Aircraft Structures
Design Example


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Goal

- From
  - Specifications
  - Preliminary design
- **Design the rear fuselage of a two-seat trainer/semi-aerobatic aircraft**
  - Stringers and Frames
  - Skin thickness
Goal (2)

• Specifications and preliminary design
  – Permit to calculate forces acting on the airplane
  – Fuselage has to resist the induced loading

• Step 1: Forces acting on the airplane are
  – Aerodynamic forces (→ flight envelope)
  – Thrust
  – Self-weight

• Step 2: The rear fuselage consists into
  – Stringers
  – Frames
  – Skin
  – Rivets

• All these elements must be calculated to resist the stresses induced by the forces (bending moments, torques and shearing)
Step 1 – Loading acting on the rear fuselage

Aerodynamic forces and self-weight
Specifications: flight envelope

- Required flight envelope
  - $n_1 = 6.28$
  - $V_D = 183.8 \text{ m/s}$
  - $V_C = 0.8V_D = 147.0 \text{ m/s}$
  - $n_2 = 0.75 \ n_1 = 4.71$
  - $n_3 = 0.5 \ n_1 = 3.14$

- Representative values for an aerobatic aircraft?

- Further requirements
  - Additional pitching acceleration allowed at any point of envelope
    
    $$ \left(20 + \frac{475}{W}\right) \frac{n}{V} \ [\text{rad/s}^2] $$
    
    Weight $W$ in [kN]
  - For asymmetric flight: angle of yaw allowed at any point of envelope
    
    $$ \psi = 0.7n_1 + \frac{457.2}{V_D} \ [\text{degrees}] $$
    
    The angle of yaw increases the overall pitching moment coefficient of the aircraft by $0.0015 / \text{degree of yaw}$
Data - Aircraft

- **Fully loaded aircraft**
  - Weight $W = 37.43 \text{ kN}$
  - Moment of inertia about center of gravity $G$: $I_\theta = 22\,235 \text{ kg\cdot m}^2$
  - Positions of $G$ and of the body drag centers known (see figure)

- **Body drag coefficients**
  - $C_{D,B}$ (engine on) = 0.01583
  - $C_{D,B}$ (engine off) = 0.0576

- **Engine**
  - Maximum horse power: 905 hp
  - Propeller efficiency: 90 %
Data - Wing

- **Geometry/Aerodynamics**
  - Span $b=14.07\ m$
  - Gross area $S = 29.64\ m^2$
  - Aerodynamic mean chord $c = 2.82\ m$
  - Lift and drag coefficients
    - See picture
- **Pitching moment**
  - $C_M = -0.238\ C_L$
  - Angle of incidence
    - Between fuselage axis and root chord
    - $-1.5^\circ$
    - Additional pitching moment coefficient
      - $-0.036$
Data - Tailplane

- **Geometry/Aerodynamics**
  - Span $b_t = 6.55$ m
  - Gross area $S_t = 8.59$ m$^2$
  - Aerodynamic centre $P$

- **Yaw → asymmetry of the slipstream → asymmetric load on the tailplane**
  - Resulting torque
    $$\frac{0.00125\rho V^2 S_t b_t \psi}{\sqrt{1 - M^2}}$$
    - $M$ is the mach number
    - $\psi$ is in degree

Section CC

Loading induced by yaw on the tailplane
Data - Fin

- **Geometry/Aerodynamics**
  - Height $h_F = 1.65$ m
  - Area $S_F = 1.80$ m²
  - Aspect-ratio $A_F = h_F^2/S_F = 1.5$
  - Lift-curve slope $a_1$
    - $a_1 = \frac{5.5A}{A + 2}$
    - With $A$ the aspect ratio of an equivalent wing: $A = \frac{(2h_F)^2}{2S_F} = 2\frac{h_F^2}{S_F} = 2A_F = 3$
    - $a_1 = 3.3$

- **In yawed flight of angle $\psi$**
  - Fin has also an incidence of $\psi$
  - Aerodynamic loading
    - $F_{\text{fin}} = \frac{1}{2} \rho V^2 S_F a_1 \psi$
    - Where $\psi$ is in rad

- **Centre of pressure of the fin**
  - 1.13 m above the axis of the fuselage
  - 3.7 m aft section AA
Initial calculation – Flight envelope

**Values**
- $n_1 = 6.28$
- $V_D = 183.8 \text{ m/s}$
- $V_C = 0.8V_D = 147.0 \text{ m/s}$
- $n_2 = 0.75 n_1 = 4.71$
- $n_3 = 0.5 n_1 = 3.14$

