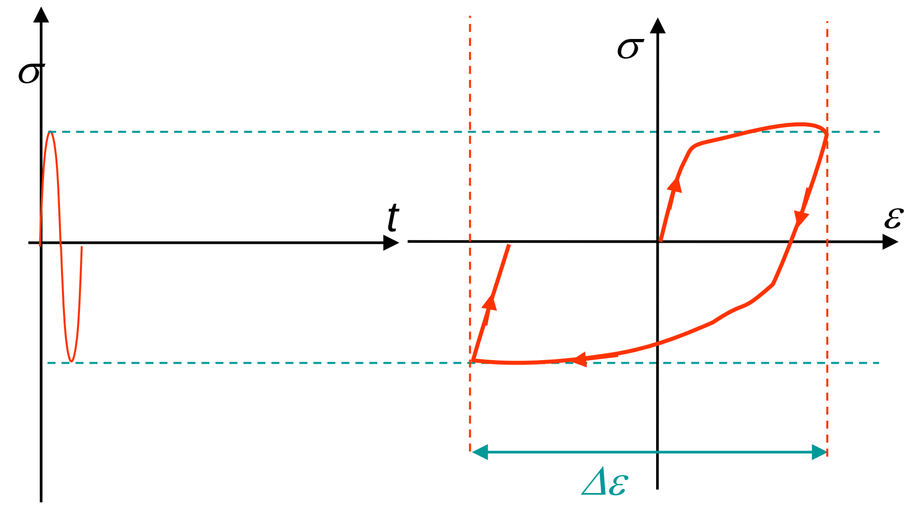
- Fatigue hardening/softening with accommodation

Controlled $\Delta\varepsilon$ = cst

Cyclic softening



Cyclic hardening

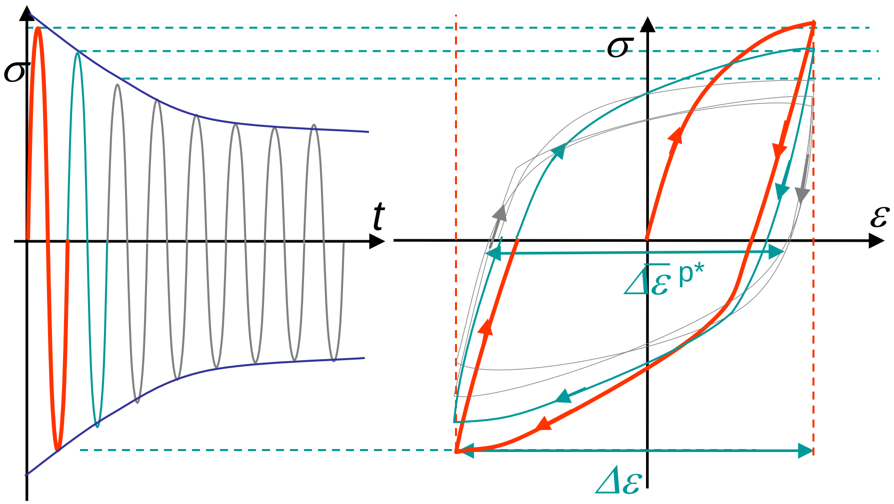


Controlled $\Delta\sigma$ = cst

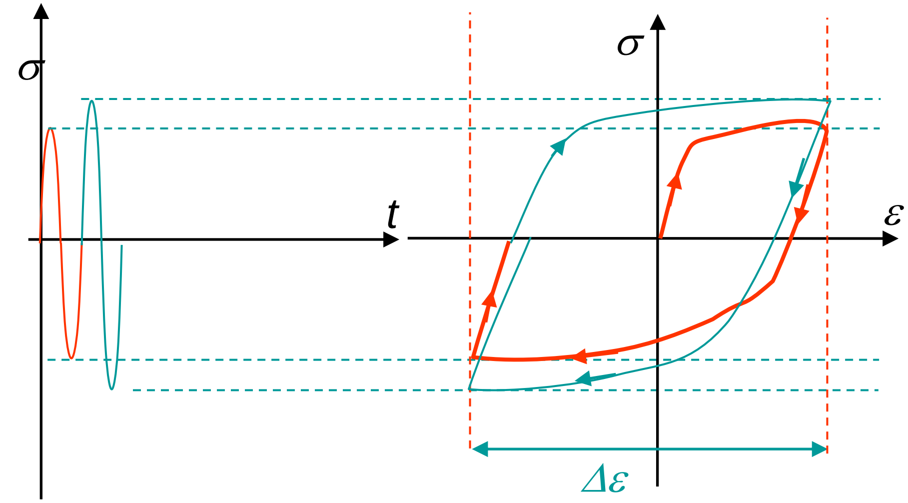
- Fatigue hardening/softening with accommodation

Controlled $\Delta\varepsilon$ = cst

Cyclic softening



Cyclic hardening



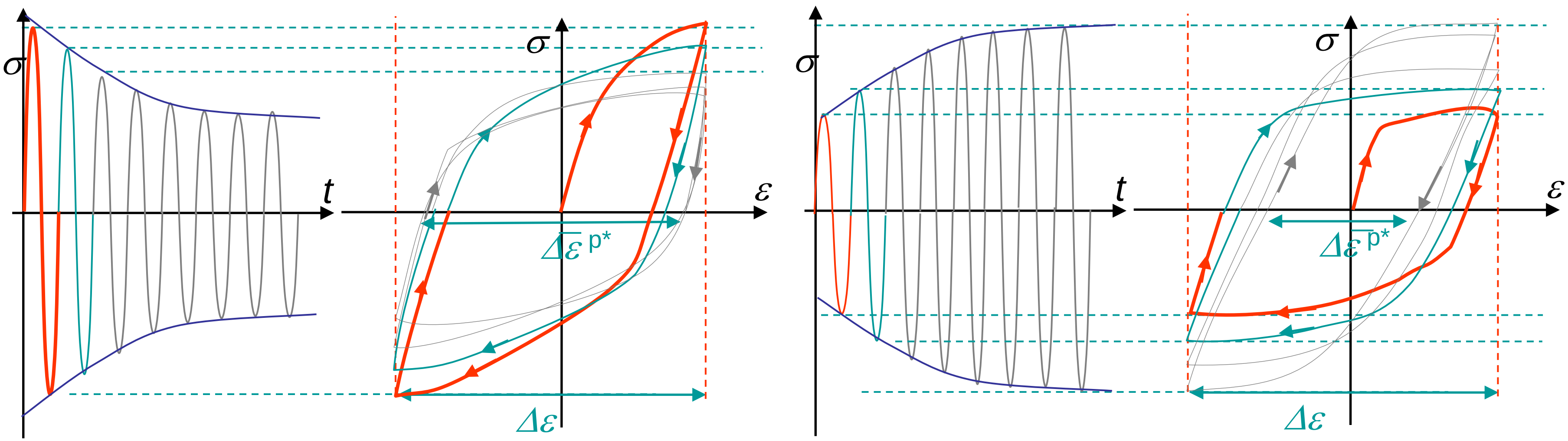
Controlled $\Delta\sigma$ = cst

- Fatigue hardening/softening with accommodation

Cyclic softening

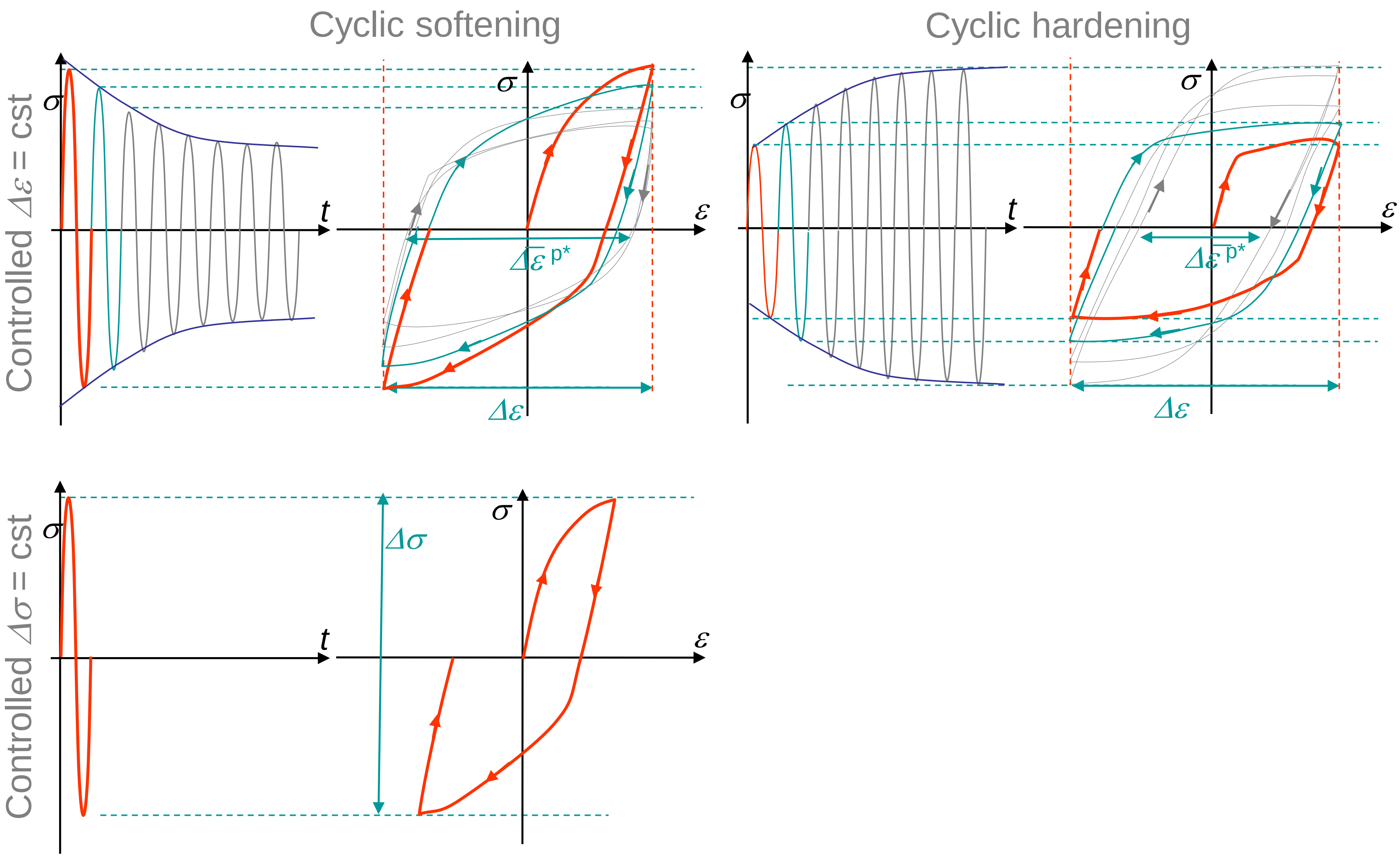
Cyclic hardening

Controlled $\Delta\varepsilon$ = cst



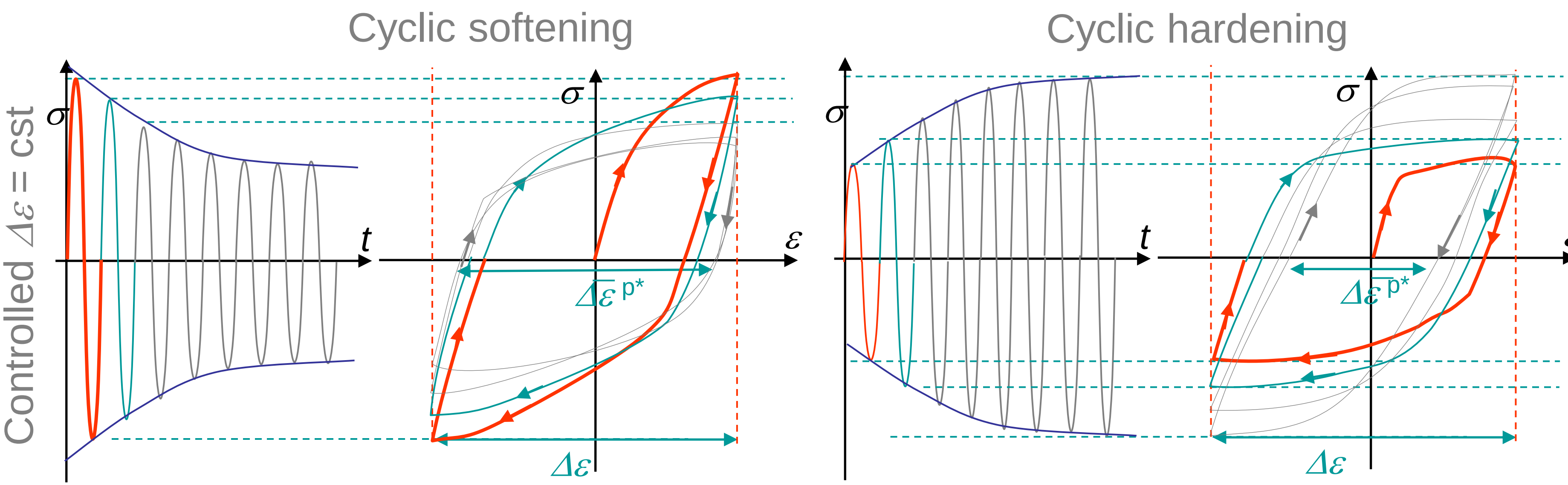
Controlled $\Delta\sigma$ = cst

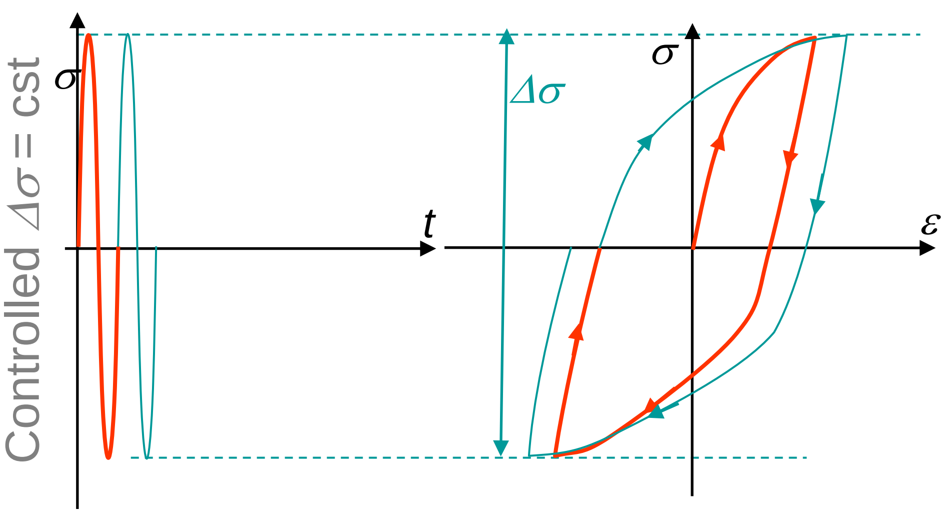
- Fatigue hardening/softening with accommodation



Material behavior: Elasto-plasticity under cyclic loading

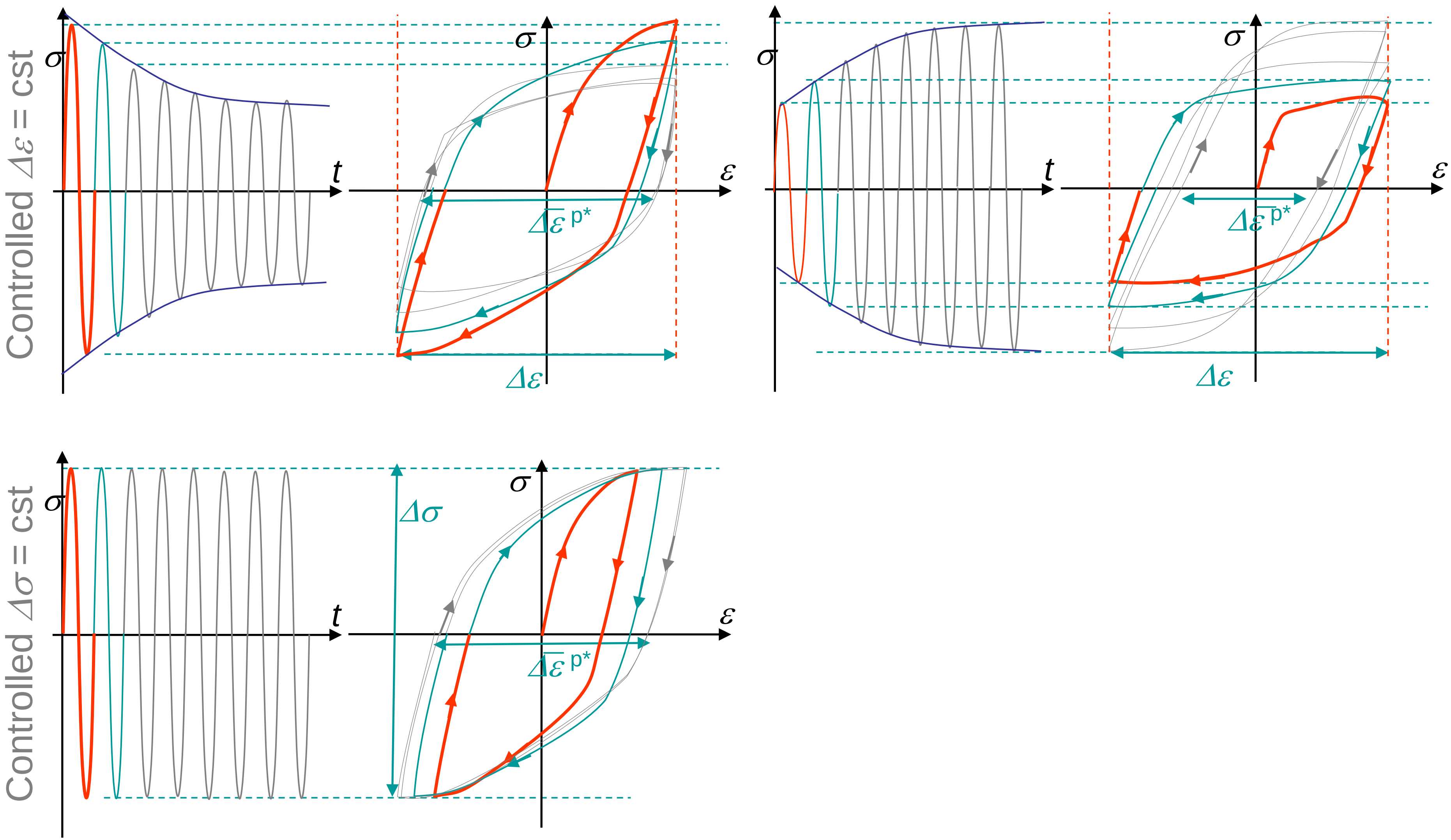
- Fatigue hardening/softening with accommodation



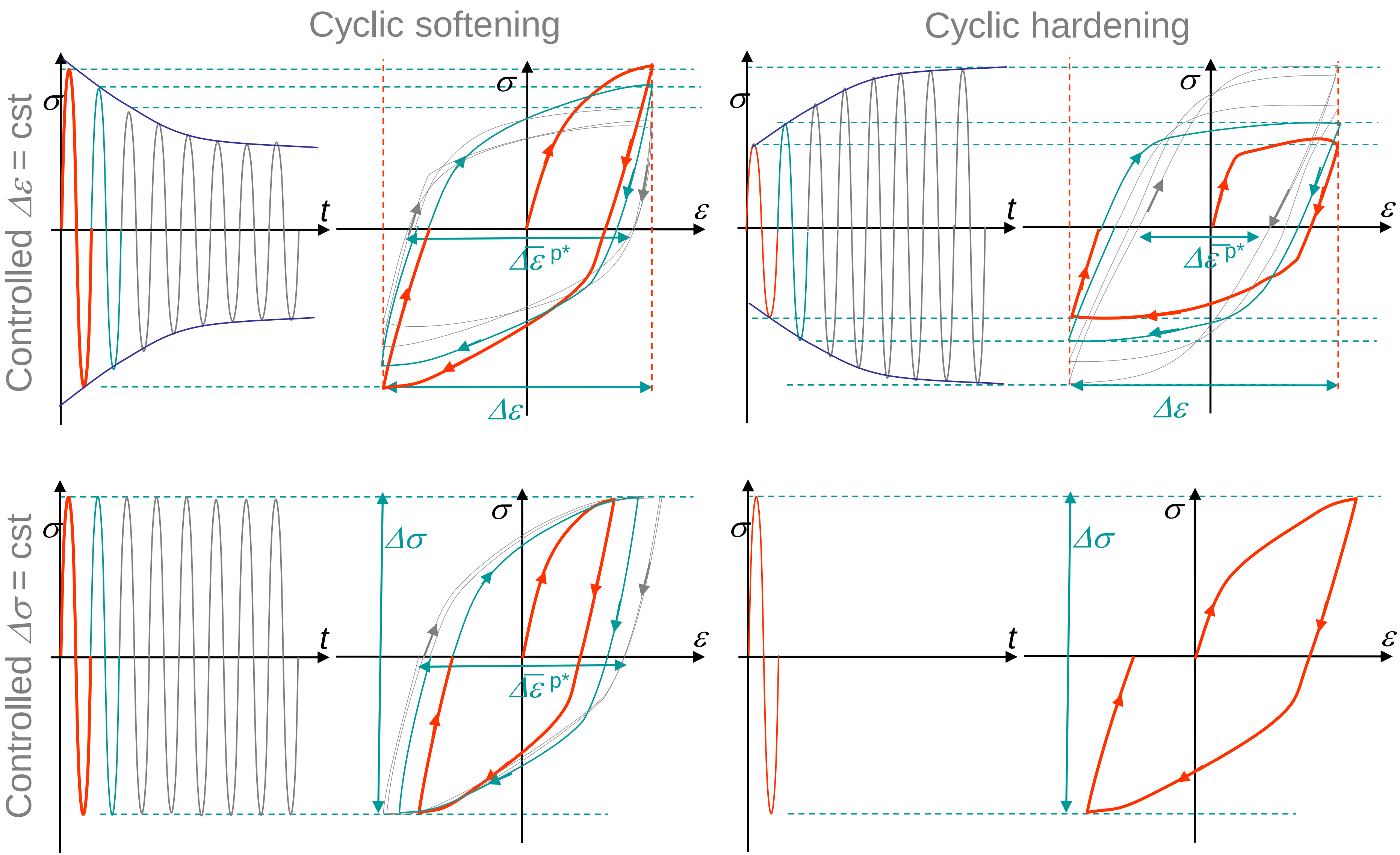


- Fatigue hardening/softening with accommodation

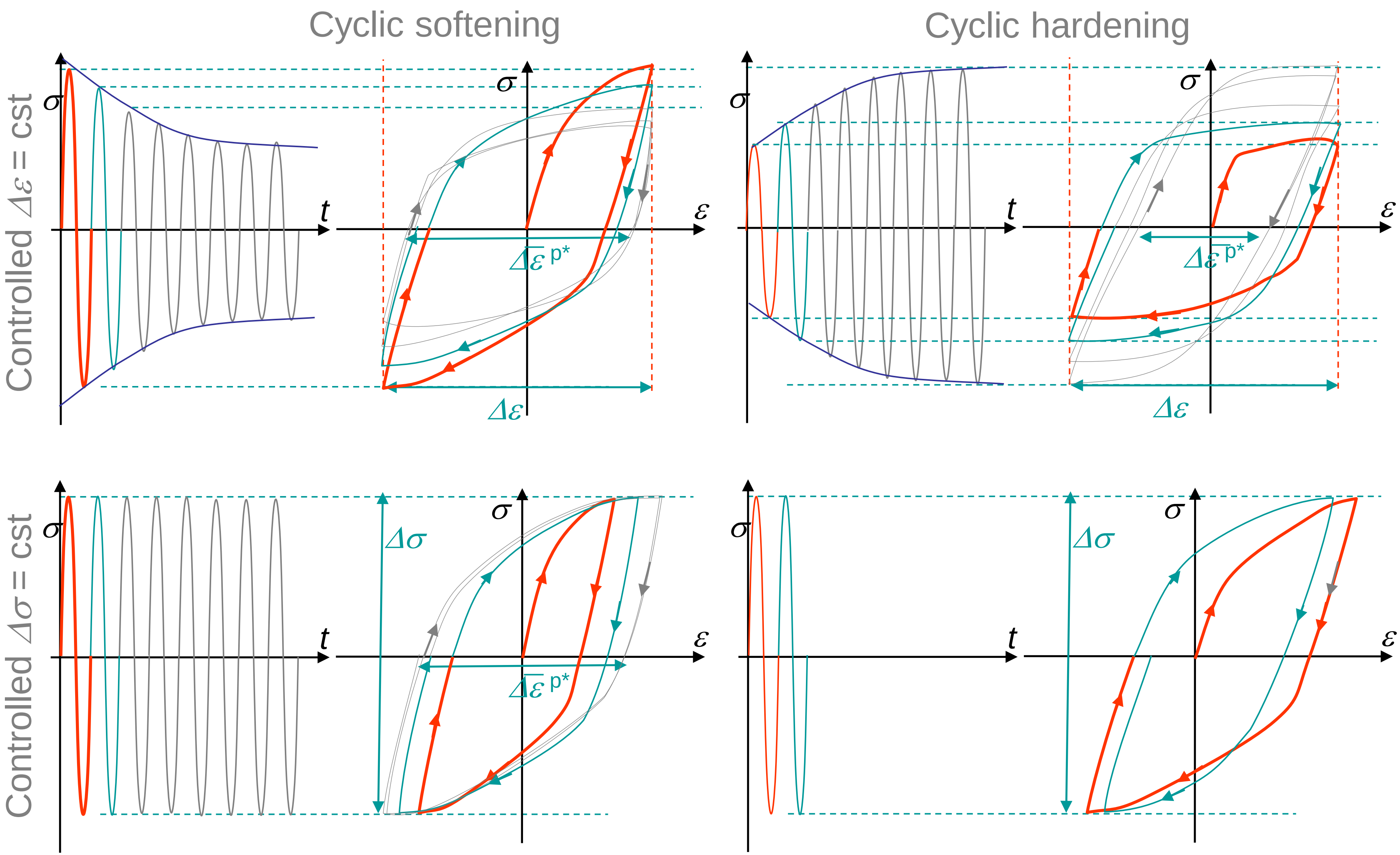
Cyclic softening | Cyclic hardening



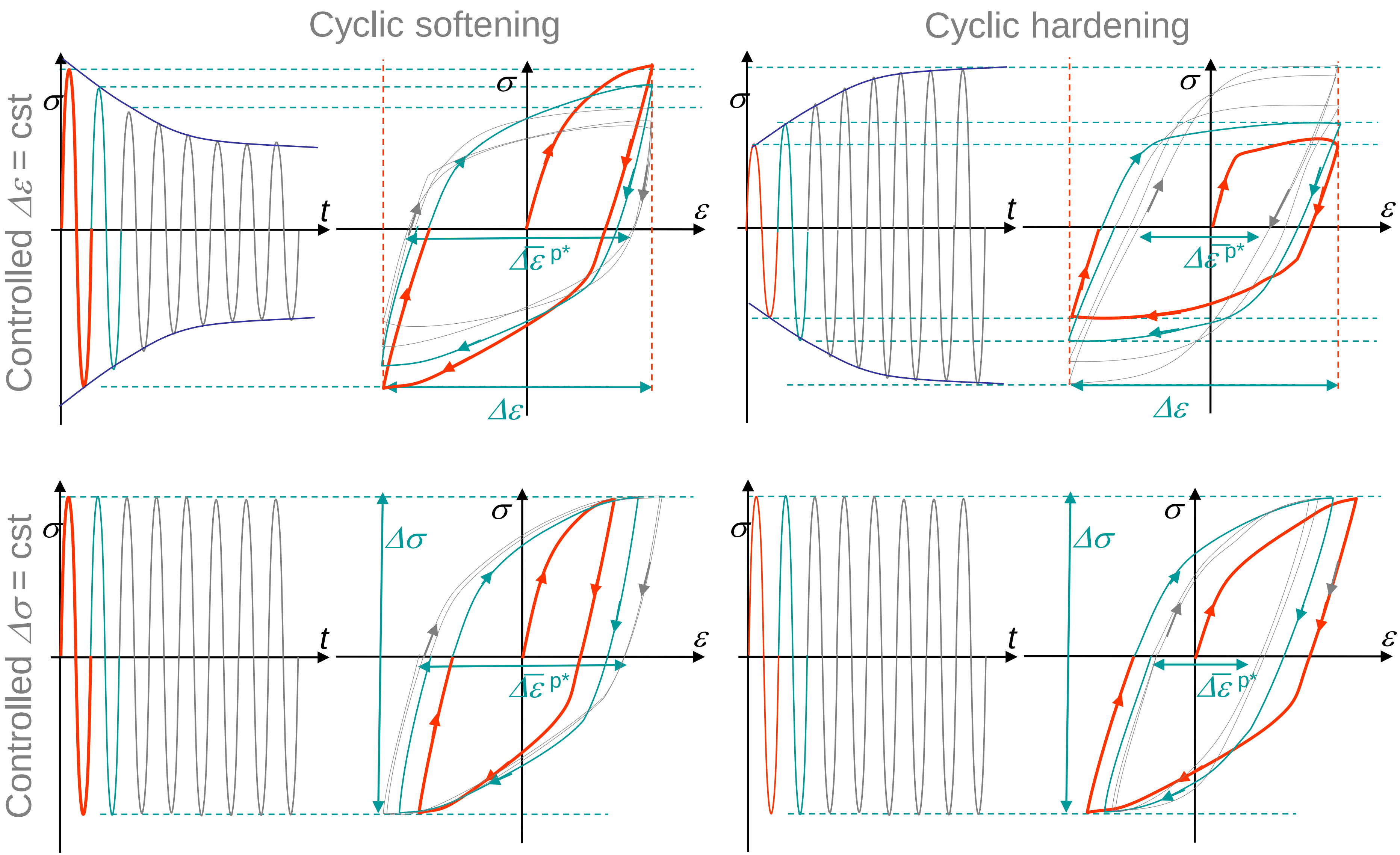
- Fatigue hardening/softening with accommodation



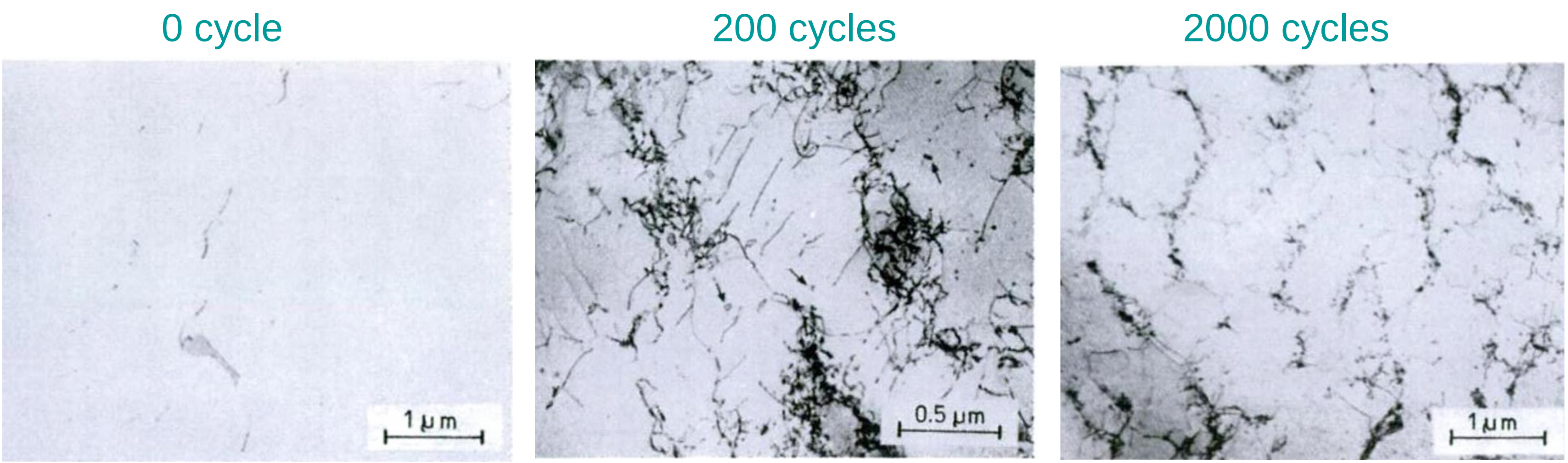
- Fatigue hardening/softening with accommodation



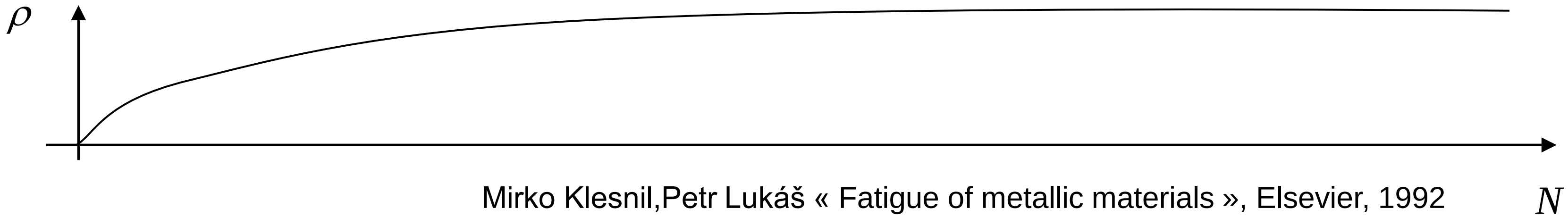
- Fatigue hardening/softening with accommodation



- Fatigue Hardening
  - Usually when initially low dislocation density (annealed materials)
  - Example: poly-crystalline iron under low amplitude cyclic loading



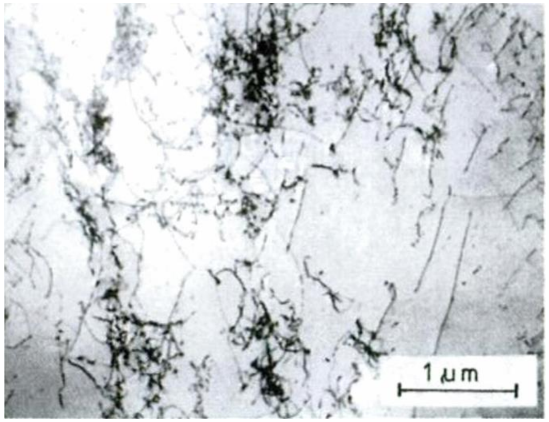
- Dislocations density increases until saturation
- Saturation obtained after a few hundred cycles and no further detectable changes



