Fracture mechanics, damage and fatigue (MECA0058-1)

January 2026

1st question

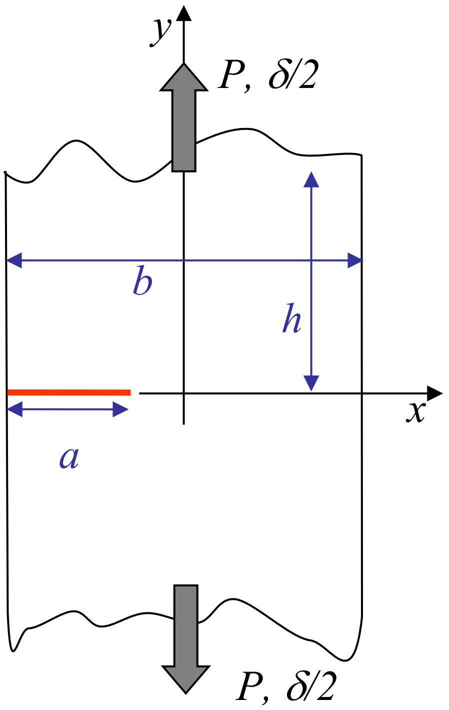

Figure 1: Schematics of SECP

Table 1: Parameters of the SECP

	Steel ASTM a470-7 (Cr-Mo-V)
h [m]	0.20
a [m]	0.01875
b [m]	0.03
Thickness t [mm]	10
Toughness K_{IC} [MPa.m$^{1/2}$]	150
Young E [GPa]	207
Yield σ^0 [MPa]	600
Poisson ν [-]	0.3
Power law α [-]	1
Power law n [-]	10

The single-edge cracked plate specimen illustrated in **Figure 1** is put under tension. The dimensions and material properties are reported in the **Table 1**. The elasto-plastic material is idealized by a power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}} . \tag{1}$$

In the elastic regime, the mode I stress intensity factor K_I is given as

$$K_I = \sigma\sqrt{\pi a} F\left(\frac{a}{b}\right) , \tag{2}$$

where the correcting factor $F\left(\frac{a}{b}\right)$ can be approximated by

$$F\left(\frac{a}{b}\right) = \sqrt{\frac{2b}{\pi a}\tan\left(\frac{\pi a}{2b}\right)} \frac{0.752+2.02\left(\frac{a}{b}\right)+0.37\left(1-\sin\left(\frac{\pi a}{2b}\right)\right)^3}{\cos\left(\frac{\pi a}{2b}\right)} . \tag{3}$$

In the elasto-plastic regime, the limit force **per unit thickness** P_0, is given by

$$P_0 = \begin{cases} 1.455\left[\sqrt{1+\left(\frac{a}{c}\right)^2}-\frac{a}{c}\right]c\sigma_p^0 \text{ for plane strain state;} \\ 1.072\left[\sqrt{1+\left(\frac{a}{c}\right)^2}-\frac{a}{c}\right]c\sigma_p^0 \text{ for plane stress state.} \end{cases} \quad (4)$$

where $c = b - a$ is the ligament. The ratio P/P_0 is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{P}{P_0}\right)^2}, \quad (5)$$

of the plastic zone

$$r_p = \begin{cases} \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane strain state;} \\ \frac{1}{\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane stress state,} \end{cases} \quad (6)$$

that is used to evaluate the effective crack length

$$a_{\text{eff}} = a + \eta r_p. \quad (7)$$

The plastic part of the J-integral is obtained from

$$J_p = \frac{\alpha {\sigma_p^0}^2}{E} a \frac{c}{b}\ h_1\left(\frac{a}{b}, n\right)\left(\frac{P}{P_0}\right)^{n+1}, \quad (8)$$

where the function h_1 is tabulated in **Table 2**.

You are requested:

A) To evaluate the load P (**per unit thickness**) leading to crack propagation considering the Linear Elastic Fracture Mechanics framework;

B) To evaluate the load P leading to crack propagation considering the Small Scale Yielding solution: Linear Elastic Fracture Mechanics framework corrected with the effective crack size (7);

C) Considering that the load P found in B) is applied to the sample, using the Non-Linear Fracture Mechanics framework, to evaluate the total J integral and to conclude on the crack initiation;

D) If needed, and based on the solution of C), to apply a correction on the load found in B) and to reevaluate the total J integral to get closer to the limit load (given the non-linearity of the system, you are not requested to find the exact limit load, only one iteration using a reasonable guess on the limit load is enough);

E) To comment on the validity of the developments and on the difference found in the limit load using the 3 approaches.