**$V_S^A$ (stalling speed on point A)?**
- Assumption: Lift from wing only

- From $C_L = \frac{nW}{\frac{1}{2} \rho V^2 S}$

  $$V_s = \left( \frac{2nW}{\rho S C_{L, \text{max}}} \right)^{1/2}$$

  $$= \sqrt{n} \left( \frac{2 \times 37.43 \times 10^3}{1.226 \times 29.64 \times 1.38} \right)^{1/2}$$

  $$= 38.6 \sqrt{n}$$

  $$V_S^A = 38.6 \sqrt{n_1} = 38.6 \sqrt{6.28} = 96.7 \text{ m/s}$$
Balancing out calculations

• Loads are calculated for various critical points of the flight envelope
  – Case A
    • Point A
    • Engine on
  – Case A*
    • Point A
    • Engine off
  – Case C
    • Point C
    • Engine off
  – Case D_1
    • Point D_1
    • Engine off
  – Case D_2
    • Point D_2
    • Engine off
Balancing out calculations – Case A – point A/engine on

- **Data (point A)**
  - $n_1 = 6.28$
  - $V = V_s^A = 96.7 \text{ m/s}$
  - $C_{L,\text{max}} = 1.38$
  - $\alpha_{L,\text{max}}^0 = 18^\circ$
  - $C_{D,W} = 0.149$
Balancing out calculations – Case A – point A/engine on

- From measures on drawing
  - Or math, see next slide
Balancing out calculations – Case A – point A/engine on

- Math

\[ d = \sqrt{0.45^2 + 0.98^2} = 1.078 \text{ m} \]

\[ \alpha = \arctan \frac{0.45}{0.98} = 24.7^\circ \]

\[ l = d \cos (24.7^\circ - 18^\circ) = 1.07 \text{ m} \]
Balancing out calculations – Case A – point A/engine on

• Methodology
  – Forces that can be directly calculated
    • Trust: $T$ from engine
    • $nW$: $n$ & $W$ are known
    • Drag (body $B$, wings $W$) from $V$ and drag coefficients
    • Pitching moment $M$ from the pitching moment coefficients and the angle of yaw $\psi$
    • Pitching moment acceleration
  – Force equilibrium, and moment equilibrium
    • 2 Equations involving $P$, $L$
  – Find other forces acting on the rear fuselage
    • Tailplane torque, fin load, fin load torque, total torque
Balancing out calculations – Case A – point A/engine on

- **Trust of the engine**
  - **Data**
    - Maximum horse power 905
    - Propeller efficiency $\eta = 90\%$
    - $1\ [\text{hp}] = 746\ [\text{W}]$
  - **Thrust**
    - $T = \frac{\eta \cdot hp \cdot 746}{V} = \frac{0.9 \times 905 \times 746}{96.7} = 6284\ [\text{N}]$
Balancing out calculations – Case A – point A/engine on

- **Weight**
  - **Data**
    - Fully loaded weight \( W = 37.43 \text{ kN} \)
  - **Loaded weight**
    - \( nW = 6.28 \times 37.43 \times 10^3 = 235060 \text{ [N]} \)
Balancing out calculations – Case A – point A/engine on

• Drag
  – Data
    • $C_{D,W} = 0.149$
    • $C_{D,B} = 0.01583$
    • $V = 96.7 \text{ m/s}$
    • $S = 29.64 \text{ m}^2$
  – Wing drag
    • $D_W = \frac{1}{2} C_{D,W} \rho V^2 S = 26091 \text{ [N]}$
  – Body drag
    • $D_B = \frac{1}{2} C_{D,B} \rho V^2 S = 2690 \text{ [N]}$
Balancing out calculations – Case A – point A/engine on

• Pitching moment
  – Data
    • \( n_1 = 6.28 \)
    • \( V_D = 183.8 \text{ m/s} \)
    • \( V = 96.7 \text{ m/s} \)
    • Fully loaded weight \( W = 37.43 \text{ kN} \)
  – Pitching moment coefficient
    • Maximum for the maximum yaw angle allowed during maneuver
    • Maximum angle of yaw allowed
      \[
      \psi = 0.7n_1 + \frac{457.2}{V_D} \quad [\text{degrees}]
      \]
      \[
      = 0.7 \times 6.28 + \frac{457.2}{183.8} = 6.9 \text{ [°]}
      \]
    • Pitching moment coefficient
      \[
      C_M = -0.238C_L - 0.036 - 0.0015\psi
      \]
      \[
      = -0.238 \times 1.38 - 0.036 - 0.0015 \times 6.9 = -0.375
      \]
  – Maximum pitching acceleration allowed
    • \( \ddot{\theta} = \left( 20 + \frac{475}{W} \right) \frac{n}{V} = \left( 20 + \frac{475}{37.43} \right) \frac{6.28}{96.7} = 2.12 [\text{rad/s}^2] \)
Balancing out calculations – Case A – point A/engine on

