Fracture mechanics, damage and fatigue (MECA0058-1)
January 2025

1st question

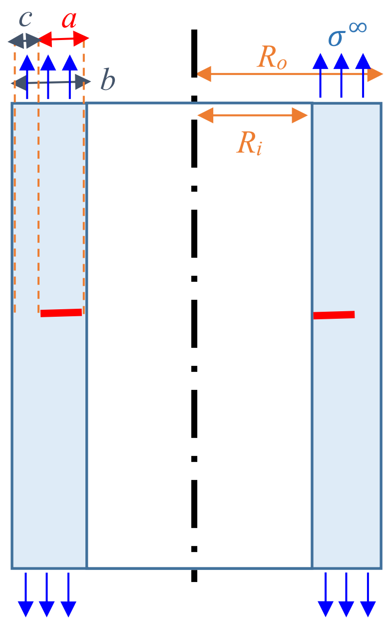

Figure 1: cylinder under uniaxial tension

Table 1: Properties of the cylinder under uniaxial tension

	Steel
Internal radius R_i [m]	1.20
External radius R_o [m]	1.32
Crack a [m]	0.03
Young E [GPa]	210
Yield σ_p^0 [MPa]	620
Poisson ν [-]	0.3
Hardening exponent n [-]	10
Hardening parameter α [–]	1
Toughness K_{IC} [MPa$\sqrt{m}$]	150
Tearing T_R [-]	10
Load σ^∞	350 MPa

We consider a cylinder under uniaxial tension, see Figure 1, with the dimensions reported in the Table 1, and submitted to a loading σ^∞, see Table 1. The results of a tension test on the material are also reported in this table. The material follows a power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}} \tag{1}$$

In the **elastic regime**, the stress intensity factor K_I is given as

$$K_I = \sigma^\infty \sqrt{\pi a}\, F\left(\frac{a}{b}, \frac{b}{R_i}\right), \tag{2}$$

where the correction F is tabulated in the non-linear fracture mechanics handbook, see also Table 2.

In the **elasto-plastic regime**, the limit load, is given by

$$P_0 = \frac{2}{\sqrt{3}}\pi\sigma_p^0(R_o^2 - R_c^2) \text{ with } R_c = R_i + a. \tag{3}$$

The ratio P/P_0, with $P = \sigma^\infty\pi(R_o^2 - R_i^2)$ the applied load, is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{P}{P_0}\right)^2}, \tag{4}$$

of the plastic zone

$$r_p = \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane strain state,} \tag{5}$$

that is used to evaluate the effective crack length

$$a_{\mathrm{eff}} = a + \eta r_p. \tag{6}$$

The plastic part of the J-integral is obtained from

$$J_p = \frac{\alpha {\sigma_p^0}^2}{E} c\frac{a}{b}\ h_1\left(\frac{a}{b}, n; \frac{b}{R_i}\right)\left(\frac{P}{P_0}\right)^{n+1}, \tag{7}$$

where the function h_1 is tabulated in the non-linear fracture mechanics handbook, see Table 4-Table 5.

You are requested, considering the loading force σ^∞:

A) To evaluate whether the crack will propagate using the Linear Elastic Fracture Mechanics framework;
B) To evaluate whether the crack will propagate using the Small Scale Yielding solution (Linear Elastic Fracture Mechanics framework corrected with the effective crack size (6));
C) To evaluate whether the crack will propagate using the Non-Linear Fracture Mechanics framework;
D) In the case of crack propagation, to assess the crack growth stability using the Non-Linear Fracture Mechanics framework;
E) To comment on the validity of the developments.

Table 2: F, V_1, V_2 for the cylinder under uniaxial tension

F, V_1 and V_2 for a circumferentially cracked cylinder in tension.

		a/b = 1/8	a/b = 1/4	a/b = 1/2	a/b = 3/4
b/R_i = 1/5	F	1.16	1.26	1.61	2.15
	V_1	1.49	1.67	2.43	3.76
	V_2	0.117	0.255	0.743	1.67
b/R_i = 1/10	F	1.19	1.32	1.82	2.49
	V_1	1.55	1.76	2.84	4.72
	V_2	0.180	0.290	0.885	2.09
b/R_i = 1/20	F	1.22	1.36	2.03	2.89
	V_1	1.59	1.81	3.26	5.99
	V_2	0.220	0.315	1.04	2.74

Table 4: h_1, h_2, h_3 for the cylinder under uniaxial tension, $\frac{b}{R_i} = \frac{1}{5}$

h_1, h_2 and h_3 for a circumferentially cracked cylinder in tension with b/R_i = 1/5.