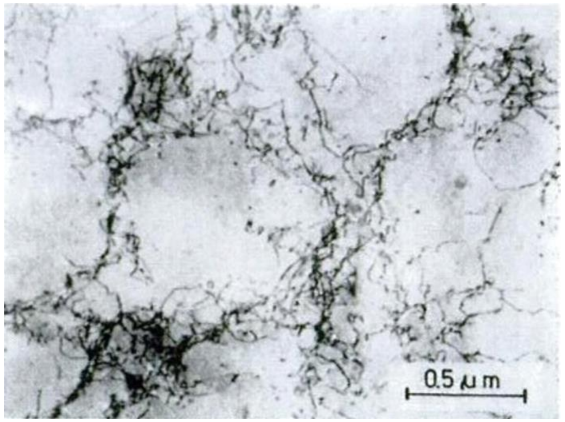
Mirko Klesnil,Petr Lukáš « Fatigue of metallic materials », Elsevier, 1992

- Fatigue softening
  - Usually when initially high dislocation density (cold worked materials)
  - Example: poly-crystalline iron under high amplitude cyclic loading

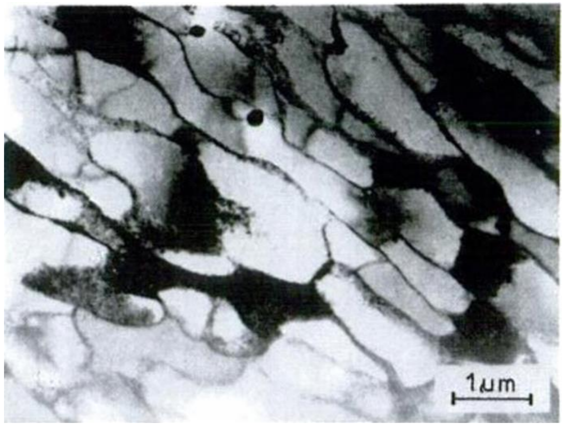
1 cycle



100 cycles

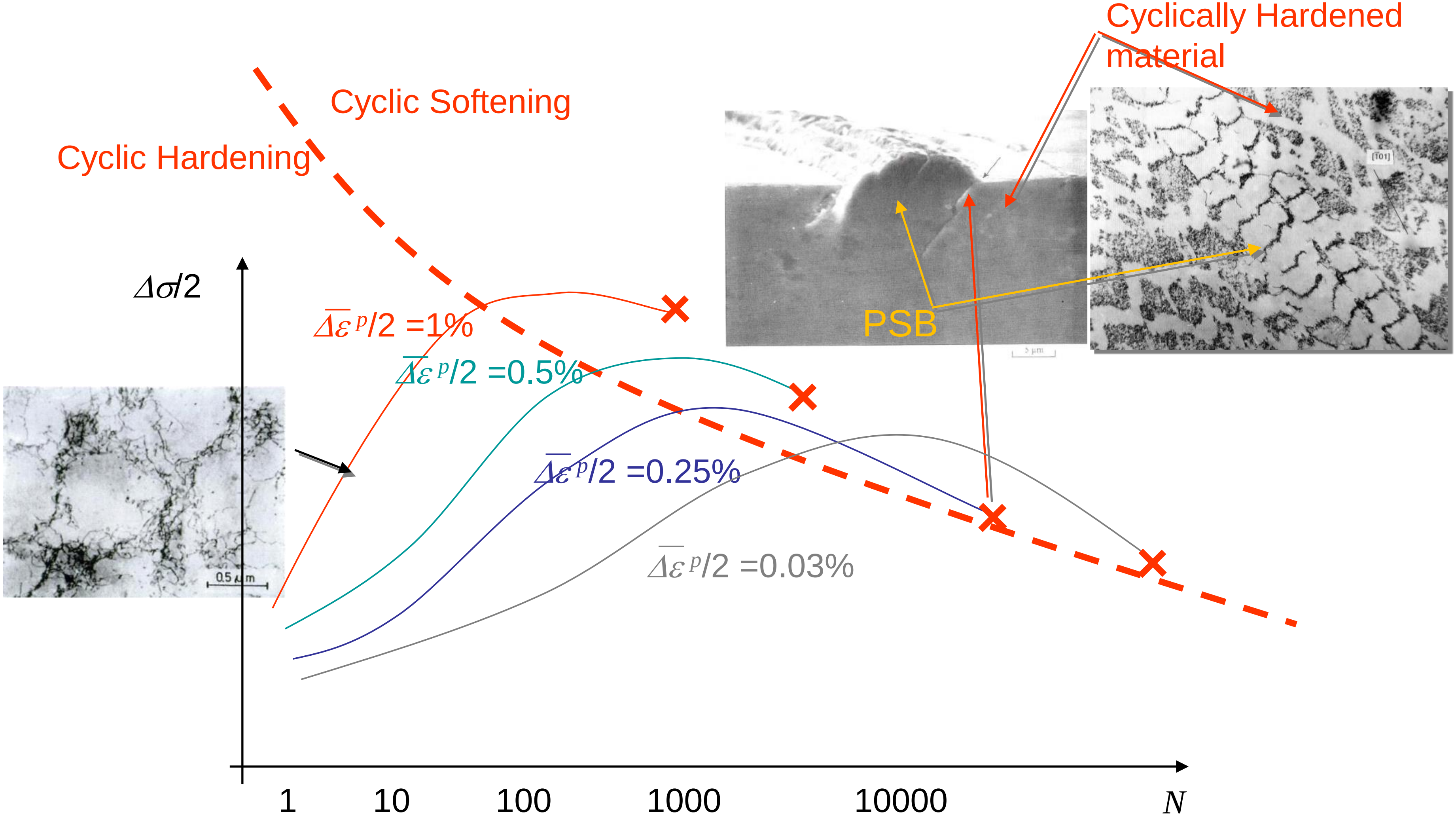


1000 cycles



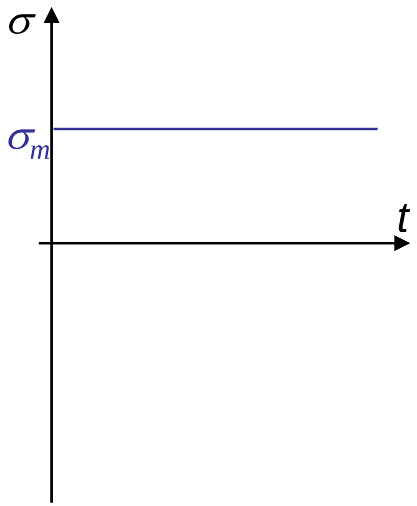
- Dislocations localize to form cells (hardening process)
- If increase of amplitude (at constant Δ deformation)
  - Cells break down to form persistent slip bands (PSB)
  - Localization of slips in PSB
  - Softening
  - Origin of cracks

- Fatigue hardening followed by softening
  - High amplitude cyclic loading

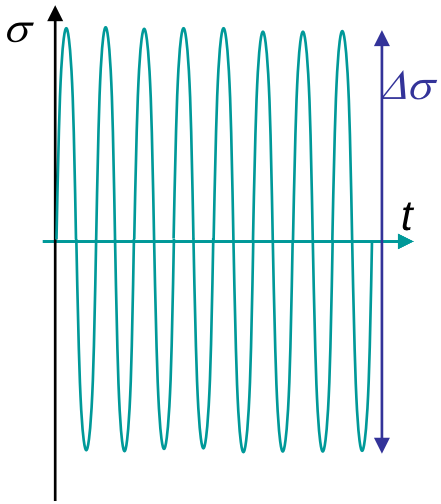


- Combination of structural and cyclic loading

Primary loading

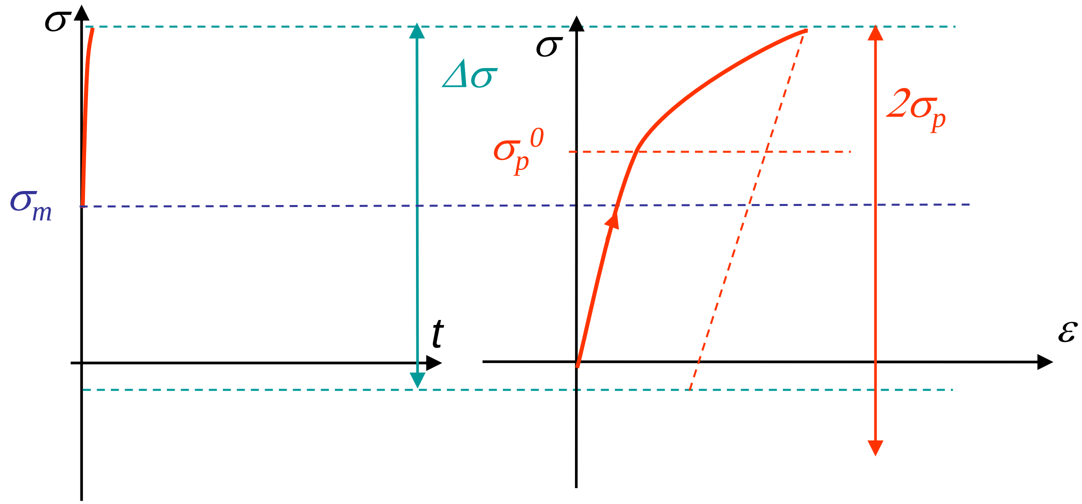


Secondary loading



- Examples
  - Pressure vessels
  - Structures subjected to flow/wind with σ_m & $\Delta\sigma$ due to
    - Nominal stress
    - Flow fluctuation
- Stabilized state?

- Combination of structural and cyclic loading under controlled σ
  - Elastic adaptation
    - For reduced $\Delta\sigma$
    - Isotropic or/and linear kinematic hardening
    - Adaptation in 1 cycle



- For slightly higher $\Delta\sigma$
- Isotropic and (linear) kinematic hardening
- Adaptation after a few cycles

- Combination of structural and cyclic loading under controlled σ
  - Elastic adaptation
    - For reduced $\Delta\sigma$
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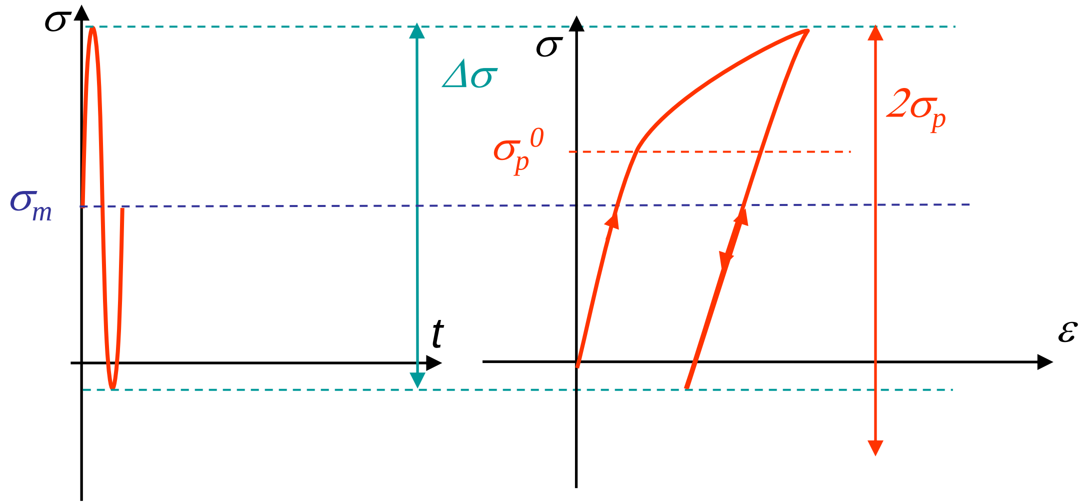

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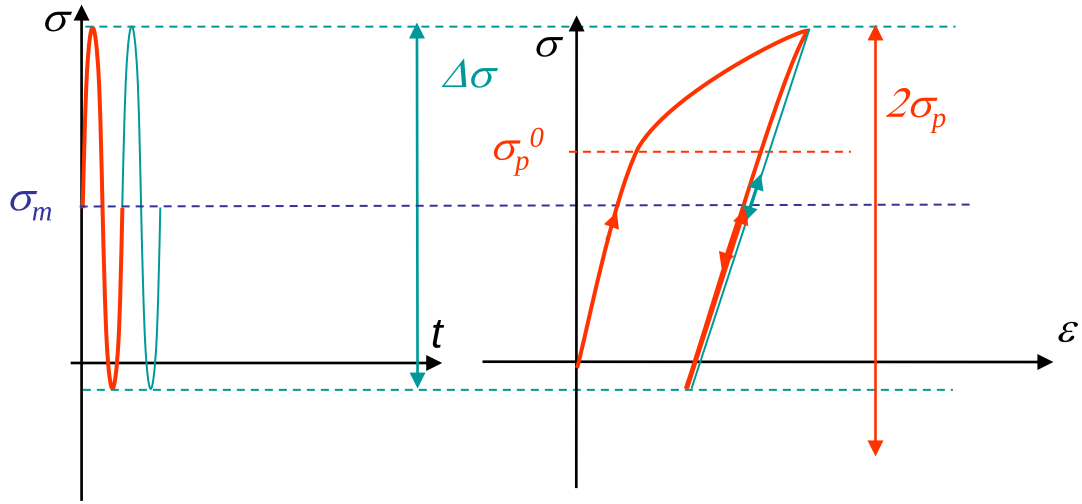

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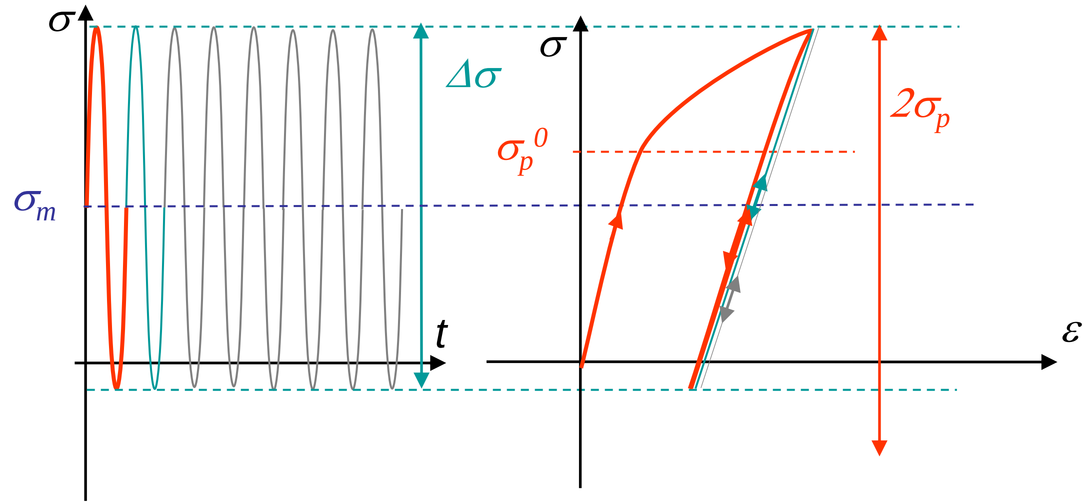

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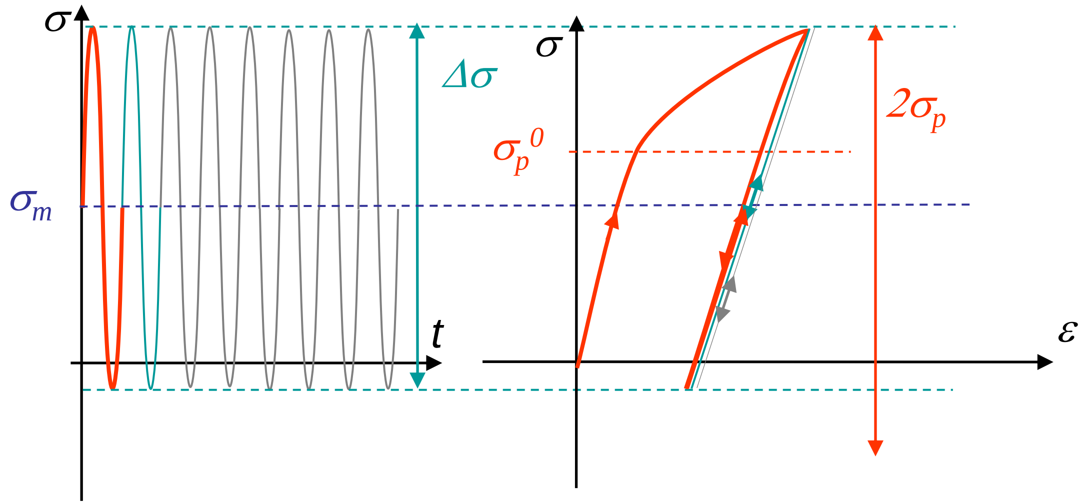

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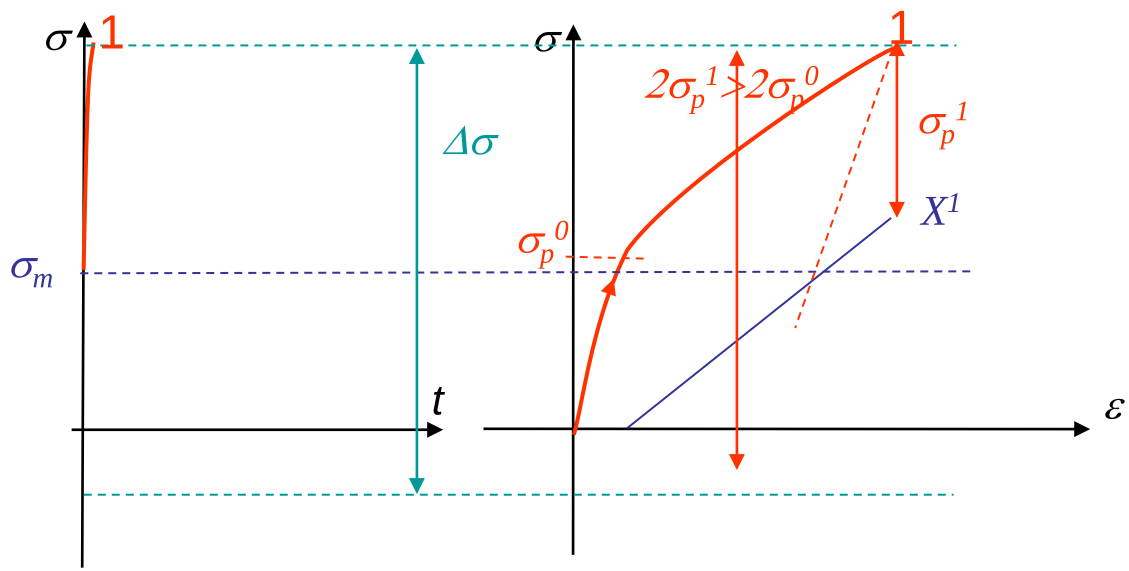

Material behavior: Elasto-plasticity under cyclic loading

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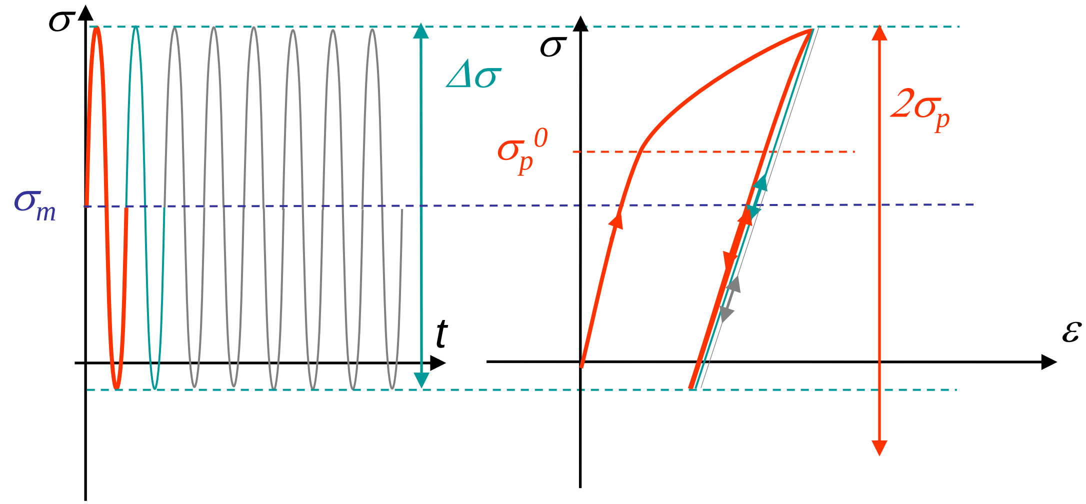

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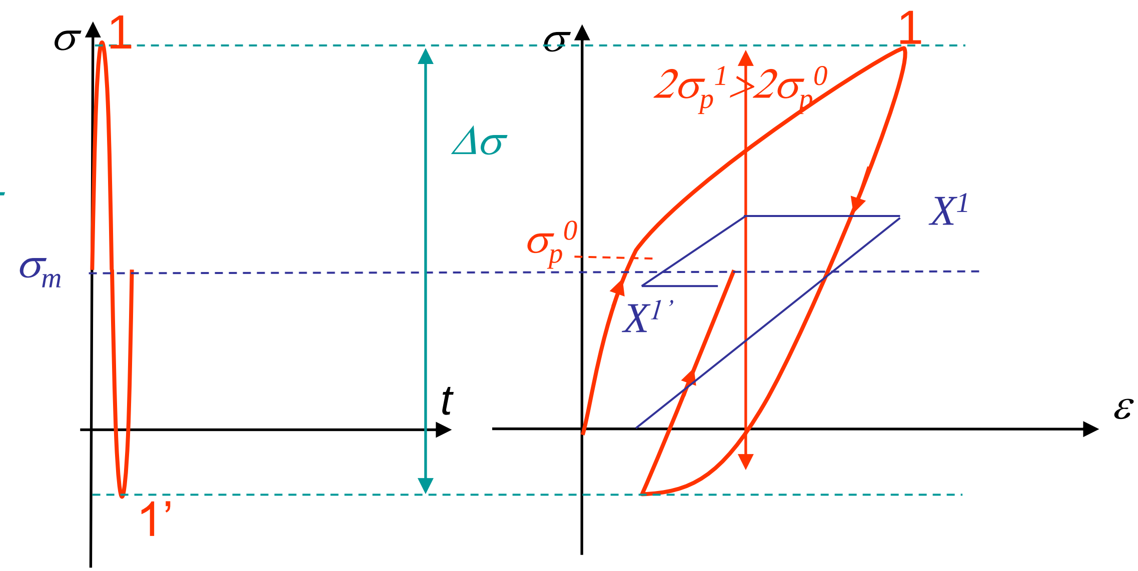

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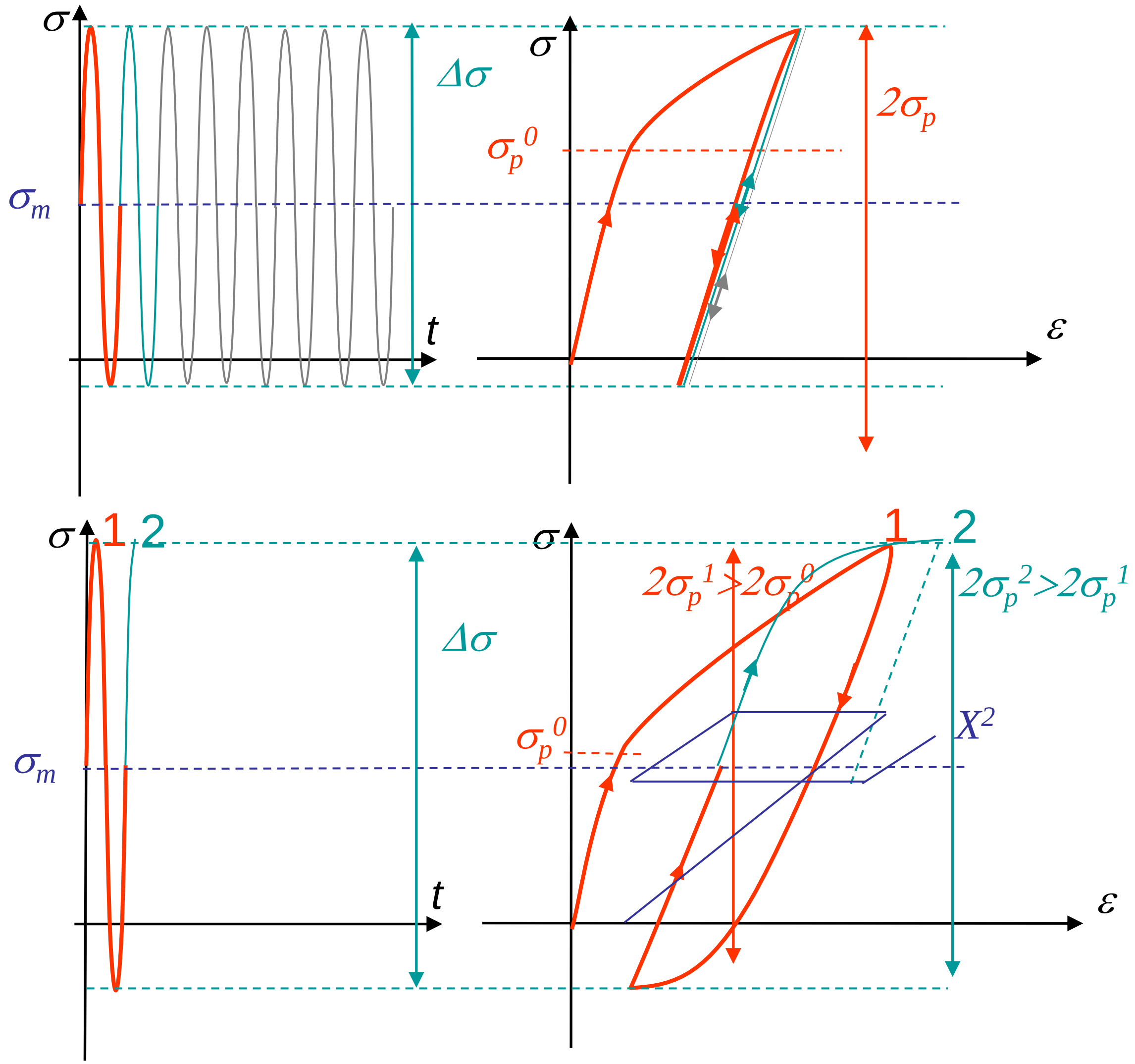

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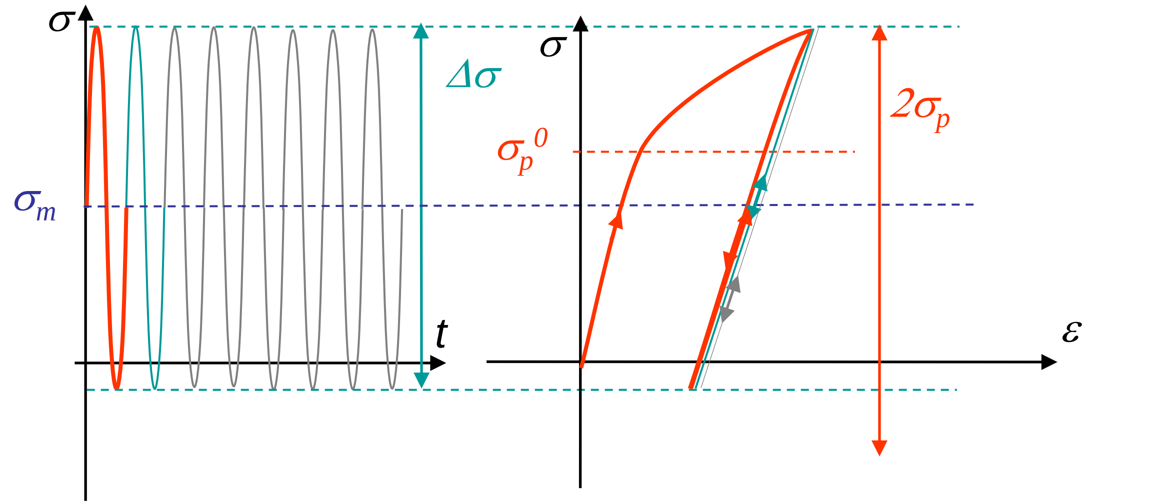

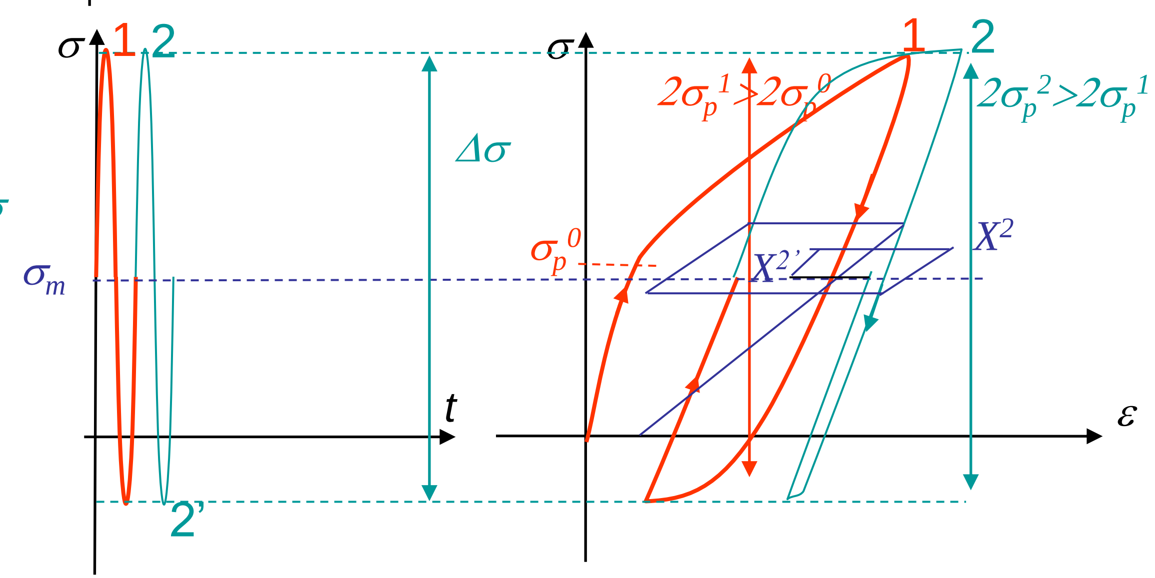

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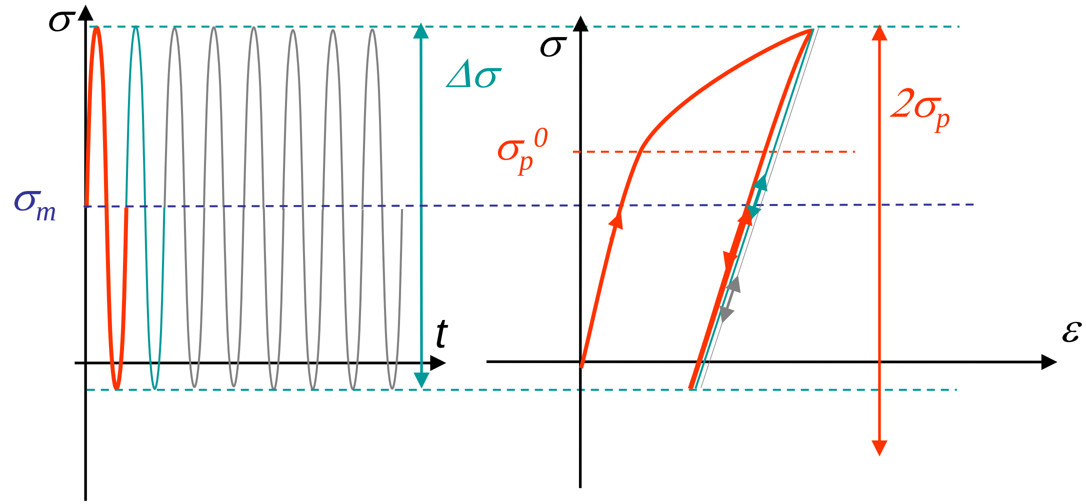