• Pitching moment (2)
  – Data
    • $V = 96.7\, \text{m/s}$
    • $S = 29.64\, \text{m}^2$
    • MAC: $c = 2.82\, \text{m}$
  – Pitching moment coefficient $C_M = -0.375$
  – Pitching moment
    
    \[
    M = C_M \frac{1}{2} \rho V^2 Sc
    \]
    
    \[
    = -0.375 \frac{1}{2} \times 1.226 \times 96.7^2 \times 29.64 \times 2.82 = -179,669\, [\text{Nm}]
    \]
Balancing out calculations – Case A – point A/engine on

- **Moments about G**

  \[ 1.07L - 0.18T + 0.04D_B - 0.12D_W - 6.28P + M = I_\theta \times \dot{\theta} \]

  \[ 1.07L - 0.18 \times 6284 + 0.04 \times 2690 - 0.12 \times 26901 - 6.28P - 179669 = \frac{22235 \times 2.12}{5.86P = L - 216090} \]

- **Vertical equilibrium**

  \[ L + P = nW - T \sin(18^\circ - 1.5^\circ) = 235060 - 6284 \sin 16.5^\circ = 233275 \]

  \[ L + P = 233275 \]
Balancing out calculations – Case A – point A/engine on

- 2 equations, 2 unknowns
  - $5.86P = L - 216090$
  - $L + P = 233275$

\[
P = 2505 \text{ [N]}
L = 230770 \text{ [N]}
\]

- Remark: for other cases, $\alpha$ is not known
  - Requires iterations on $\alpha$ in order to determine $C_L$
  - Should also be done here as $V_S$ was computed using $C_L$ of wing (10% error)
Balancing out calculations – Case A – point A/engine on

• Tailplane torque
  – Due to asymmetric slipstream (yaw)
  – Data
    • $b_t = 6.55 \text{ m}$
    • $S_t = 8.59 \text{ m}^2$
    – $M_{\text{tail}} = \frac{0.00125}{\sqrt{1 - \text{Mach}^2}} \rho V^2 S_t b_t \psi$
    – $M_{\text{tail}} = \frac{0.00125}{\sqrt{1 - (96.7/340.8)^2}} \times 1.226 \times 96.7^2 \times 8.59 \times 6.55 \times 6.9$
    – $M_{\text{tail}} = 5802 \text{ [N} \cdot \text{m}]$

• Fin load
  – Due to yaw
  – Data
    • $S_F = 1.80 \text{ m}^2$
    • $a_1 = 3.3$
    – $F_{\text{fin}} = \frac{1}{2} \rho V^2 S_F a_1 \psi$
    – $F_{\text{fin}} = \frac{1}{2} \times 1.226 \times 96.7^2 \times 1.8 \times 3.3 \times (6.9 \frac{\pi}{180}) = 4100 \text{ [N]}$
Balancing out calculations – Case A – point A/engine on

- Total torque (rear fuselage)
  
  \[ M_{\text{fus}} = M_{\text{tail}} + F_{\text{fin}} \times 1.13 = 5802 + 4100 \times 1.13 = 10\,435 \, [Nm] \]
Balancing out calculations - End

- **Summary**
  - Other cases follow the same method

<table>
<thead>
<tr>
<th>Case</th>
<th>$n$ [-]</th>
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</table>
Fuselage loads – Preliminary choices

- **Fuselage**
  - Stringers
  - Frames
  - Skin

- **Frames**
  - Un-pressurized fuselage $\leftrightarrow$ frames will not support significant loads
  - Frames are required to maintain the fuselage shape $\leftrightarrow$ nominal in size

- **Combination of stringer and skin will resist self-weight and aerodynamic loads**
  - Shear forces
  - Bending moments
  - Torques
Fuselage loads – Preliminary choices

• Geometrical data

• Circular cross-section
  – Simple to fabricate
  – Simple to design
  – Will meet the design requirements

• Possible arrangement
  – 24 stringers arranged symmetrically spaced at
    • Section AA: ~168 mm
    • Section BB: ~96 mm
  – All stringers have the same cross-sectional area
Fuselage loads – Data

- **Material:** Aluminum alloy
  - Stingers & skin
  - 0.1 % proof stress: 232.5 [N/mm²] = 232.5 [MPa]
  - Shear strength: 145.5 [N/mm²] = 145.5 [MPa]

- **Self-weight: Assumptions**
  - Fuselage weight is from 4.8 to 8.0 % of the total weight
  - Tailplane/fin assembly is from 1.2 to 2.5 % of the total weight
  - Half of the fuselage weight is aft of the section AA

\[
W_{\text{rear fuselage}} = \frac{1}{2} \times 37.43 \times 10^3 \times 6.4\% = 1198 [N]
\]
\[
W_{\text{tailplane+fin}} = 37.43 \times 10^3 \times 1.8\% = 674 [N]
\]

- The weight distribution varies proportionally to the skin surface area
• Assumptions on geometry
  – Rear fuselage is uniformly tapered

\[ \text{Area}_{skin} = \frac{1}{2} \pi (D_{max} + D_{min}) L \]

\[ = \frac{1}{2} \pi (1.28 + 0.1) 4.57 = 9.91 \, [m^2] \]

