University of Liège
Aerospace & Mechanical Engineering

Aircraft Structures
Design Example

From “Aircraft Structures for engineering students, T. H. G. Megson, Butterworth-Heinemann, An imprint of Elsevier Science, 2003, ISBN 0 340 70588”

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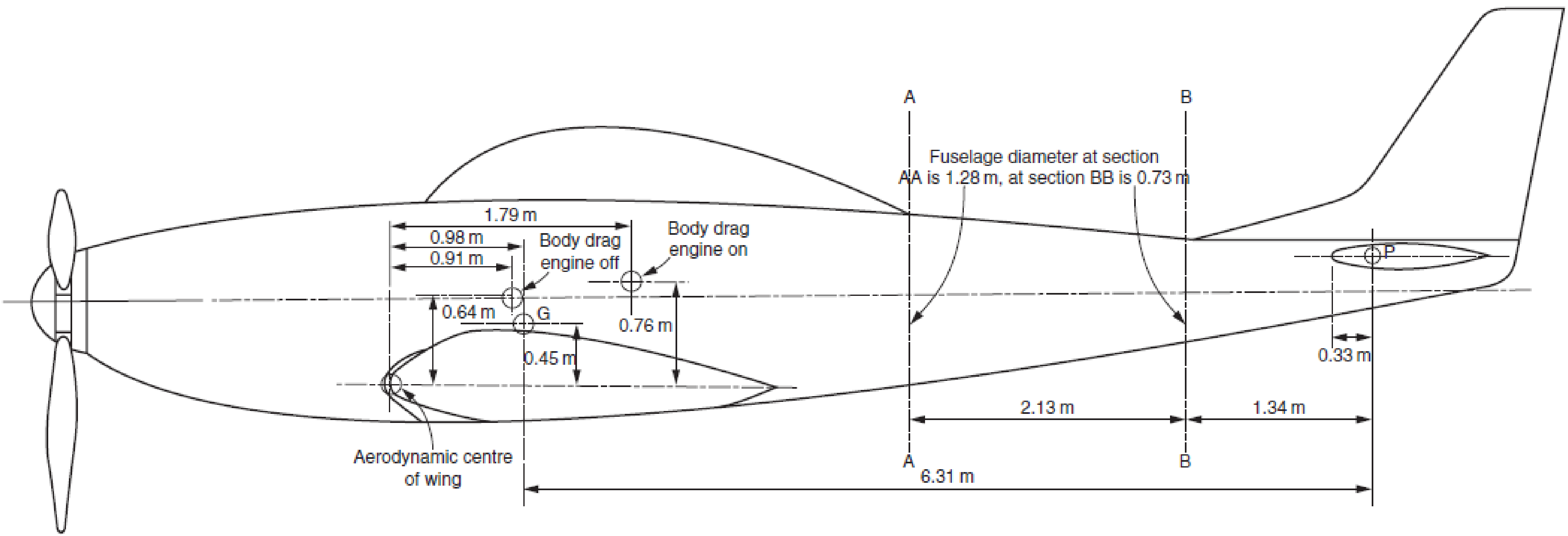
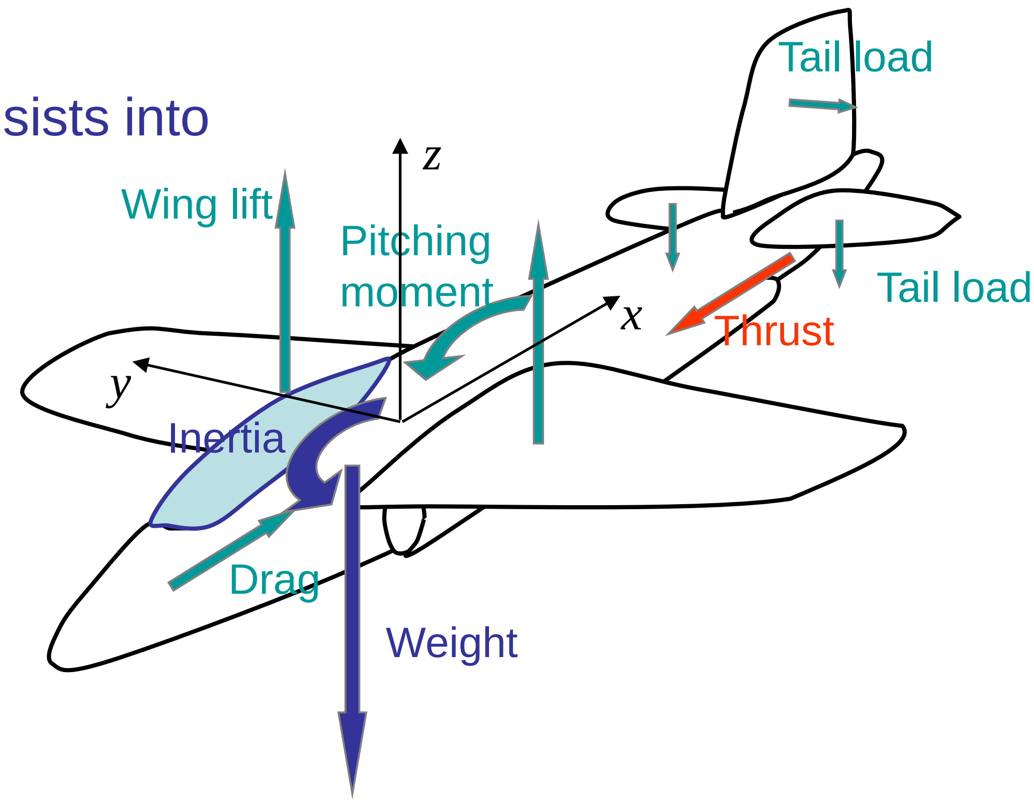
Goal

