Fracture mechanics, Damage & Fatigue (MECA0058-1)
January 2024

First question

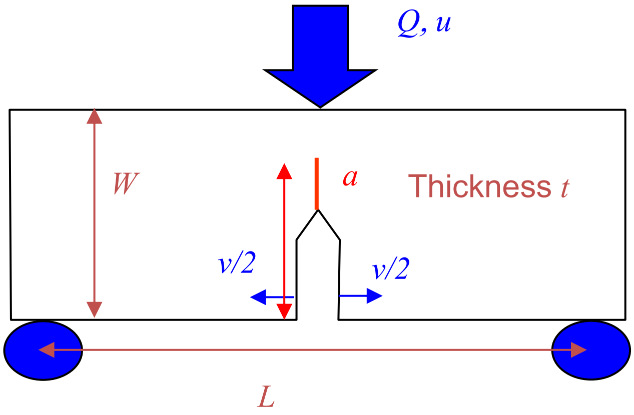

Figure 1: SENB specimen geometry

Table 1: SENB specimen properties

Properties	Values
Width W	0.04 m
Length L	0.16 m
Thickness t	0.03 m
Young E	210 GPa
Yield σ_p^0	600 MPa
Poisson ν [-]	0.3
Hardening exponent n	10
Hardening parameter α	1

One wants to measure the critical J value of a ductile material. To this end Single Edge Notched Bending (SENB) specimens are manufactured, see Figure 1, with the dimensions reported in the Table 1. The results of a tension test on the material are also reported in this table. The material follows a power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}} \tag{1}$$

In the **elastic regime**, following the norm ASME E399-90, the evolution of the crack mouth opening v_e, and of the stress intensity factor K in terms of the loading force Q and of the crack size a, were calibrated using the finite element method.

For the crack mouth opening, one has in elasticity

$$\begin{cases} U = \dfrac{1}{1+\sqrt{\frac{4E' t v_e W}{LQ}}} \\ \dfrac{a}{W} = 0.9997 - 3.95U + 2.982U^2 - 3.214U^3 + 51.52U^4 - 113U^5 \end{cases} \tag{2}$$

or equivalently

$$\frac{v_e}{Q} = \frac{6La}{E' t W^2}\left[0.76 - 2.28\frac{a}{W} + 3.87\left(\frac{a}{W}\right)^2 - 2.04\left(\frac{a}{W}\right)^3 + \frac{0.66}{\left(1-\frac{a}{W}\right)^2}\right] \tag{3}$$

The loading point displacement reads in elasticity

$$\frac{u_e}{Q} = \frac{6L^2}{4\,E' t W^2}\left(\frac{\frac{a}{W}}{1-\frac{a}{W}}\right)^2 \left[5.58 - 19.57\frac{a}{W} + 36.82\left(\frac{a}{W}\right)^2 - 34.94\left(\frac{a}{W}\right)^3 + 12.77\left(\frac{a}{W}\right)^4\right] \tag{4}$$

For the stress intensity factor, one has in elasticity

$$K_I = \frac{QL}{tW^{\frac{3}{2}}} \; 3\sqrt{a/W} \; \frac{1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left(2.15-3.93\frac{a}{W}+2.7\left(\frac{a}{W}\right)^2\right)}{2\left(1+\frac{2a}{W}\right)\left(1-\frac{a}{W}\right)^{\frac{3}{2}}} \tag{5}$$

In the **elasto-plastic regime**, the limit load in plane strain is given by

$$Q^0 = 0.728\sigma_p^0 \frac{2t(W-a)^2}{L} \tag{6}$$

The ratio Q/Q^0, with Q the applied load, is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{Q}{Q^0}\right)^2}, \tag{7}$$

of the plastic zone

$$r_p = \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane strain state,} \tag{8}$$

that is used to evaluate the effective crack length

$$a_{\text{eff}} = a + \eta r_p. \tag{9}$$

The plastic part of the crack mouth opening displacement reads

$$v_p = \frac{\alpha\sigma_p^0}{E} a\, h_2\left(\frac{a}{W}, n\right)\left(\frac{Q}{Q^0}\right)^n \tag{10}$$

the plastic part of the loading point displacement reads

$$u_p = \frac{\alpha\sigma_p^0}{E} a\, h_3\left(\frac{a}{W}, n\right)\left(\frac{Q}{Q^0}\right)^n \tag{11}$$

and the plastic part of the J-integral reads

$$J_p = \frac{\alpha{\sigma_p^0}^2}{E}(W-a)\, h_1\left(\frac{a}{W}, n\right)\left(\frac{Q}{Q^0}\right)^{n+1} \tag{12}$$

The function h_1, h_2 and h_3 are tabulated in the non-linear handbook, see Table 2.