– CC is a section midway AA and BB.
– The center of gravity of the tailplane/fin assembly has been estimated to be 4.06 m from the section AA on a line parallel to the fuselage centre line
Fuselage loads – Data

- **Data**
  - Weight of rear fuselage: 1198 [N]
  - Skin area of rear fuselage: 9.91 [m²]

- **Self-weight / m of the fuselage**

  \[
  \text{weight/m} = \frac{W_{\text{rear fuselage}} \times \pi \times D}{Area_{\text{skin}}}
  \]

  \[
  \begin{align*}
  \text{weight/m}_{AA} &= \frac{1}{9.91} \times 1198 \times \pi \times 1.28 = 486.1 \text{ [N/m]} \\
  \text{weight/m}_{CC} &= \frac{1}{9.91} \times 1198 \times \pi \times 1.01 = 383.6 \text{ [N/m]} \\
  \text{weight/m}_{BB} &= \frac{1}{9.91} \times 1198 \times \pi \times 0.73 = 277.2 \text{ [N/m]} \\
  \text{weight/m}_{DD} &= \frac{1}{9.91} \times 1198 \times \pi \times 0.1 = 38.0 \text{ [N/m]}
  \end{align*}
  \]
Fuselage loads – Shear force and bending moment due to self-weight

- **Self-weight induces**
  - Shear forces : SF
  - Bending moments : BM

- **SF and BM are calculated by equilibrium (‘MNT’)**

![Diagram of fuselage loads with dimensions and forces](image)

- Shear force : \( SF \)
- Bending moment : \( BM \)

\[
\text{Resultant} = W_{\text{rear fuselage}} = 1198 \text{ [N]}
\]
Fuselage loads – Shear force and bending moment due to self-weight

- Transform self-weight into
  - A triangular repartition: $q_1(x)$
  - And a constant linear force: $q_2$

\[ q_1(x) \]

\[ W_{\text{tailplane+fin}} = 674 \, [N] \]

\[ q_2 = 38 \, [N/m] \]

\[ 4.57 \, m \]

\[ 4.06 \, m \]
Fuselage loads – Shear force and bending moment due to self-weight

- **Effect of load factor**

- The self weight is multiplied by the load factor $n$
  - Forces are not applied in the cross section plane
  - Forces are $(\alpha-1.5^\circ)$ –inclined with this section
    - Will be multiplied by $\cos(\alpha-1.5^\circ)$ later
  - Rigorously, an axial loading should also be considered
- Distance along fuselage axis is multiplied by $\cos(\alpha-1.5^\circ)$
  - When computing bending moment
- We do not compute the fuselage compression
  - Should be done and risk of buckling avoided
Fuselage loads – Shear force and bending moment due to self-weight

- Reactions at section AA

\[ SF_A = n \times (Q_1 + Q_2 + W_{tailplane+fin}) \]
\[ = n \times \left( \frac{1}{2} \times 448.1 \times 4.57 + 38.0 \times 4.57 + 674 \right) = 1872n \quad [N] \]

\[ W_{tailplane+fin} = 674 \quad [N] \]

\[ q_2 = 38 \quad [N/m] \]

\[ BM_A = n \cos(\alpha - 1.5^\circ) \times \left[ 4.06 \times W_{tailplane+fin} + \frac{1}{3} \times 4.57 \times Q_1 + \frac{1}{2} \times 4.57 \times Q_2 \right] \]
\[ = 4693n \cos(\alpha - 1.5^\circ) \quad [Nm] \]
Fuselage loads – Shear force and bending moment due to self-weight

- Reactions at section CC

\[ SF_C = n \times (Q_1 + Q_2 + W_{\text{tailplane+fin}}) \]
\[ = n \times \left( \frac{1}{2} \times 345.6 \times 3.51 + 38.0 \times 3.51 + 674 \right) = 1409n \ [N] \]

\[ BM_C = n \cos (\alpha - 1.5^\circ) \times \left[ (4.06 - 1.065) \times W_{\text{tailplane+fin}} + \frac{1}{3} \times 3.51 \times Q_1 + \frac{1}{2} \times 3.51 \times Q_2 \right] \]
\[ = 2959n \cos (\alpha - 1.5^\circ) \ [Nm] \]
Fuselage loads – Shear force and bending moment due to self-weight

- **Reactions at section BB**

\[
\begin{align*}
SF_B &= n \times (Q_1 + Q_2 + W_{\text{tailplane+fin}}) \\
&= n \times \left(\frac{1}{2} \times 239.2 \times 2.44 + 38.0 \times 2.44 + 674\right) = 1059n \ [N] \\
BM_B &= n \cos (\alpha - 1.5^\circ) \times \left[(4.06 - 2.13) \times W_{\text{tailplane+fin}} + \frac{1}{3} \times 2.44 \times Q_1 + \frac{1}{2} \times 2.44 \times Q_2\right] \\
&= 1651n \cos (\alpha - 1.5^\circ) \ [Nm]
\end{align*}
\]
Total shear forces, bending moments and torque