- From
  - Specifications
  - Preliminary design
- Design the rear fuselage of a two-seat trainer/semi-aerobatic aircraft
  - Stringers and Frames
  - Skin thickness



Goal (2)

- Specifications and preliminary design
  - Permit to calculate forces acting on the airplane
  - Fuselage has to resist the induced loading
- Step 1: Forces acting on the airplane are
  - Aerodynamic forces (→ flight envelope)
  - Thrust
  - Self-weight
- Step 2: The rear fuselage consists into
  - Stringers
  - Frames
  - Skin
  - Rivets
- All these elements must be calculated to resist the stresses induced by the forces (bending moments, torques and shearing)

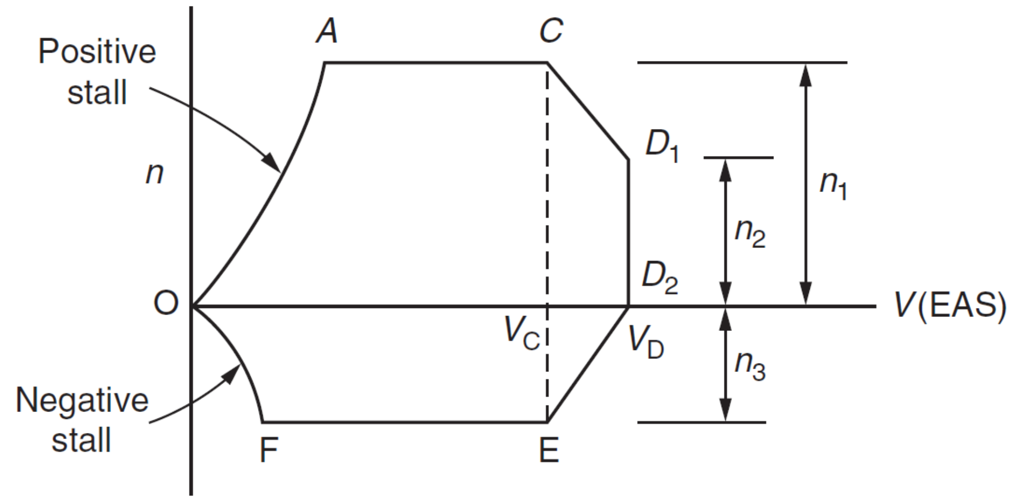


Step 1 – Loading acting on the rear fuselage

Aerodynamic forces and self-weight

