Fracture mechanics, Damage & Fatigue (MECA0058-1)

January 2022

First question

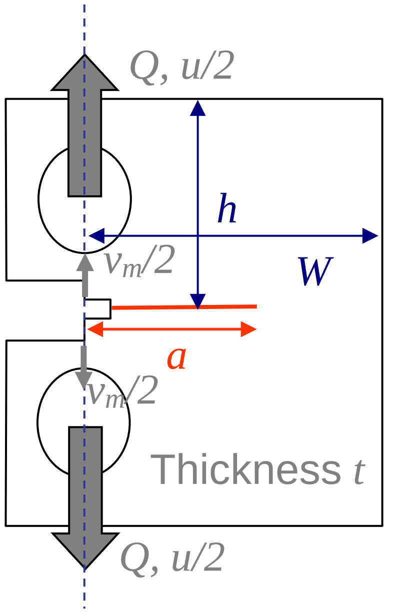

Figure 1: Compact Test Specimen (CTS)

Table 1 : Properties of the CTS

	Steel
h [m]	0.036
a [m]	0.02
W [m]	0.08
Thickness t [m]	0.03
Toughness K_{IC} [MPa.m$^{1/2}$]	150
Young E [GPa]	210
Yield σ_p^0 [MPa]	400
Poisson ν [-]	0.3
Power law α [-]	1
Power law n [-]	10
Tearing T_R [-]	25

A compact test specimen (CTS) is submitted to a loading Q, see Figure 1. The displacement at the loading pins is denoted by u, and the crack mouth opening is denoted by v_m. The CTS is made of steel, see material properties in Table 1. The steel material follows the power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}} \tag{1}$$

In the elastic regime, following the norm ASME E399-90, the evolution of the crack mouth opening $v_{m,e}$, and of the stress intensity factor K in terms of the loading force Q and of the crack size a, were calibrated using the finite element method.

For the crack mouth opening, one has in elasticity

$$\begin{cases} U = \dfrac{1}{1+\sqrt{\dfrac{E' t v_{m,e}}{Q}}} \\ \dfrac{a}{W} = 1 - 4.5U + 13.157U^2 - 172.551U^3 + 879.944U^4 - 1514.671U^5 \end{cases} \tag{2}$$

and

$$\frac{v_{m,e}}{Q} = \frac{1}{E' t}\left[\frac{19.75}{\left(1-\frac{a}{W}\right)^2}\right]\left[0.5 + 0.192\frac{a}{W} + 1.385\left(\frac{a}{W}\right)^2 - 2.919\left(\frac{a}{W}\right)^3 + 1.842\left(\frac{a}{W}\right)^4\right] \tag{3}$$

For the stress intensity factor, one has in elasticity

$$K_I = \frac{Q}{tW^{\frac{1}{2}}} \frac{\left(2+\frac{a}{W}\right)\left(0.886+4.64\frac{a}{W}-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.6\left(\frac{a}{W}\right)^4\right)}{\left(1-\frac{a}{W}\right)^{\frac{3}{2}}} \tag{4}$$

In the elasto-plastic regime, the limit load in plane strain is given by

$$Q^0 = 1.455\sigma_p^0(W-a)t\left[\sqrt{\left(\frac{2a}{W-a}\right)^2+\frac{4a}{W-a}+2}-\left(\frac{2a}{W-a}+1\right)\right] \quad (5)$$

The ratio Q/Q^0, with Q the applied load, is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{Q}{Q^0}\right)^2}, \quad (6)$$

of the plastic zone

$$r_p = \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane strain state,} \quad (7)$$

that is used to evaluate the effective crack length

$$a_{\text{eff}} = a + \eta r_p. \quad (8)$$

The plastic part of the crack mouth opening displacement reads

$$v_p = \frac{\alpha\sigma_p^0}{E} a\, h_2\left(\frac{a}{W}, n\right)\left(\frac{Q}{Q^0}\right)^n \quad (9)$$

and the plastic part of the J-integral reads

$$J_p = \frac{\alpha{\sigma_p^0}^2}{E}(W-a)\, h_1\left(\frac{a}{W}, n\right)\left(\frac{Q}{Q^0}\right)^{n+1} \quad (10)$$

The function h_1 and h_2 are tabulated in the non-linear handbook and are reported in Table 2.