- Resultant forces in each section
  - Example section AA

\[
\begin{align*}
T_y^{AA} &= -F_{\text{fin}} \\
T_z^{AA} &= (SF_A - P) \cos (\alpha - 1.5^\circ) \\
M_y^{AA} &= BM_A - P \times 3.47 \text{ m} \cos (\alpha - 1.5^\circ) \\
M_z^{AA} &= F_{\text{Fin}} \times 3.7 \text{ m} \\
M_{x,\text{section}} &= -M_{\text{tail}} - 1.13F_{\text{fin}} = -M_{\text{fus}}
\end{align*}
\]
- **Total shear forces, bending moments and torque**

**Resultant forces in each section (2)**

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- **Example case A, section AA**
  - $SF_A = 1872 \, n \, [N] \, & \, BM_A = 4693 \, n \, \cos(\alpha-1.5^\circ) \, [Nm]$

\[
\begin{align*}
T_{y}^{AA} &= -F_{\text{fin}} = -4100 \, \text{N} \\
T_{z}^{AA} &= (SF_A - P) \cos (\alpha - 1.5^\circ) = (1872 \times 6.28 - 2505) \cos 16.5^\circ = 8580 \, \text{N} \\
M_{y}^{AA} &= BM_A - P \times 3.47 \, \text{m} \ \cos (\alpha - 1.5^\circ) \\
&= (4693 \times 6.28 - 2505 \times 3.47) \cos 16.5^\circ = 20774 \, \text{Nm} \\
M_{z}^{AA} &= F_{\text{Fin}} \times 3.7 \, \text{m} = 4100 \times 3.7 = 15174 \, \text{Nm} \\
M_{\text{section}} &= -M_{\text{tail}} - 1.13F_{\text{fin}} = -M_{\text{fus}} = -10439 \, \text{Nm}
\end{align*}
\]
Total shear forces, bending moments and torque

- **Table of sections loading**
  - $SF_A = 1872 \text{ } n \text{ [N]}$ & $BM_A = 4693 \text{ } n \cos(\alpha-1.5^\circ) \text{ [Nm]}$
  - $SF_C = 1409 \text{ } n \text{ [N]}$ & $BM_C = 2959 \text{ } n \cos(\alpha-1.5^\circ) \text{ [Nm]}$
  - $SF_B = 1059 \text{ } n \text{ [N]}$ & $BM_B = 1651 \text{ } n \cos(\alpha-1.5^\circ) \text{ [Nm]}$

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Step 2 – Design of the rear fuselage

Stringers, frames, skin and rivets
Fuselage design calculation – Material and approach

• 2 types of design are possible
  – Elastic design
    • Allowed stress = 0.1% proof stress / $s$
    • $s$ : safety factor = 1.5
  – Ultimate load design
    • Ultimate stresses
    • Actual load multiplied by an ultimate load factor

• For linear systems : both designs give the same results

• We use the elastic design as 0.1 proof stress is known

\[
\sigma_{max} = \frac{\sigma_{0.1\%}}{s} = \frac{232.5}{1.5} = 155 \text{ MPa}
\]
\[
\tau_{max} = \frac{\tau_{strength}}{s} = \frac{145.5}{1.5} = 97 \text{ MPa}
\]
Stringers section: Data

• Frames
  – Un-pressurized fuselage frames will not support significant loads
  – Frames are required to maintain the fuselage shape nominal in size

• Circular cross-section
  – 24 stringers arranged symmetrically and spaced at around
    • Section AA – $l \sim 168$ mm
    • Section BB – $l \sim 96$ mm
  – All stringers have the same cross-sectional area
Stringers section

- **Direct stress**
  - Induced by $M_z$ and $M_y$
  - Obtained from $\sigma_{xx} = \frac{M_y}{I_{yy}} \hat{z} - \frac{M_z}{I_{zz}} y$ (as $I_{yz} = 0$)

- **Unknown**
  - $B$: the area of the stringers in a section
  - $B$ should be chosen such that
    \[ \sigma_{xx} \leq \sigma_{max} = 155 \text{ MPa} \]

- **Second moments of area**
  \[
  I_{zz} = I_{yy} = \sum_{i=1}^{24} B \times z_i^2
  \]
  \[
  = 4BD^2 \left( 0.1294^2 + 0.25^2 + 0.353^2 + 0.433^2 + 0.483^2 + \frac{0.5^2}{2} \right)
  \]
  \[
  = 3BD^2
  \]
**Stringers section**

- **Values of** $M_x$ **and** $M_y$
  
  - **Stress**
    
    \[ \sigma_{xx} = \frac{M_y}{I_{yy}} \bar{z} - \frac{M_z}{I_{zz}} y \]
  