Specifications: flight envelope

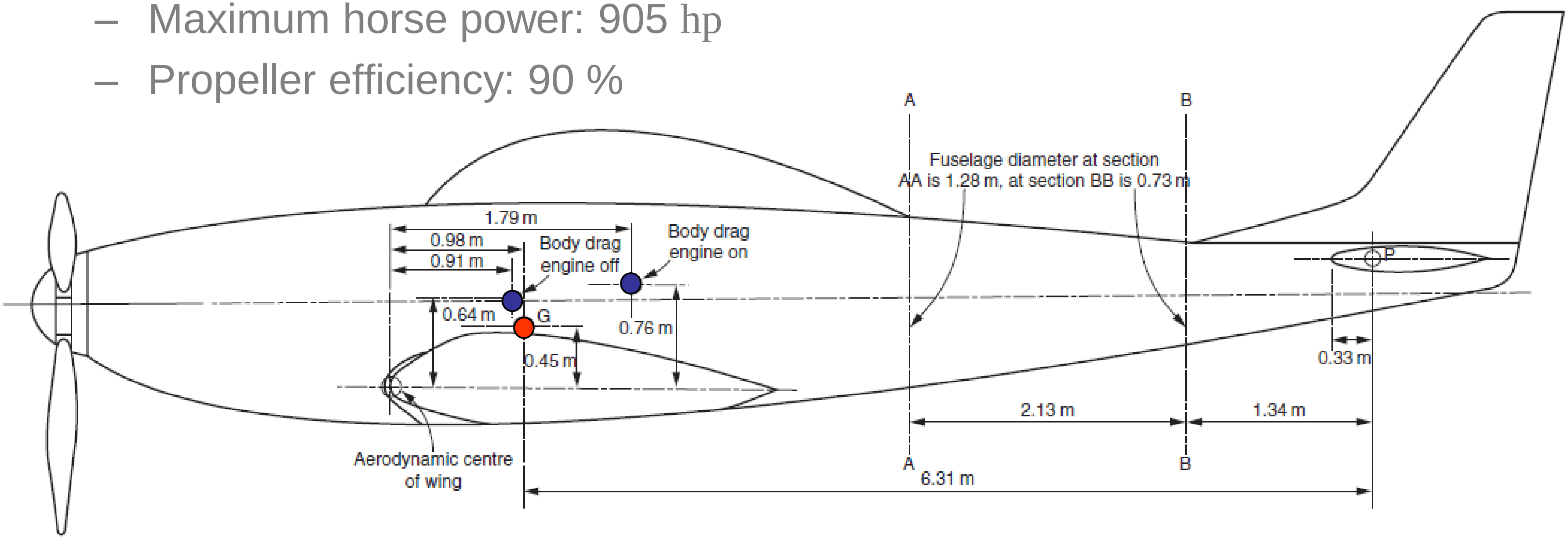
- Required flight envelope
  - $n_1 = 6.28$
  - $V_D = 183.8$ m/s
  - $V_C = 0.8V_D = 147.0$ m/s
  - $n_2 = 0.75\, n_1 = 4.71$
  - $n_3 = 0.5\, n_1 = 3.14$
- Representative values for an aerobatic aircraft ?



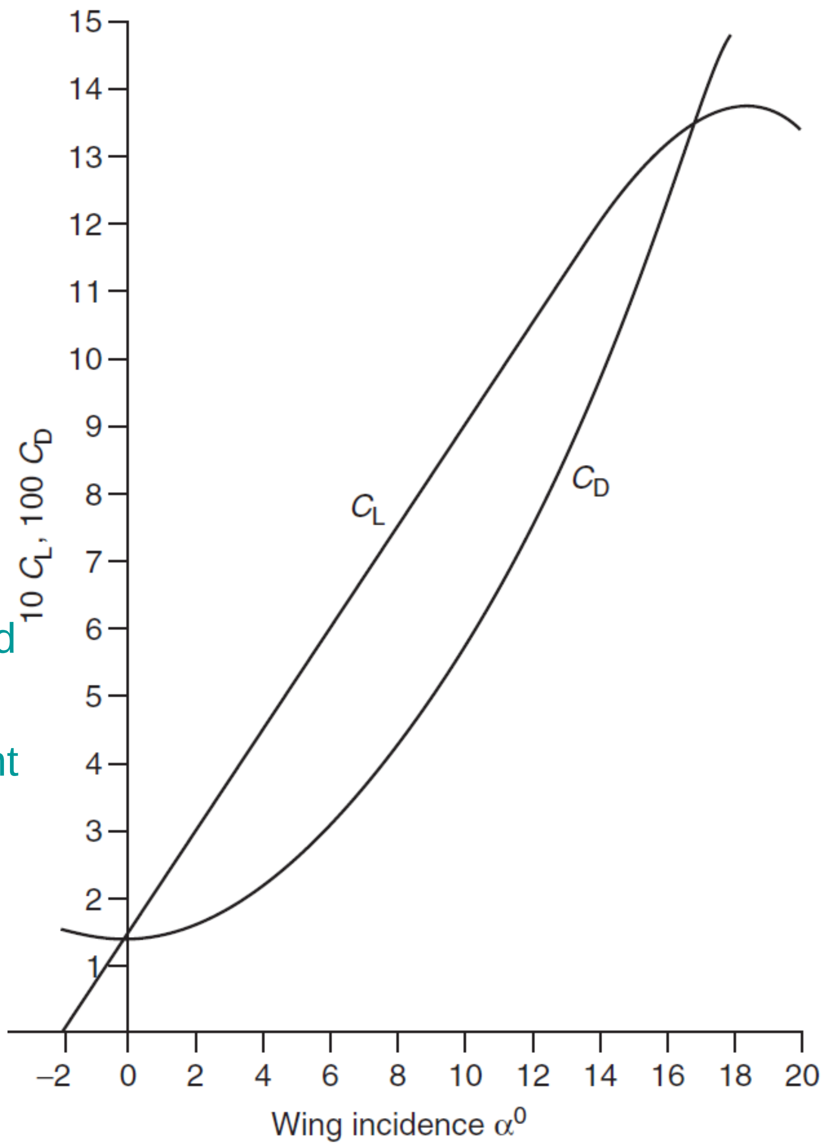
- Further requirements
  - Additional pitching acceleration allowed at any point of envelope
    - $\left(20 + \frac{475}{W}\right)\frac{n}{V} \quad [rad/s^2]$ Weight W in [kN]
  - For asymmetric flight: angle of yaw allowed at any point of envelope
    - $\psi = 0.7n_1 + \frac{457.2}{V_D} \quad [degrees]$
    - The angle of yaw increases the overall pitching moment coefficient of the aircraft by -0.0015 / degree of yaw