		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10
a/b = 1/8	h_1	3.78	5.00	5.94	7.54	8.99	11.1
	h_2	4.56	5.55	6.37	7.79	9.10	11.0
	h_3	0.369	0.700	1.07	1.96	3.04	4.94
a/b = 1/4	h_1	3.88	4.95	5.64	6.49	6.94	7.22
	h_2	4.40	5.12	5.57	6.07	6.28	6.30
	h_3	0.673	1.25	1.79	2.79	3.61	4.52
a/b = 1/2	h_1	4.40	4.78	4.59	3.79	3.07	2.34
	h_2	4.36	4.30	3.91	3.00	2.26	1.55
	h_3	1.33	1.93	2.21	2.23	1.94	1.46
a/b = 3/4	h_1	4.12	3.03	2.23	1.546	1.30	1.11
	h_2	3.46	2.19	1.36	0.638	0.436	0.325
	h_3	1.54	1.39	1.04	0.686	0.508	0.366

Table 3: h_1, h_2, h_3 for the cylinder under uniaxial tension, $\frac{b}{R_i} = \frac{1}{10}$

h_1, h_2 and h_3 for a circumferentially cracked cylinder in tension with b/R_i = 1/10.

		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10
a/b = 1/8	h_1	4.00	5.13	6.09	7.69	9.09	11.1
	h_2	4.71	5.63	6.45	7.85	9.09	10.9
	h_3	0.548	0.733	1.13	2.07	3.16	5.07
a/b = 1/4	h_1	4.17	5.35	6.09	6.93	7.30	7.41
	h_2	4.58	5.36	5.84	6.31	6.44	6.31
	h_3	0.757	1.35	1.93	2.96	3.78	4.60
a/b = 1/2	h_1	5.40	5.90	5.63	4.51	3.49	2.47
	h_2	4.99	5.01	4.59	3.48	2.56	1.67
	h_3	1.555	2.26	2.59	2.57	2.18	1.56
a/b = 3/4	h_1	5.18	3.78	2.57	1.59	1.31	1.10
	h_2	4.22	2.79	1.67	0.725	0.48	0.300
	h_3	1.86	1.73	1.26	0.775	0.561	0.360

Table 5: h_1, h_2, h_3 for the cylinder under uniaxial tension, $\frac{b}{R_i} = \frac{1}{20}$

h_1, h_2 and h_3 for a circumferentially cracked cylinder in tension with $b/R_i = 1/20$.

		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10
a/b = 1/8	h_1	4.04	5.23	6.22	7.82	9.19	11.1
	h_2	4.82	5.69	6.52	7.90	9.11	10.8
	h_3	0.680	0.759	1.17	2.13	3.23	5.12
a/b = 1/4	h_1	4.38	5.68	6.45	7.29	7.62	7.65
	h_2	4.71	5.56	6.05	6.51	6.59	6.39
	h_3	0.818	1.43	2.03	3.10	3.91	4.69
a/b = 1/2	h_1	6.55	7.17	6.89	5.46	4.13	2.77
	h_2	5.67	5.77	5.36	4.08	2.97	1.88
	h_3	1.80	2.59	2.99	2.98	2.50	1.74
a/b = 3/4	h_1	6.64	4.87	3.08	1.68	1.30	1.07
	h_2	5.18	3.57	2.07	0.808	0.472	0.316
	h_3	2.36	2.18	1.53	0.772	0.494	0.330

2nd question

Table 6: Geometry of the DCB test.

	DCB sample
h [mm]	2.5
L [mm]	250
Thickness t [mm]	20
Manufactured crack a_d [mm]	60
Initial crack a_0 [mm]	65

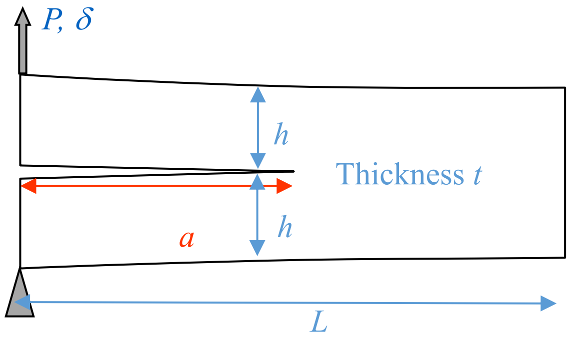

Figure 2: Schematics of the DCB test.

