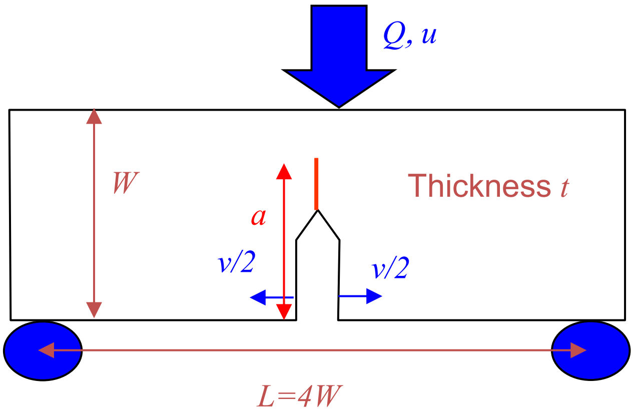
Fracture mechanics, Damage & Fatigue (MECA0058-1)
January 2020

First question



Steel SENB specimen	
Width W [m]	0.08
Thickness t [m]	0.03
Crack length a [m]	0.03
Young's modulus E [GPa]	210
Yield σ_p^0 [MPa]	400
Poisson ratio ν [-]	0.3
Hardening exponent n [-]	3
Hardening parameter α [–]	1
Toughness K_{IC} [MPa$\sqrt{m}$]	150

Single Edge Notched Bending (SENB) specimens are typically used in order to measure the critical J value of a ductile material. We consider the specimen illustrated in the Figure, with the dimensions reported in the Table. The results from a tension test on the material are also reported in this table. The material follows a power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}}. \tag{1}$$

In the elastic regime, following the norm ASME E399-90, the stress intensity factor reads

$$K_I = \frac{QL}{tW^{\frac{3}{2}}}\ 3\sqrt{a/W}\ \frac{1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left(2.15-3.93\frac{a}{W}+2.7\left(\frac{a}{W}\right)^2\right)}{2\left(1+\frac{2a}{W}\right)\left(1-\frac{a}{W}\right)^{\frac{3}{2}}}. \tag{2}$$

In the elasto-plastic regime, the limit load in plane strain is given by

$$Q^0 = 0.728\sigma_p^0\frac{2t(W-a)^2}{L}. \tag{3}$$

The ratio Q/Q^0, with Q the applied load, is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{Q}{Q^0}\right)^2}, \tag{4}$$

of the plastic zone

$$r_p = \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \quad \text{for plane strain state,} \tag{5}$$

that is used to evaluate the effective crack length

$$a_{\text{eff}} = a + \eta r_p. \tag{6}$$

The plastic part of the J-integral reads

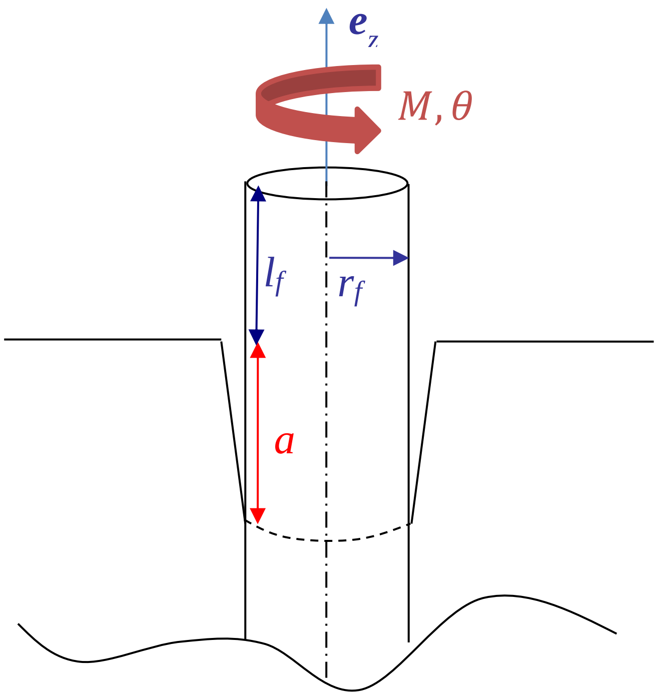
$$J_p = \frac{\alpha {\sigma_p^0}^2}{E} (W - a)\, h_1\left(\frac{a}{W}, n\right) \left(\frac{Q}{Q^0}\right)^{n+1}. \tag{7}$$

The function h_1 is tabulated in the non-linear handbook.