Data - Aircraft

- Fully loaded aircraft
  - Weight W = 37.43 kN
  - Moment of inertia about center of gravity G: I_θ = 22 235 $\mathrm{kg{\cdot}m^2}$
  - Positions of G and of the body drag centers known (see figure)
- Body drag coefficients
  - $C_{D,B}$ (engine on) = 0.01583
  - $C_{D,B}$ (engine off) = 0.0576
- Engine
  - Maximum horse power: 905 hp
  - Propeller efficiency: 90 %



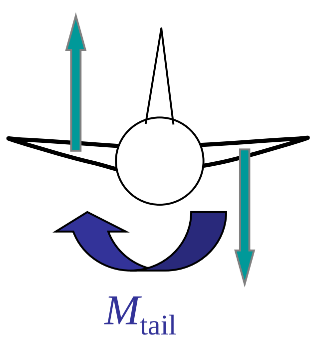
- Geometry/Aerodynamics
  - Span b=14.07 m
  - Gross area S = 29.64 m^2
  - Aerodynamic mean chord c = 2.82 m
  - Lift and drag coefficients
    - See picture
- Pitching moment
  - C_M = -0.238 C_L
  - Angle of incidence
    - Between fuselage axis and root chord
    - -1.5°
    - Additional pitching moment coefficient -0.036



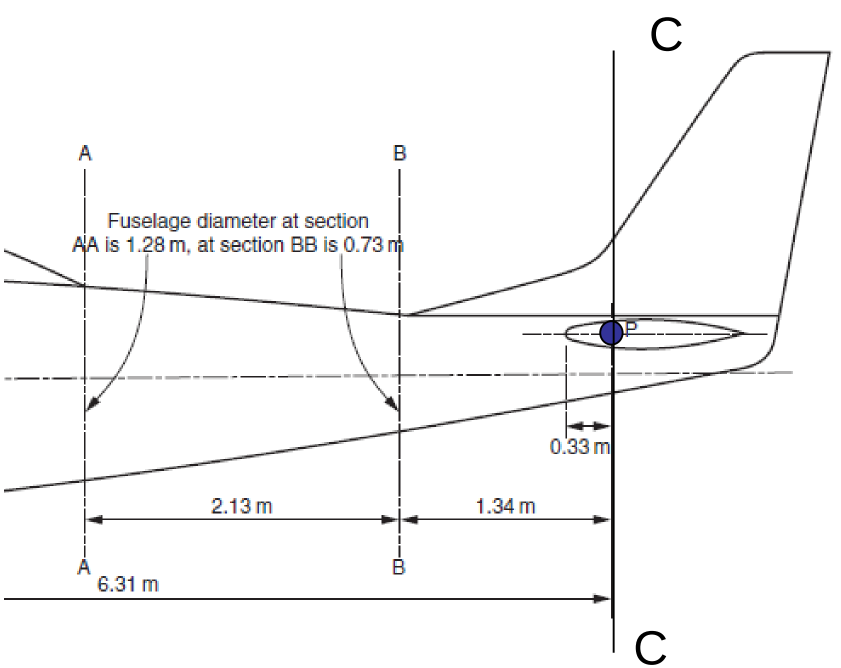
Data - Tailplane

- Geometry/Aerodynamics
  - Span b_t = 6.55 m
  - Gross area S_t = 8.59 m^2
  - Aerodynamic centre P
- Yaw → asymmetry of the slipstream → asymmetric load on the tailplane
  - Resulting torque $\frac{0.00125}{\sqrt{1-M^2}}\rho V^2 S_t b_t \psi$
    - M is the mach number
    - & ψ is in degree