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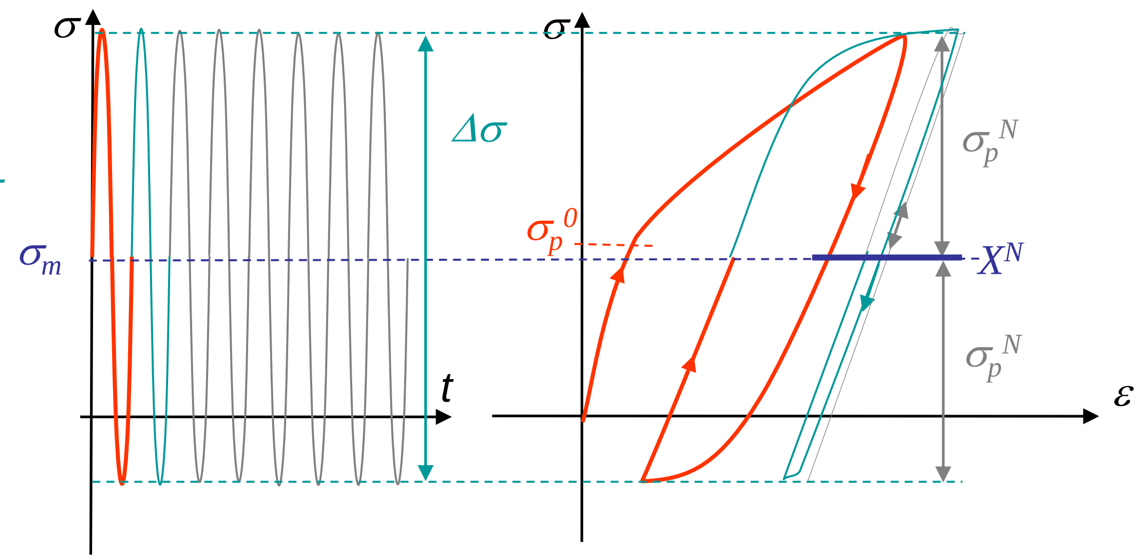
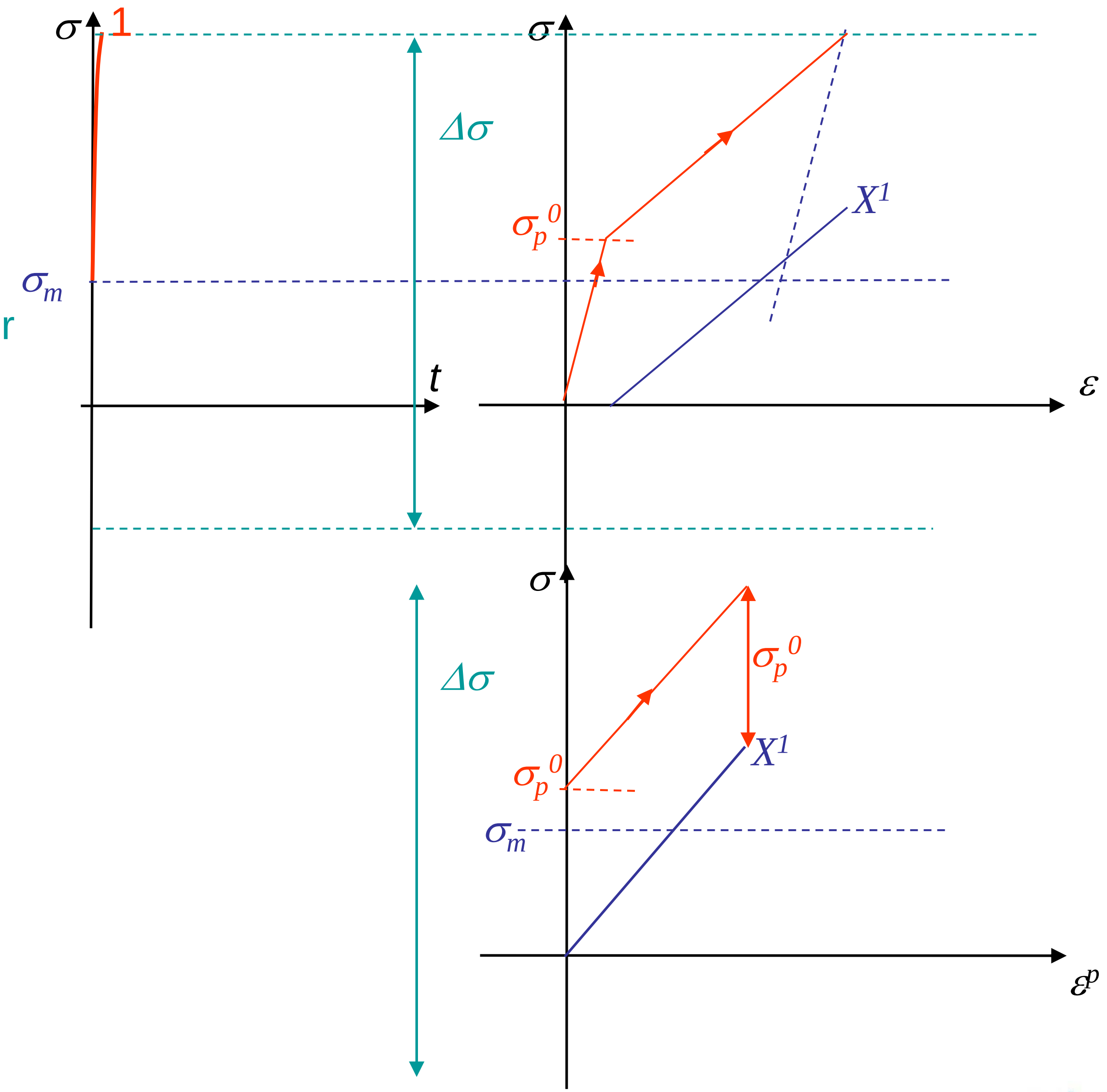


- Combination of structural and cyclic loading under controlled σ (2)
  - Accommodation
    - Large $\Delta\sigma$
    - Can be modeled by linear kinematic hardening (accommodation after 1 cycle)



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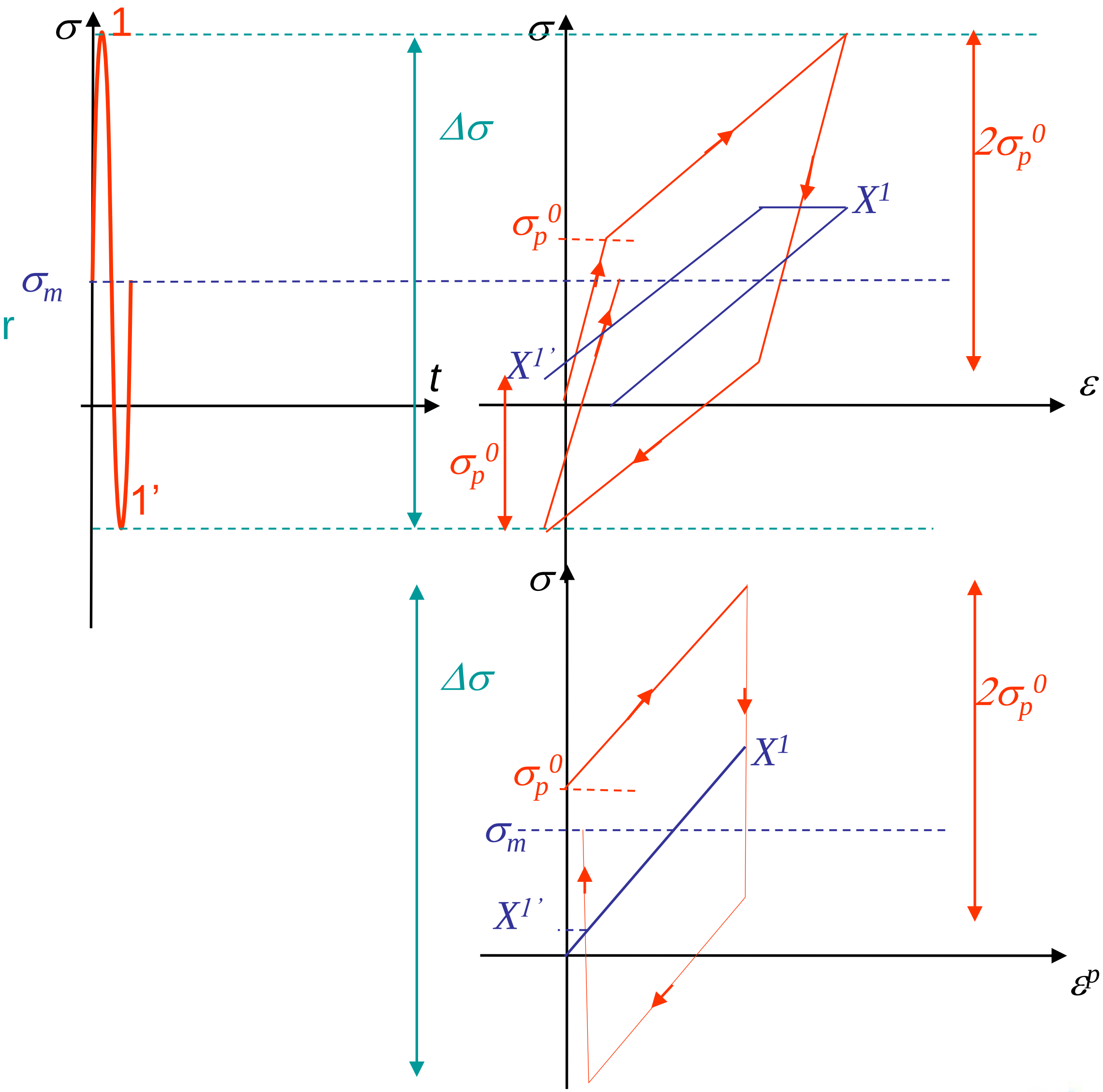

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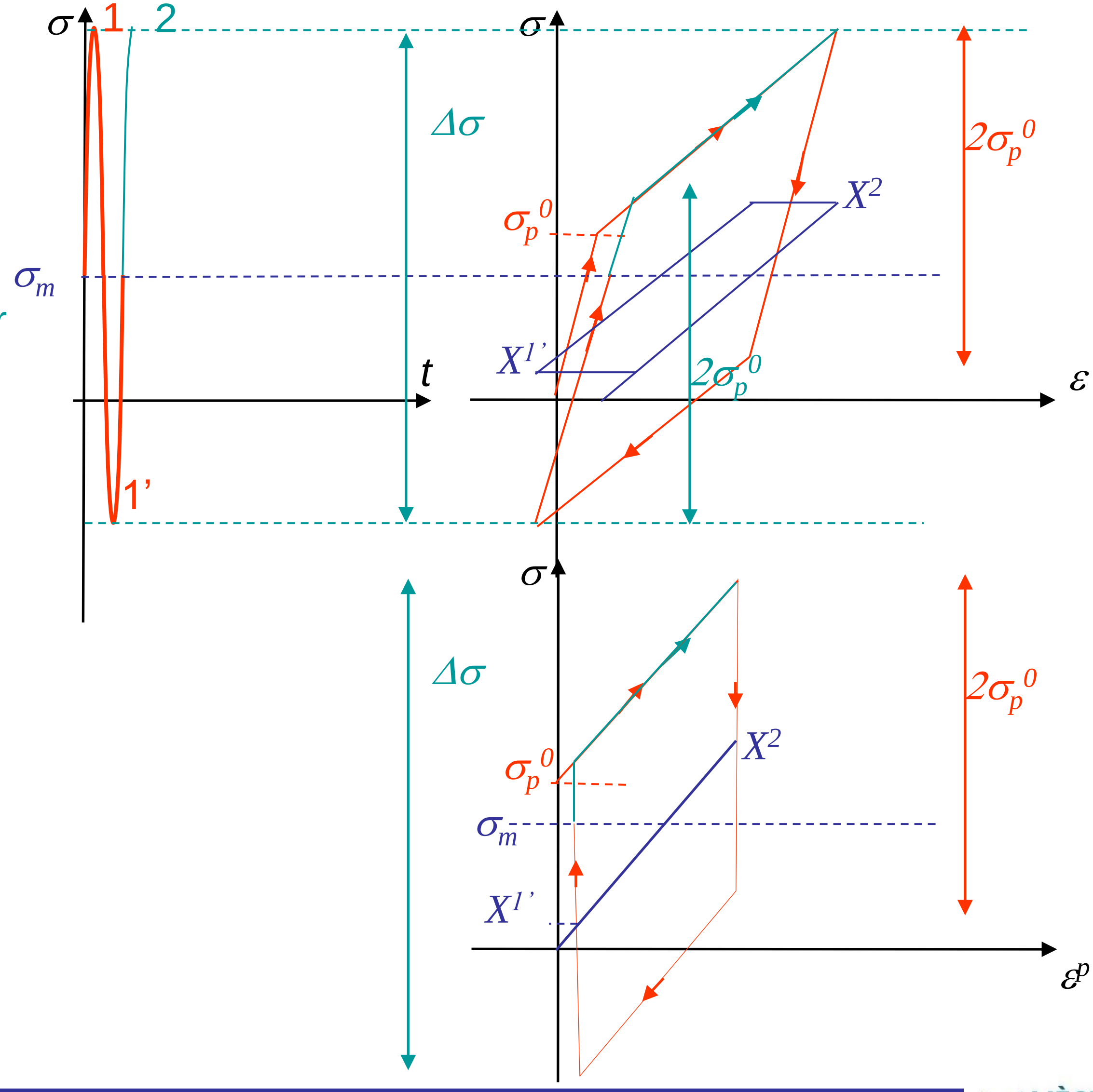

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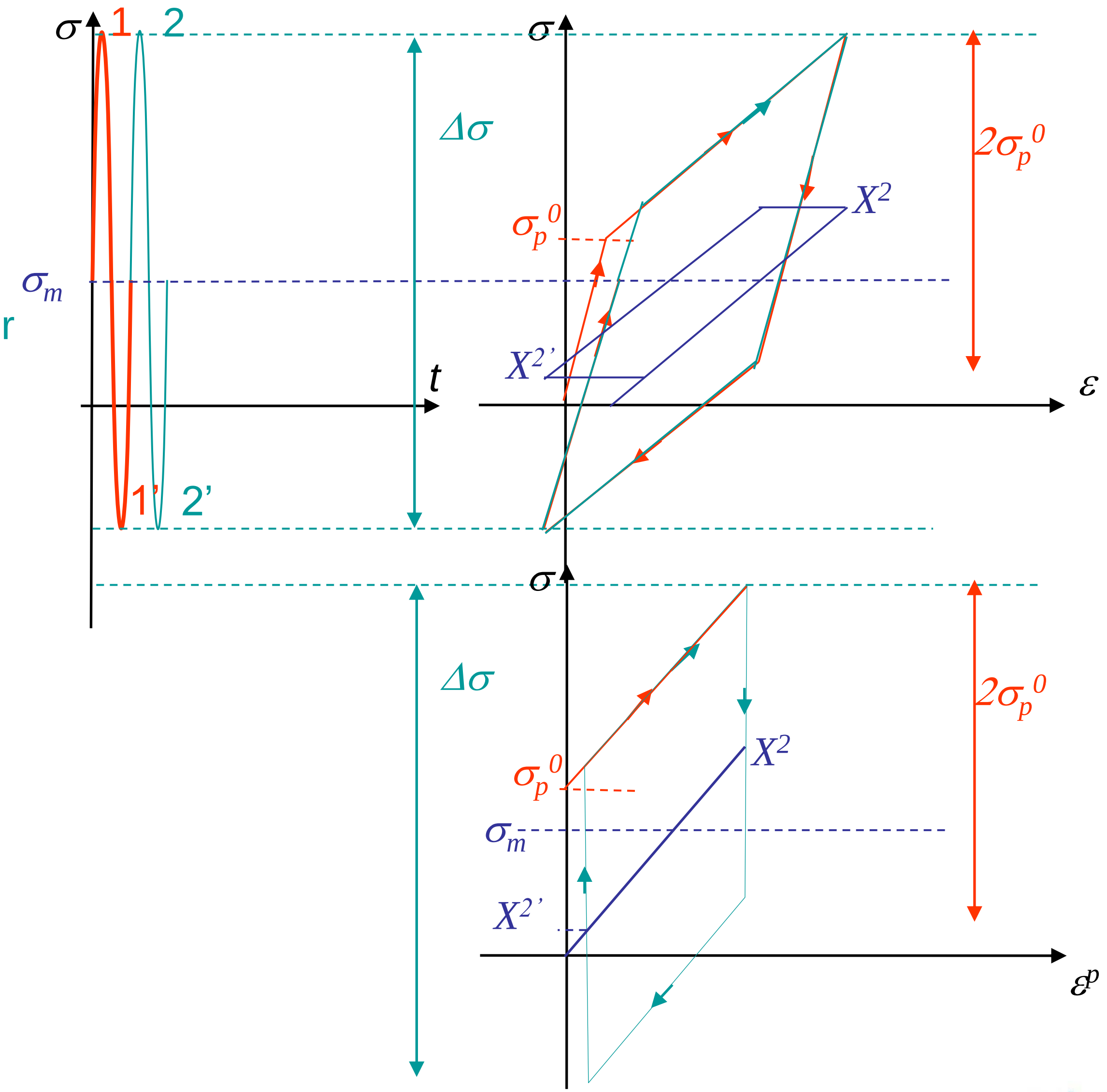

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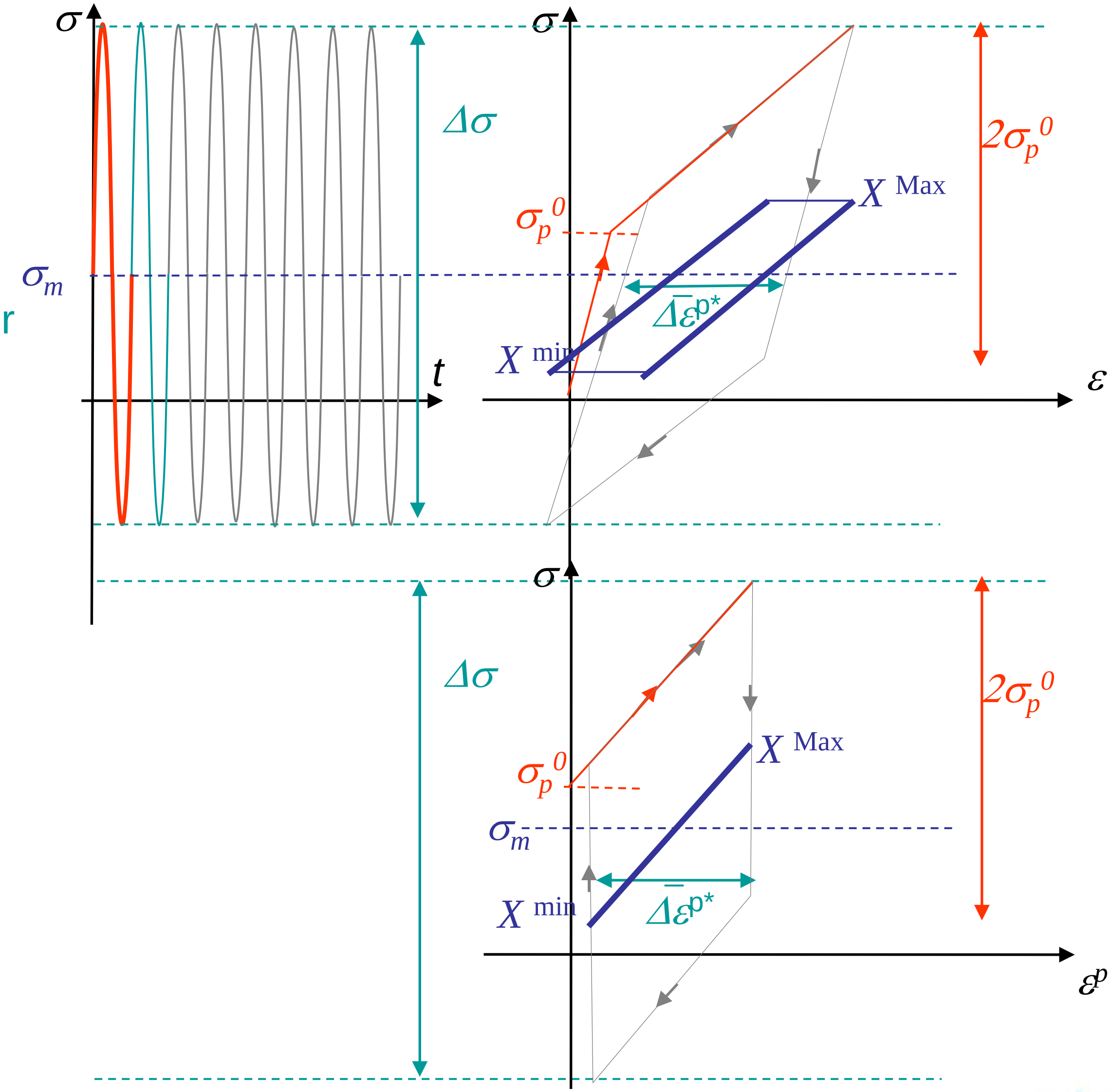
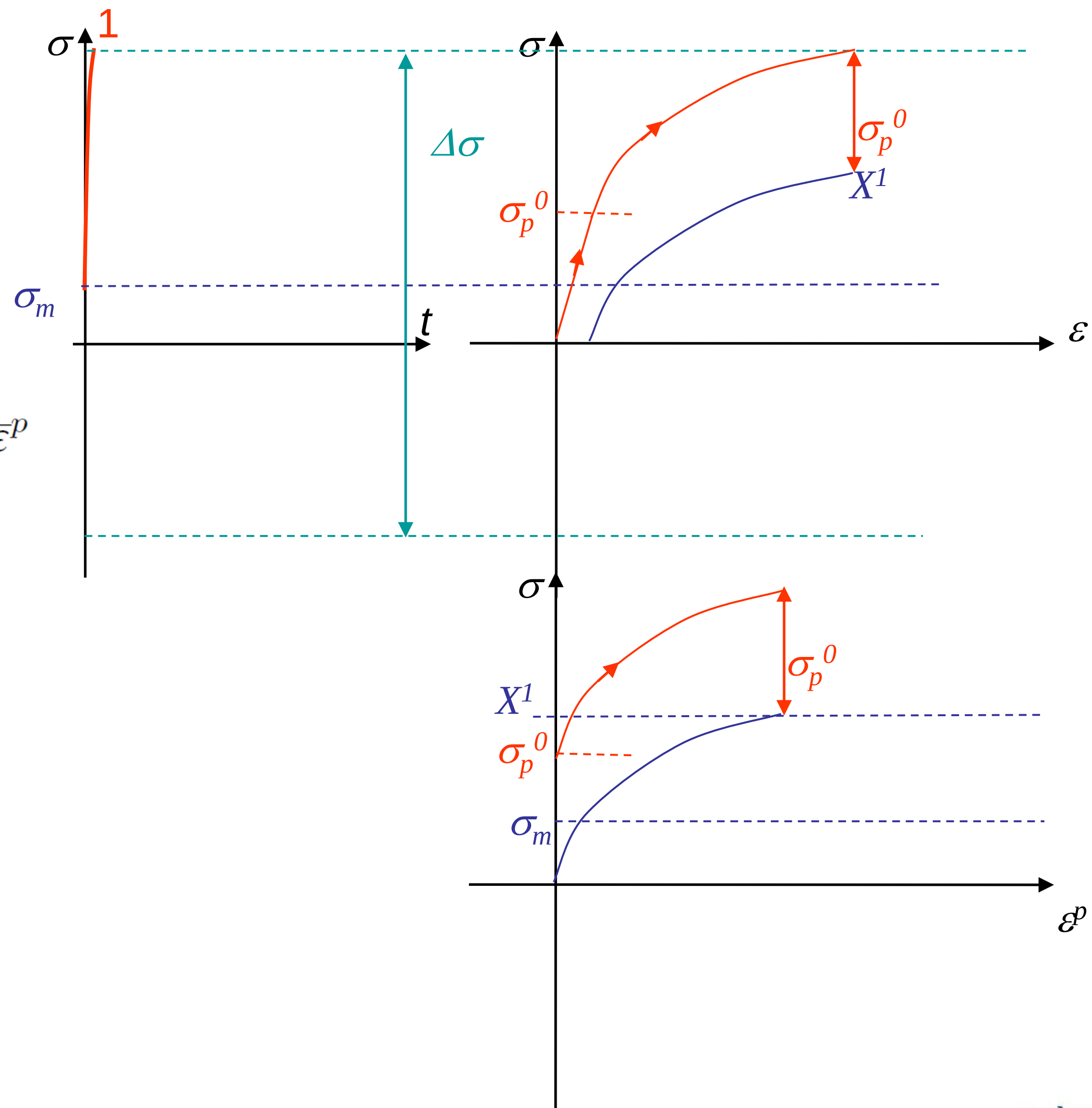
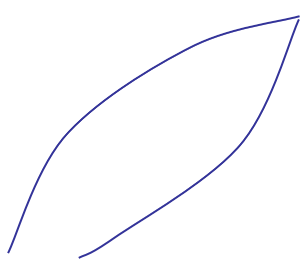


- Combination of structural and cyclic loading under controlled σ (3)
  - Ratcheting (or Rochet)
    - Large $\Delta\sigma$ & $\sigma_m \neq 0$
    - Threatens life
    - Non-linear kinematic hardening leads to ratcheting

$$d\mathbf{X} = \frac{2}{3} C d\boldsymbol{\varepsilon}^p - \gamma \mathbf{X} d\bar{\varepsilon}^p$$

If $|\boldsymbol{X}|^{\text{min}} \neq |\boldsymbol{X}|^{\text{Max}}$ curves of back-stress are not reversible





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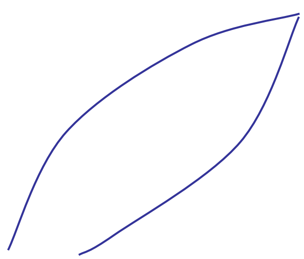

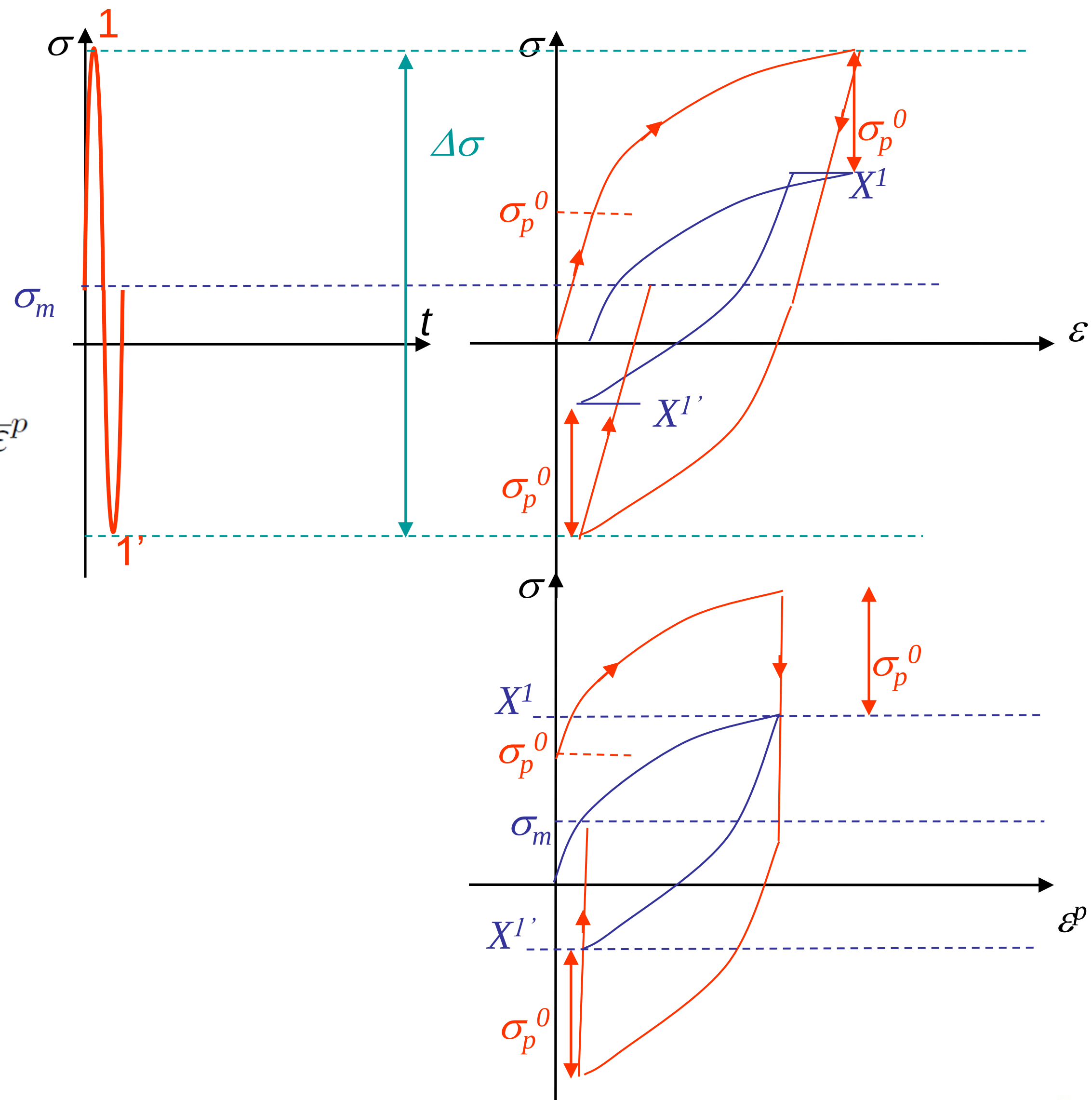

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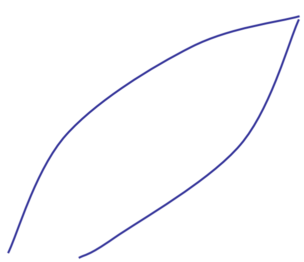

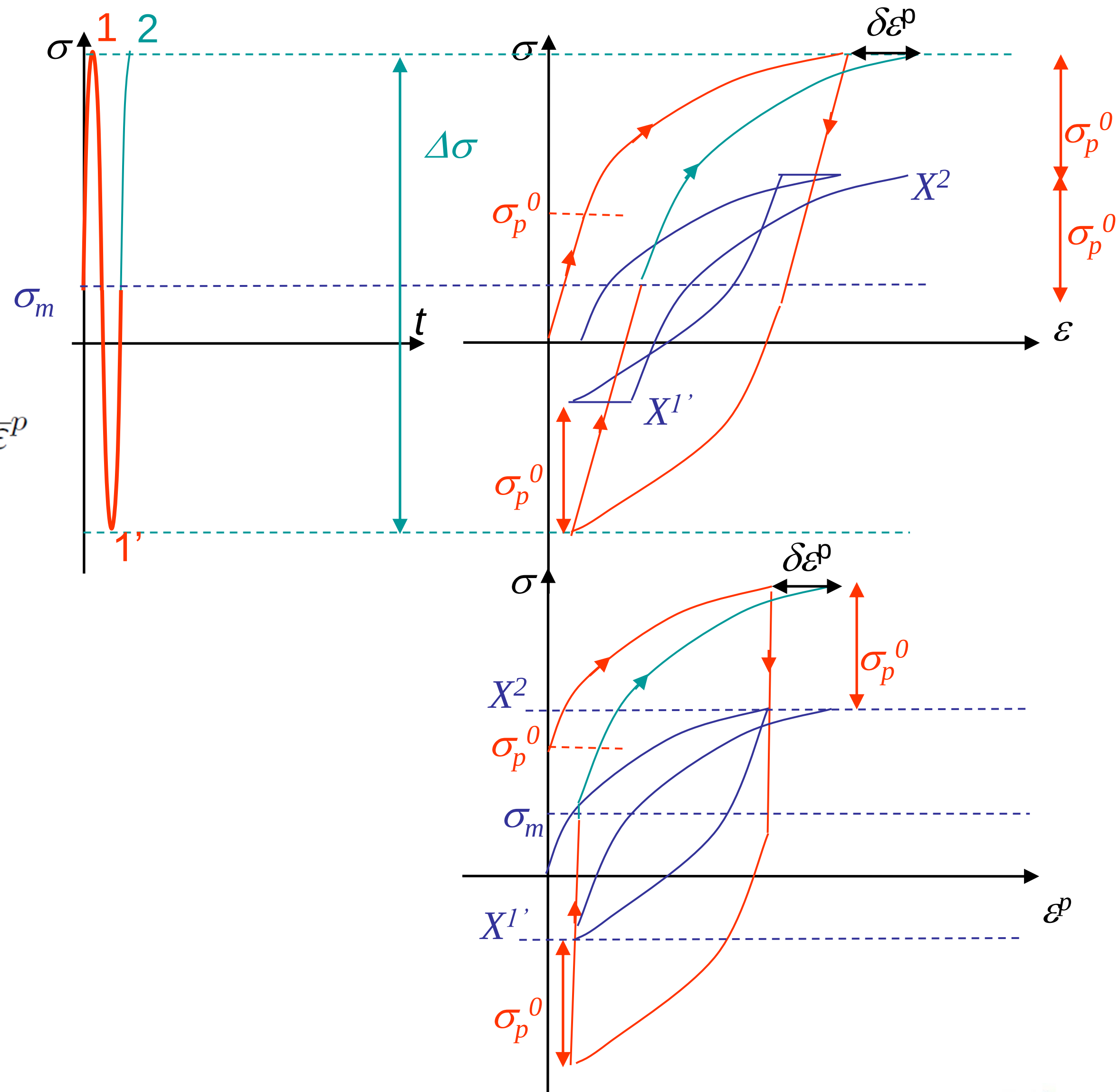

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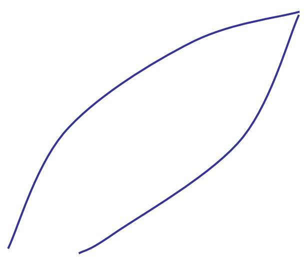