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Table 2: Plastic coefficients of the SECP

h_1, h_2, h_3 and h_5 for SECP in plane strain under remote uniform tension.

a/b		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
a/b = 1/8	h_1	4.95	6.93	8.57	11.5	13.5	16.1	18.1	19.9	21.2
	h_2	5.25	6.47	7.56	9.46	11.1	12.9	14.4	15.7	16.8
	h_3	26.6	25.8	25.2	24.2	23.6	23.2	23.2	23.5	23.7
	h_5	0.0	0.558	0.807	1.26	1.62	1.98	2.19	2.33	0.0
a/b = 1/4	h_1	4.34	4.77	4.64	3.82	3.06	2.17	1.55	1.11	0.712
	h_2	4.76	4.56	4.28	3.39	2.64	1.81	1.25	0.875	0.552
	h_3	10.3	7.64	5.87	3.70	2.48	1.50	0.970	0.654	0.404
	h_5	0.0	1.14	1.11	0.833	0.604	0.375	0.237	0.153	0.0894
a/b = 3/8	h_1	3.88	3.25	2.63	1.68	1.06	0.539	0.276	0.142	0.0595
	h_2	4.54	3.49	2.67	1.57	0.946	0.458	0.229	0.116	0.048
	h_3	5.14	2.99	1.90	0.923	0.515	0.240	0.119	0.060	0.0246
	h_5	0.0	1.43	1.10	0.643	0.380	0.179	0.0879	0.0442	0.0181
a/b = 1/2	h_1	3.40	2.30	1.69	0.928	0.514	0.213	0.0902	0.0385	0.0119
	h_2	4.45	2.77	1.89	0.954	0.507	0.204	0.0854	0.0356	0.0110
	h_3	3.15	1.54	0.912	0.417	0.215	0.085	0.0358	0.0147	0.00448
	h_5	0.0	1.60	1.11	0.562	0.300	0.121	0.0511	0.0213	0.00657
a/b = 5/8	h_1	2.86	1.80	1.30	0.697	0.378	0.153	0.0625	0.0256	0.0078
	h_2	4.37	2.44	1.62	0.0806	0.423	0.167	0.0671	0.0272	0.00823
	h_3	2.31	1.08	0.681	0.329	0.171	0.067	0.0268	0.0108	0.00326
	h_5	0.0	1.80	1.21	0.604	0.318	0.126	0.0509	0.0207	0.00626
a/b = 3/4	h_1	2.34	1.61	1.25	0.769	0.477	0.233	0.116	0.059	0.0215
	h_2	4.32	2.52	1.79	1.03	0.619	0.296	0.146	0.0735	0.0267
	h_3	2.02	1.10	0.765	0.435	0.262	0.125	0.0617	0.0312	0.0113
	h_5	0.0	2.17	1.55	0.895	0.539	0.258	0.127	0.0639	0.0232
a/b = 7/8	h_1	1.91	1.57	1.37	1.10	0.925	0.702			
	h_2	4.29	2.75	2.14	1.55	1.23	0.921			
	h_3	2.01	1.27	0.988	0.713	0.564	0.424			
	h_5	0.0	2.601	2.203	1.47	1.16	0.875			

h_1, h_2, h_3 and h_5 for SECP in plane stress under remote uniform tension.