Section CC

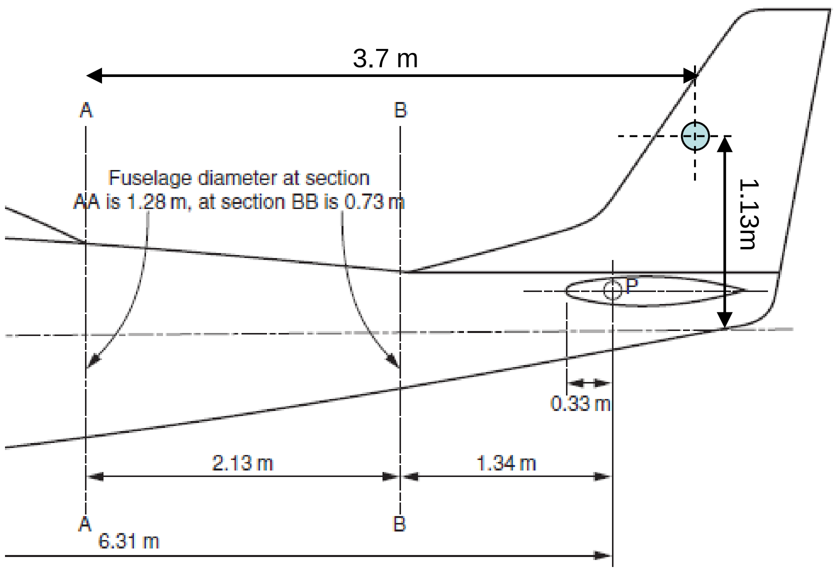


Loading induced by yaw on the tailplane



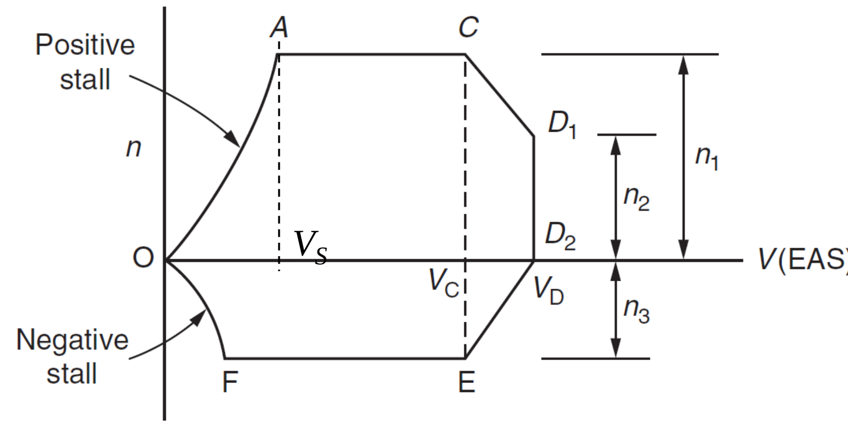
- Geometry/Aerodynamics
  - Height h_F = 1.65 m
  - Area S_F = 1.80 m^2
  - Aspect-ratio $A_F = h_F^2/S_F$ = 1.5
  - Lift-curve slope a_1
    - $a_1 = \dfrac{5.5A}{A+2}$
    - With A the aspect ratio of an equivalent wing: $A = (2h_F)^2/2S_F = 2\, h_F^2/S_F = 2A_F = 3$

    $\Longrightarrow$ a_1 = 3.3
- In yawed flight of angle ψ
  - Fin has also an incidence of ψ
  - Aerodynamic loading
    - $F_{\text{fin}} = \dfrac{1}{2}\rho V^2 S_F a_1 \psi$
    - Where ψ is in rad
- Centre of pressure of the fin
  - 1.13 m above the axis of the fuselage
  - 3.7 m aft section AA



Initial calculation – Flight envelope

- Values
  - $n_1 = 6.28$
  - $V_D = 183.8$ m/s
  - $V_C = 0.8V_D = 147.0$ m/s
  - $n_2 = 0.75\, n_1 = 4.71$
  - $n_3 = 0.5\, n_1 = 3.14$