You are requested to evaluate the maximum force Q leading to crack propagation. Toward this aim, you are requested to follow the steps

1) To evaluate the force Q leading to crack propagation considering the Linear Elastic Fracture Mechanics framework;
2) To evaluate the force Q leading to crack propagation considering the Small Scale Yielding solution (Linear Elastic Fracture Mechanics framework corrected with the effective crack size (6));
3) Using the Non-Linear Fracture Mechanics framework, to verify whether the solution obtained in 2) is the correct one;
4) Following the conclusion of 3), to do one correction (one iteration) on the force Q to find the value leading to crack propagation with the Non-Linear Fracture Mechanics framework;
5) To explain the differences in the results obtained with the three different methods and to comment on the validity of the developments.

Second question



Fibers	
r_f [mm]	0.01
Longitudinal Young's modulus E_f [GPa]	200
Transverse shear modulus μ_f [GPa]	16
Tensile strength σ_f^0 [MPa]	1600

Matrix	
Young's modulus E_m [GPa]	3.2
Shear modulus μ_m [GPa]	1.185
Tensile strength σ_m^0 [MPa]	80

In order to characterize the bonding in composite materials, a twisting of a partially debonded semi-infinite single fiber embedded in a semi-infinite matrix is considered; properties are reported in the above Tables. Some assumptions are made:

1) The undebonded fiber emerges from the matrix on a length l_f, see Figure here above;
2) An initial part of the fiber is debonded on a length a, all around the fiber circumference;
3) As a consequence of the semi-infinite nature of the matrix, one can assume that both the stress and strain states below the debonded part vanish;
4) The twist rate $\theta_{,z} = \frac{\partial\theta}{\partial z} = \frac{\theta}{l_f+a}$ is assumed to be constant on the free and debonded parts of the fiber (i.e. on the length $l_f + a$), with θ the twist angle at fiber extremity;
5) A torque M is applied at the free extremity of the fiber, which is assumed to be constant on the free and debonded parts of the fiber (i.e. on the length $l_f + a$);
6) Linear elasticity is assumed.

You are requested to evaluate the critical energy release rate with the following methodology

1) To determine the fracture mode and to express the surface of the associated crack;
2) To derive the relationship between the twist-angle θ and the torque M;
3) To evaluate the internal energy (in Joules, not the density) of the fiber in terms of the symbols given in the Figure here above and in the Tables here above (no values substitution):
    a. In the case of a given twist-angle θ (and not in terms of the torque M);
    b. In the case of a given torque M (and not in terms of the twist-angle θ);
4) To evaluate the work of external forces (in Joules) in terms of the symbols given in the Figure here above and in the Tables here above (no values substitution):
    a. In the case of a given twist-angle θ (and not in terms of the torque M);
    b. In the case of a given torque M (and not in terms of the twist-angle θ);

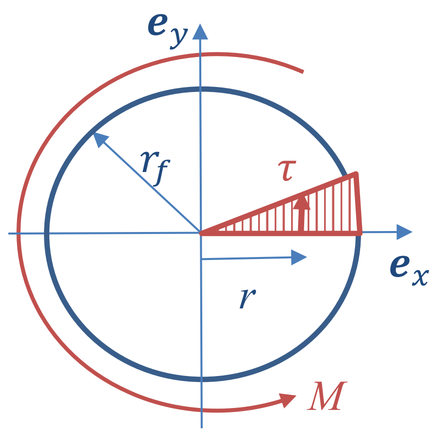
5) To express the energy release rate in terms of the symbols given in the Figure here above and in the Tables here above (no values substitution):
   a. In the case of a given twist-angle θ (and not in terms of the torque M);
   b. In the case of a given torque M (and not in terms of the twist-angle θ);
6) If the crack starts to growth for a torque of $M = 10^{-6}$ Nm, to evaluate the value of the critical energy release rate related to the failure mode.

Tips

1) As a reminder, the torsion equations in a circular rod are
   a. Shear stress τ – shear angle γ – shear strain ε_{xy} relationships

$$\tau = \mu\gamma = 2\mu\varepsilon_{xy} \text{ and } \gamma = r\,\theta_{,z} = r\frac{\partial\theta}{\partial z}, \qquad (1)$$

with μ the shear modulus and $\theta, z = \frac{\partial\theta}{\partial z}$ the twist rate (uniform on the section).



   b. Twist moment M – shear stress τ relationship

$$M = \int_S r\tau\, dS\,, \qquad (2)$$

with S the cross section of rod

   c. Internal energy density (in Joules **per unit volume**) under pure shearing

$$U = \frac{1}{2}\boldsymbol{\sigma}:\boldsymbol{\varepsilon} = \frac{1}{2}\left(\tau\varepsilon_{xy} + \tau\varepsilon_{yx}\right) = \frac{1}{2}\gamma\tau. \qquad (3)$$

2) As a reminder, in the case of an axisymmetric distribution, the integral in polar coordinates reads

$$\int_S \cdot\, dS = 2\pi\int_0^{r_f} \cdot\, r\, dr. \qquad (4)$$