  - **Worst case**
    
    - Case D1 (dive)
    - $M_z$ and $M_y$ have same sign
    - $y$ and $z$ of opposite sign

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Stringers section

- Calculate $B\sigma_{xx}$
  
  $\sigma_{xx} = \frac{M_y}{3BD^2}z - \frac{M_z}{3D^2}y$

  $B\sigma_{xx} = \frac{M_y}{3D^2}z - \frac{M_z}{3D^2}y$

- For each
  - Section
    - $M$ & $D$ change
  - Stringer
    - $y$ & $z$ change
  - Determine minimal value of stringers’ area $B$ such that

  $\sigma_{xx} \leq \sigma_{max} = 155 \text{ MPa}$

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**Max. $B\sigma_{xx}$ [kN]** 8.94 7.97 6.41

**Min. $B$ [mm²]** 57.7 51.4 41.4
Stringers and frames

- From calculation $B_{\text{min}}(\text{AA}) > B_{\text{min}}(\text{CC}) > B_{\text{min}}(\text{CC})$
  - Lighter stringer between CC and BB

- type A stringers: $B > 57.7 \text{ mm}^2$
- type B stringers: $B > 51.4 \text{ mm}^2$
Stringers and frames

- **Stringer type “Z-section”**

Type A stringer: $B = 58.1 > 57.7 \text{ mm}^2$

Type B stringer: $B = 51.9 > 51.4 \text{ mm}^2$
Frames

- Non load bearing
- Must be of sufficient size to be connected with stringers (AA/BB/CC sections)
  - Use of brackets
- Must be of sufficient size to allow to be cut
  - So stringers can pass through them
Skin thickness

- Skin must resist shear flow due to
  - Shear loads $T_y$, $T_z$ &
  - Torque $M_x$

- Calculate the shear flow
  - At each boom there is a discontinuity

\[
q^{i+1} - q^i = -\frac{T_z}{I_{yy}} B_i z_i - \frac{T_y}{I_{zz}} B_i y_i
\]

\[
= -\frac{T_z}{3D^2} z_i - \frac{T_y}{3D^2} y_i
\]

As stringers have constant area in one section and as $I_{yy} = I_{zz} = 3BD^2$
Skin thickness – Shear flow

• Shear flow due to $T_z$
  – Following equations $q^{i+1} - q^i = -\frac{T_z}{3D^2} z_i$

\[
\begin{align*}
q^{1.2} &= q^{24.1} - \frac{T_z}{3D^2} \times 0.1294D = q^{24.1} - 0.043 \frac{T_z}{D} \\
q^{2.3} &= q^{1.2} - \frac{T_z}{3D^2} \times 0.25D = q^{24.1} - 0.126 \frac{T_z}{D} \\
q^{3.4} &= q^{2.3} - \frac{T_z}{3D^2} \times 0.353D = q^{24.1} - 0.244 \frac{T_z}{D} \\
q^{4.5} &= q^{3.4} - \frac{T_z}{3D^2} \times 0.433D = q^{24.1} - 0.388 \frac{T_z}{D} \\
q^{5.6} &= q^{4.5} - \frac{T_z}{3D^2} \times 0.483D = q^{24.1} - 0.549 \frac{T_z}{D} \\
q^{6.7} &= q^{5.6} - \frac{T_z}{3D^2} \times 0.5D = q^{24.1} - 0.716 \frac{T_z}{D} \\
\end{align*}
\]

– By symmetry (no torque)
  • $q^{6.7} = - q^{5.6}$

$$q^{24.1} - 0.716 \frac{T_z}{D} = -q^{24.1} + 0.549 \frac{T_z}{D} \iff q^{24.1} = \frac{0.633 \times T_z}{D}$$
Skin thickness – Shear flow

- Shear flow due to $T_z$ (2)
  - Final form of $qD/T_z$
Skin thickness – Shear flow

- Shear flow due to $T_y$
  - Following equations
    \[
    q^{i+1} - q^i = -\frac{T_y}{3D^2} y_i
    \]
    \[
    \begin{align*}
    q^{1,2} &= q^{24,1} - \frac{T_y}{3D^2} \times 0.483D = q^{24,1} - 0.161 \frac{T_y}{D} \\
    q^{2,3} &= q^{1,2} - \frac{T_y}{3D^2} \times 0.433D = q^{24,1} - 0.305 \frac{T_y}{D} \\
    q^{3,4} &= q^{2,3} - \frac{T_y}{3D^2} \times 0.353D = q^{24,1} - 0.4232 \frac{T_y}{D} \\
    q^{4,5} &= q^{3,4} - \frac{T_y}{3D^2} \times 0.25D = q^{24,1} - 0.507 \frac{T_y}{D} \\
    q^{5,6} &= q^{4,5} - \frac{T_y}{3D^2} \times 0.1294D = q^{24,1} - 0.55 \frac{T_y}{D}
    \end{align*}
    \]
  - But we also have
    \[
    q^{24,1} = q^{23,24} - \frac{T_y}{3D^2} \times 0.5D = q^{23,24} - 0.1667 \frac{T_y}{D}
    \]
  - By symmetry (no torque)
    \[
    q^{23,24} = -q^{24,1} \quad \Rightarrow \quad 2q^{24,1} = -0.1667 \frac{T_y}{D} \quad \Rightarrow \quad q^{24,1} = -\frac{0.0833 \times T_y}{D}
    \]
Shear flow due to $T_y$ (2)