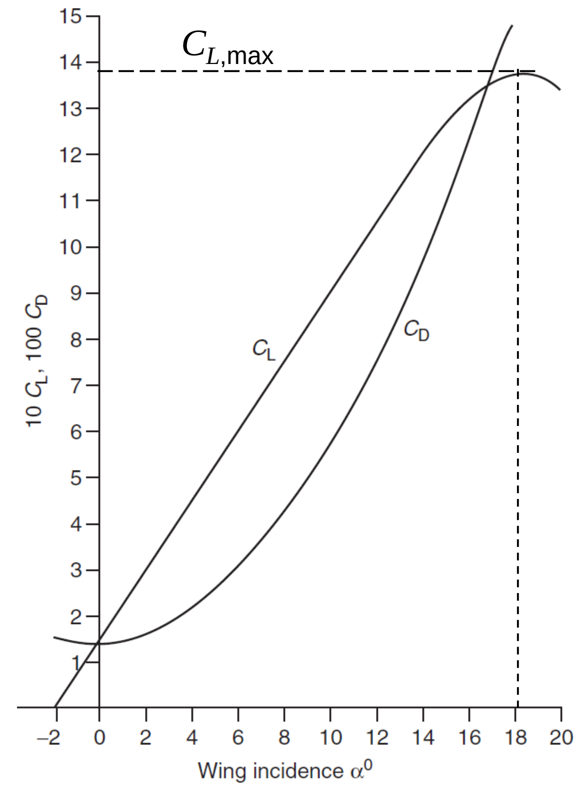


- $V_S^{\,A}$ (stalling speed on point A)?
  - Assumption: Lift from wing only
  - From $$C_L = \frac{nW}{\frac{1}{2}\rho V^2 S}$$

$$\Rightarrow \quad V_s = \left(\frac{2nW}{\rho S C_{L,\,\max}}\right)^{1/2}$$

$$= \sqrt{n}\left(\frac{2 \times 37.43\ 10^3}{1.226 \times 29.64 \times 1.38}\right)^{1/2}$$