Table 2 : Tabulated plastic coefficient for the CTS

h_1, h_2 and h_3 for the compact specimen in plane strain.

a/b		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
a/b = 1/4	h_1	2.23	2.05	1.78	1.48	1.33	1.26	1.25	1.32	1.57
	h_2	17.9	12.5	11.7	10.8	10.5	10.7	11.5	12.6	14.6
	h_3	9.85	8.51	8.17	7.77	7.71	7.92	8.52	9.31	10.9
a/b = 3/8	h_1	2.15	1.72	1.39	0.970	0.693	0.443	0.276	0.176	0.098
	h_2	12.6	8.18	6.52	4.32	2.97	1.79	1.10	0.686	0.370
	h_3	7.94	5.76	4.64	3.10	2.14	1.29	0.793	0.494	0.266
a/b = 1/2	h_1	1.94	1.51	1.24	0.919	0.685	0.461	0.314	0.216	0.132
	h_2	9.33	5.85	4.30	2.75	1.91	1.20	0.788	0.530	0.317
	h_3	6.41	4.27	3.16	2.02	1.41	0.888	0.585	0.393	0.236
a/b = 5/8	h_1	1.76	1.45	1.24	0.974	0.752	0.602	0.459	0.347	0.248
	h_2	7.61	4.57	3.42	2.36	1.81	1.32	0.983	0.749	0.485
	h_3	5.52	3.43	2.58	1.79	1.37	1.00	0.746	0.568	0.368
a/b = 3/4	h_1	1.71	1.42	1.26	1.033	0.864	0.717	0.575	0.448	0.345
	h_2	6.37	3.95	3.18	2.34	1.88	1.44	1.12	0.887	0.665
	h_3	4.86	3.05	2.46	1.81	1.45	1.11	0.869	0.686	0.514
a/b → 1	h_1	1.57	1.45	1.35	1.18	1.08	0.950	0.850	0.730	0.630
	h_2	5.39	3.74	3.09	2.43	2.12	1.80	1.57	1.33	1.14
	h_3	4.31	2.99	2.47	1.95	1.79	1.44	1.26	1.07	0.909

You are requested

A) To evaluate the force Q leading to crack propagation considering the Linear Elastic Fracture Mechanics framework;
B) To evaluate the force Q leading to crack propagation considering the Small Scale Yielding solution (Linear Elastic Fracture Mechanics framework corrected with the effective crack size (8));
C) Considering that the force Q found in B) is applied to the sample, using the Non-Linear Fracture Mechanics framework, to evaluate the total J integral and to conclude on the crack initiation;
D) Considering that the force Q found in B) is applied to the sample, using the Non-Linear Fracture Mechanics framework, to assess the crack growth stability;
E) To comment on the validity of the developments.

Second question

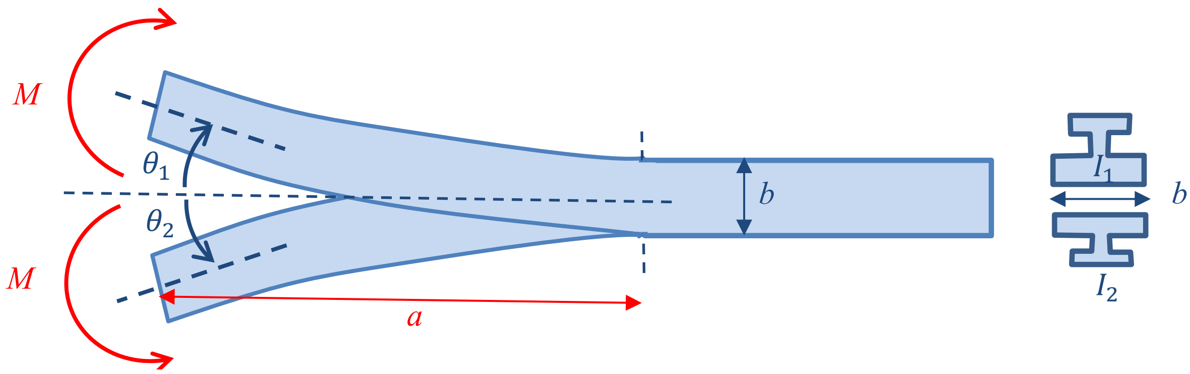

Figure 2: Two-beam system

Two beams of the same material, see Figure 2, with Young modulus E and Poisson ratio ν, and with respective symmetric cross sections of inertia I_1 and I_2, are welded on top of each other on their width b. The two beams deformations are unconstrained on a surface $a \times b$ and each beam extremity is submitted to a bending moment M yielding deflection angles θ_1 and θ_2, respectively for each beam.