- Final form of $qD/T_y$
Skin thickness – Shear flow caused by torque

- Shear flow due to torque

\[ q_T = \frac{M_x}{2A} \]
\[ = \frac{M_x}{2(\pi D^2/4)} \]
\[ = \frac{0.637 \times M_x}{D^2} \]
Skin thickness – Shear flow caused by torque

- **Maximum shear flow**
  - As $T_z > 0$, $T_y < 0$ & $M_x < 0$
  - Maximum shear flow is in a skin panel between stringers 12 and 18
    - Example: $q_{12,13} = -0.633\frac{T_z}{D} + 0.083\frac{T_y}{D} + 0.637\frac{M_x}{D^2}$
Skin Thickness – Maximum shear flow

- Maximum shear flow (2)
  - Equations

\[
\begin{align*}
q_{12\ 13} &= -0.633 \frac{T_z}{D} + 0.083 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q_{13\ 14} &= -0.590 \frac{T_z}{D} + 0.244 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q_{14\ 15} &= -0.507 \frac{T_z}{D} + 0.389 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q_{15\ 16} &= -0.389 \frac{T_z}{D} + 0.507 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q_{16\ 17} &= -0.244 \frac{T_z}{D} + 0.590 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q_{17\ 18} &= -0.083 \frac{T_z}{D} + 0.633 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2}
\end{align*}
\]
### Skin Thickness – Maximum shear flow

- Maximum shear flow (3)
  - Critical case: D1

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Skin Thickness – Maximum shear flow

- **Computation**
  - See Table

- **Minimum skin thickness**
  - From

\[
\frac{q_{\text{max}}}{t} \leq \tau_{\text{max}}
\]

\[
\frac{q_{\text{max}}}{\tau_{\text{max}}} \leq t
\]

\[t = \frac{65}{97} = 0.67 \text{ mm}\]

- But must support rivets
  - 1mm skin thickness is chosen

### Table

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<td>-41 kN·m</td>
<td>-40 kN·m</td>
<td>-40 kN·m</td>
</tr>
</tbody>
</table>

### Panels

<table>
<thead>
<tr>
<th>(n^o)</th>
<th>(q \text{ [kN} \cdot \text{m}^{-1}])</th>
<th>(q \text{ [kN} \cdot \text{m}^{-1}])</th>
<th>(q \text{ [kN} \cdot \text{m}^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-13</td>
<td>-22.04</td>
<td>-32.18</td>
<td>-57.07</td>
</tr>
<tr>
<td>13-14</td>
<td>-23.96</td>
<td>-34.64</td>
<td>-60.45</td>
</tr>
<tr>
<td>14-15</td>
<td>-25.33</td>
<td>-36.47</td>
<td>-63.03</td>
</tr>
<tr>
<td>15-16</td>
<td>-26.04</td>
<td>-37.55</td>
<td>-64.56</td>
</tr>
<tr>
<td>16-17</td>
<td>-26.05</td>
<td>-37.82</td>
<td>-65.02</td>
</tr>
<tr>
<td>17-18</td>
<td>-25.36</td>
<td>-37.26</td>
<td>-64.35</td>
</tr>
</tbody>
</table>

Max. \(q \text{ [kN} \cdot \text{m}^{-1}]\) | -26.1 | -37.8 | -65.0 |
Min. \(t \text{ [mm]}\) | 0.27 | 0.39 | 0.67 |
Rivets size – Skin/stringers

• What are the forces acting on the rivet?
  – At a stringer, we found for $T_z$
    • $q_{i+1} - q_i = -\frac{T_z}{3D^2} z_i$
    • This corresponds to
      – The shear flow balanced by
      – All the rivets linking the skin to the stringer
  – Therefore the shear load per unit stringer length acting on the rivets fixing the skin to the stringer $i$ is
    • $R_i = \left( -\frac{T_y}{3D^2} y_i - \frac{T_z}{3D^2} z_i \right)$
    • Maximum between stringers 6 and 12
      – $T_y < 0$ & $T_z > 0$ $\rightarrow$ $y < 0$ & $z > 0$
      – Critical case is still D1
    – Remark: the torque does not lead to a discontinuity in the shear flow
### Results