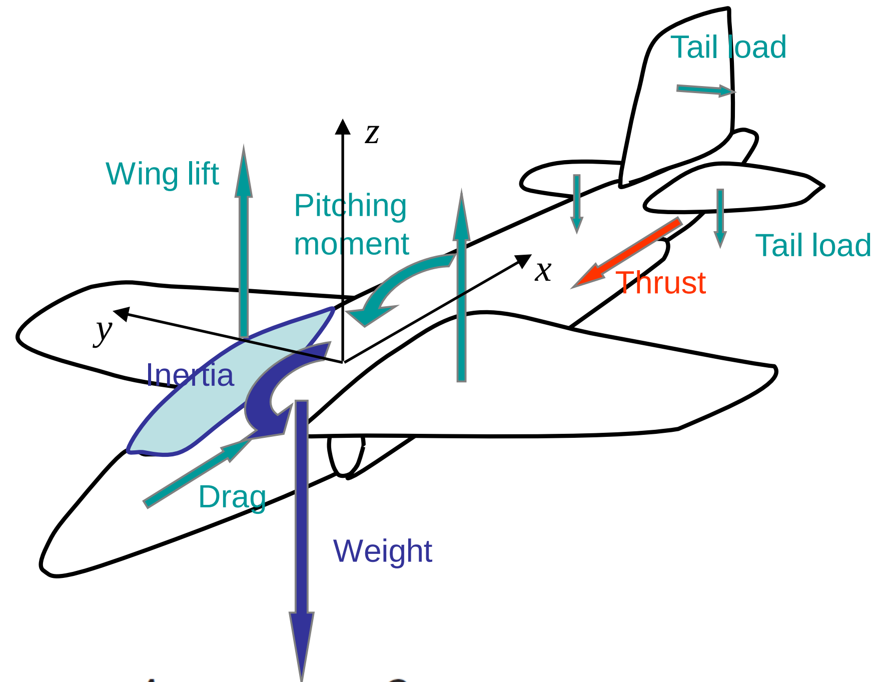
$$= 38.6\sqrt{n}$$

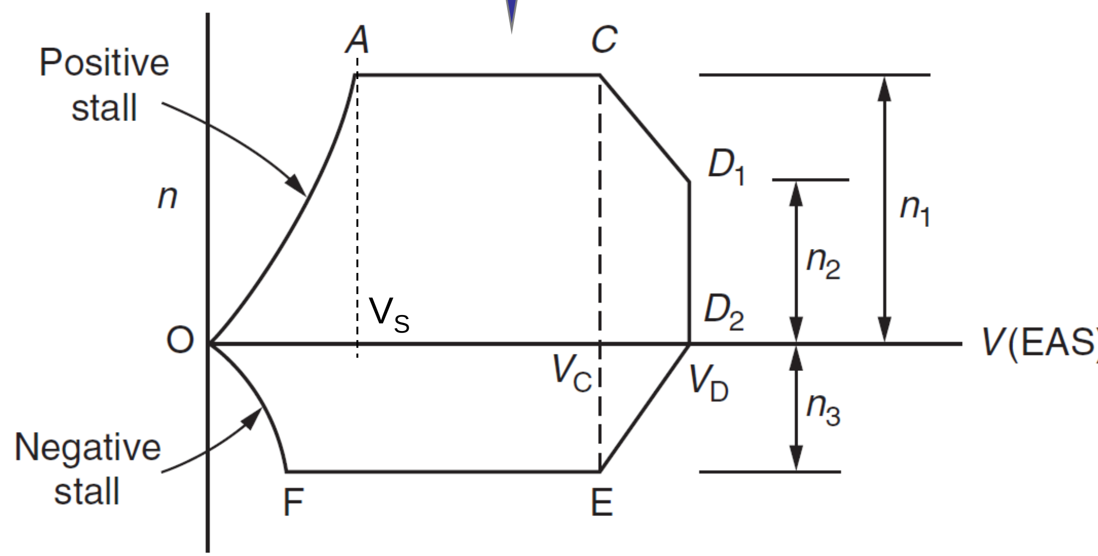


$$\Rightarrow \quad V_s^{A} = 38.6\sqrt{n_1} = 38.6\sqrt{6.28} = 96.7\,\mathrm{m/s}$$

Balancing out calculations

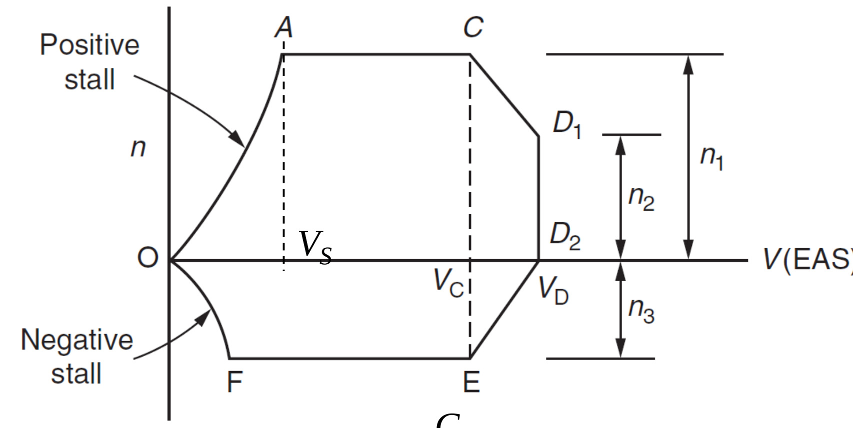
- Loads are calculated for various critical points of the flight envelope
  - Case A
    - Point A
    - Engine on
  - Case A*
    - Point A
    - Engine off
  - Case C
    - Point C
    - Engine off
  - Case D_1
    - Point D_1
    - Engine off
  - Case D_2
    - Point D_2
    - Engine off