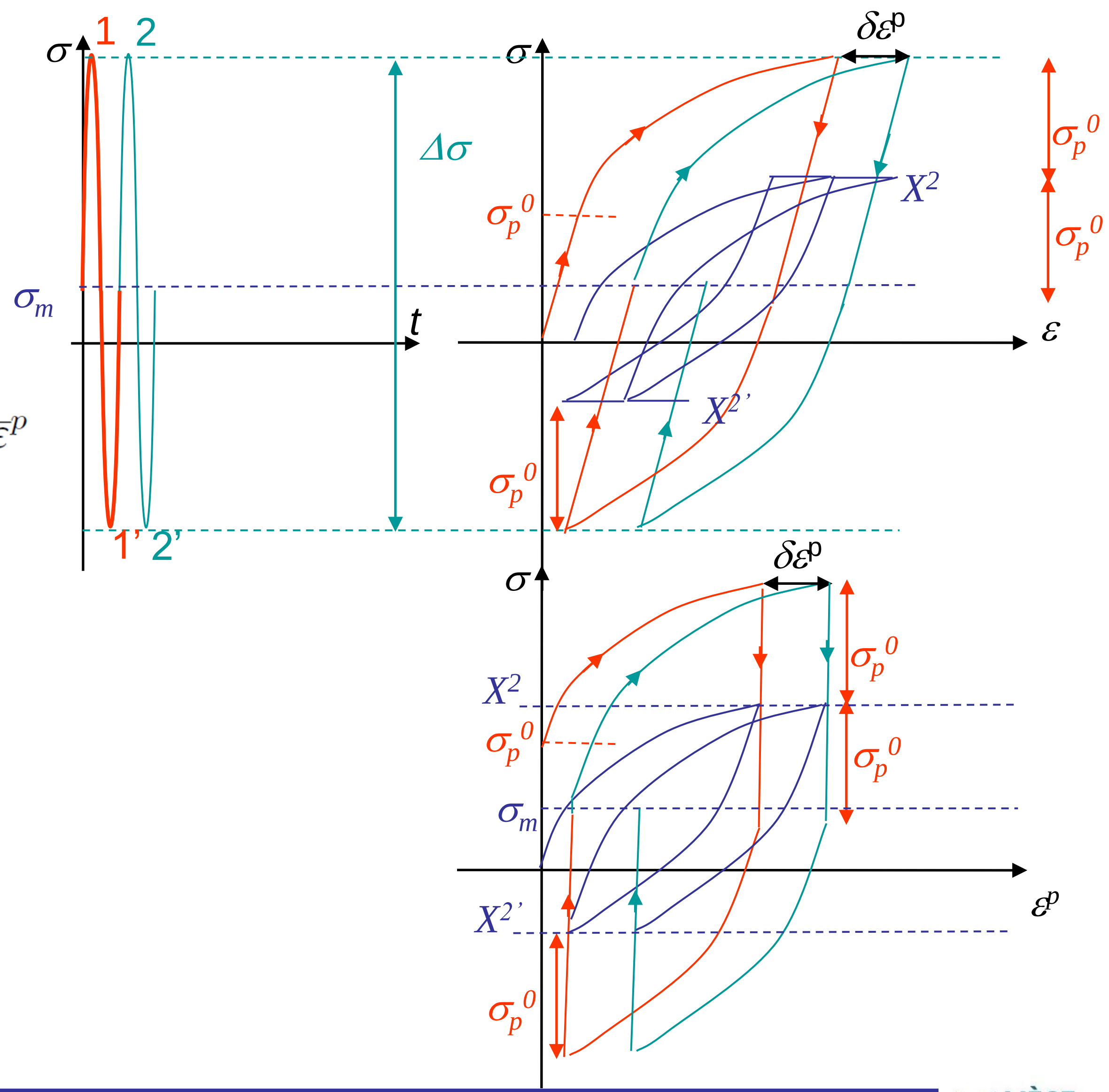

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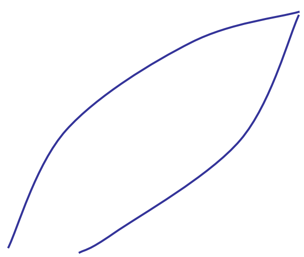
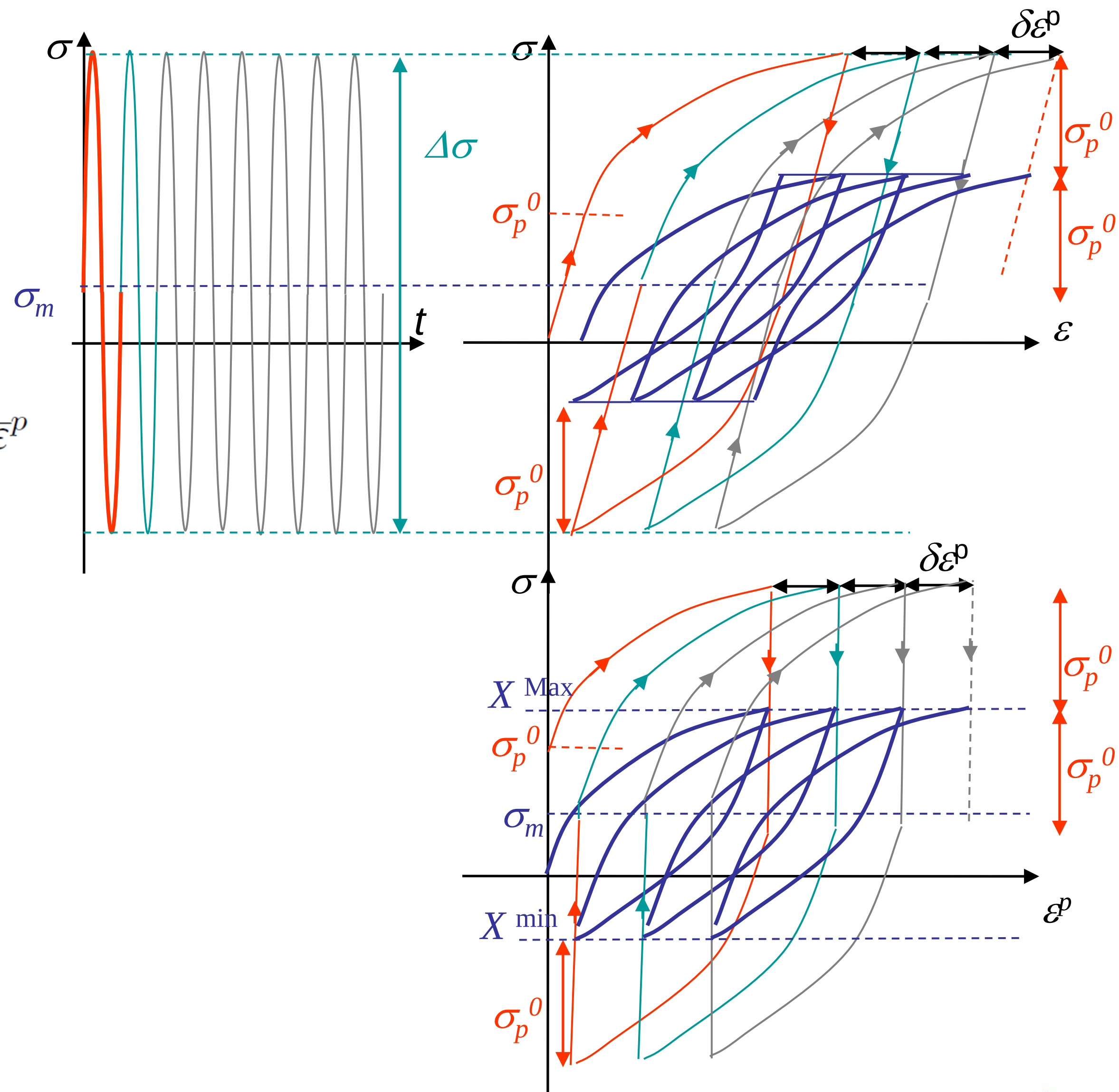
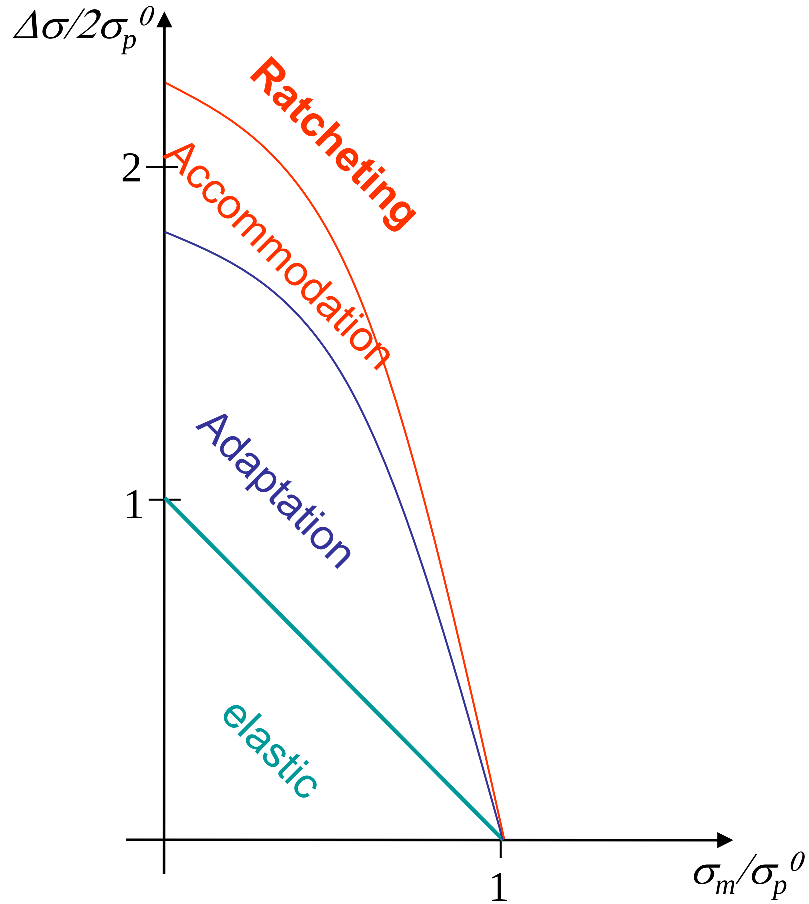




- Combination of structural and cyclic loading under controlled σ (4)
  - Bree diagram
    - For a given material
    - Remarks
      - Non-linear kinematic hardening always predicts ratcheting (if $\sigma_m \neq 0$)
      - Linear kinematic hardening never predicts ratcheting

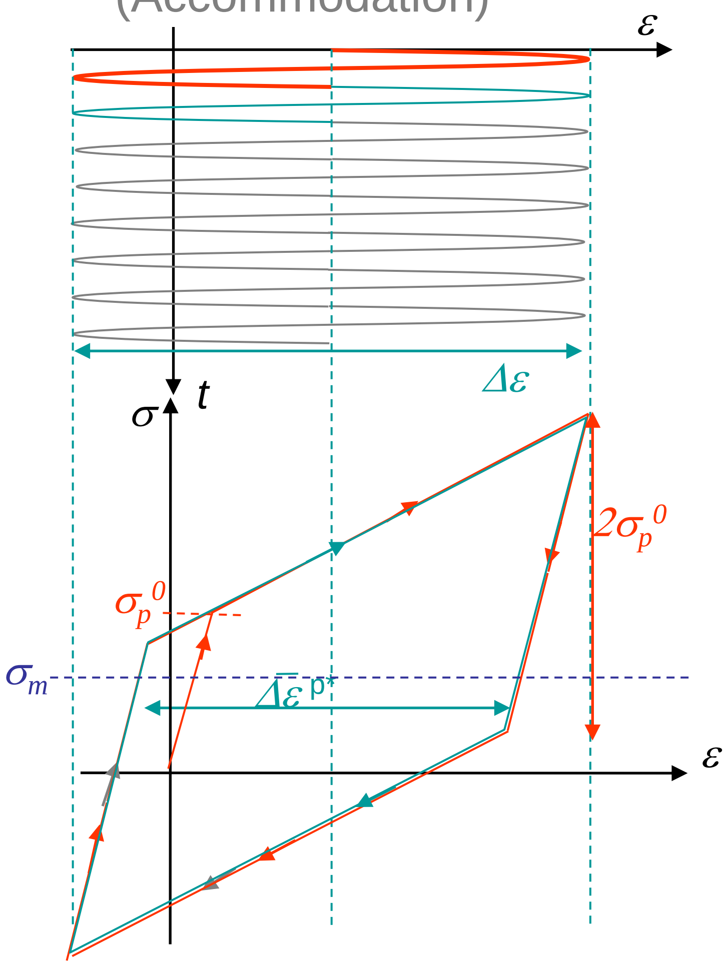
  ⟹ Laws should be enhanced
      - Threshold
      - Chaboche (1991)



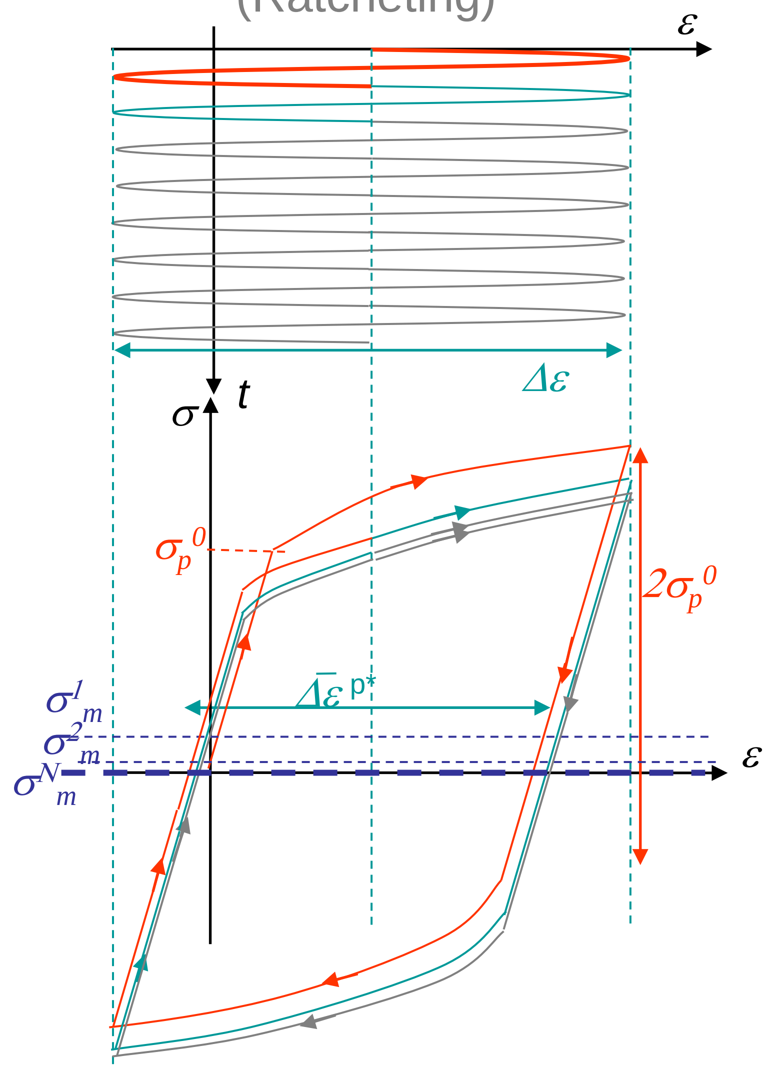
Material behavior: Elasto-plasticity under cyclic loading

- Combination of structural and cyclic loading under controlled ε

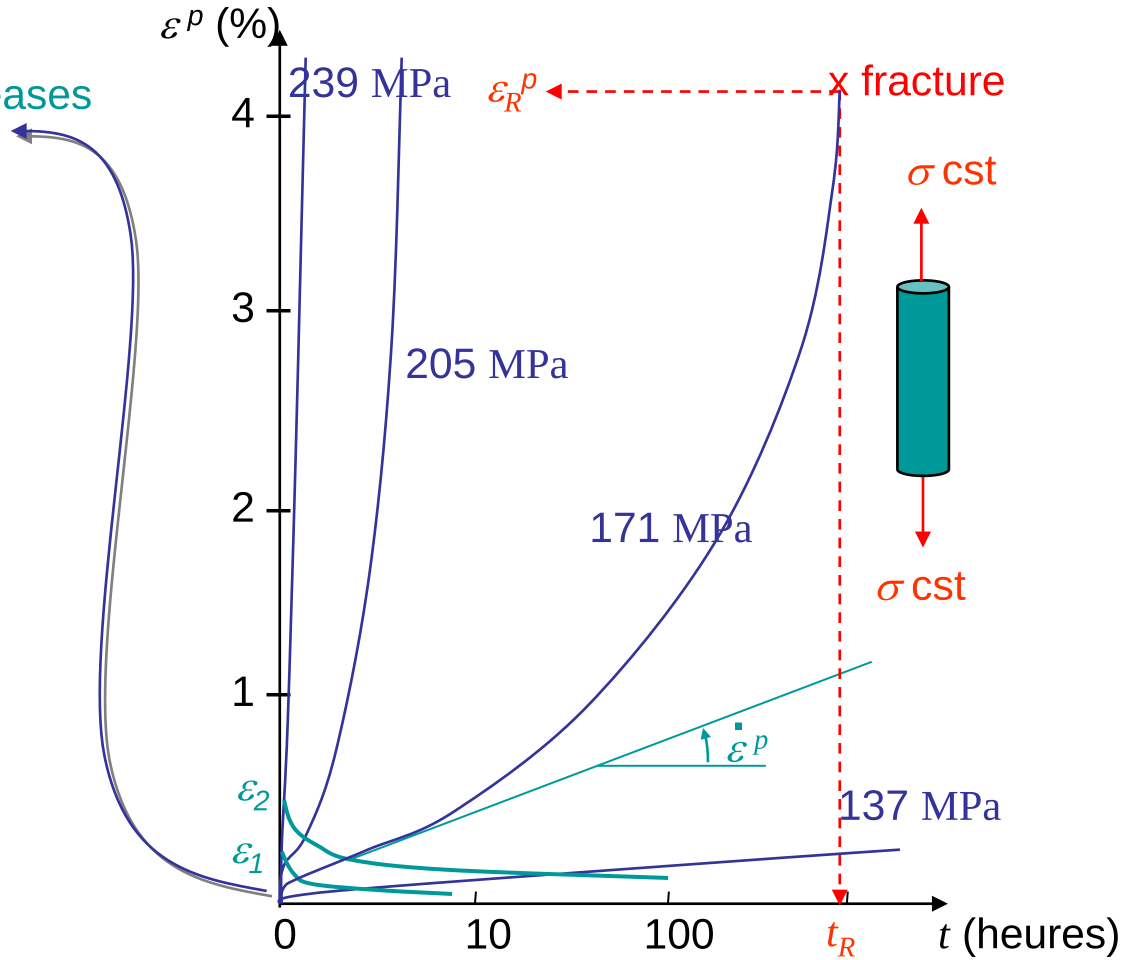
No mean stress relaxation
(Accommodation)



Mean stress relaxation
(Ratcheting)



- Different stages
  - Example: IN100 & T = 1000°C under different constant tensile stresses
  - Primary creep
    - Low strain ($\varepsilon < \varepsilon_1$)
    - Initially high $\dot{\varepsilon}^p$ but decreases with hardening



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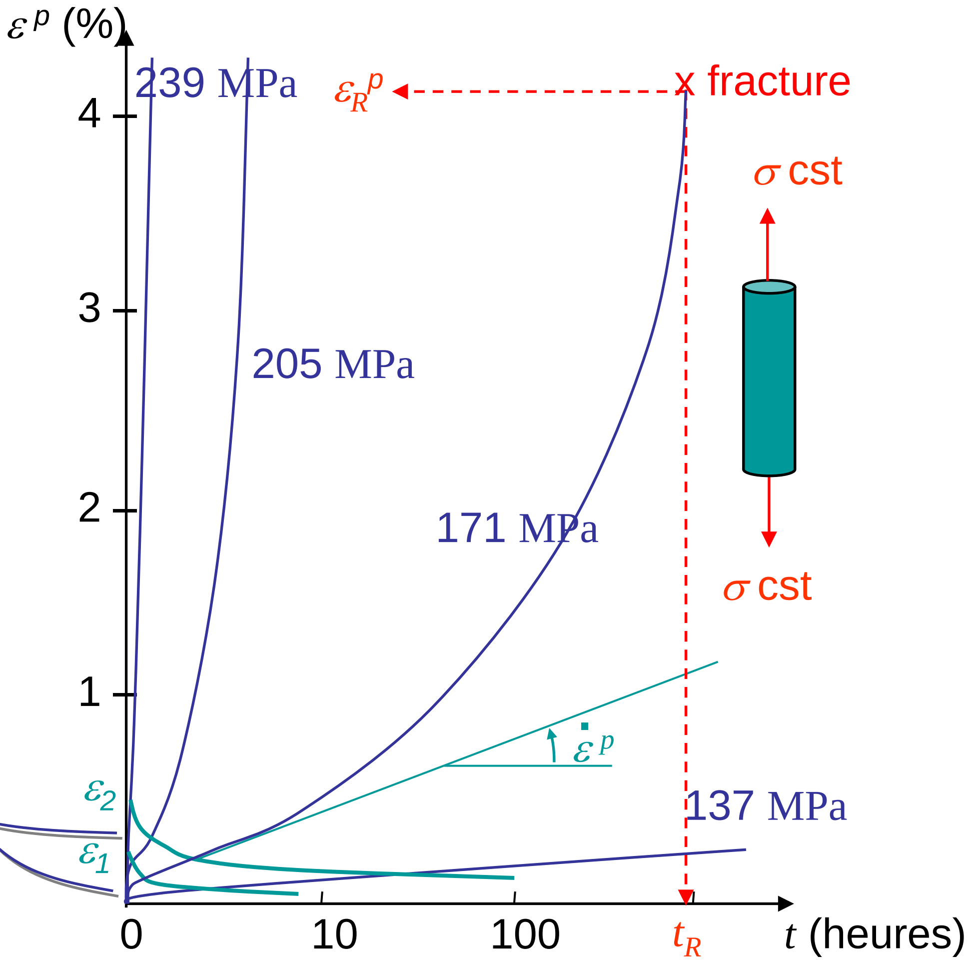
  - Secondary creep
    - For $\varepsilon_1 < \varepsilon < \varepsilon_2$
    - Nearly constant $\dot{\varepsilon}^p$
    - Balance between hardening and thermal softening
    - Dislocation slip & climb (see next slide)



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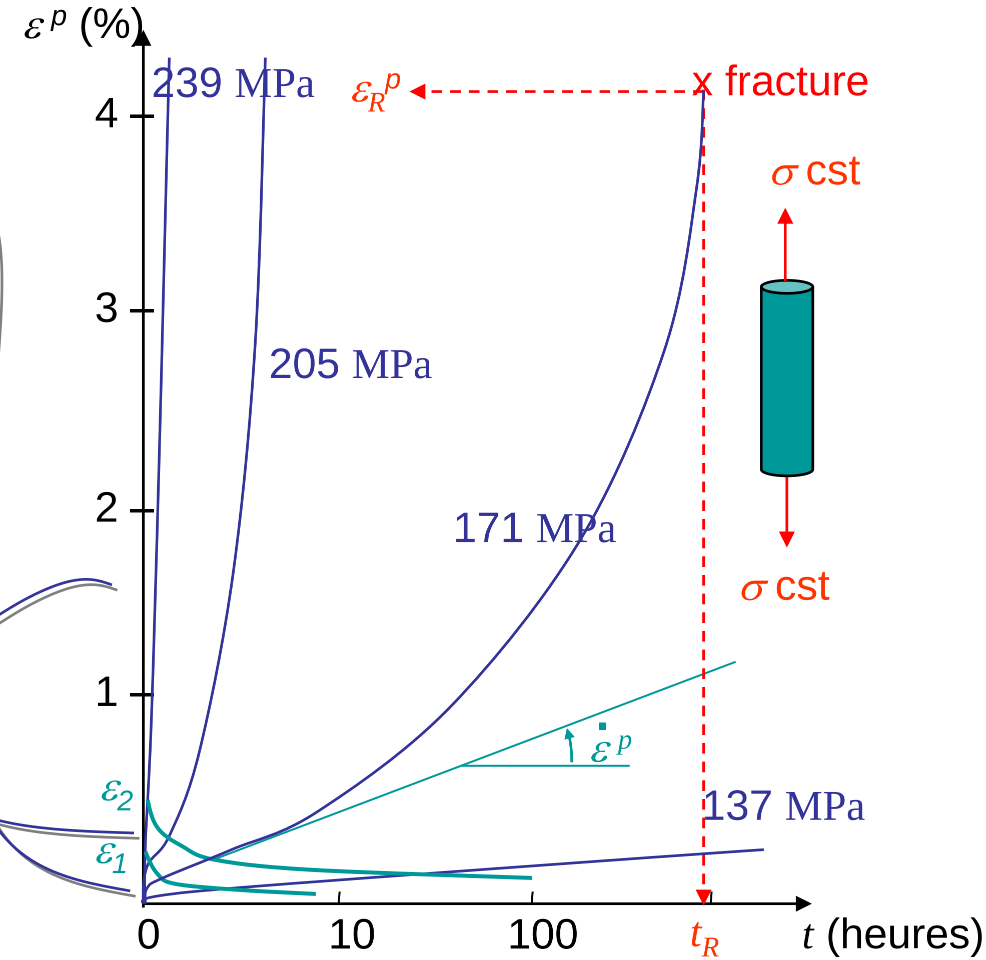
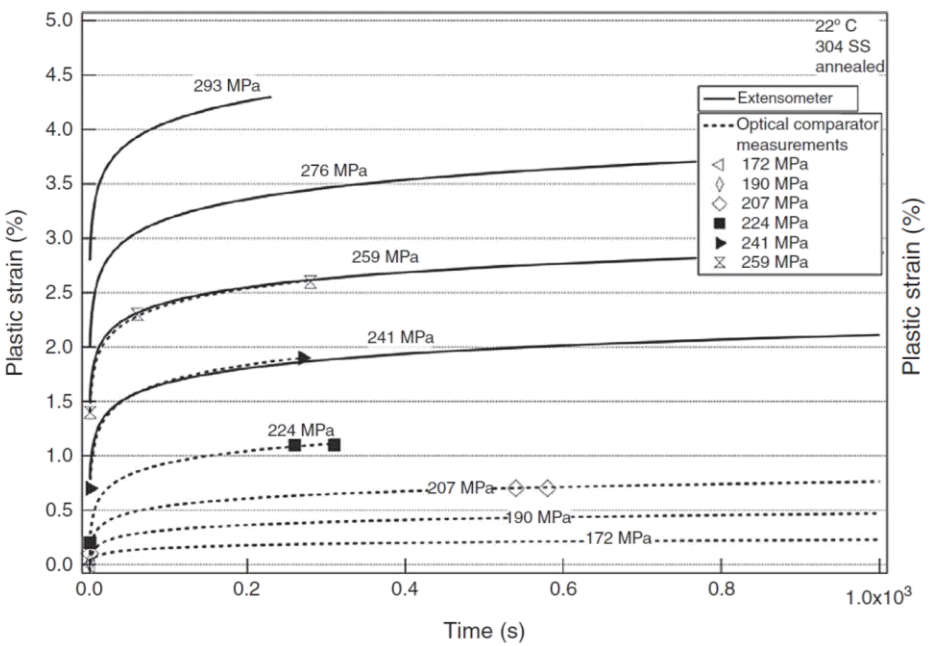
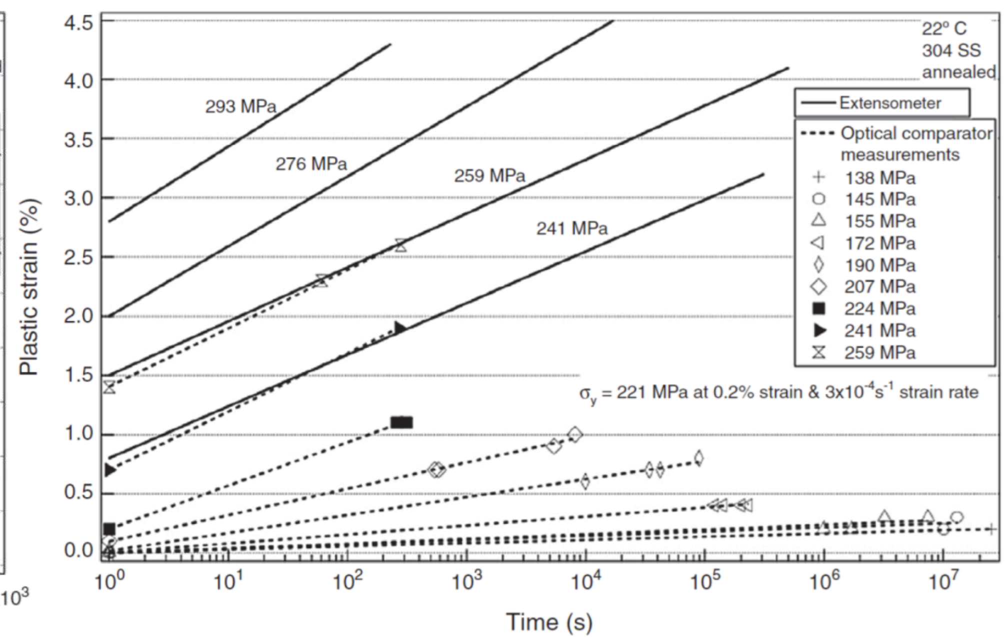
    - For $\varepsilon_1 < \varepsilon < \varepsilon_2$
    - Nearly constant $\dot{\varepsilon}^p$
    - Balance between hardening and thermal softening
    - Dislocation slip & climb (see next slide)
  - Tertiary creep
    - For $\varepsilon > \varepsilon_2$
    - Increases of $\dot{\varepsilon}^p$
    - Begin of damage
      - Grain-boundary sliding
      - Voids formation



- Primary creep
  - Dislocations accumulate (pile-up) at barriers (grain boundaries …)

    → Hardening
  - Primary creep can exist at temperature below 30% T_m
    - E.g. annealed 304 stainless steel
    - Described by $\dot{\varepsilon}^p = \frac{\alpha}{t}$

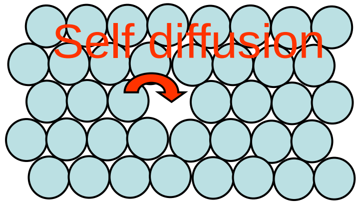


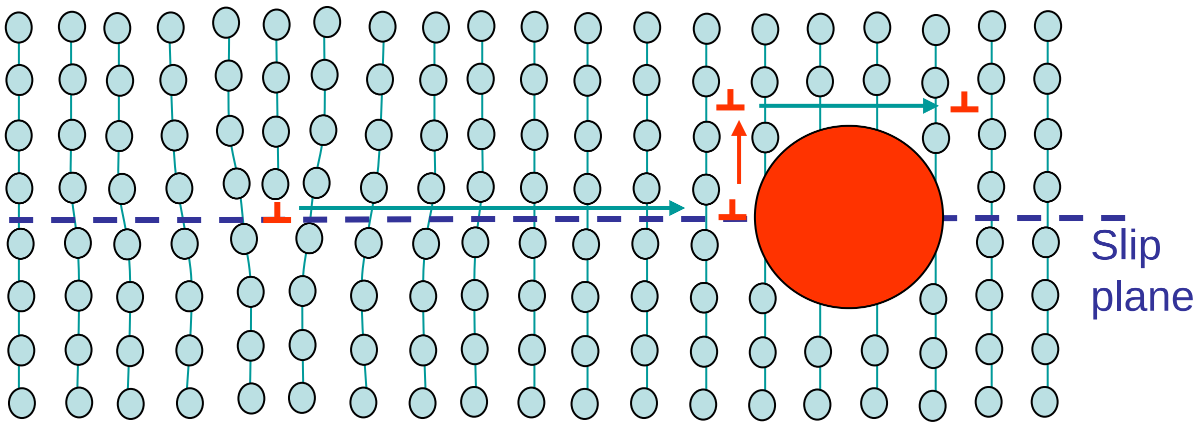


Michael E. Kassner, Kamia Smith, Low temperature creep plasticity, Journal of Materials Research and Technology, Volume 3, Issue 3, 2014, Pages 280-288, https://doi.org/10.1016/j.jmrt.2014.06.009.