a/b		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
a/b = 1/8	h_1	3.58	4.55	5.06	5.30	4.96	4.14	3.29	2.60	1.92
	h_2	5.15	5.43	6.05	6.01	5.47	4.46	3.48	2.74	2.02
	h_3	26.1	21.6	18.0	12.7	9.24	5.98	3.94	2.72	2.0
	h_5	0.296	0.490	0.627	0.748	0.720	0.586	0.450	0.345	0.255
a/b = 1/4	h_1	3.14	3.26	2.92	2.12	1.53	0.960	0.615	0.400	0.230
	h_2	4.67	4.30	3.70	2.53	1.76	1.05	0.656	0.419	0.237
	h_3	10.1	6.49	4.36	2.19	1.24	0.630	0.362	0.224	0.123
	h_5	0.904	1.05	0.932	0.631	0.433	0.258	0.160	0.103	0.0583
a/b = 3/8	h_1	2.81	2.37	1.94	1.37	1.01	0.677	0.474	0.342	0.226
	h_2	4.47	3.43	2.63	1.69	1.18	0.762	0.524	0.372	0.244
	h_3	5.05	2.65	1.60	0.812	0.525	0.328	0.223	0.157	0.102
	h_5	1.73	1.40	1.10	0.720	0.510	0.332	0.229	0.164	0.108
a/b = 1/2	h_1	2.46	1.67	1.25	0.776	0.510	0.286	0.164	0.0956	0.0469
	h_2	4.37	2.73	1.91	1.09	0.694	0.380	0.216	0.124	0.0607
	h_3	3.10	1.43	0.871	0.461	0.286	0.155	0.088	0.0506	0.0247
	h_5	2.41	1.58	1.12	0.652	0.417	0.229	0.130	0.0748	0.0395
a/b = 5/8	h_1	2.07	1.41	1.105	0.755	0.551	0.363	0.248	0.172	0.107
	h_2	4.30	2.55	1.84	1.16	0.816	0.523	0.353	0.242	0.150
	h_3	2.27	1.13	0.771	0.478	0.336	0.215	0.146	0.100	0.0616
	h_5	3.02	1.88	1.36	0.861	0.606	0.388	0.262	0.179	0.111
a/b = 3/4	h_1	1.70	1.14	0.910	0.624	0.447	0.280	0.181	0.118	0.0670
	h_2	4.24	2.47	1.81	1.15	0.798	0.490	0.314	0.203	0.115
	h_3	1.98	1.09	0.784	0.494	0.344	0.211	0.136	0.0581	0.0496
	h_5	3.44	2.12	1.56	0.986	0.686	0.421	0.270	0.174	0.0987
a/b = 7/8	h_1	1.38	1.11	0.962	0.792	0.677	0.574			
	h_2	4.22	2.68	2.08	1.54	1.27	1.04			
	h_3	1.97	1.25	0.969	0.716	0.591	0.483			
	h_5	3.91	2.53	1.96	1.45	1.19	0.973			

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Table 3: Cracked support properties

Properties	Values
Radius W	0.05 m
Thickness t	0.0207 m
Young E	70 GPa
Yield σ_p^0	400 MPa
Poisson ν [-]	0.3
Toughness K_{IC}	30 MPa $m^{1/2}$

Table 4: Finite elements prediction of cracked support displacement for Q = 100 N

Crack length a [mm]	Load-point displacement q [mm]
12	6.640×10^{-4}
16	8.787×10^{-4}
20	13.12×10^{-4}
24	19.37×10^{-4}
28	29.75×10^{-4}
32	43.85×10^{-4}
36	82.49×10^{-4}

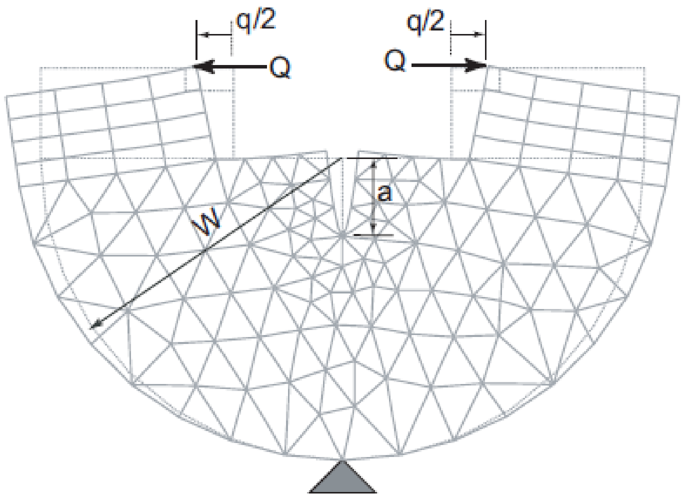

Figure 2: Schematics of the cracked support

A cracked circular support is analyzed using the finite element method, see **Figure 2**. The dimensions of the sample and the elastic properties of the material are reported in the **Table 3**.

The finite elements simulations conducted with a loading Q = 100 N for different crack sizes lead to the load-point displacements reported in the **Table 4**.

You are requested

A) Using numerical approximations, to represent the evolution (in a table or on a graph) of the energy release rate vs. the crack-length (for Q = 100 N).

B) Using these results to represent the evolution (in a table or on a graph) of the stress intensity factor vs. the crack-length (for Q = 100 N).

C) If the real circular support is cracked and exhibits a compliance at the loading points of 25×10^{-3} mm/kN, to compute the maximum load Q_{max} that can be applied to the support.

D) To justify the validity of this last result.

E) Assuming the real sample is loaded under a constant load $1.1 \times Q_{max}$ to determine whether the crack would propagate in a stable or unstable way for a constant Toughness K_{IC}.