In order to determine the mode I critical energy release rate G_{CI} related to composite laminate delamination, a Double Cantilever Beam (DCB) test is performed, see the geometry reported in Figure 2 and Table 6. The specimens were cut from an autoclave consolidated unidirectional laminate panel at 0deg, with an initially delaminated zone of length a_d obtained using adhesive.

The sample is first loaded up to crack initiation before being unloaded so that the delamination test is performed from a new initial crack length a_0 whose crack front is not affected by the manufacturing process. When applying the displacement δ on the specimen, the evolution of the loading P is recorded as well as the evolution of the crack length a, see Figure 3.

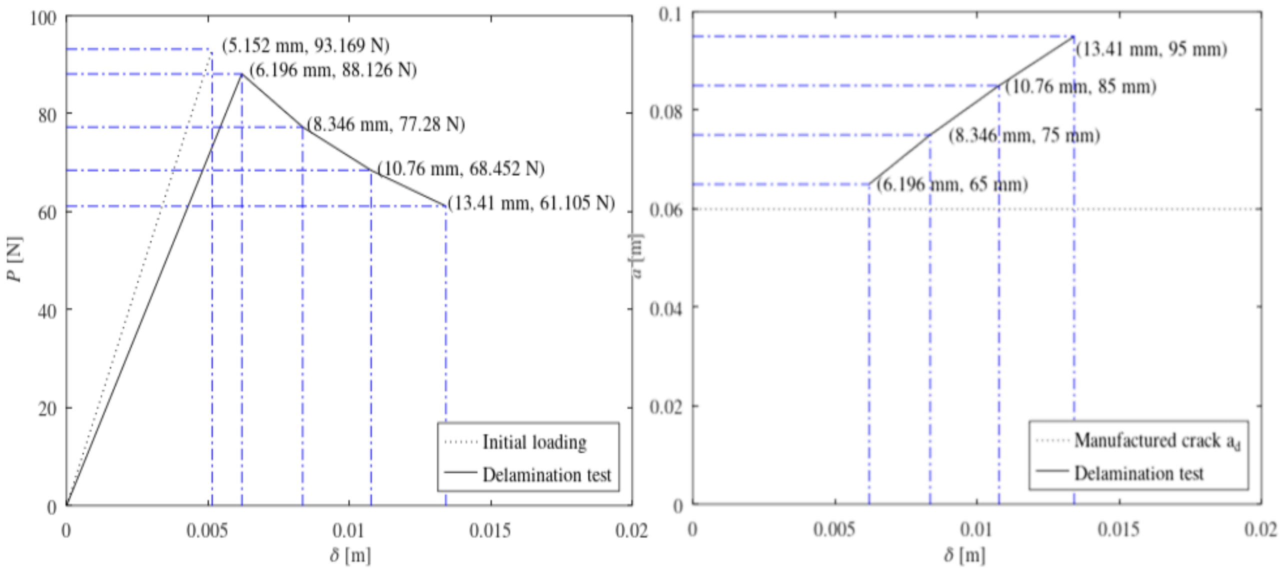

Figure 3: Delamination test: (left) evolution of the loading in terms of the displacement; (right) evolution of the crack length in terms of the displacement

The double cantilever beam theory leads to an expression of the displacement δ in terms of the load P as

$$\delta = \frac{8Pa^3}{Eth^3}, \tag{1}$$

where E is an equivalent Young's modulus along the beam axis.

You are requested to evaluate the evolution of the mode I critical energy release rate G_{CI} related to the delamination of the composite laminate, with respect to the crack advance. To do so, you are invited to follow the following steps:

A) Using Eq. (1), show that the critical energy release rate G_{CI} can be expressed in terms of the thickness t, P reached during crack growth, the crack length a, and a parameter α defined as

$$\alpha = \frac{\left(\frac{\delta}{P}\right)^{1/3}}{a}. \tag{2}$$

B) For the different (5) recorded values in Figure 3, and using the expression of A) compute successively
   a. The value α;
   b. The critical energy release rate G_{CI}.

C) What can you conclude from these values evolutions with the crack size, with in particular
   a. Why is it convenient to express the critical energy release rate G_{CI} in terms of the parameter α ?
   b. How can you explain the evolution of the parameter α with the crack length a?
   c. How can you explain the evolution of G_{CI} with the crack length a, what is the relevant value for a practical application?

D) Using the toughness locus represented by Figure 3, evaluate the critical energy release rate G_{CI} directly from the figure information and from the thickness t. In particular
   a. Justify theoretically your approach;
   b. Evaluate the critical energy release rate G_{CI} directly from the figure by considering two discrete recorded values (2 out of 5) of your choice;
   c. Compare to the previously computed values.