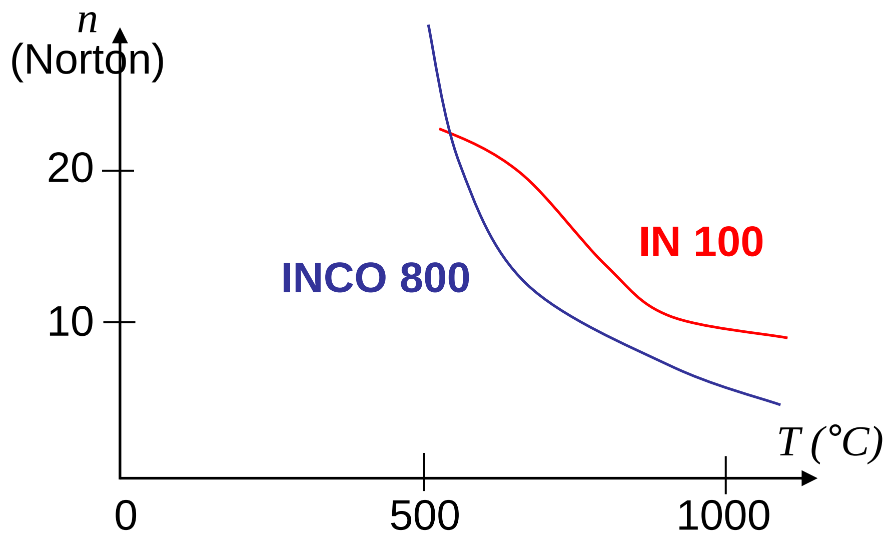
- Secondary creep: dislocation slip and climb
  - If atoms
    - Have enough energy (Temperature and strain energy)
    - Are in close packed lattice

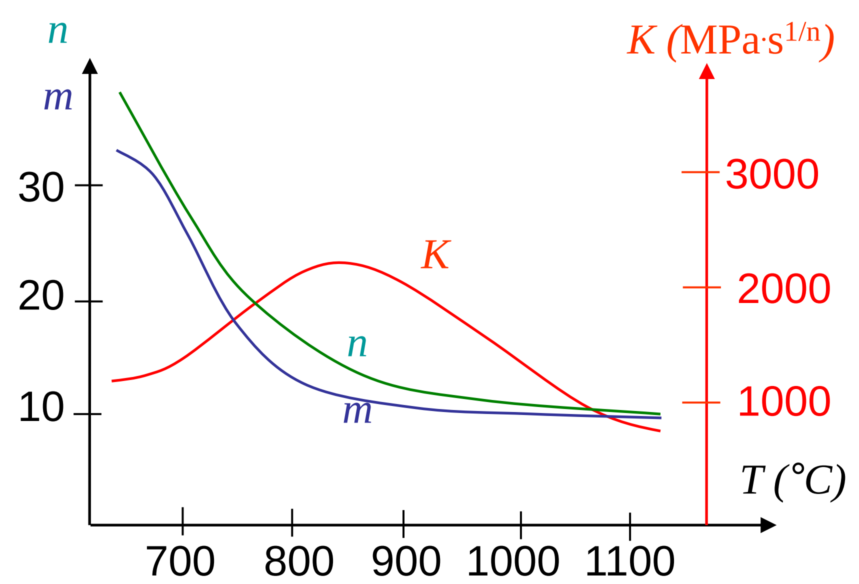
    They self-diffuse and fill vacancies
  - This corresponds for a blocked dislocation
    - To climb normal to its slip plane by atoms self-diffusion
    - To lie on an unobstructed slip plane
    - To move and to annihilate with a dislocation of opposite sign



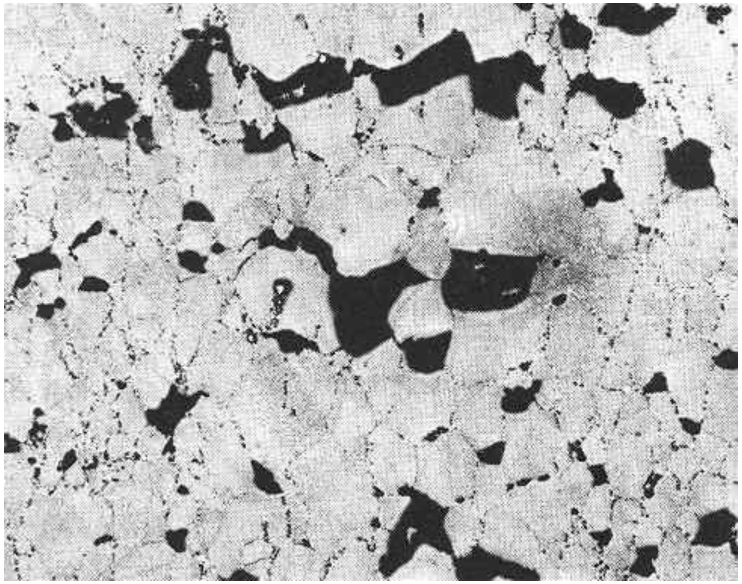


- Secondary creep
  - Constant $\dot{\varepsilon}^p$
  - Norton law
    - $\dot{\boldsymbol{\varepsilon}}^p = \left(\frac{\sigma}{\lambda}\right)^n$
  - Lemaître law
    - $\dot{\boldsymbol{\varepsilon}}^p = \left(\frac{\sigma}{K\left(\bar{\varepsilon}^p\right)^{\frac{1}{m}}}\right)^n$

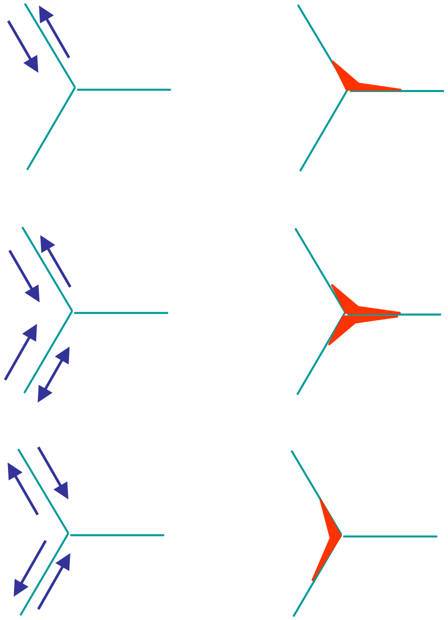




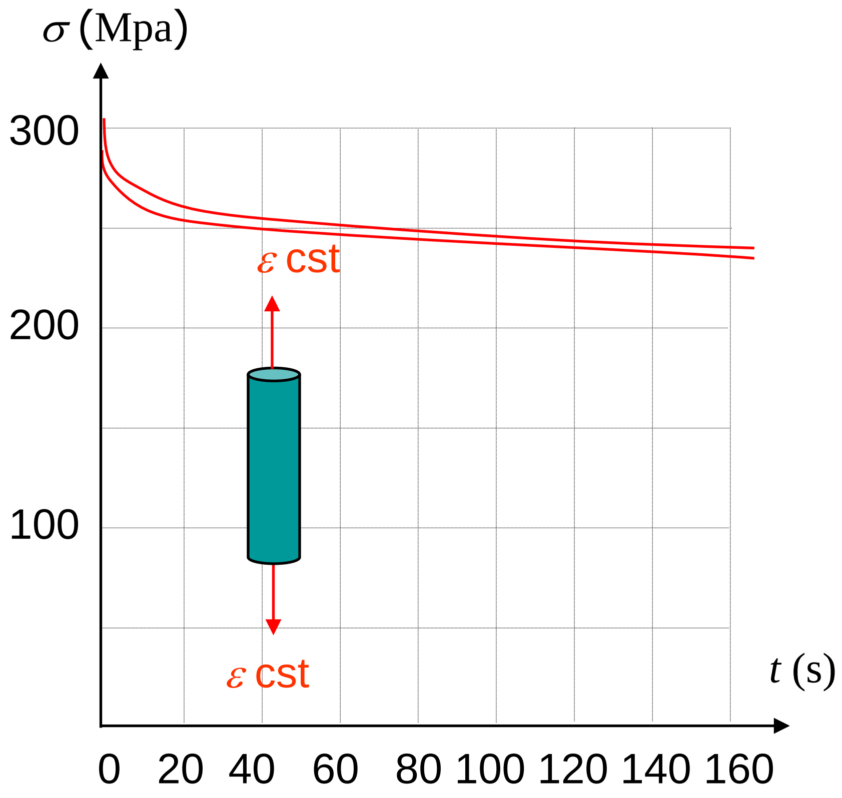
- Tertiary creep
  - Grain boundary sliding is activated
  - Formation of voids under shearing



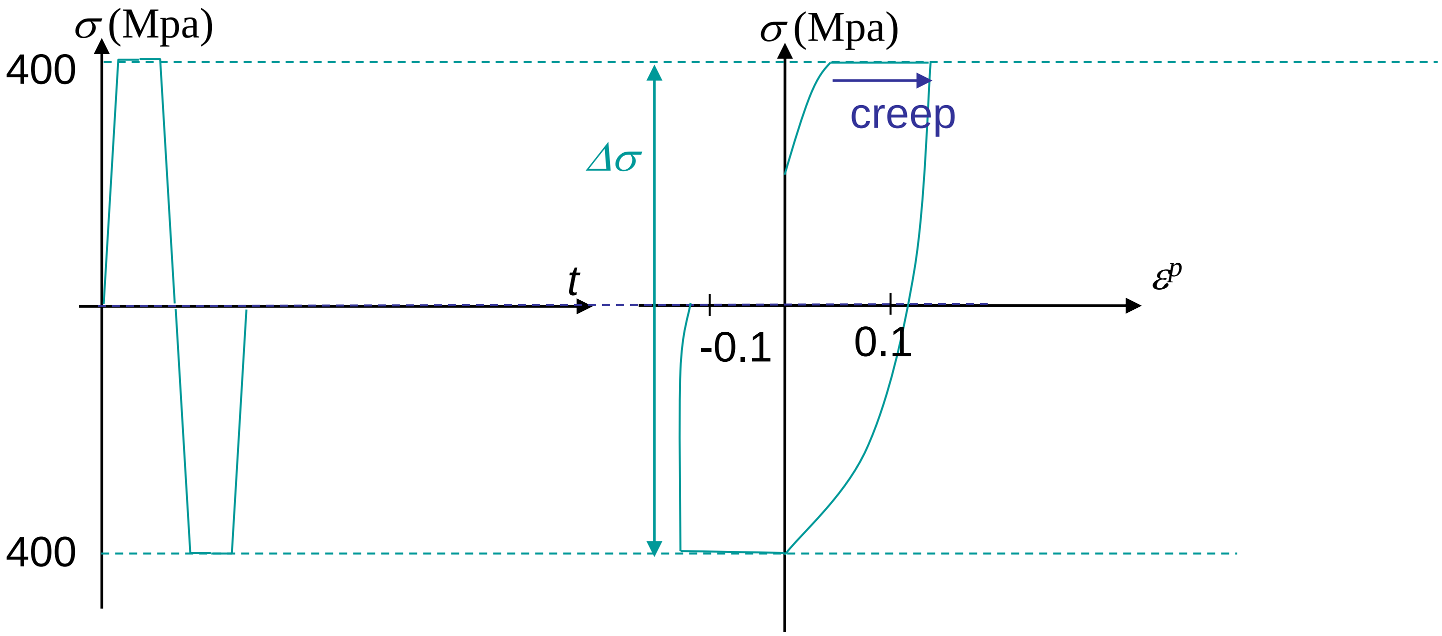
Nimonic 80A, 1023 K and 154 MPa



- Recovery: strain driven test
  - Quenched AU4G alloy
  - At 200 °C

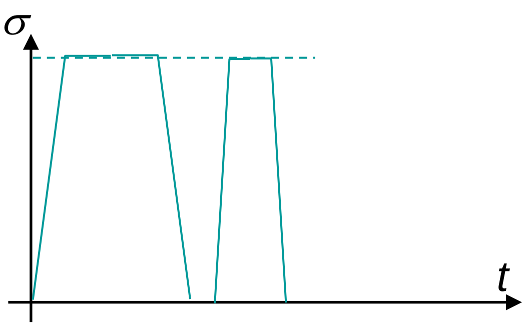


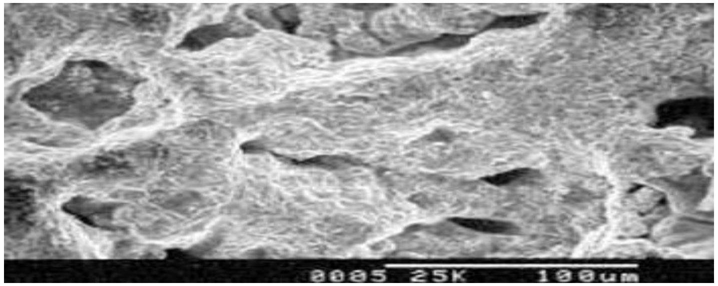
- Effect of creep during cyclic loading under constant $\Delta\sigma$
  - IN 100 Super alloy, first cycle with holding time, 900°C



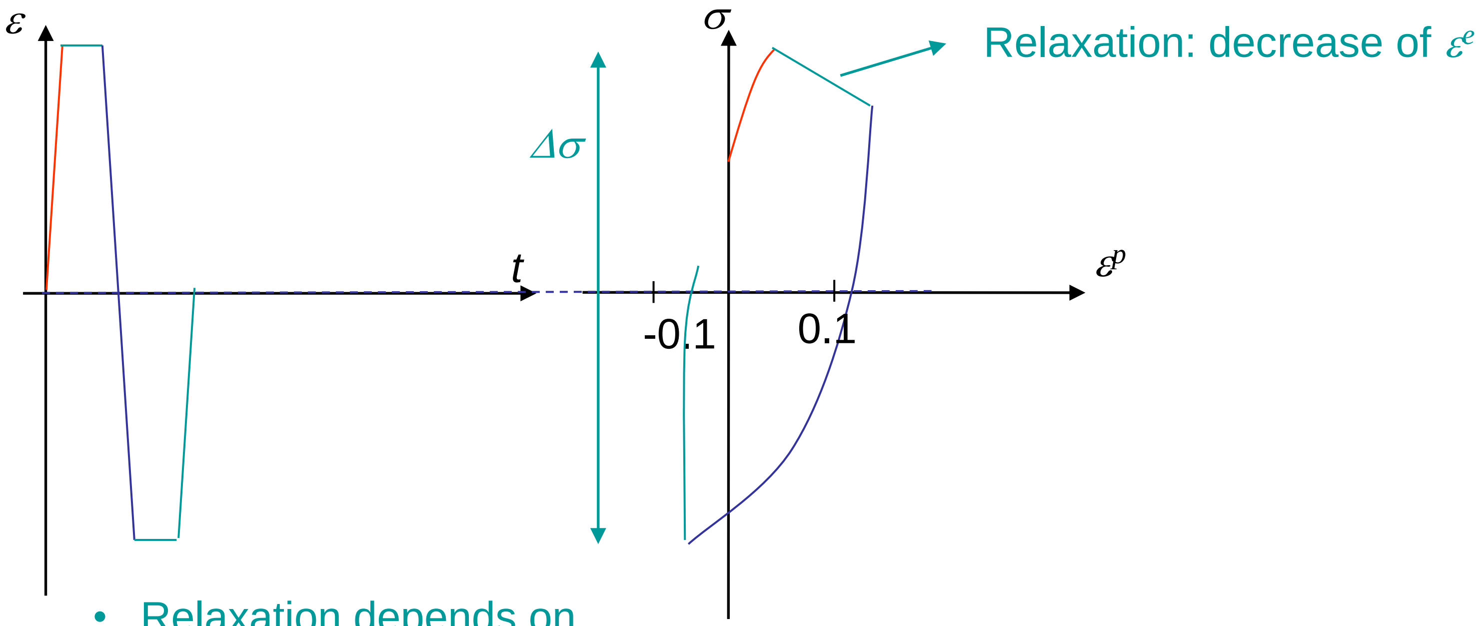
- Creep depends on
  - Holding time
  - Temperature
  - Frequency

- Example of creep during cyclic loading under constant $\Delta\sigma$
  - HP turbine blade
    - Long working phases at high temperature (axial loading)
    - Long rest phases at room T°
    - Inter-crystalline fracture mode and voids



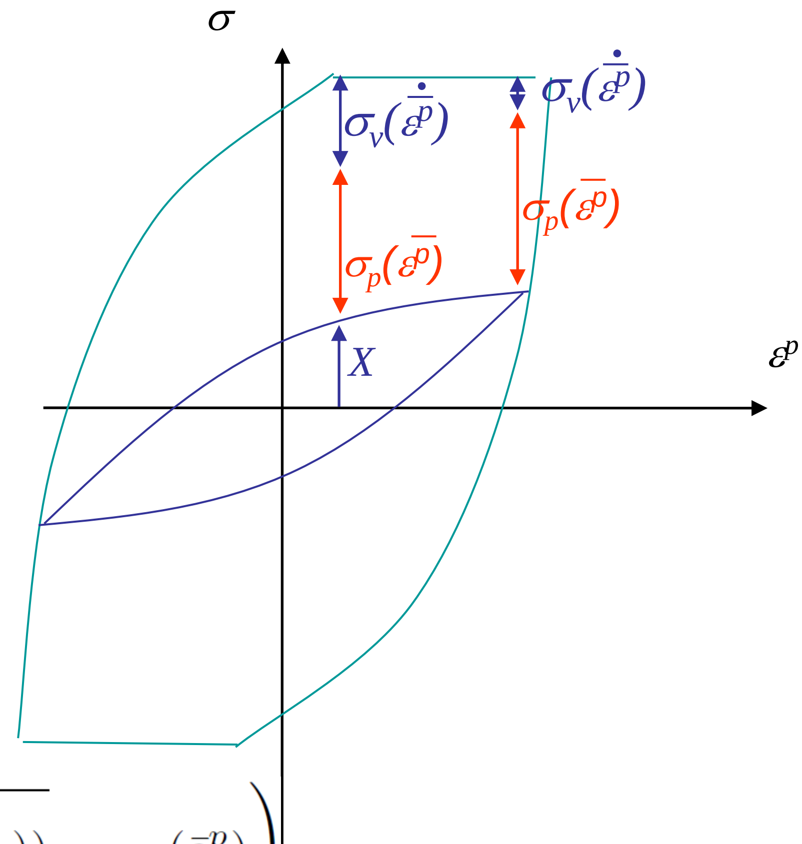


- Effect of creep during cyclic loading under constant $\Delta\varepsilon$
  - IN 100 Super alloy, first cycle with holding time, 900°C



- Relaxation depends on
  - Holding time
  - Temperature
  - Frequency
- Creep and relaxation can be modeled by considering
  - Kinematic hardening
  - Visco-plasticity

- Model of creep during cyclic loading
  - Stabilized cycle
    - During holding time
    - $\bar{\varepsilon}^p$ increases (so σ_p)
    - $\dot{\bar{\varepsilon}}^p$ decreases (so σ_v)
    - α increases
  - Model requires
    - (Isotropic) Kinematic hardening
    - Visco-plasticity
  - Lemaître & Chaboche
    - Use a dissipation potential



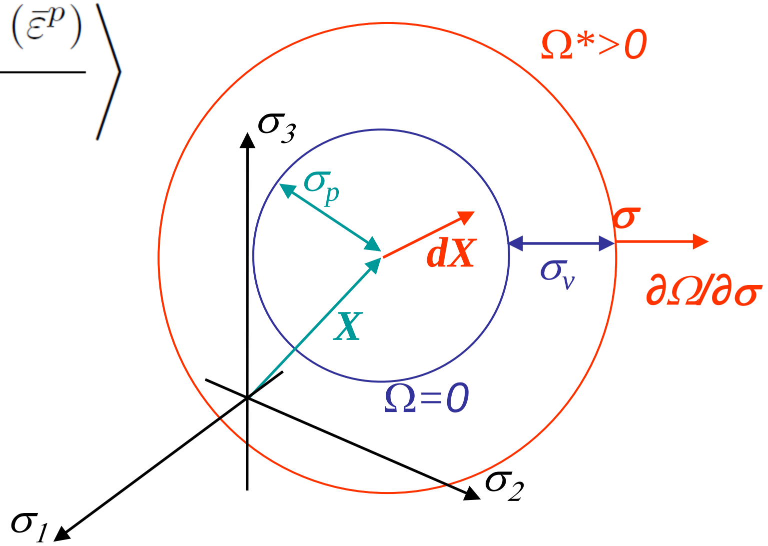
$$\underbrace{\Omega\left(\sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\left(\boldsymbol{\alpha}\right)\right):\left(\mathbf{s}-\mathbf{X}\left(\boldsymbol{\alpha}\right)\right)}-\sigma_p\left(\bar{\varepsilon}^p\right)\right)}$$

If <0, there should not be a plastic flow ⇒ If •>0: <•> = •, if not <•> =0

$$\Rightarrow \quad \Omega = \frac{K}{n+1}\left\langle\frac{\sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\right):\left(\mathbf{s}-\mathbf{X}\right)}-\sigma_p\left(\bar{\varepsilon}^p\right)}{K}\right\rangle^{n+1}$$

- Visco-plasticity model
  - Plastic flow direction & amplitude?
    - Dissipation potential

$$\Omega = \frac{K}{n+1}\left\langle \frac{\sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{X}):(\mathbf{s}-\mathbf{X})} - \sigma_p(\bar{\varepsilon}^p)}{K} \right\rangle^{n+1}$$



- Normality $\dot{\boldsymbol{\varepsilon}}^p = \frac{\partial \Omega}{\partial \boldsymbol{\sigma}}$
  - Let $J_2 = \sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{X}):(\mathbf{s}-\mathbf{X})}$

$$\Rightarrow \quad \dot{\boldsymbol{\varepsilon}}^p = \frac{3}{2}\frac{\partial \Omega}{\partial J_2}\frac{\mathbf{s}-\mathbf{X}}{J_2}$$

  - As $\dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p} = \frac{\partial \Omega}{\partial J_2}$

$$\Rightarrow \quad \dot{\boldsymbol{\varepsilon}}^p = \frac{3}{2}\dot{\bar{\varepsilon}}^p \frac{\mathbf{s}-\mathbf{X}}{\sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{X}):(\mathbf{s}-\mathbf{X})}}$$

- Visco-plasticity model
  - Plastic flow direction & amplitude
    - $$\Omega = \frac{K}{n+1}\left\langle \frac{\sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\right):\left(\mathbf{s}-\mathbf{X}\right)} - \sigma_p\left(\bar{\varepsilon}^p\right)}{K} \right\rangle^{n+1}$$
    - Hardening laws

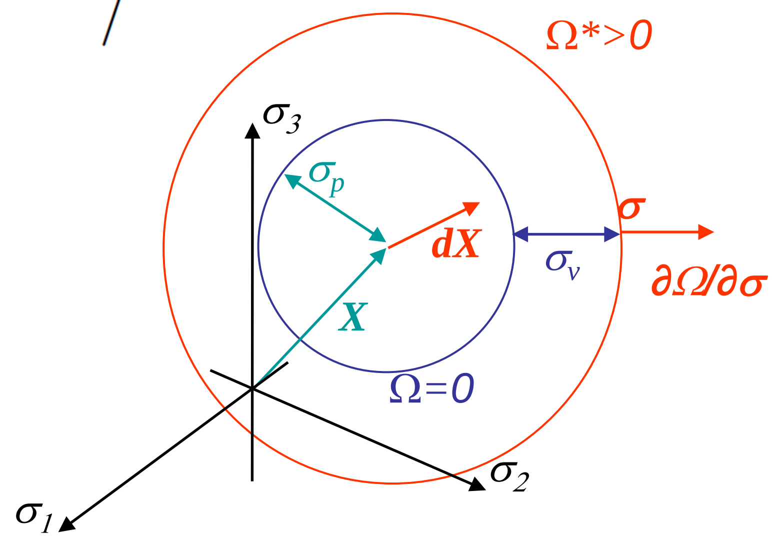
$$\begin{cases} \dot{\mathbf{X}} = \frac{2}{3}C\dot{\boldsymbol{\varepsilon}}^p - \gamma \mathbf{X} d\dot{\bar{\varepsilon}}^p \\ \sigma_p = \sigma_p^0 + Q\left(1 - e^{-b\bar{\varepsilon}^p}\right) \end{cases}$$

    - Parameters are temperature dependent
    - Normality $\dot{\boldsymbol{\varepsilon}}^p = \frac{\partial \Omega}{\partial \boldsymbol{\sigma}}$

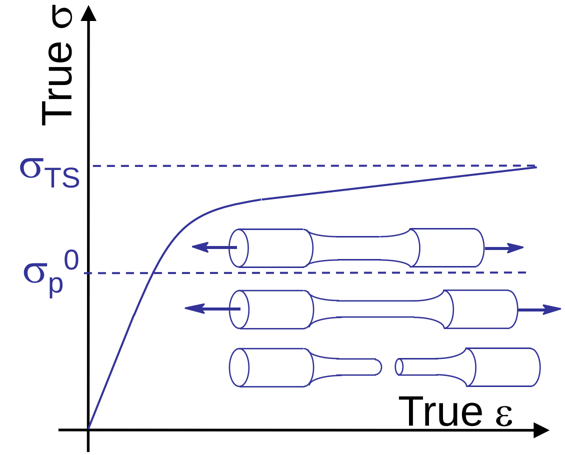
$$\Rightarrow \dot{\bar{\varepsilon}}^p = \left\langle \frac{\sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\right):\left(\mathbf{s}-\mathbf{X}\right)} - \sigma_p}{K} \right\rangle^{n}$$

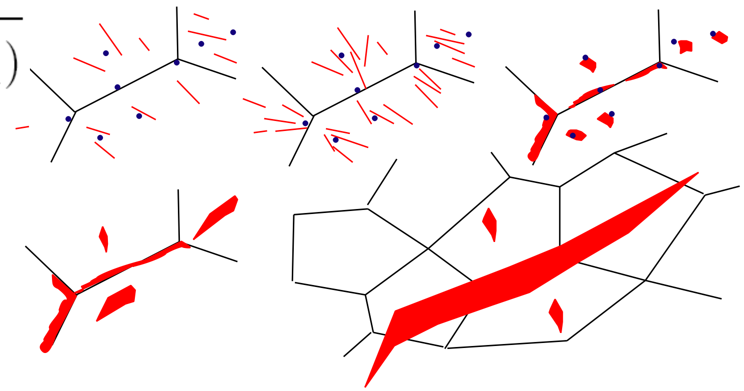
    - Relation viscous stress – strain rate

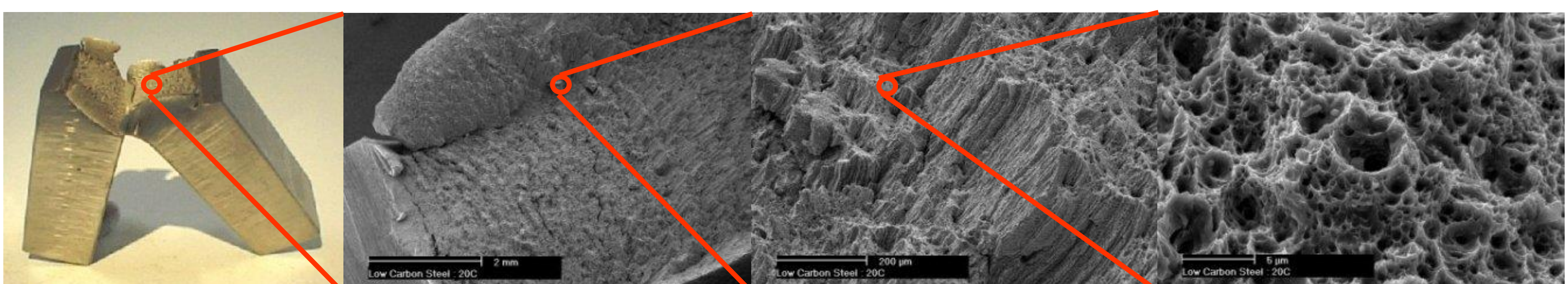
$$\Rightarrow J_2 = \sqrt{\frac{3}{2}\left(\mathbf{s}-\mathbf{X}\right):\left(\mathbf{s}-\mathbf{X}\right)} = \sigma_p^0 + Q\left(1 - e^{-b\bar{\varepsilon}^p}\right) + \underbrace{K\dot{\bar{\varepsilon}}^{p\,\frac{1}{n}}}_{\sigma_v}$$



- Failure mechanism of ductile material
  - Plastic deformations prior to (macroscopic) failure of the specimen
    - Dislocations motion ⟹ void nucleation around inclusions ⟹ micro cavity coalescence ⟹ crack growth
    - Griffith criterion $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$ should be replaced by $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\left(2\gamma_s + W_{\mathrm{pl}}\right)}$

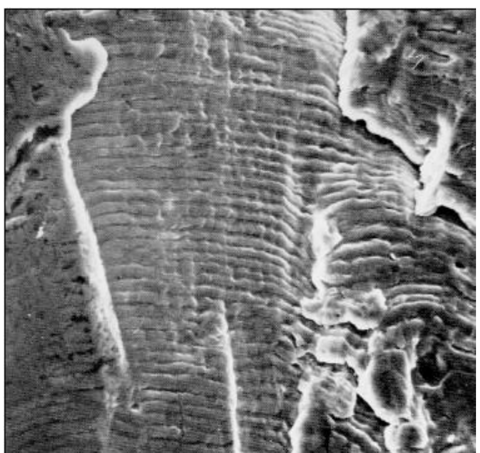


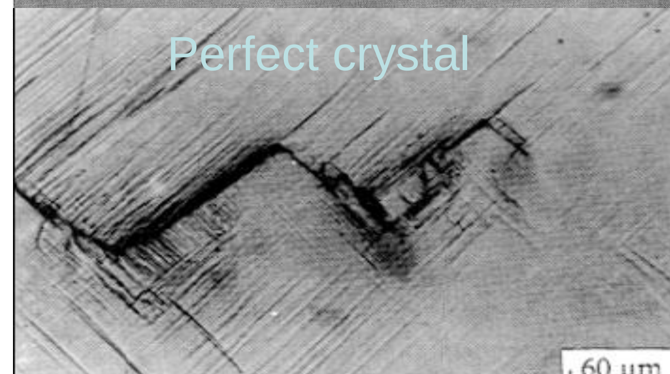


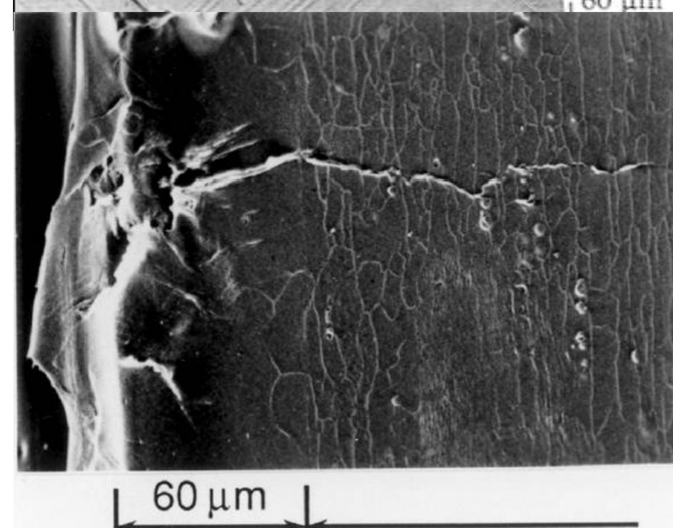


- Failure mechanism by fatigue
  - Crack nucleation at persistent slip bands
  - Stage I crack growth
    - Along slip planes
  - Stage II crack growth
    - Across several grains
      - Along a slip plane in each grain,
      - Straight ahead macroscopically
    - Striation of the failure surface: corresponds to the cycles

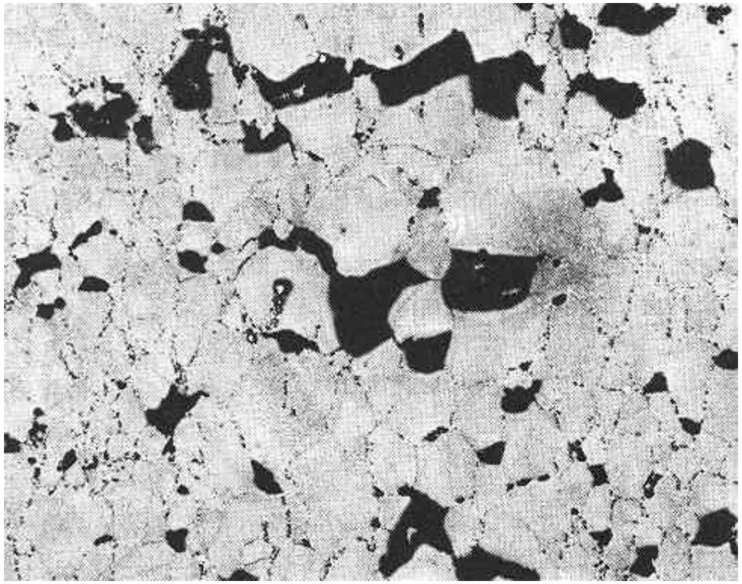
Fracture mechanics



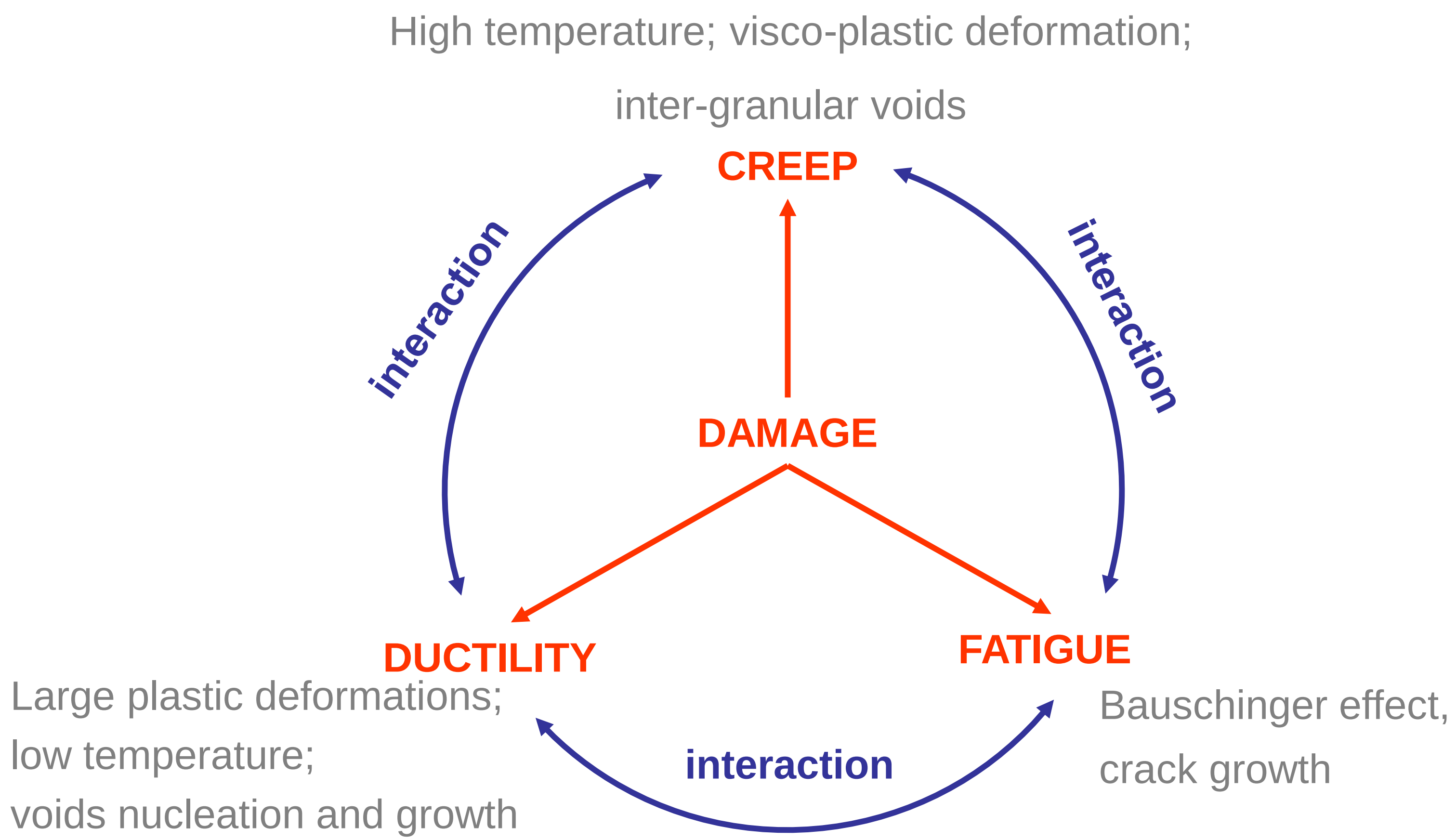




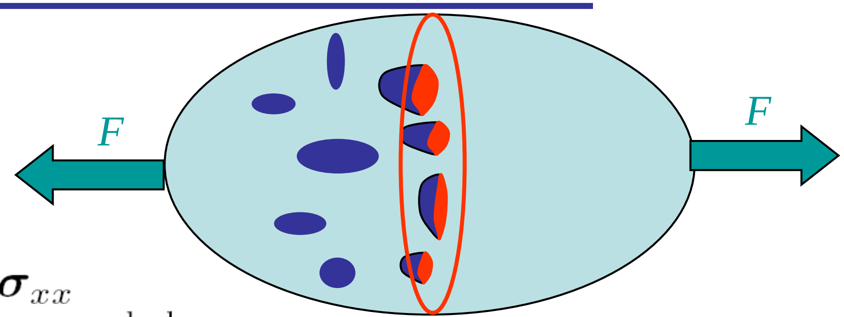
- Failure mechanism by creep
  - Inter-granular void formation



- Interaction between damage sources

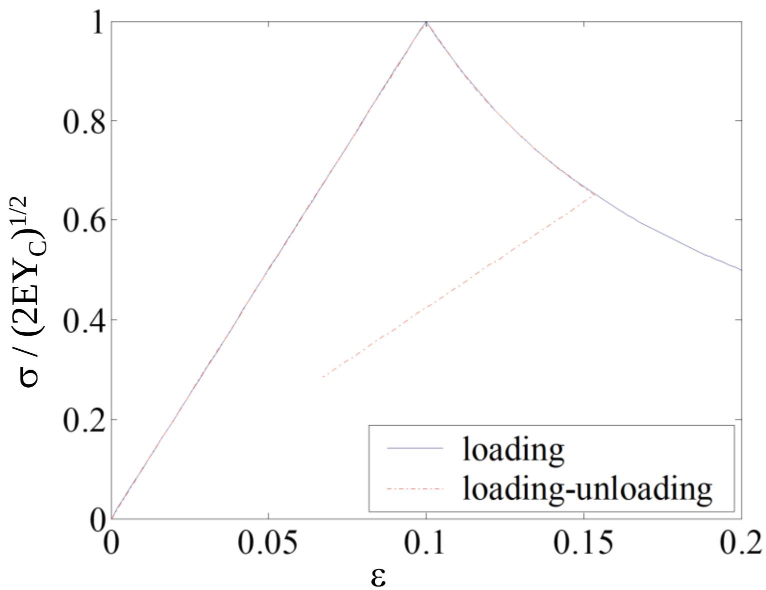


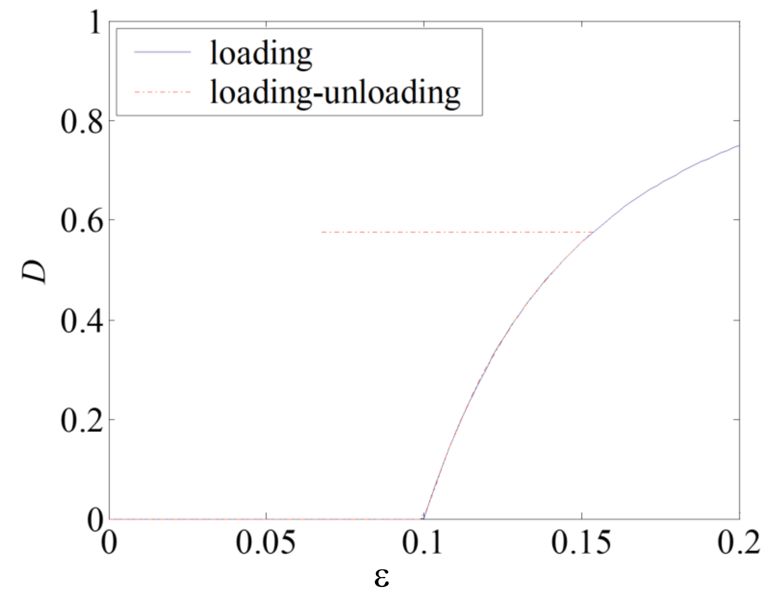
- Safe-life design: evaluation of a resulting damage



- Model (1D)
  - Change of elastic properties
    - Virgin section S $\Longrightarrow$ $\boldsymbol{\sigma}_{xx}^{\text{virgin}} = \frac{F}{S} = \boldsymbol{\sigma}_{xx}$
    - Damage of the surface is defined as $D = \frac{S^{\text{holes}}}{S}$
    - So the effective (or damaged) surface is actually $\hat{S} = S - S^{\text{holes}} = (1 - D)\, S$
    - And so the effective stress is $\hat{\boldsymbol{\sigma}}_{xx} = \frac{F}{S\,(1 - D)} = \frac{\boldsymbol{\sigma}_{xx}}{1 - D}$ Apparent macroscopic stress
  - Resulting deformation
    - Hooke's law is still valid if it uses the effective stress $\boldsymbol{\varepsilon}_{xx} = \frac{\hat{\boldsymbol{\sigma}}}{E} = \frac{\boldsymbol{\sigma}_{xx}}{E\,(1 - D)}$
    - So everything is happening as if Hooke's law was multiplied by (1-D)
  - Isotropic 3D linear elasticity $\boldsymbol{\sigma} = (1 - D)\,\mathcal{H} : \boldsymbol{\varepsilon}$
  - Failure criterion: $D=D_C$, with $0 < D_C < 1$
- But how to evaluate D in ductility, safe-life design etc ?

