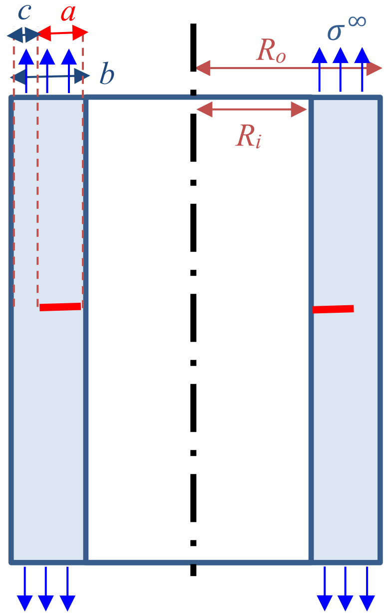
Fracture mechanics, Damage & Fatigue (MECA0058-1)
January 2019

First question



	Steel
Internal radius R_i [m]	1.20
External radius R_o [m]	1.32
Crack a [m]	0.03
Young E [GPa]	210
Yield σ_p^0 [MPa]	620
Poisson ν [-]	0.3
Hardening exponent n [-]	10
Hardening parameter α [–]	1
Toughness K_{IC} [MPa$\sqrt{m}$]	40 (low temperature) 180 (room temperature)

One wants to assess the safety of a cylinder under uniaxial tension in which an extended internal circumferential crack has been found. The dimensions and material properties are reported in the Table. The elasto-plastic material is idealized by a power law

$$\sigma_e = \sigma_p^0 \left(\frac{E\varepsilon}{\alpha\sigma_p^0}\right)^{\frac{1}{n}}. \tag{3}$$

In the elastic regime, the stress intensity factor K_I is given in terms as

$$K_I = \sigma^\infty \sqrt{\pi a}\, F\left(\frac{a}{b}, \frac{b}{R_i}\right), \tag{4}$$

where the correction F is tabulated in the non-linear fracture mechanics handbook.

In the elasto-plastic regime, the limit load, is given by

$$P_0 = \frac{2}{\sqrt{3}}\pi\sigma_p^0(R_o^2 - R_c^2) \text{ with } R_c = R_i + a. \tag{6}$$

The ratio P/P_0, with $P = \sigma^\infty\pi(R_o^2 - R_i^2)$ the applied load, is used to evaluate the fraction

$$\eta = \frac{1}{2}\frac{1}{1+\left(\frac{P}{P_0}\right)^2}, \tag{7}$$

of the plastic zone

$$r_p = \frac{1}{3\pi}\left[\frac{n-1}{n+1}\right]\left(\frac{K_I}{\sigma_p^0}\right)^2 \text{ for plane strain state,} \tag{8}$$

that is used to evaluate the effective crack length

$$a_{\mathrm{eff}} = a + \eta r_p. \tag{9}$$

The plastic part of the J-integral is obtained from

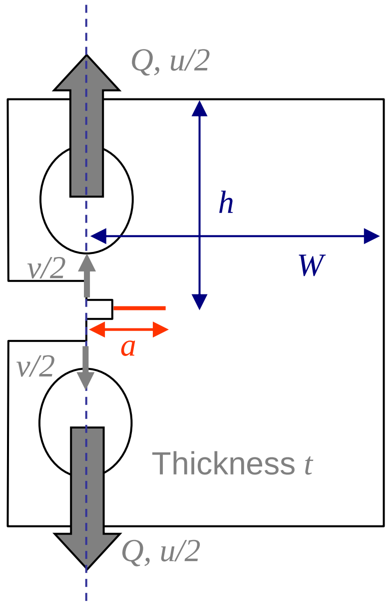
$$J_p = \frac{\alpha {\sigma_p^0}^2}{E} c \frac{a}{b} \; h_1\left(\frac{a}{b}, n; \frac{b}{R_i}\right) \left(\frac{P}{P_0}\right)^{n+1}, \tag{10}$$

where the function h_1 is tabulated in the non-linear fracture mechanics handbook.

You are requested to assess the crack propagation of the sample on which an axial stress σ^∞ is applied when working at low temperature and at room temperature. Toward this aim, you are requested to follow the steps

A) To determine the limit stress σ^∞ yielding failure using the Linear Elastic Fracture Mechanics (LEFM) approach (for both environmental conditions) ;
B) To verify (for both environmental conditions) this limit stress using the Small Scale Yielding (SSY) solution (Linear Elastic Fracture Mechanics framework corrected with the effective crack size (9)), and, if needed, to perform one iteration on this stress and to deduce an approximation of the limit stress σ^∞ yielding failure using the SSY approach;
C) To verify (for both environmental conditions) whether the stress obtained using the SSY approach in B) is safe in the context of the Non-Linear Fracture Mechanics (NLFM) framework, and, if needed, to perform one iteration on this stress σ^∞ in order to find the correct limit pressure yielding failure using NLFM;
D) To explain the differences in the results obtained with the three different methods and for both environmental conditions, and to comment on the validity of the developments.

<u>Second question</u>



	Aluminum Alloy
h [m]	0.045
W [m]	0.075
Thickness t [mm]	25
Young E [GPa]	71
Yield σ^0 [MPa]	430
Poisson ν [-]	0.33

Toughness of aluminum alloy is tested measured using the Compact Tension (CT) test specimen, see Figure and Table here above.

The test was calibrated using the Finite Element method by considering cracked bodies of different lengths and loaded with Q = 12 kN. Results are reported in the Table beside.

A CT specimen is first pre-cracked by fatigue and then loaded until Q=12 kN, without propagating the crack. The displacement measured at loading pin is u=0.3215 mm. Finally, the specimen is loaded up to failure and a load Q=24.5 kN is found at crack propagation.

CT calibration test for $Q = 12$ kN	
a [mm]	u [mm]
22.5	0.150
26.25	0.180
30	0.218
33.775	0.267
37.5	0.330
41.25	0.415
45	0.536
48.75	0.715
52.5	0.997

You are requested

A) To evaluate the critical energy release rate of the tested aluminum alloy;
B) To deduce the toughness of the alloy;
C) To justify the validity of the measurements.