The specimen is first submitted to a cyclic loading, at low stress so that it remains elastic, to initiate an initial crack which extends up to the size a_0.

The sample is then loaded to measure the critical J_{IC}. Before the crack propagates the sample is partly unloaded with an apparent stiffness (in terms of load vs. crack mouth opening) of 194.1 kN/mm, before being loaded again. After the crack propagates, the sample is unloaded and reloaded five successive times at loads ranging between 58.55 kN and 61.5 kN. The relation between the applied loading Q and the crack mouth opening v is measured and represented in Figure 2.

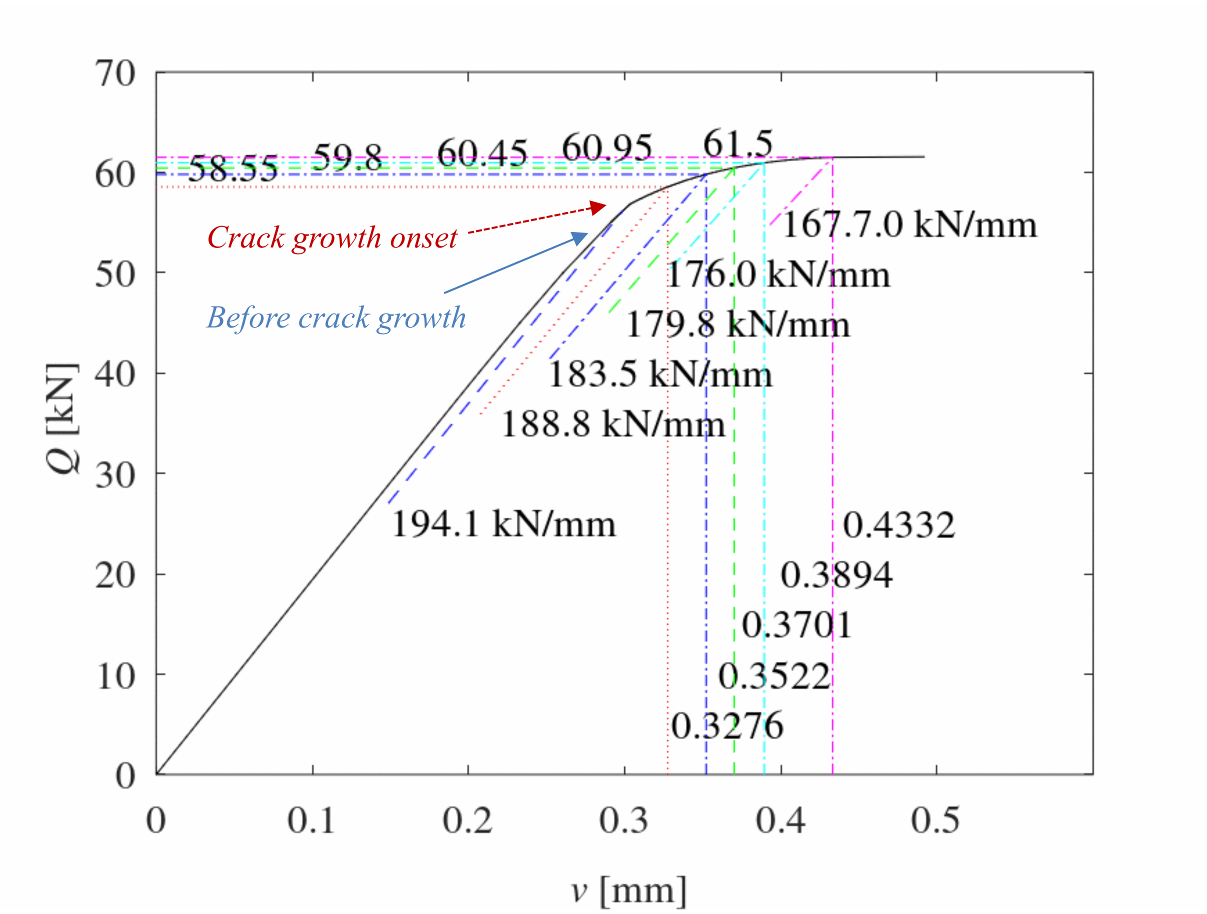

Figure 2: Specimen fracture test

You are requested:

A) To calculate the size a_0 of the crack obtained after the cyclic loading

B) For at least 2 of the sampling points after crack propagation to compute

   a. The crack size

   b. The stress intensity factor assuming Small Scale Yielding (with effective crack length)

   c. The elastic and plastic parts of the J integral (you are not requested to follow strictly the ASTM E1820 norm but to evaluate the J integral from the provided Equations 5-12)

   d. The total J integral

C) Using these values, to derive an equation of the form $J = C_1 + C_2\,\Delta a$

D) From this interpolation to evaluate

   a. J_{IC} and the corresponding toughness

   b. The tearing of the material

E) To discuss the validity of the approach

F) To assess whether one could have directly measured the toughness?

Table 2: h_1, h_2, h_3 for the SENB specimen

a/b		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
a/b = 1/8	h_1	0.936	0.869	0.805	0.687	0.580	0.437	0.329	0.245	0.165
	h_2	6.97	6.77	6.29	5.29	4.38	3.24	2.40	1.78	1.19
	h_3	3.00	22.1	20.0	15.0	11.7	8.39	6.14	4.54	3.01
a/b = 1/4	h_1	1.20	1.034	0.930	0.762	0.633	0.523	0.396	0.303	0.215
	h_2	5.80	4.67	4.01	3.08	2.45	1.93	1.45	1.09	0.758
	h_3	4.08	9.72	8.36	5.86	4.47	3.42	2.54	1.90	1.32
a/b = 3/8	h_1	1.33	1.15	1.02	0.084	0.695	0.556	0.442	0.360	0.265
	h_2	5.18	3.93	3.20	2.38	1.93	1.47	1.15	0.928	0.684
	h_3	4.51	6.01	5.03	3.74	3.02	2.30	1.80	1.45	1.07
a/b = 1/2	h_1	1.41	1.09	0.922	0.675	0.495	0.331	0.211	0.135	0.0741
	h_2	4.87	3.28	2.53	1.69	1.19	0.773	0.480	0.304	0.165
	h_3	4.69	4.33	3.49	2.35	1.66	1.08	0.669	0.424	0.230
a/b = 5/8	h_1	1.46	1.07	0.896	0.631	0.436	0.255	0.142	0.084	0.0411
	h_2	4.64	2.86	2.16	1.37	0.907	0.518	0.287	0.166	0.0806
	h_3	4.71	3.49	2.70	1.72	1.14	0.652	0.361	0.209	0.102
a/b = 3/4	h_1	1.48	1.15	0.974	0.693	0.500	0.348	0.223	0.140	0.0745
	h_2	4.47	2.75	2.10	1.36	0.936	0.618	0.388	0.239	0.127
	h_3	4.49	3.14	2.40	1.56	1.07	0.704	0.441	0.272	0.144
a/b = 7/8	h_1	1.50	1.35	1.20	1.02	0.855	0.690	0.551	0.440	0.321
	h_2	4.36	2.90	2.31	1.70	1.33	1.00	0.782	0.613	0.459
	h_3	4.15	3.08	2.45	1.81	1.41	1.06	0.828	0.649	0.486