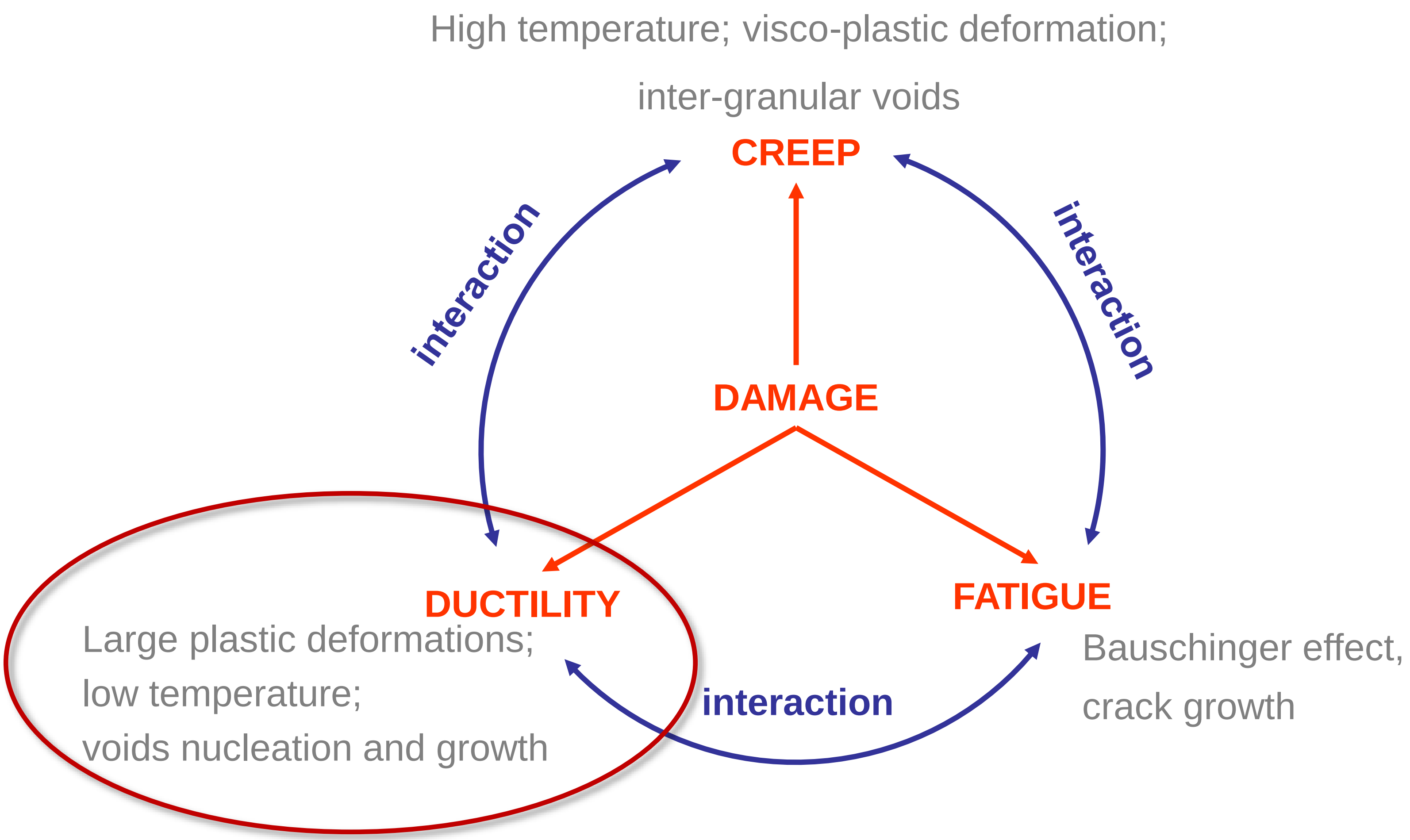
- Evolution of damage D for isotropic elasticity
  - Equations
    - Stresses $\boldsymbol{\sigma} = (1-D)\,\mathcal{H} : \boldsymbol{\varepsilon}$
    - Example of damage criterion $f\left(\boldsymbol{\varepsilon},\, D\right) = (1-D)\,\dfrac{\boldsymbol{\varepsilon} : \mathcal{H} : \boldsymbol{\varepsilon}}{2} - Y_C \leq 0$
      - Y_C is an energy related to a deformation threshold
    - There is a time history $f\dot{D} = 0$
      - Either damage is increased if $f = 0$
      - Or damage remains the same if $f < 0$
  - Example for Y_C such that damage appears for $\varepsilon = 0.1$





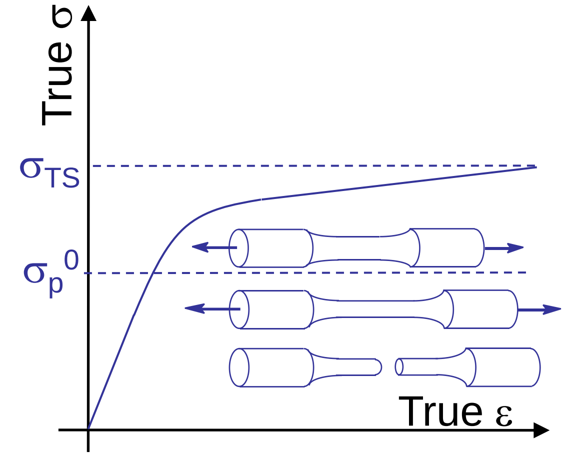
- But for ductile materials plasticity is important as it induces the damage

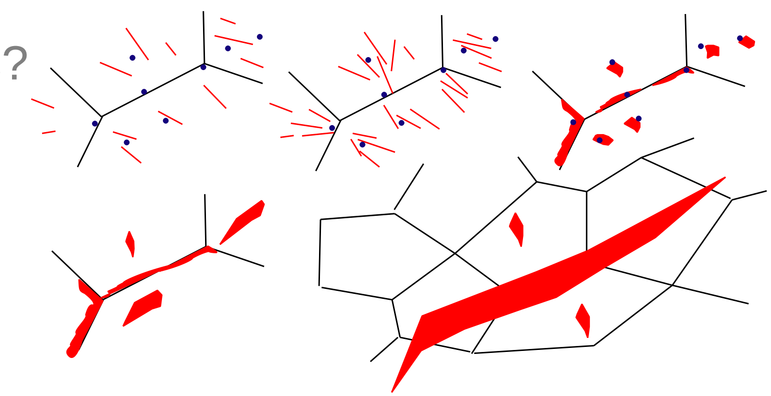
- Interaction between damage sources

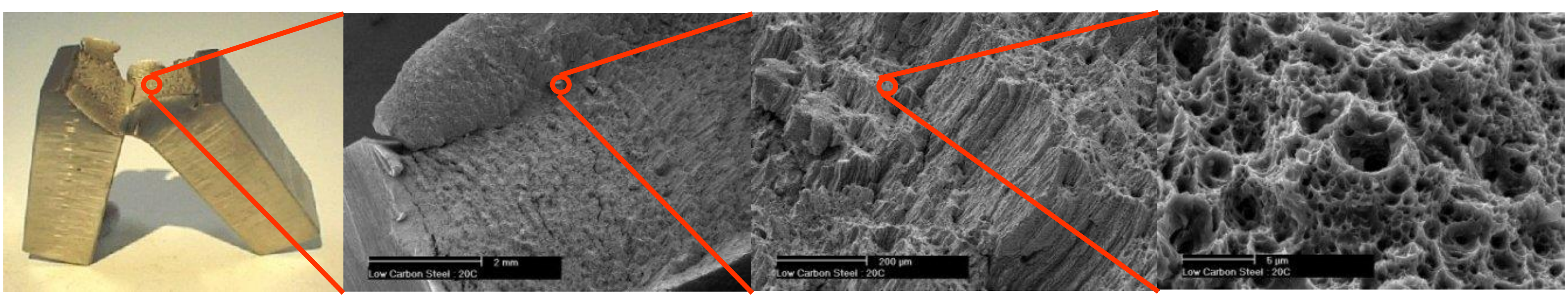


Ductile failure

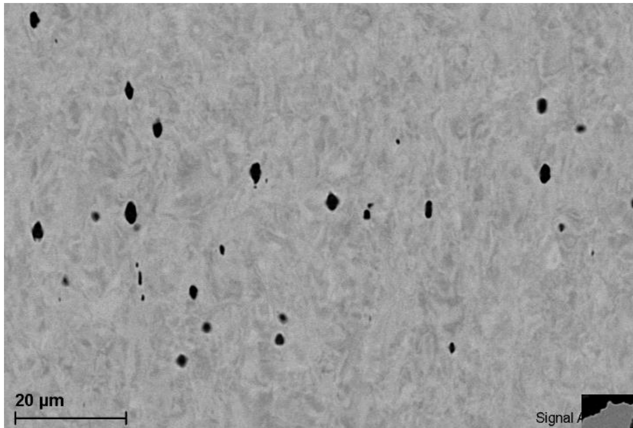
- Failure mechanism
  - Plastic deformations prior to (macroscopic) failure of the specimen
    - Dislocations motion ⟹ void nucleation around inclusions ⟹ micro cavity coalescence ⟹ crack growth
  - How to account for these complex effects?



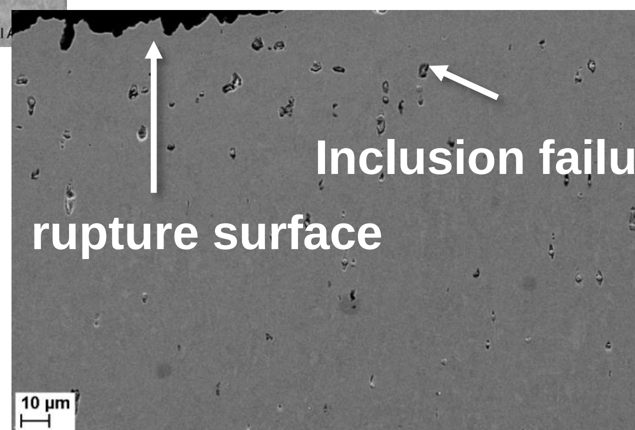




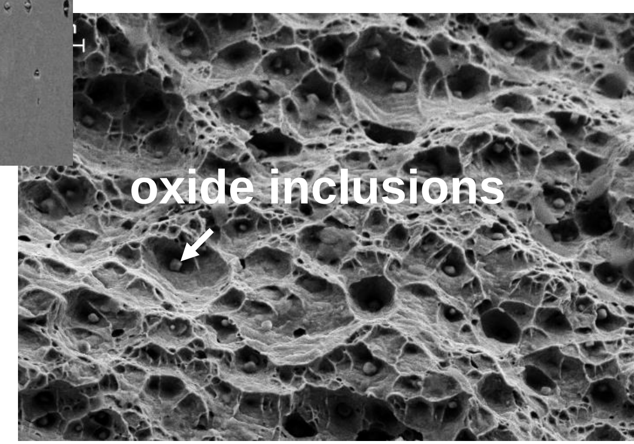
- Ductile failure mechanism



Porosity free
Initial inclusions

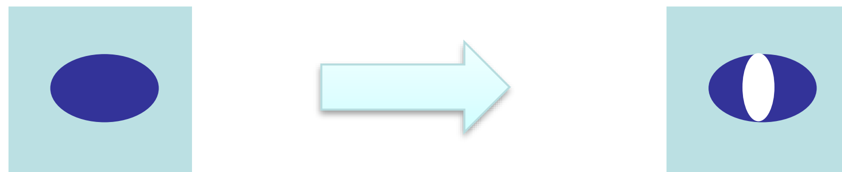


Nucleation from
inclusion failure

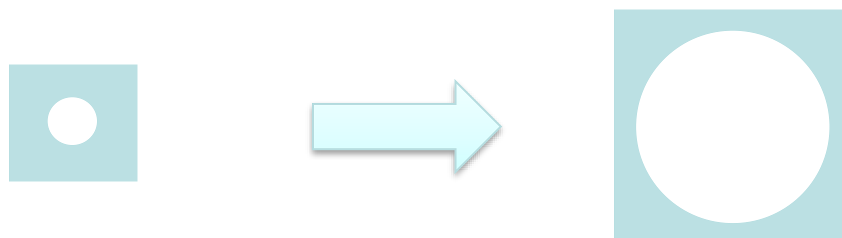


Tests performed at UCL by Antoine Hilhorst

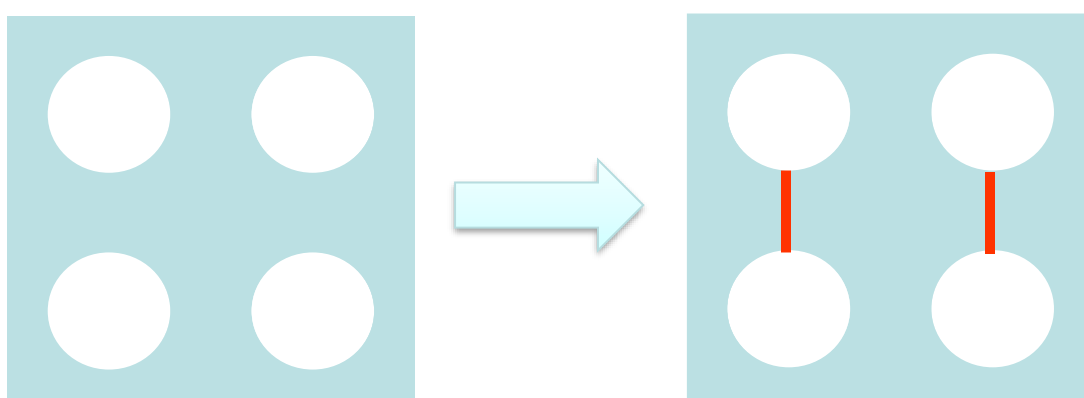
- Ductile failure mechanism (2)
  - Void nucleation (dislocations, particle/matrix decohesion, particle cracking…)



  - Void growth of existing voids (because of plastic incompressibility)



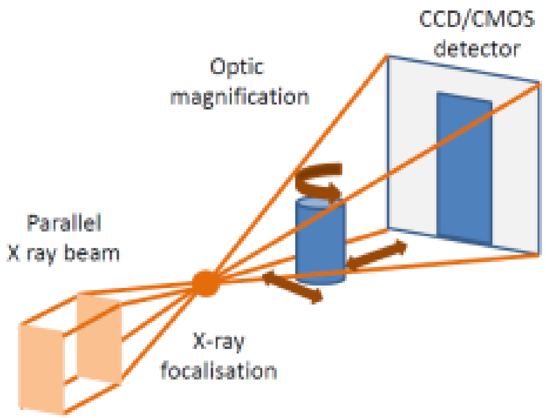
  - Void coalescence (crack growth by shrinking of ligaments between voids)



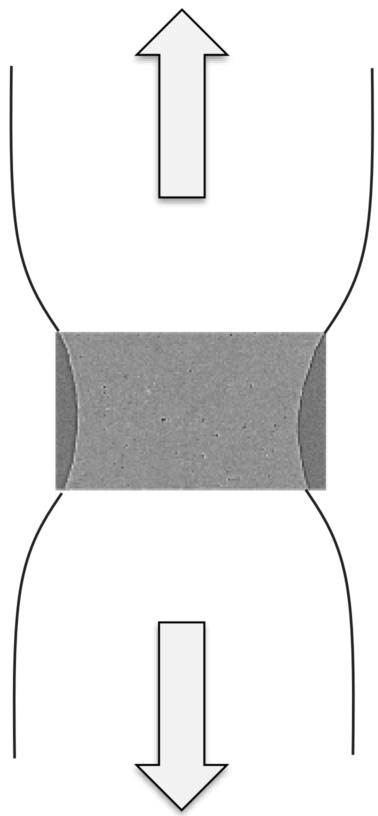
Ductile failure

- Ductile failure: complex coalescence scenarios
  - What does happen inside a « ductile » material under large strain ?

X-ray tomography of in-situ tensile tests
= scanner for materials



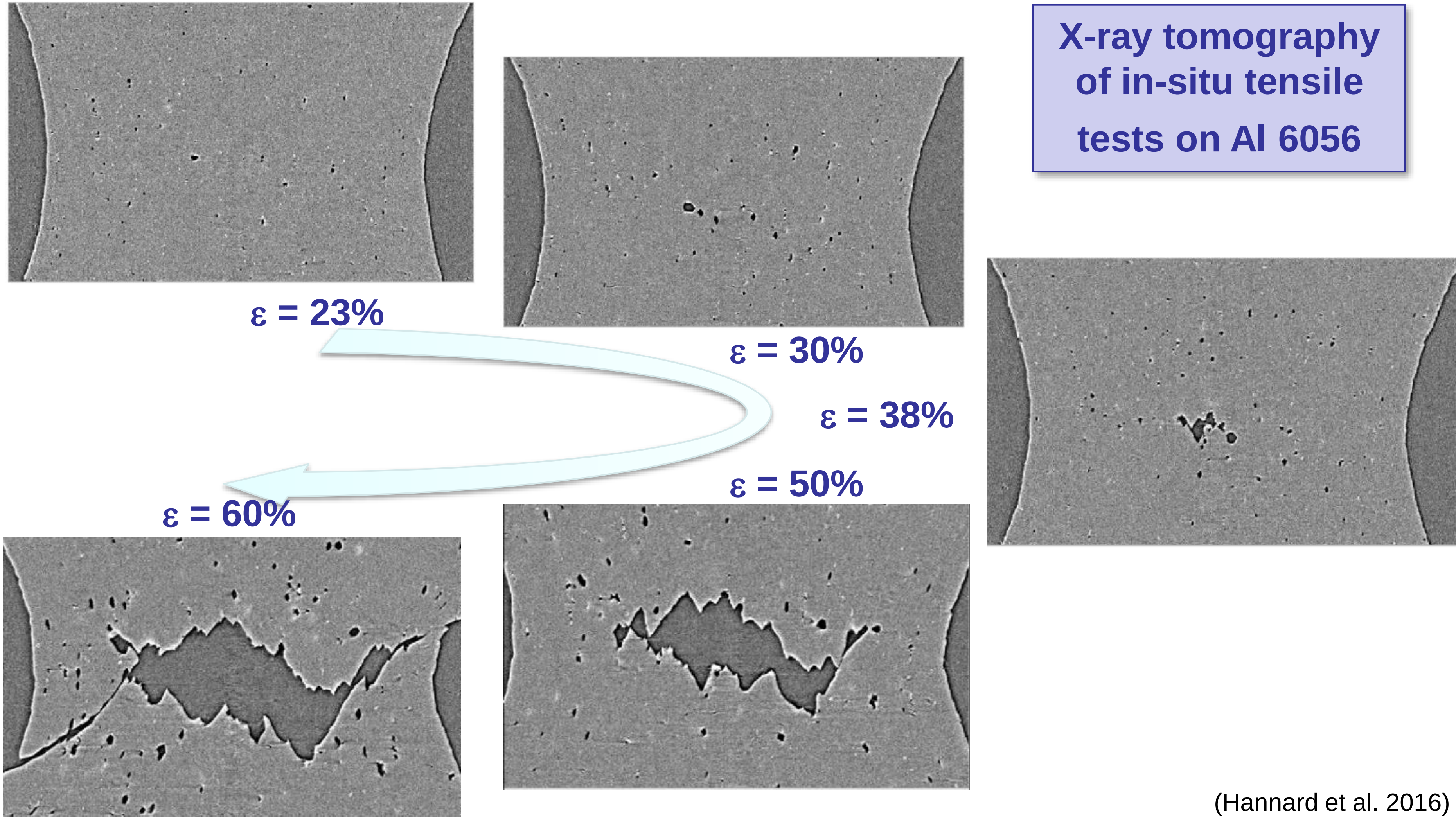
Tests performed at the ESRF synchrotron
in Grenoble by F. Hannard (Ph. D. UCL)
collaboration with Dr. E. maire INSA Lyon



(Hannard et al. 2016)

- Ductile failure: complex coalescence scenarios (2)
  - What does happen inside a « ductile » material under large strain ?

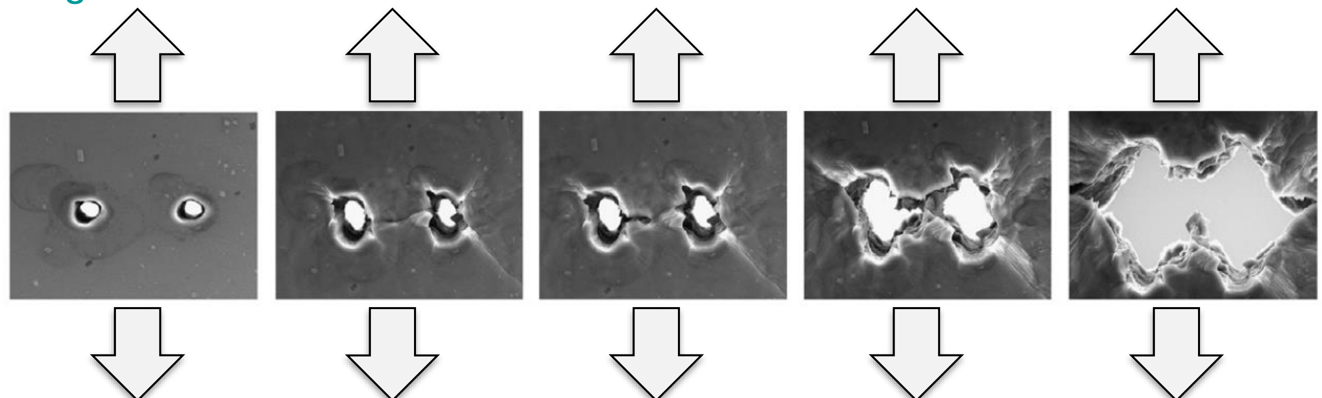
X-ray tomography of in-situ tensile tests on Al 6056



(Hannard et al. 2016)

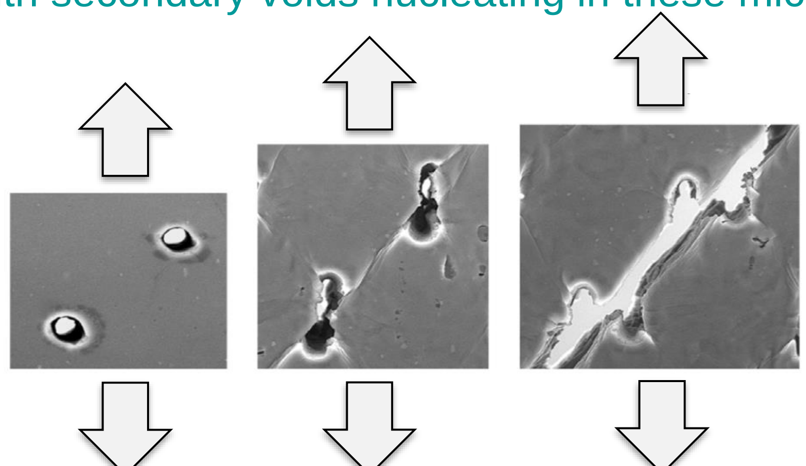
- Ductile failure: complex coalescence scenarios (3)
  - Localization band perpendicular to the main loading direction
    - Shrinking of ligaments between voids

Internal necking coalescence



  - Micro shear bands inclined to the main loading direction
    - Joining primary voids
    - Possibly with secondary voids nucleating in these micro bands

Shear driven coalescence



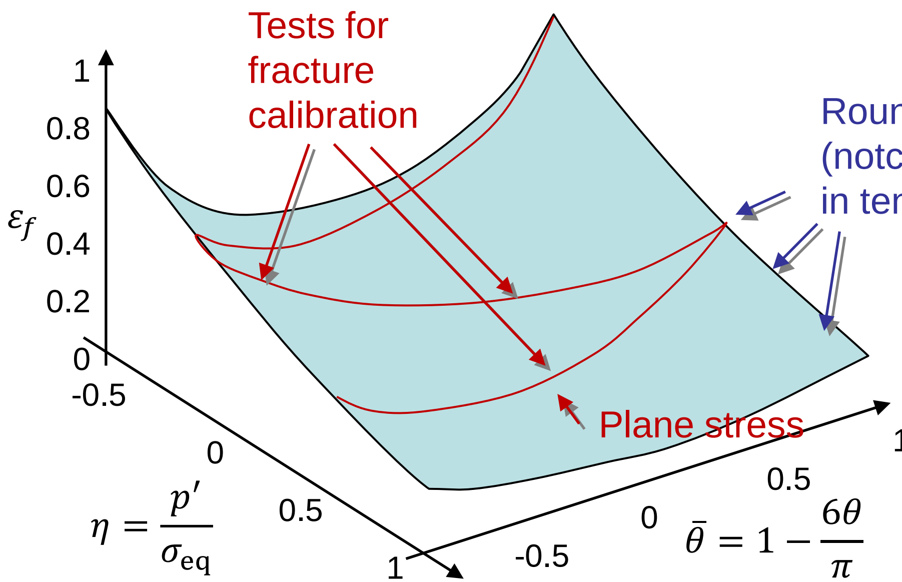
(Weck & Wilkinson 2008)

- Ductile failure: stress-state dependent fracture strain
  - Stress triaxiality dependent

$$\eta = \frac{p'}{\sigma_{\mathrm{eq}}} \in [-\infty \ \infty] \qquad p' = \frac{\mathrm{tr}\,(\boldsymbol{\sigma})}{3} \qquad \sigma_{\mathrm{eq}} = \sqrt{\frac{3}{2}\mathrm{dev}\,(\boldsymbol{\sigma}) : \mathrm{dev}\,(\boldsymbol{\sigma})}$$

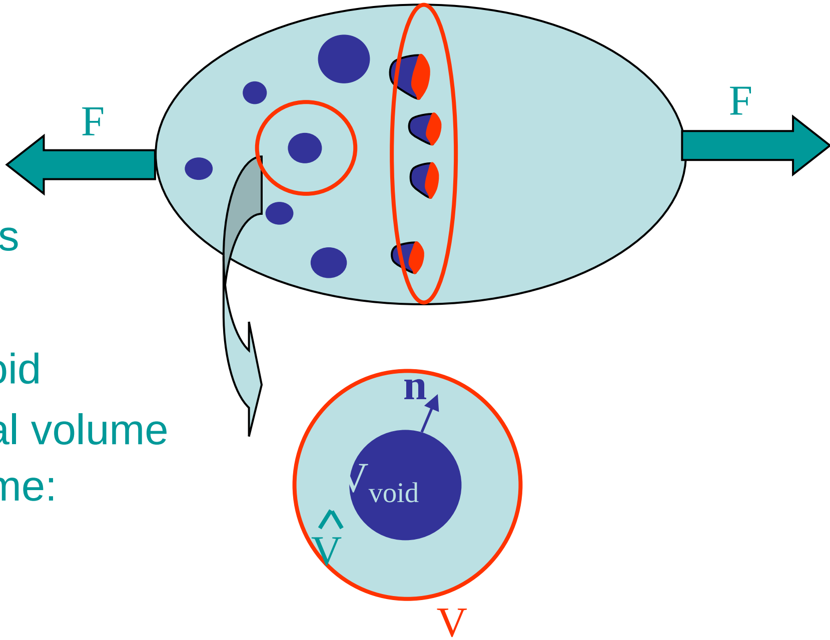
  - Lode dependent

$$\theta = \frac{1}{3}\arccos\left(\frac{27J_3}{2\sigma_{\mathrm{eq}}^3}\right) \qquad J_3 = \det\left(\mathrm{dev}\,(\boldsymbol{\sigma})\right)$$



Fracture locus for 2024-T351 aluminum alloy

(Bai & Wierzbicki 2010)



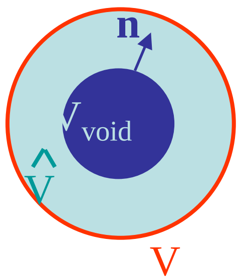
- Gurson's model, 1977
  - Assumptions
    - Given a rigid-perfectly-plastic material with already existing spherical microvoids
    - Extract a statistically representative sphere V embedding a spherical microvoid
      - Porosity: fraction of voids in the total volume and thus in the representative volume:

$$f_V = \frac{V_{\text{void}}}{V} = 1 - \frac{\hat{V}}{V}$$

with $\hat{V}$ the material part of the volume

      - Material rigid-perfectly plastic $\Longrightarrow$ elastic deformations negligible
  - Define
    - Macroscopic strains and stresses: ε & σ
    - Microscopic strains and stresses: $\hat{\varepsilon}$ & $\hat{\sigma}$
    - Link the 2 scales

- Gurson's model, 1977 (2)
  - Macroscopic strains



$$\dot{\varepsilon} = \frac{1}{V}\int_V \dot{\hat{\varepsilon}} dV = \frac{1}{V}\int_{\hat{V}} \dot{\hat{\varepsilon}} dV + \frac{1}{V}\int_{V_{\text{void}}} \dot{\hat{\varepsilon}} dV$$

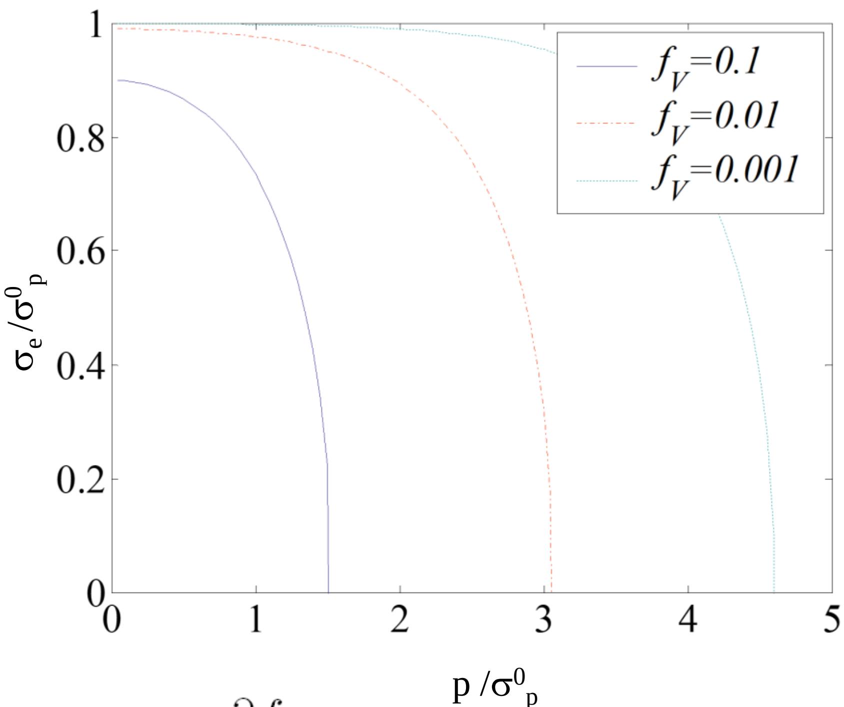
  - Stresses
    - In $\hat{V}$ microscopic stresses $\hat{\sigma}$ are related to the microscopic deformations $\hat{\varepsilon}$
      - In terms of an energy rate $\hat{\boldsymbol{\sigma}} = \frac{\partial \dot{\hat{W}}}{\partial \dot{\hat{\boldsymbol{\varepsilon}}}}$
    - As the energy rate has to be conserved $\dot{W}\left(\dot{\boldsymbol{\varepsilon}}\right) = \frac{1}{V}\int_{\hat{V}} \dot{\hat{W}}\left(\dot{\hat{\boldsymbol{\varepsilon}}}\right) dV$

$$\Longrightarrow \boldsymbol{\sigma} = \frac{\partial \dot{W}}{\partial \dot{\boldsymbol{\varepsilon}}} = \frac{1}{V}\int_{\hat{V}} \frac{\partial \dot{\hat{W}}}{\partial \dot{\hat{\boldsymbol{\varepsilon}}}} : \frac{\partial \dot{\hat{\boldsymbol{\varepsilon}}}}{\partial \dot{\boldsymbol{\varepsilon}}} dV = \frac{1}{V}\int_{\hat{V}} \hat{\boldsymbol{\sigma}} : \frac{\partial \dot{\hat{\boldsymbol{\varepsilon}}}}{\partial \dot{\boldsymbol{\varepsilon}}} dV$$