You are requested

A) To evaluate the internal energy of the system subjected to the loading M in terms of I_1, I_2, b, M, E, a, and to deduce the energy release rate of the system in terms of I_1, I_2, b, M, E, a;
B) To evaluate the compliance of the system subjected to the loading M in terms of I_1, I_2, b, E, a, and to deduce the energy release rate of the system in terms of I_1, I_2, b, M, E, a;
C) To evaluate the stress intensity factor of the system;
D) To discuss the stability of the crack;
E) In the case of rectangular cross sections $h_1 \times b = 0.01 \times 0.02\ \mathrm{m}^2$ and $h_2 \times b = 0.005 \times 0.02\ \mathrm{m}^2$, for the values of $E = 71$ GPa, $\nu = 0.22$, $a = 0.25$ mm and a critical stress intensity factor of $30\ \mathrm{MPa} \cdot \mathrm{m}^{\frac{1}{2}}$, to evaluate the maximum bending moment M that can be applied.

<u>Reminder of beam bending theory</u>

For a beam under pure bending (constant bending moment M), see Figure 3:

1) The curvature κ is constant and is related to the neutral axis deflection u and to the bending moment M through
$$\frac{1}{\kappa} = \frac{\partial^2 u(x)}{\partial x^2} = \frac{M}{EI} = cst \qquad (1)$$
where E is the Young's modulus and I the cross-section inertia.
2) The deflection angle reads
$$\theta(x) = \frac{\partial u(x)}{\partial x} \qquad (2)$$
3) The inertia with the **origin** taken at the **section center of inertia,** reads
$$I = \int_A z^2 dzdy; \qquad (3)$$
4) The stress distribution of a symmetric profile of uniform Young modulus, with the **origin** taken at the **section center of inertia,** reads
$$\sigma_{xx} = \frac{Mz}{I} \qquad (4)$$
5) The internal energy **per unit beam length** of a symmetric profile of uniform Young's modulus reads
$$e_{int} = \frac{M^2}{2EI} \qquad (5)$$

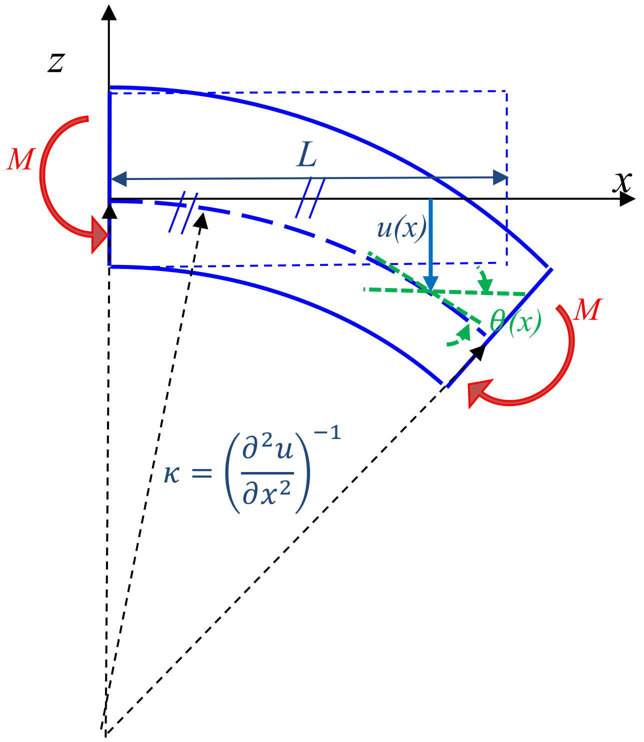

Figure 3: Beam under pure bending