Second question

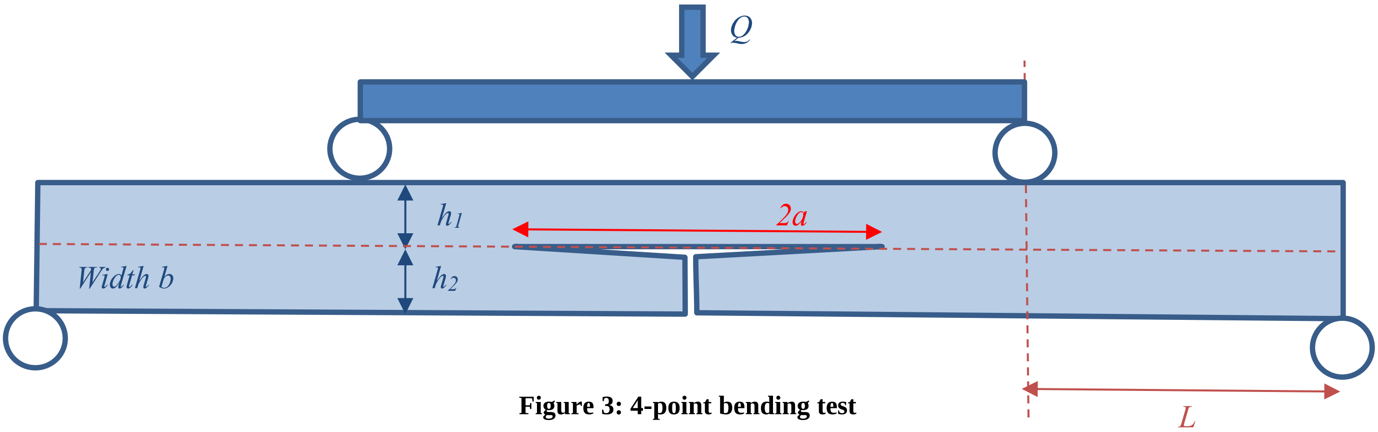

Figure 3: 4-point bending test

Two beams of the same material, with Young modulus E and with respective rectangular cross sections $h_1 \times b$ and $h_2 \times b$, are glued on top of each other and submitted to a 4-point bending test, see Figure 3. The particularity of this test is to achieve a constant bending moment $M = \frac{LQ}{2}$ between the two loading hinges. However, the lower beam is fractured in its center, leading to a centered delamination of size $2a \times b$ between the two beams.

Using the **energetic approach (variation of potential energy resulting from a crack propagation)**, you are requested to evaluate the energy release rate, with in details:

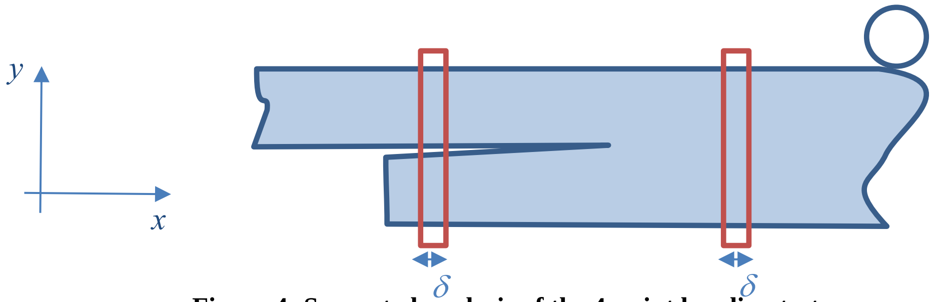

Figure 4: Suggested analysis of the 4-point bending test

A) Considering the strips of material of width δ lying on both sides of the crack tip, see Figure 4, which are assumed to be far enough from the crack tip and loading points so that the pure bending expressions hold, you are requested to evaluate the internal strain energy in each of the two strips.

B) Using the results of A), you are requested to evaluate the energy release rate of the 4-point bending test in terms of h_1, h_2, b, M, E, a and to explain and detail how you reach this expression;

C) You are requested to discuss on the stability of the crack;

D) In the case of rectangular cross sections $h_1 \times b = 0.01 \times 0.02\ \mathrm{m}^2$ and $h_2 \times b = 0.005 \times 0.02\ \mathrm{m}^2$, $a = 0.005\ \mathrm{m}$, $L = 0.2\ \mathrm{m}$ and a critical stress intensity factor of $30\ \mathrm{MPa} \cdot \mathrm{m}^{\frac{1}{2}}$, you are requested to evaluate the maximum load Q that can be applied.

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Reminder of beam theory

1) In the pure bending case, the stress distribution of a symmetric profile of uniform Young modulus, with the **origin** taken at the **section center of inertia,** reads $\sigma_{xx} = -\frac{My}{I}$ with $I = \int_A y^2 dy$;
2) In the pure bending case, the internal energy **per unit beam length** of a symmetric profile of uniform Young's modulus, with the **origin** taken at the **section center of inertia,** reads $e_{int} = \frac{M^2}{2EI}$ with $I = \int_A y^2 dA$
3) Verify the units.