  - Gurson solved these equations
    - For a rigid-perfectly-plastic microscopic behavior
    - Which leads to a new macroscopic yield function depending on the porosity

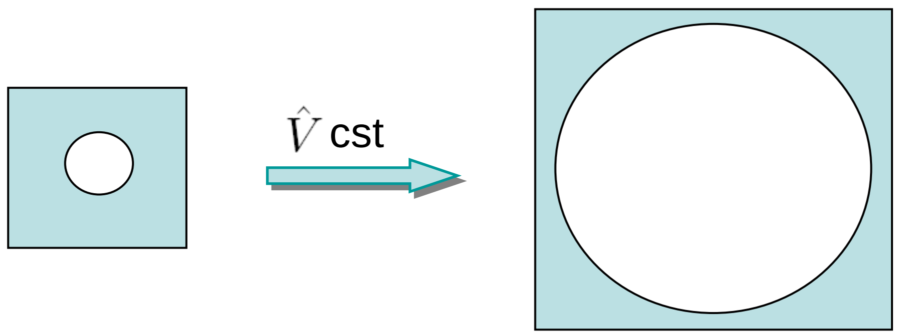
$$f\left(\boldsymbol{\sigma}\right) = \left(\frac{\sigma_e}{\sigma_p^0}\right)^2 + 2 f_V \cosh \frac{\operatorname{tr}\left(\boldsymbol{\sigma}\right)}{2\sigma_p^0} - f_V^2 - 1 \leq 0$$

- Gurson's model, 1977 (3)
  – Shape of the new yield surface $f\left(\boldsymbol{\sigma}\right) = \left(\frac{\sigma_e}{\sigma_p^0}\right)^2 + 2 f_V \cosh \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{2\sigma_p^0} - f_V^2 - 1 \leq 0$



  – Normal flow $\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}$
  – What remains to be defined is the evolution of the porosity f_V

- Gurson's model, 1977 (4)
  - Evolution of the porosity f_V
    - Volume of material $\hat{V} = (1 - f_V)\, V$ is constant as
      - Elastic deformations are neglected
      - Plastic deformations are isochoric



    - Void porosity and macroscopic volume variation are linked

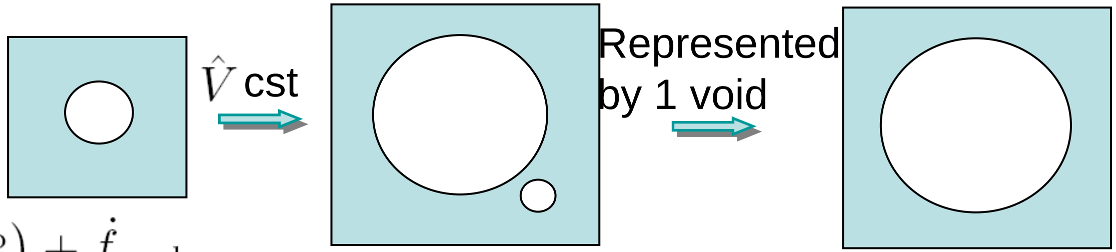
$$0 = (1 - f_V)\, dV - df_V\, V \Longrightarrow \dot{f}_V = (1 - f_V)\, \frac{\dot{V}}{V}$$

    - But volume variation can be expressed from the deformation tensor

$$\frac{dV}{V} = \mathrm{tr}\,(d\boldsymbol{\varepsilon}) = \mathrm{tr}\,(d\boldsymbol{\varepsilon}^{\mathrm{p}})$$ as elastic deformations are neglected

- Gurson's model, 1977 (5)
  - Eventually
    - The porosity is actually not an independent internal variable
    - Yield surface $f\left(\boldsymbol{\sigma}\right) = \left(\frac{\sigma_e}{\sigma_p^0}\right)^2 + 2 f_V \cosh \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{2\sigma_p^0} - f_V^2 - 1 \leq 0$
    - Normal flow $\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\lambda} \frac{\partial f}{\partial \boldsymbol{\sigma}}$ with $\dot{f}_V = \left(1 - f_V\right) \mathrm{tr}\left(\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}\right)$
  - Assumptions were
    - Rigid perfectly-plastic material
    - Initial porosity (no void nucleation)
    - No voids interaction
    - No voids coalescence
  - More evolved models have been developed to account for
    - Hardening
    - Voids nucleation, interactions and coalescences

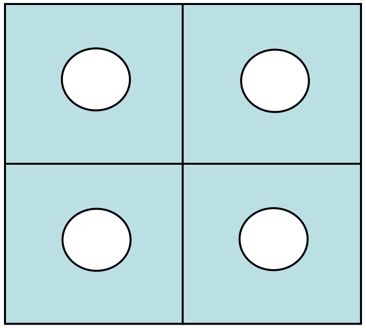
- Hardening
  - Yield criterion $f(\boldsymbol{\sigma}) = \left(\frac{\sigma_e}{\sigma_p^0}\right)^2 + 2 f_V \cosh \frac{\mathrm{tr}(\boldsymbol{\sigma})}{2\sigma_p^0} - f_V^2 - 1 \leq 0$ remains valid but one has to account for the hardening of the matrix $\Longrightarrow \sigma_p^0 \rightarrow \sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)$
    - In this expression, the equivalent plastic strain of the matrix $\hat{\bar{\varepsilon}}^{\mathrm{p}}$ is used instead of the macroscopic one $\bar{\varepsilon}^{\mathrm{p}}$
  - Values related to the matrix and the macroscopic volume are dependant as the dissipated energies have to match $\Longrightarrow (1 - f_V)\,\sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)\dot{\hat{\bar{\varepsilon}}}^{\mathrm{p}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}$
- Voids nucleation
  - Increase rate of porosity results from
    - Matrix incompressibility
    - Creation of new voids



$\Longrightarrow \dot{f}_V = (1 - f_V)\,\mathrm{tr}\left(\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}\right) + \dot{f}_{\mathrm{nucl}}$

- The nucleation rate can be modeled as strain controlled $\Longrightarrow \dot{f}_{\mathrm{nucl}} = A\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)\dot{\hat{\bar{\varepsilon}}}^{\mathrm{p}}$

- Voids interaction
  - 1981, Tvergaard
    - In Gurson model a void is considered isolated
    - The presence of neighboring voids decreases the maximal loading as the stress distribution changes

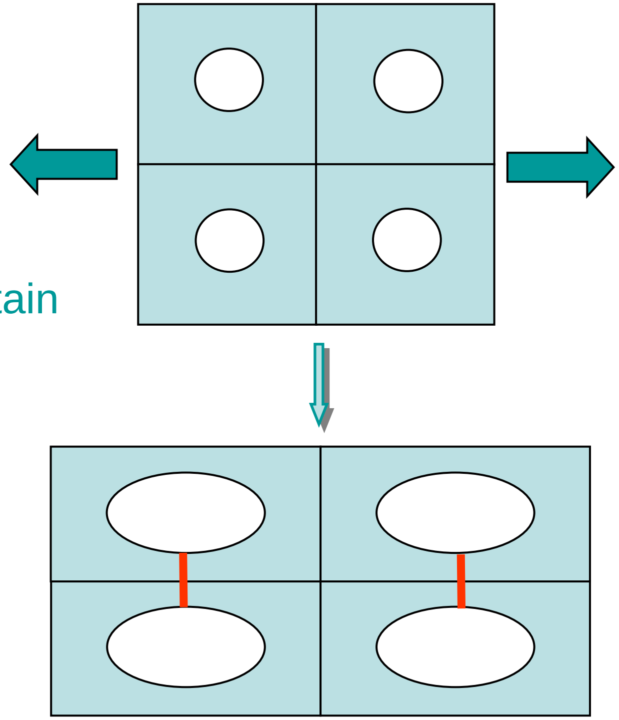


$$f(\boldsymbol{\sigma}) = \left(\frac{\sigma_e}{\sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)}\right)^2 + 2 f_V \cosh \frac{\mathrm{tr}(\boldsymbol{\sigma})}{2\sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)} - f_V^2 - 1 \leq 0$$

$$f(\boldsymbol{\sigma}) = \left(\frac{\sigma_e}{\sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)}\right)^2 + 2 q f_V \cosh \frac{\mathrm{tr}(\boldsymbol{\sigma})}{2\sigma_p\left(\hat{\bar{\varepsilon}}^{\mathrm{p}}\right)} - q^2 f_V^2 - 1 \leq 0$$

- With $1 < q < 2$ depending on the hardening exponent

- Voids coalescence
  - 1984, Tvergaard & Needleman
    - When two voids are close ($f_V \sim f_C$), the material loses capacity of sustaining the loading
    - If f_V is still increased, the material is unable to sustain any loading

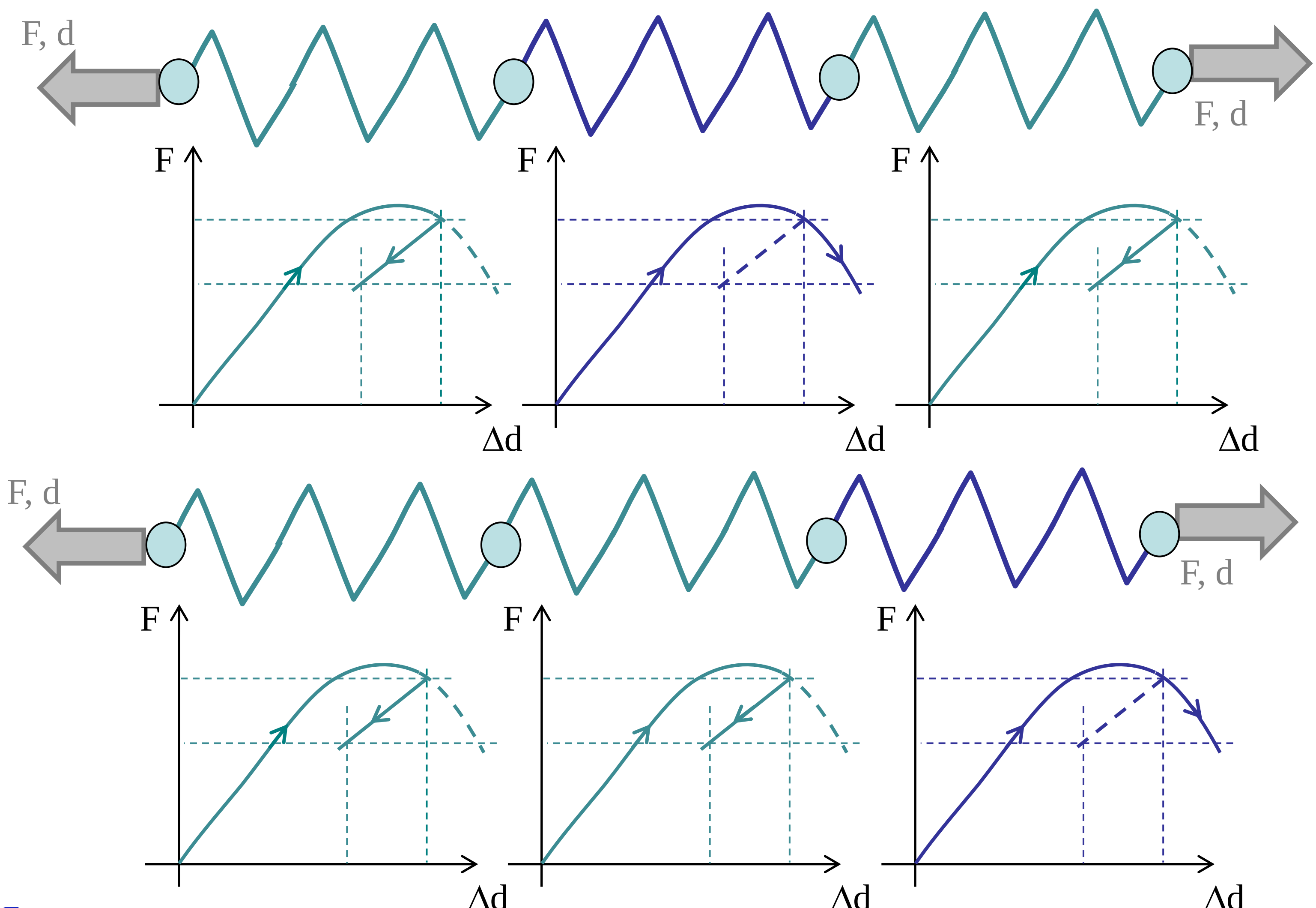


$$f(\boldsymbol{\sigma}) = \left(\frac{\sigma_e}{\sigma_p\left(\hat{\varepsilon}^{\mathrm{p}}\right)}\right)^2 + 2 f_V \cosh \frac{\operatorname{tr}(\boldsymbol{\sigma})}{2\sigma_p\left(\hat{\varepsilon}^{\mathrm{p}}\right)} - f_V^2 - 1 \leq 0$$

$$f(\boldsymbol{\sigma}) = \left(\frac{\sigma_e}{\sigma_p\left(\hat{\varepsilon}^{\mathrm{p}}\right)}\right)^2 + 2 q f_V^* \cosh \frac{\operatorname{tr}(\boldsymbol{\sigma})}{2\sigma_p\left(\hat{\varepsilon}^{\mathrm{p}}\right)} - q^2 {f_V^*}^2 - 1 \leq 0$$

- with $f_V^* = \begin{cases} f_V & \text{if } f_V < f_C \\ f_C + \frac{\frac{1}{q} - f_C}{f_F - f_C}\left(f_V - f_C\right) & \text{if } f_V > f_C \end{cases}$

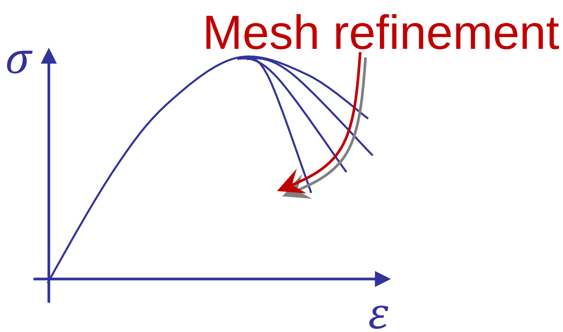
- Softening response
  – Loss of solution uniqueness → mesh dependency

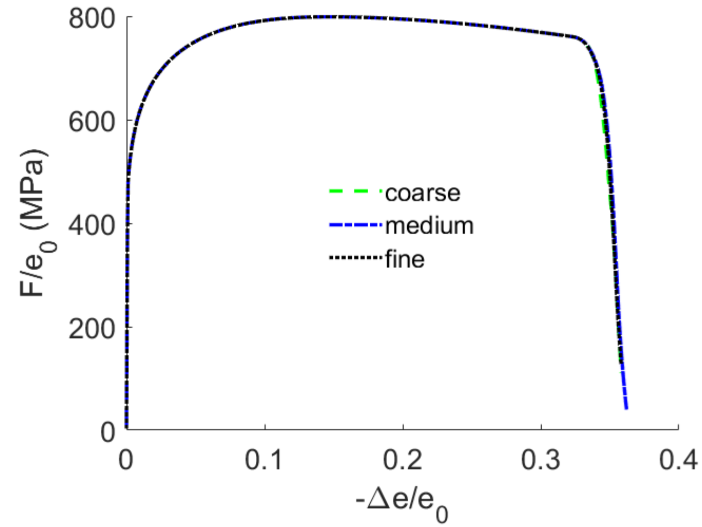


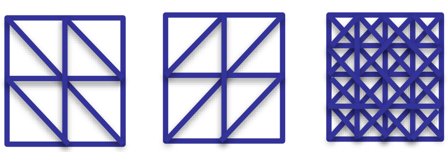
- Softening response (2)

Local Gurson

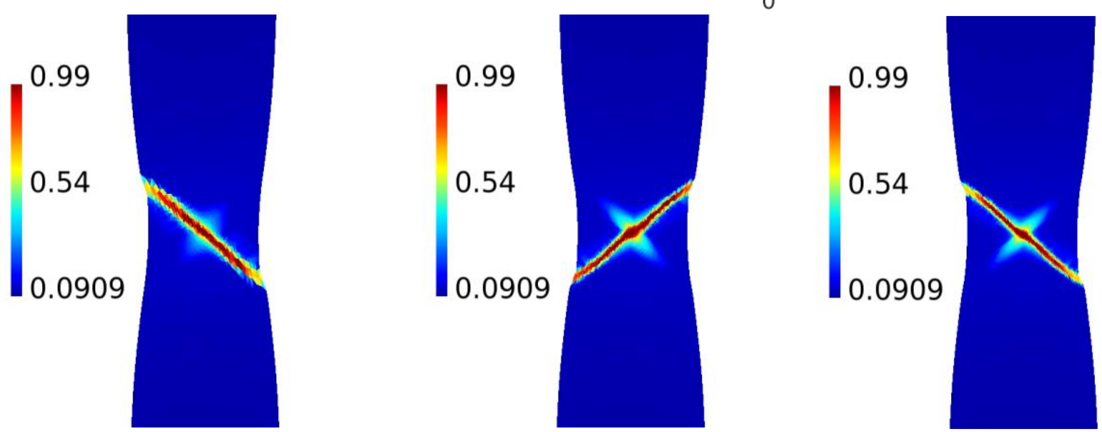
Non-local Gurson





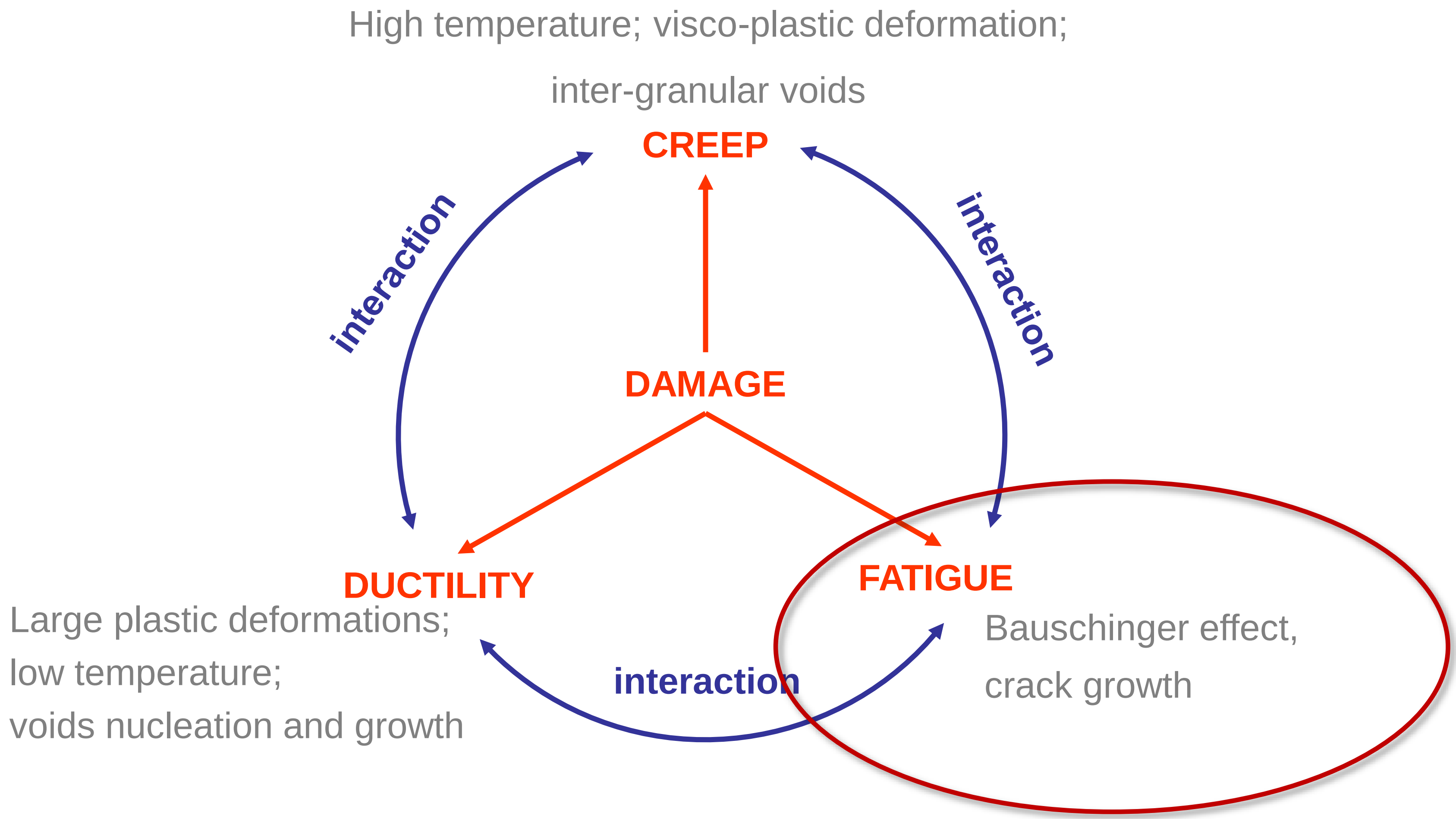


The numerical results change without convergence

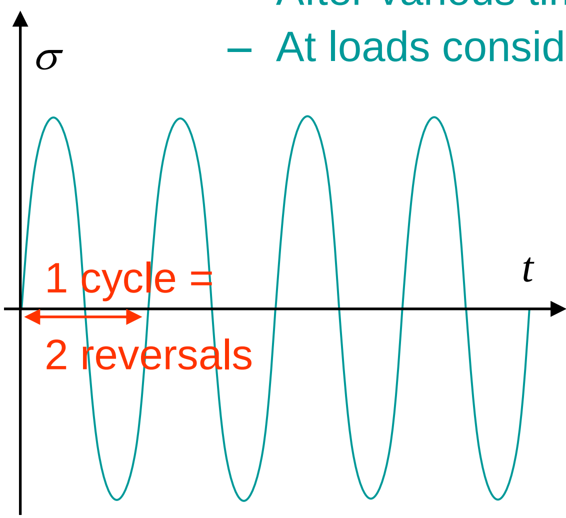


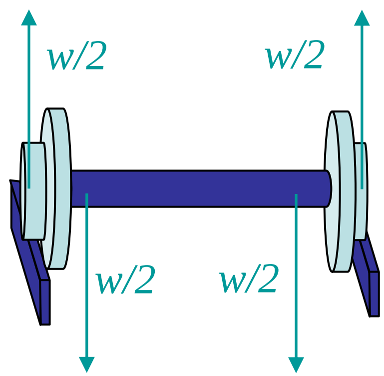
Coarse Medium Fine

- Interaction between damage sources



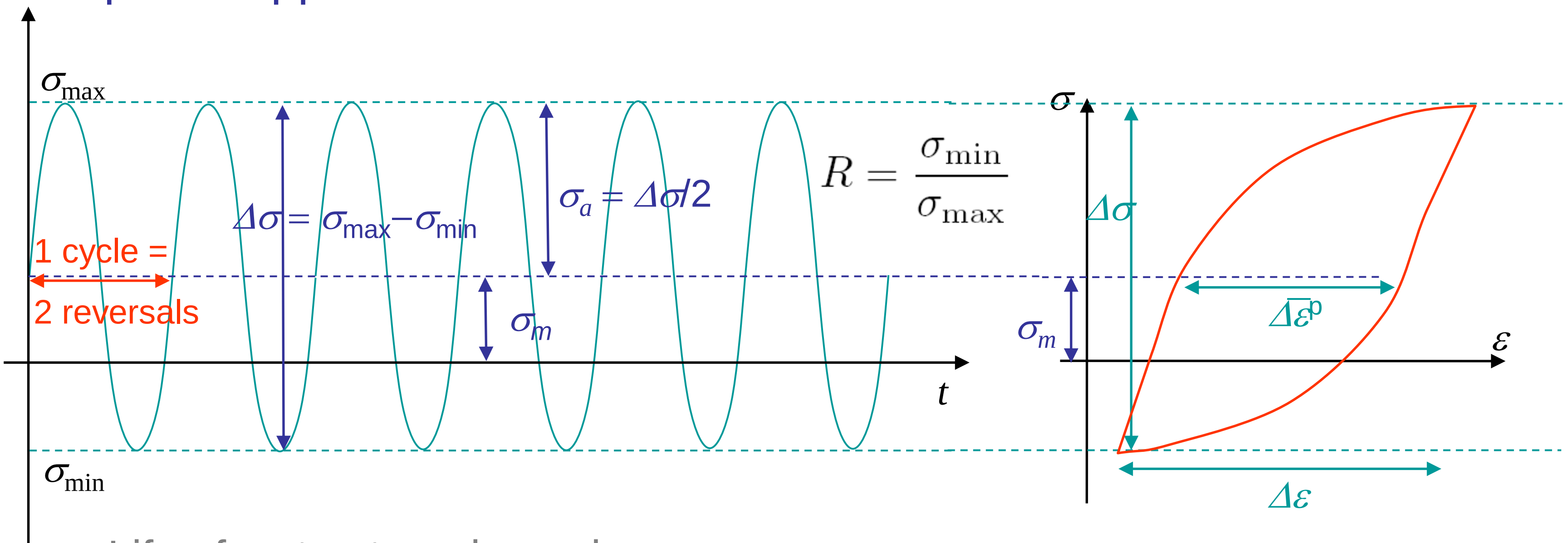
- Design with stresses lower than
  - Elastic limit (σ_p^0) or
  - Tensile strength (σ_{TS})
- ~1860, Wöhler
  - Technologist in the German railroad system
  - Studied the failure of railcar axles
    - Failure occurred
      - After various times in service
      - At loads considerably lower than expected





    - Failure due to cyclic loading/unloading
  - « Total life » approach
    - **Empirical** approach of fatigue

- Empirical approach

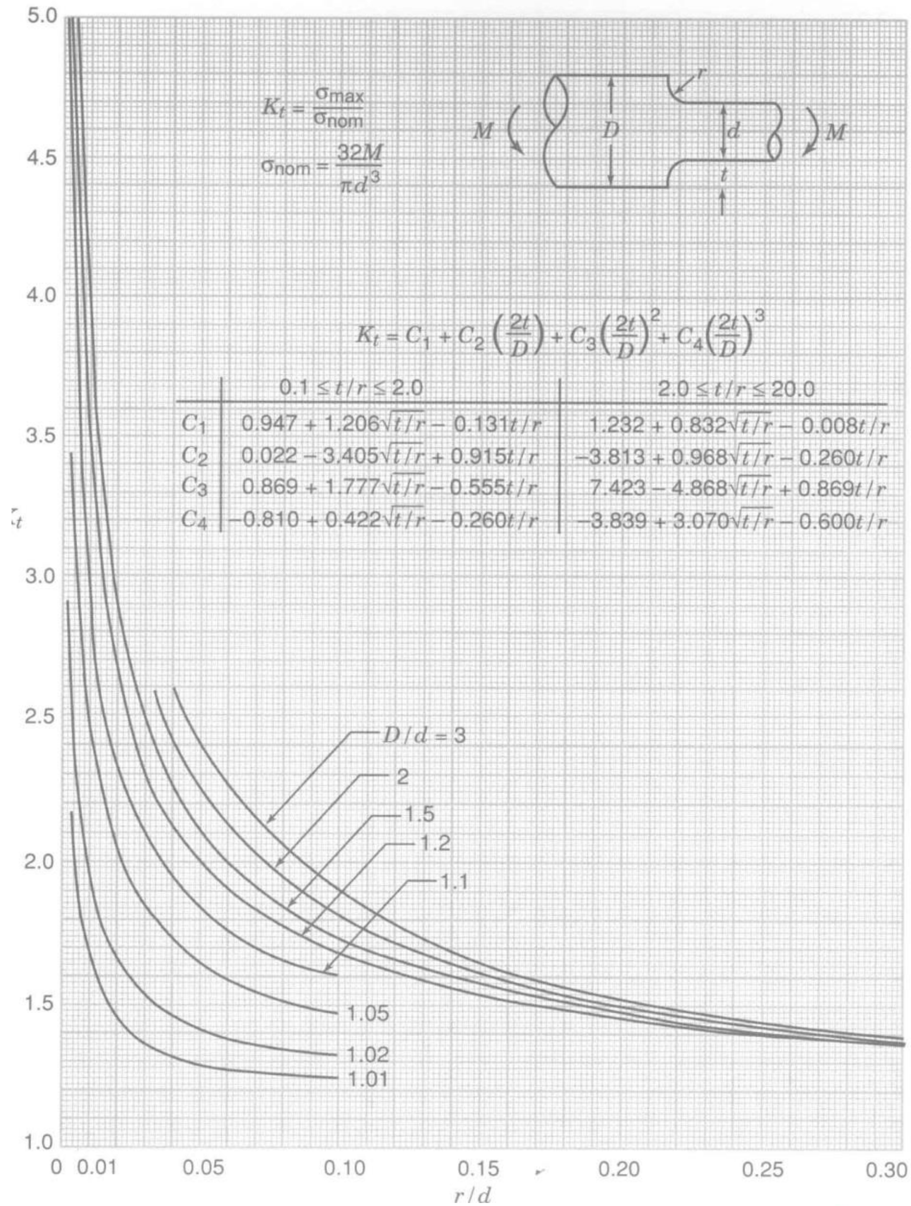


– Life of a structure depends on
  - Minimal & maximal stresses: σ_{min} & σ_{max} & mean stress: $\sigma_m=(\sigma_{max} + \sigma_{min})/2$
  - Amplitude: $\sigma_a = \Delta\sigma/2 = (\sigma_{max} - \sigma_{min})/2$
  - Plastic increment $\Delta\varepsilon^p$
  - Load Ratio: $R = \sigma_{min} / \sigma_{max}$
    – See lecture on crack propagation
  - Under particular environmental conditions (humidity, high temperature, …):
    – Frequency of cycle
    – Shape of cycle (sine, step, …)

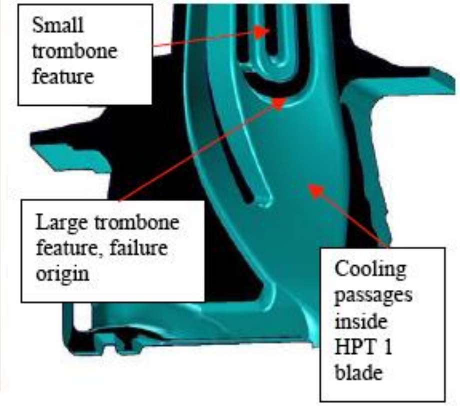
creep

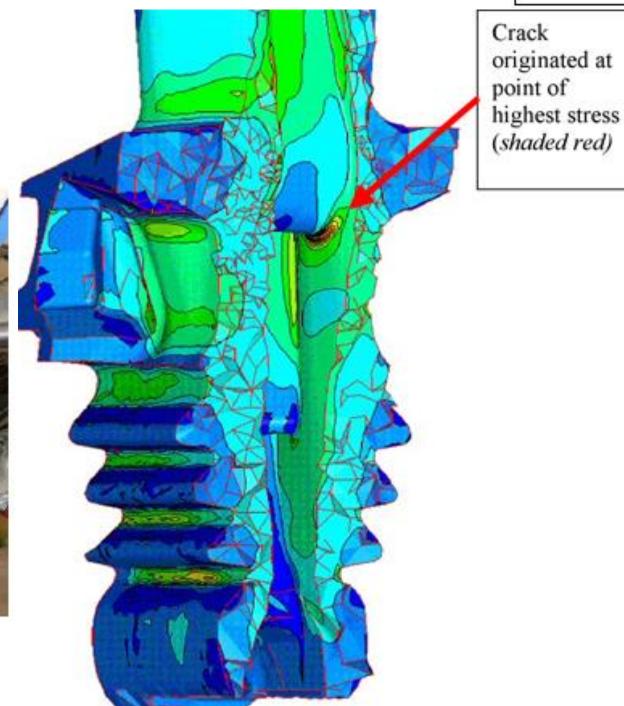
- Safe-life design
  - No crack before a determined number of cycles: life of the structure
    - At the end of the expected life the component is changed even if no failure has occurred
    - Emphasis on prevention of crack initiation
    - Approach theoretical in nature
      - Assumes initially crack free structures
  - Components of rotating structures vibrating with the flow cycles (blades)
    - Once cracks form, the remaining life is very short due to the high frequency of loading

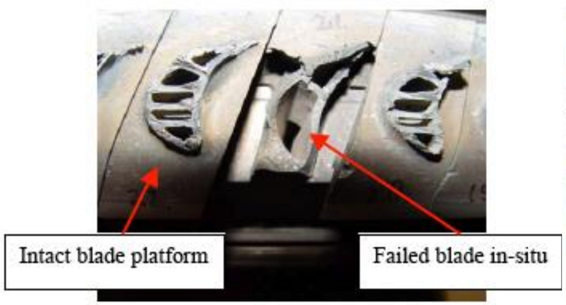
- How to estimate the time-life?
  - Based on empirical approaches
    - Macroscopic material responses (fatigue tests …)
    - Does not directly compute the microstructure evolution
  - Requires
    - Accurate knowledge of response fields
      - Stress
      - Stress concentration
        - » Factor K_t
      - Plastic strain
      - Temperature
    - Accounting for non-linear material behavior
    - Use of security factor
      - As low as possible

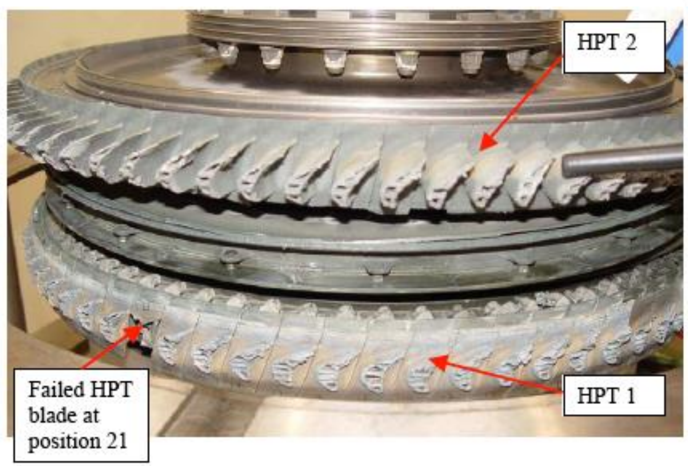


- Inaccurate estimation of plastic strain
  - Low-cycle fatigue of a blade from stage 1 of the high-pressure turbine
  - Cracking at stress concentration in the internal cooling passages
  - Blade failed and impacted other blades, which separated from platform