- Maximal load:
  - 4.91 kN per unit stringer length

<table>
<thead>
<tr>
<th>Section</th>
<th>AA</th>
<th>CC</th>
<th>BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D)</td>
<td>1.28 m</td>
<td>1.01 m</td>
<td>0.73 m</td>
</tr>
<tr>
<td>(T_y)</td>
<td>-15 kN</td>
<td>-15 kN</td>
<td>-15 kN</td>
</tr>
<tr>
<td>(T_z)</td>
<td>15 kN</td>
<td>12 kN</td>
<td>12 kN</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Stringers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n^\circ)</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

Max. \(|R|\) [kN/m]: 2.71, 3.20, 4.91
• Rivets
  – Maximal load:
    • 4.91 kN per unit stringer length
• Rivets
  – For 2.5 mm diameter countersunk rivets with skin thickness 1.0 mm
    • Allowable load in shear: 668 N
    • The number of rivets/m is given by
      \[ n = \frac{4910}{668} = 7.35 \text{ or } 8 \text{ rivets/m} \]
      • This corresponds to a rivet pitch of 125 mm
      • Too large: does not ensure structural rigidity
    – We choose 25 mm rivet pitch: 40 rivets/m
What are the forces acting between frames & stringers?

- These forces correspond to the direct stresses in the stringers
- Already calculated
- Maximal forces
  - Section AA: 9 kN
  - Section CC: 8 kN
  - Section BB: 6.4 kN

<table>
<thead>
<tr>
<th>Sect.</th>
<th>AA</th>
<th>CC</th>
<th>BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>1.28 m</td>
<td>1.01 m</td>
<td>0.73 m</td>
</tr>
<tr>
<td>$M_y$</td>
<td>42 kN·m</td>
<td>29 kN·m</td>
<td>16 kN·m</td>
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<tr>
<td>$M_z$</td>
<td>55 kN·m</td>
<td>39 kN·m</td>
<td>23 kN·m</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<tbody>
<tr>
<td>$n^\circ$</td>
</tr>
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</tr>
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<td>7</td>
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<td>10</td>
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<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

Max. $B\sigma_{xx}$ [kN] | 8.94 | 7.97 | 6.41 |
Min. $B$ [mm$^2$] | 57.7 | 51.4 | 41.4 |
• **Number of rivets between the skin and frames**
  
  - **Section AA**
    - Maximal forces: 9 kN
    - This force is resisted by the rivets between the skin and frame
    - Distance available for one stringer \( l = 0.168 \text{ m} \)
    - Resistance of one rivet: 668 N
      
      \[
      \text{pitch} = \frac{0.168}{\frac{8940}{668}} = 0.0126 \text{ m}
      \]
  
  - **Section CC**
    
    \[
    \text{pitch} = \frac{0.132}{\frac{7970}{668}} = 0.011 \text{ m}
    \]

  - **Section BB**
    
    \[
    \text{pitch} = \frac{0.096}{\frac{6410}{668}} = 0.01 \text{ m}
    \]

  - Distance between the rivets: 10 mm for all frames
Fuselage design - End

Plans and Figures
Design of the rear fuselage

- Stringers type B'
- Section BB
- Frame 5 see detail 5 (Fig. A.14(e))
- Frame 4 see detail 4 (Fig. A.14(d))
- Cut out for stringers from previous panel
- Frame 3 see detail 3 (Fig. A.14(c))
- Frame 2 see detail 2 (Fig. A.14(b))
- Frame 1 see detail 1 (Fig. A.14(a))
- Section AA
Rear fuselage: details (A14A)

For Details of Bracket
see Fig. A.14 (c)
Skins from Previous
Sections Overlap Where
Necessary
All Rivets 2.5 mm Countersunk
Except for Bracket.
Rear fuselage: details (A14C)

1.2mm Thickness
Matt.
No Off.48
Rivets 2.5mm
Mushroom

Bracket

Stringer Type 'A'

Stringer Type 'B'

Frame 3

A Elevation From Port Side

10 mm

25 mm

All Rivets 2.5 mm Countersunk
Except for Bracket Use 2.5 mm
Mushroom. Skins from Previous
Sections Overlap Where Necessary

2013-2014

Aircraft Structures: Design Example
Rear fuselage: details (A14D)

Section on AA

Stringer Type 'B'

Framo 4

Elevation on Port Side

10 mm

25 mm

All rivets 2.5 mm Countersunk.
Skins from Previous
Sections Overlap Where Necessary

Lip Cut Away
Rear fuselage: details (A14E)

- Stringer Type 'B'
- Frame 5
- Section on ABCDEA
- 10 mm
- 25 mm
- 5 mm
- 16 mm
- 6 mm
- 12.5 mm

Brackets, As Shown
Material 1.2 mm thick
No. of 24
Rivets 2.5 mm Diameter Mushroom

Skins from Previous
Sections Overlap
Where Necessary
Rivets 2.5 mm Diameter Countersunk.