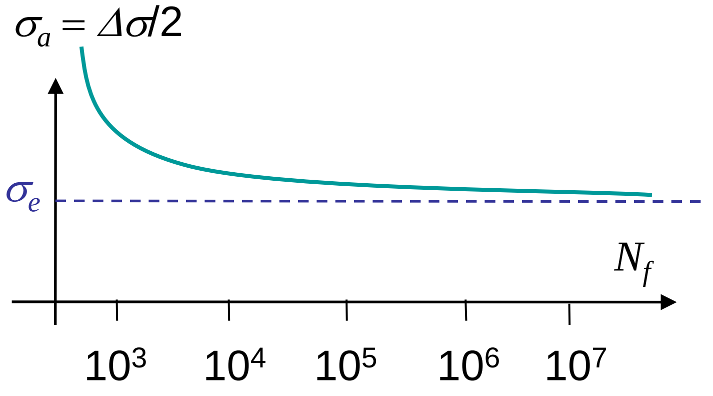




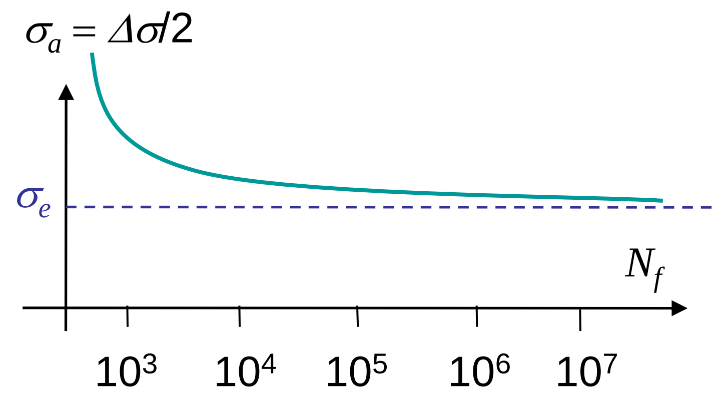
Source: Australian Transport Safety, Engine Failure, Boeing Co 717-200, VH-VQA, Near Melbourne, Victoria, Report – Final, http://www.atsb.gov.au/publications/investigation_reports/2004/aair/aair200402948.aspx

Total life approach

- First kind of total life approach: « stress life » approach
  - For high cycle fatigue
    - Structures experiencing (essentially) elastic deformations
    - Life > 10^4 cycles
  - Fatigue limit
    - For $\sigma_a < \sigma_e$ (endurance limit): infinite life (>10^7 cycles)
    - For $\sigma_a > \sigma_e$, finite life
    - Materials with fatigue limit
      - Mild/low strength steel
      - Ti-Al-Mg alloys
      - With $\sigma_e \sim [\,0.35\,;\,0.5]\,\sigma_{TS}$
    - Materials without fatigue limit
      - Al alloys
      - Mg alloys
      - High strength steels
      - Non-ferro alloys



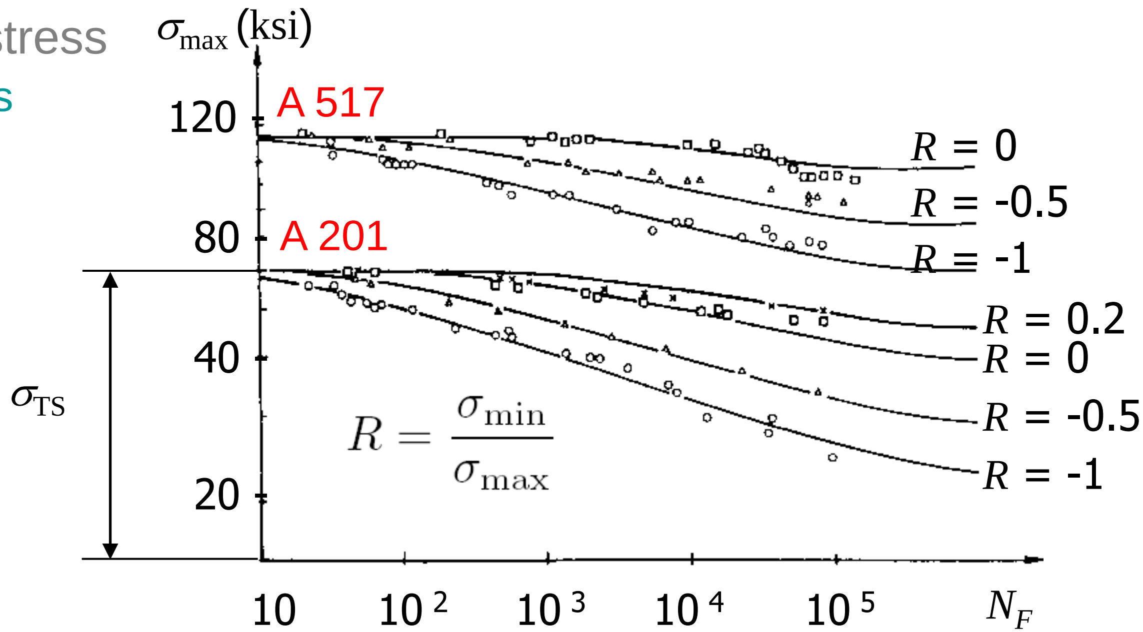
- First kind of total life approach: « stress life » approach (2)
  - Life of structure
    - Assumptions
      - $\sigma_m = 0$ &
      - N_f identical cycles before failure
    - For $\sigma_a < \sigma_e$ (endurance limit): infinite life (>10^7 cycles)
    - For $\sigma_a > \sigma_e$, finite life
    - With $\sigma_e \sim [\,0.35\,;\,0.5\,]\,\sigma_{TS}$
    - 1910, Basquin Law



$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f\,(2N_f)^b$$

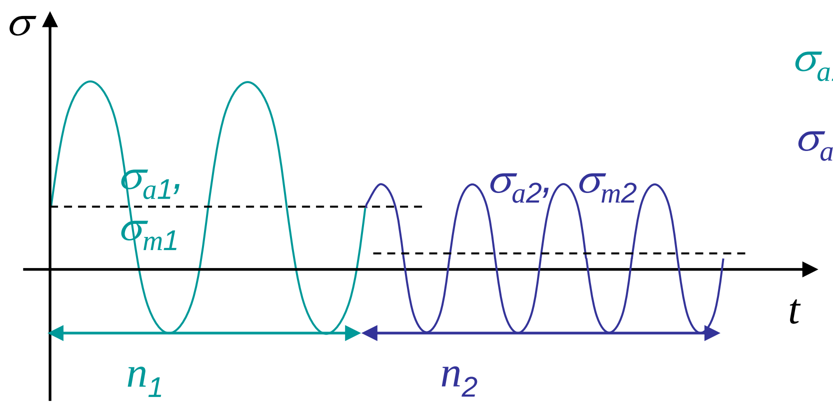
  - σ_f' fatigue coefficient (mild steel T_{amb} : ~ [1; 3] GPa)
  - b fatigue exponent (mild steel T_{amb} : ~ [-0.1; -0.06])
  - Parameters resulting from experimental tests
  - If endurance limit exists: use N_f=10^7 and $\sigma_a = \sigma_e$ in Basquin Law

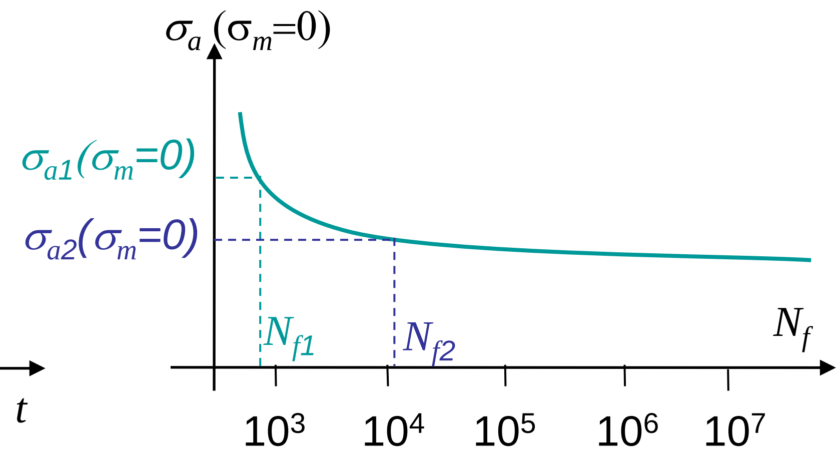
- First kind of total life approach: « stress life » approach (3)
  - Effect of mean stress
    - Wöhler curves



- For $\sigma_m \neq 0$ and N_f identical cycles, the maximal amplitude is corrected
  - Soderberg $\sigma_a = \sigma_a|_{\sigma_m=0}\left(1-\frac{\sigma_m}{\sigma_p}\right)$ conservative
  - Goodman $\sigma_a = \sigma_a|_{\sigma_m=0}\left(1-\frac{\sigma_m}{\sigma_{TS}}\right)$
  - Gerber $\sigma_a = \sigma_a|_{\sigma_m=0}\left[1-\left(\frac{\sigma_m}{\sigma_{TS}}\right)^2\right]$ only for alloys under tension

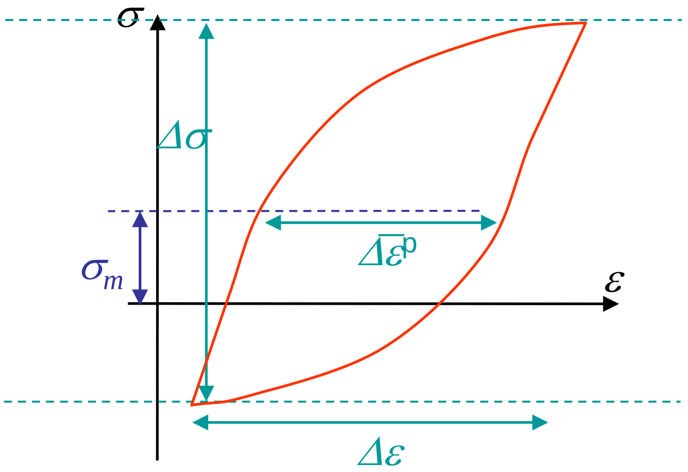
- First kind of total life approach: « stress life » approach (4)
  - Effect of varying amplitude loads





- n_i cycles of constant amplitude lead to a damage $D_i = \frac{n_i}{N_{fi}}$
- 1945, Miner-Palmgreen law
  - At fracture: $D = \sum_i D_i = D_c$
  - Does not account for the sequence in which the cycles are applied
  - Only if low variation in cycles
  - Only if pure fatigue damage

- Second kind of total life approach: « strain life » approach
  - For low cycle fatigue
    - Structures experiencing (essentially)
      - Large plastic deformations
      - Stress concentration
      - (High temperatures)
  - For N_f identical cycles before failure
    - 1954, Manson-Coffin $\frac{\Delta\bar{\varepsilon}^p}{2} = \varepsilon'_f \left(2N_f\right)^c$



      - ε_f' : fatigue ductility coefficient ~ true fracture ductility (metals)
      - c : fatigue ductility coefficient exponent ~ [-0.7 , -0.5] (metals)
      - $\Delta\bar{\varepsilon}^p$ plastic strain increment during the loading cycles

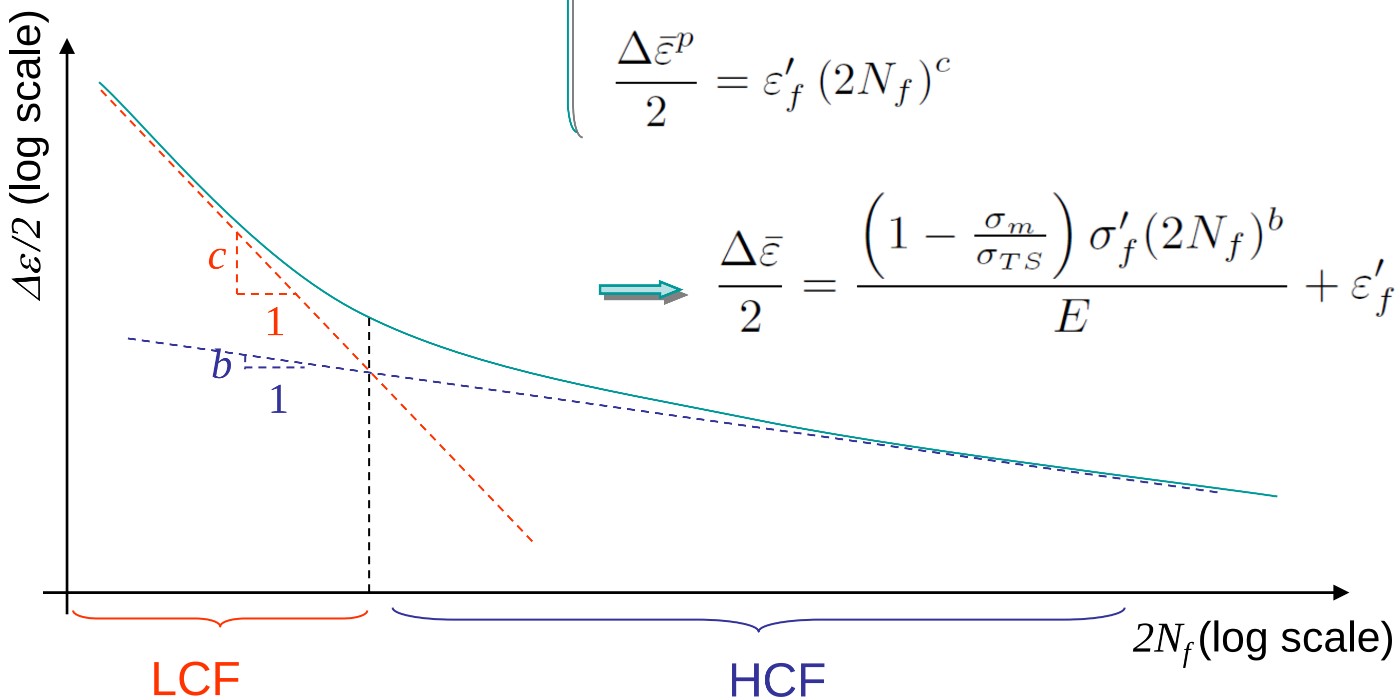
- General relation

$$\Delta\bar{\varepsilon} = \Delta\bar{\varepsilon}^e + \Delta\bar{\varepsilon}^p \Longrightarrow \Delta\bar{\varepsilon} = \frac{\Delta\sigma}{E} + \Delta\bar{\varepsilon}^p$$

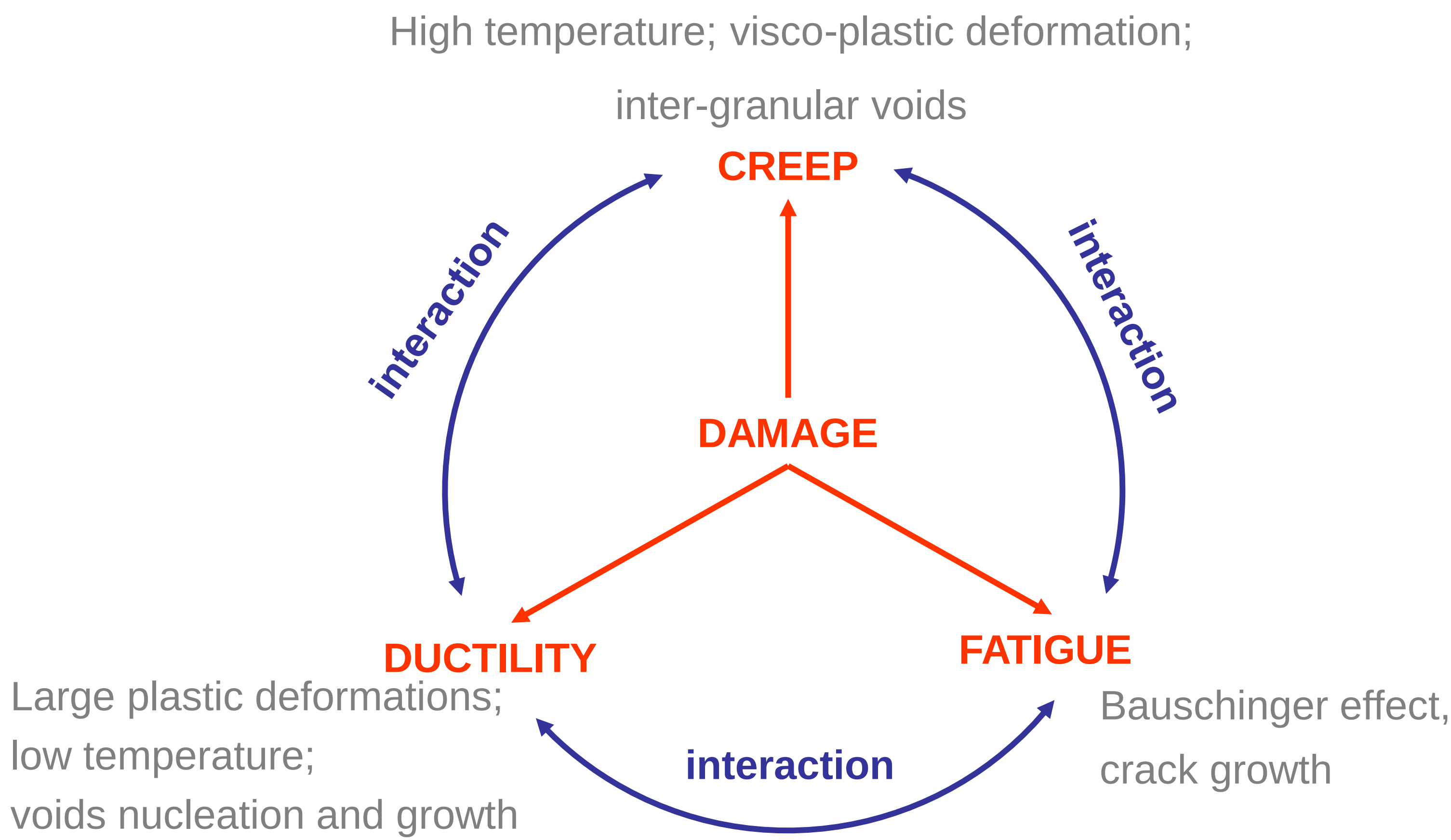
Using Goodman

$$\begin{cases} \dfrac{\Delta\sigma}{2} = \sigma_a = \sigma'_f\,(2N_f)^b \\ \sigma_a = \sigma_a\big|_{\sigma_m=0}\left(1-\dfrac{\sigma_m}{\sigma_{\text{TS}}}\right) \\ \dfrac{\Delta\bar{\varepsilon}^p}{2} = \varepsilon'_f\,(2N_f)^c \end{cases}$$

$$\Longrightarrow \frac{\Delta\bar{\varepsilon}}{2} = \frac{\left(1-\frac{\sigma_m}{\sigma_{TS}}\right)\sigma'_f(2N_f)^b}{E} + \varepsilon'_f(2N_f)^c$$

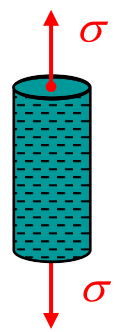


- Interaction between damage sources

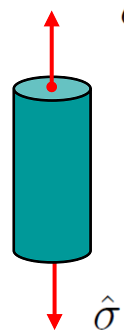


- Experiments

Damaged sample



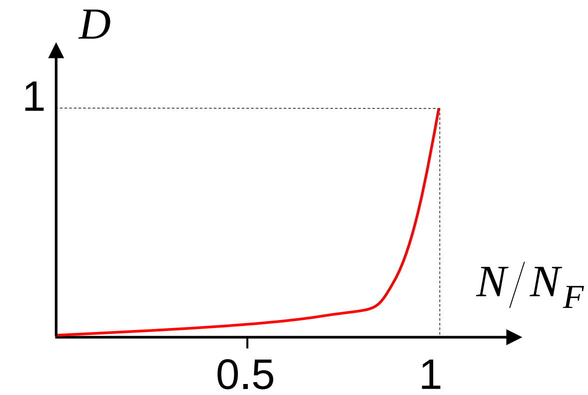
Equivalent homogenized sample



$$\hat{\sigma} = \frac{\sigma}{1-D}$$

$$0 \leq D \leq 1$$

Experimental test



- Damage (in 1D)
  - Fatigue $$dD_f = \left[1-(1-D_f)^{1-\frac{1}{b}}\right]^{\alpha(\sigma_m,\,\sigma_{\max})}\left(\frac{\sigma_a}{\sigma'(\sigma_m,\,\sigma_{TS})}\right)^{-\frac{1}{b}} 2dN$$
  - Ductility $$dD = \left\langle \frac{\sigma - \sigma_D}{(1-D)\,S} \right\rangle^{s} \frac{d\sigma}{S}$$
  - Creep (Kachanov-Rabotnov) $$\dot{D} = \left(\frac{\sigma}{A}\right)^{r}(1-D)^{-k(\sigma)}$$

Material, temperature dependent, parameters obtained from 1D experiments

- 1952, De Havilland 106 Comet 1, UK (1)
  - First jetliner, 36 passengers, pressurized cabin
    - 1954, January, flight BOAC 781 Rome-Heathrow
      - Plane G-ALYP disintegrated above the sea
      - After 1300 flights

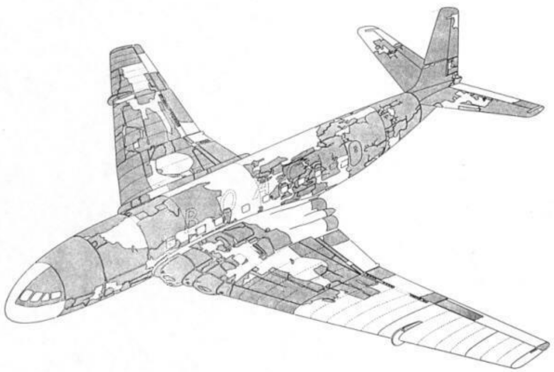


FIG. 2. DIAGRAM SHOWING AMOUNT OF WRECKAGE RECOVERED—G-ALYP.

  - Total life approach failed
    - Fuselages failed well before the design limit of 10000 cycles

Design using total life approach

- 1952, De Havilland 106 Comet 1, UK (2)
  - 1954, August, retrieve of the ALYP roof
    - Origin of failure at the communication window
    - Use of square riveted windows
    - Punched riveting instead of drill riveting
      - ⇒ Presence of initial defects

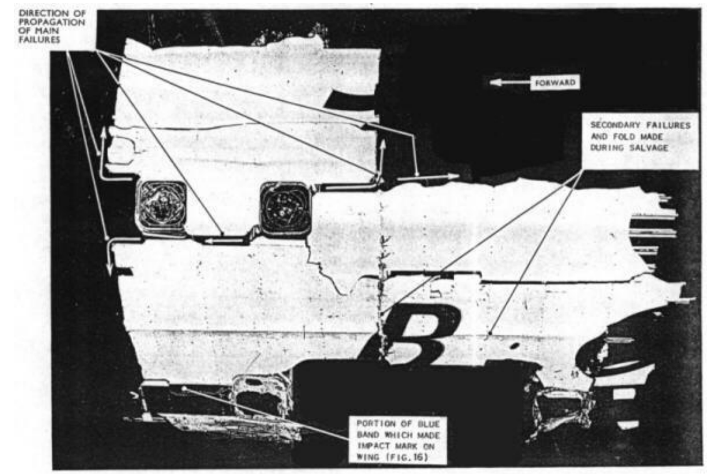


FIG. 12. PHOTOGRAPH OF WRECKAGE AROUND ADF AERIAL WINDOWS—G-ALYP.

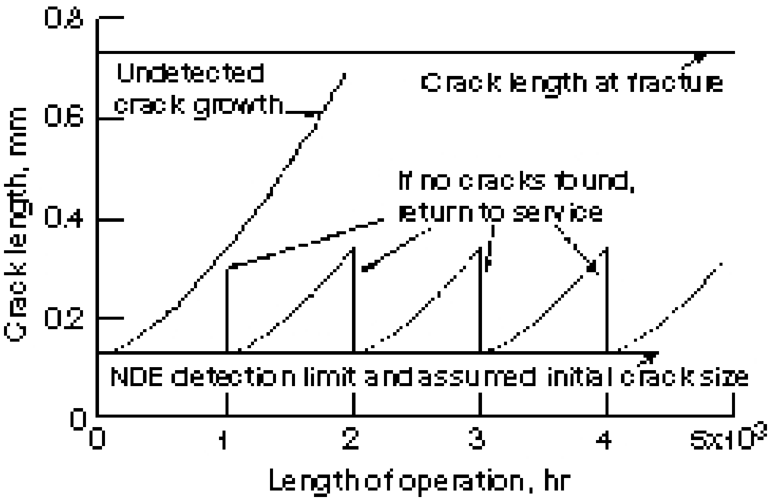
- The total life approach accounts for crack initiation in smooth specimen but does not account for inherent defects
  - The initial defects of the fuselage tested against fatigue could have hardened after the initial static test load, which was not the case with the production planes
  - Life time can be improved by
    - “Shot-peening” : surface bombarded by small spherical media
      - Residual stresses of compression in the surface layer
      - Prevents crack initiation
    - Surface polishing (to remove cracks)

Design using total life approach

- Economically inefficient
  - PW F100 (F15 & F16)
  - Using total life approach against LCF
    - All disks replaced when statistically 1 disk had a fatigue crack (a<0.75 mm)
    - Studies indicate that at least 80% of parts replaced have at least a full order of magnitude of remaining fatigue life
    - Extra cost for US Air force: > $50 000 000 /year

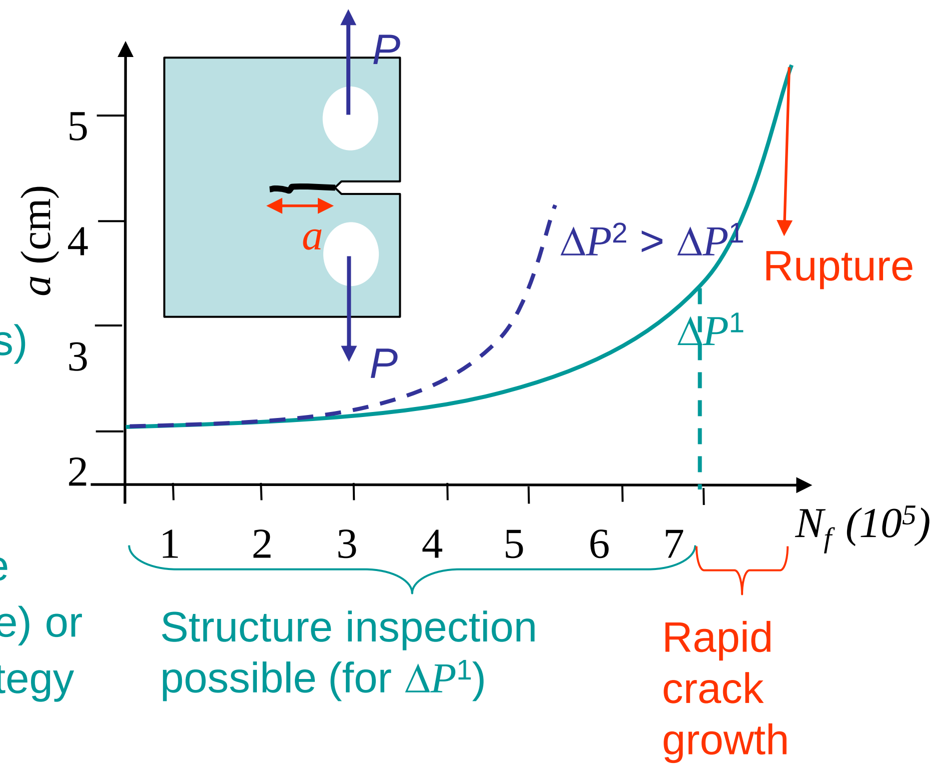
http://www.grc.nasa.gov/WWW/RT/RT1996/5000/5220bo1.htm
http://ocw.mit.edu/courses/materials-science-and-engineering/3-35-fracture-and-fatigue-fall-2003/lecture-notes/fatigue_crack_growth.pdf
(Subra Suresh)

- Economically inefficient (2)
  - Air-force starts using Retirement For Cause approach in 1986
    - Periodic nondestructive evaluation to assess the damage state of components
    - Components with no detectable cracks: returned to service
  - Allows
    - Parts with low life
      - Detected and discarded before they can cause an incident
    - Parts with high life
      - Used to their full potential
  - Basic to an RFC program
    - Calculation of crack-growth rates under the expected service loads (mechanical and thermal)
    - The results are used to define safe-use intervals between required (nondestructive) inspections

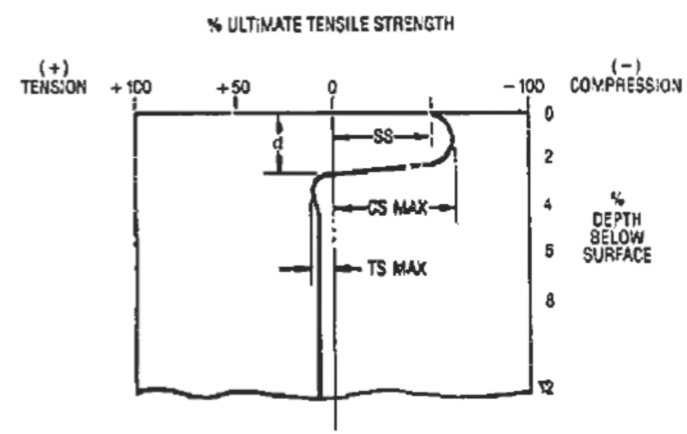


http://www.grc.nasa.gov/WWW/RT/RT1996/5000/5220bo1.htm
http://ocw.mit.edu/courses/materials-science-and-engineering/3-35-fracture-and-fatigue-fall-2003/lecture-notes/fatigue_crack_growth.pdf
(Subra Suresh)

- « Damage tolerant design »
  - Assume cracks are present from the beginning of service
  - Characterize the significance of fatigue cracks on structural performance
    - Control initial crack sizes through manufacturing processes and (non-destructive) inspections
    - Estimate crack growth rates during service (Paris-Erdogan)
    - Schedule conservative inspection intervals (e.g. every so many cycles)
    - Verify crack growth during these inspections
    - Predict end of life (a_f)
    - Remove old structures from service before predicted end-of-life (fracture) or implement repair-rehabilitation strategy
  - Non-destructive inspections
    - Optical
    - X-rays
    - Ultrasonic (reflection on crack surface)



- Shot-peened metallic material
  - Properties before shot-peening
    - Young modus E = 210 GPa
    - HCF parameters σ'_f = 1100 MPa, b = -0.08
    - LCF parameters ε'_f = 1, c = -0.63
  - Due to shot-peening
    - Compressive residual stress of 250 MPa
    - Tensile strength σ_{TS} = 500 Mpa
  - What is the life improvement due to shoot-peening?



Exercice 1: Solution

- Before shot peening
  - $$\frac{\Delta\bar{\varepsilon}}{2} = \frac{\sigma'_f (2N_f)^b}{E} + \varepsilon'_f (2N_f)^c$$

$$\Longrightarrow \frac{\Delta\bar{\varepsilon}}{2} = \frac{1100}{210000}(2N_f)^{-0.08} + (2N_f)^{-0.63} = 5.24\,10^{-3}\;(2N_f)^{-0.08} + (2N_f)^{-0.63}$$

- After shot peening
  - Residual stress acts like a compressive mean stress $\Longrightarrow \Delta\sigma \to \Delta\sigma\left(1 - \frac{\sigma_m}{\sigma_{\mathrm{TS}}}\right)$

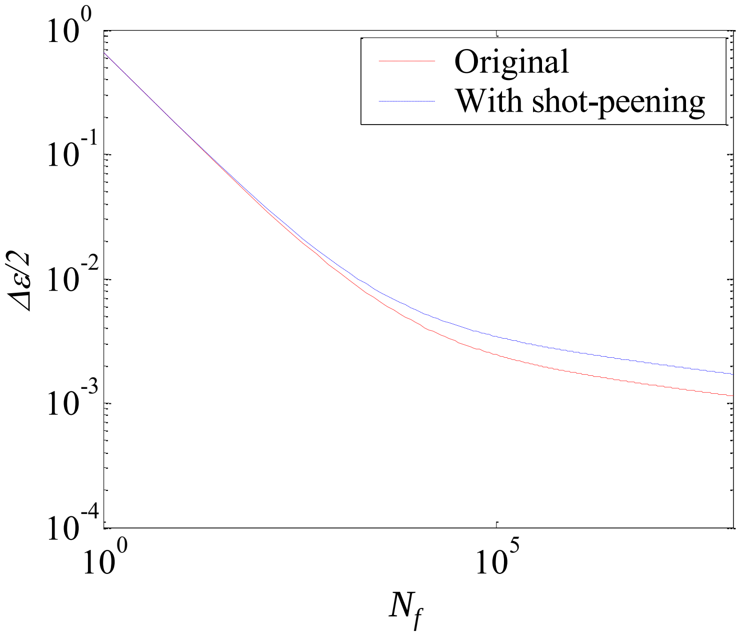
$$\Longrightarrow \frac{\Delta\bar{\varepsilon}}{2} = \frac{\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)\sigma'_f (2N_f)^b}{E} + \varepsilon'_f (2N_f)^c$$

$$\Longrightarrow \frac{\Delta\bar{\varepsilon}}{2} = \left(1 + \frac{250}{500}\right)\frac{1100}{210000}(2N_f)^{-0.08} + (2N_f)^{-0.63}$$

$$\Longrightarrow \frac{\Delta\bar{\varepsilon}}{2} = 7.86\,10^{-3}(2N_f)^{-0.08} + (2N_f)^{-0.63}$$

Exercice 1: Solution

- Comparison



  – Improve mainly the HCF regime

- References
  - Mechanics of Solid Materials. Jean Lemaître,Jean-Louis Chaboche, Cambridge University press, 1994
  - Plasticity and viscoplasticity under cyclic loadings. Jean-Louis Chaboche, ATHENS – Course MP06 – 16 – 20 March 2009