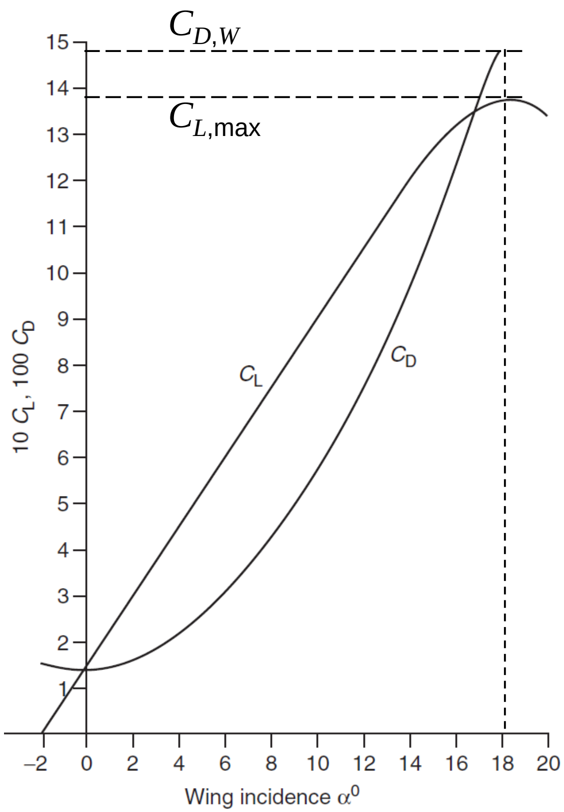




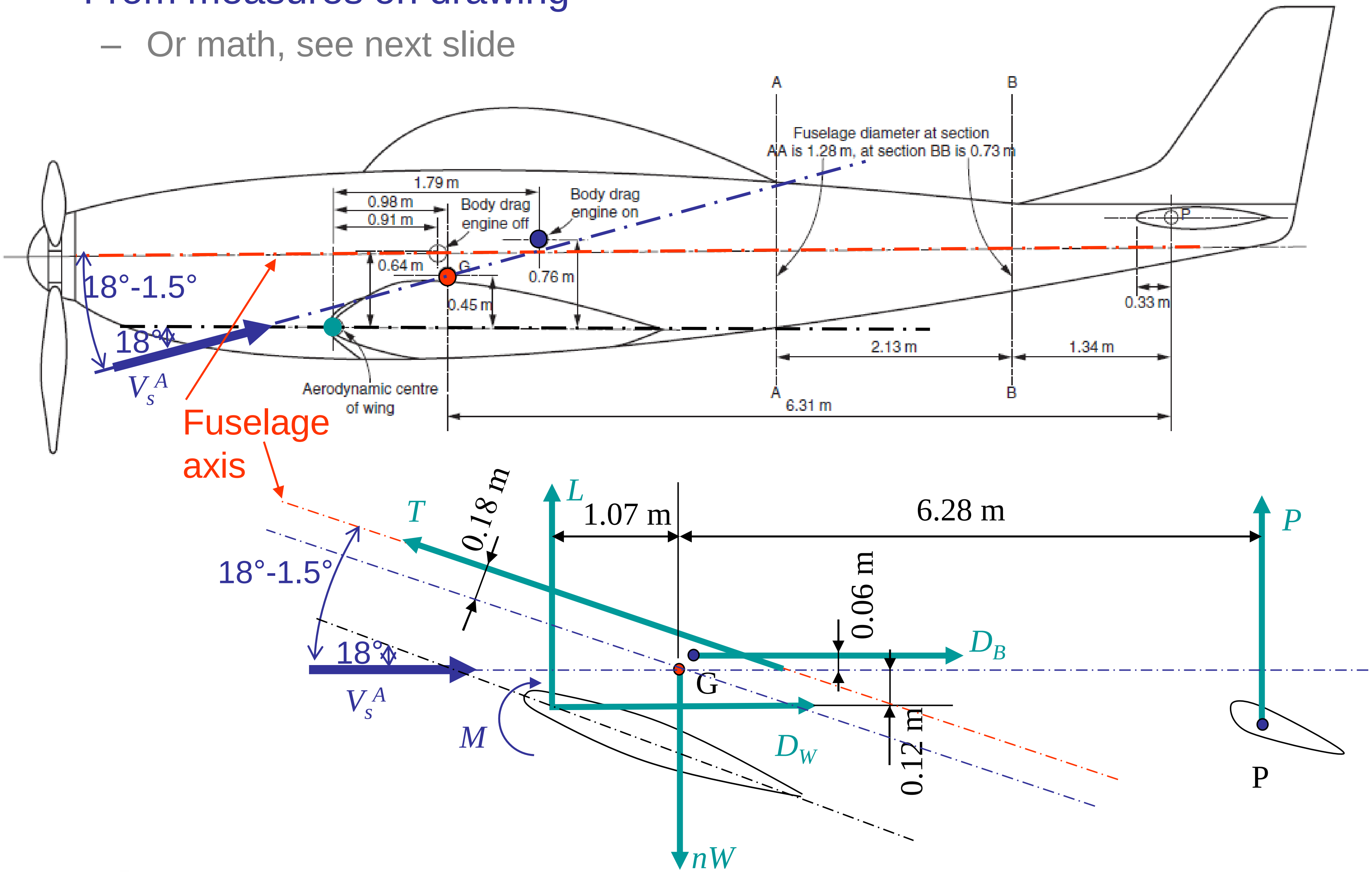
Balancing out calculations – Case A – point A/engine on

- Data (point A)
  - $n_1 = 6.28$
  - $V = V_s^A = 96.7\ \text{m/s}$
  - $C_{L,\max} = 1.38$
  - $\alpha^0_{L,\max} = 18°$
  - $C_{D,W} = 0.149$



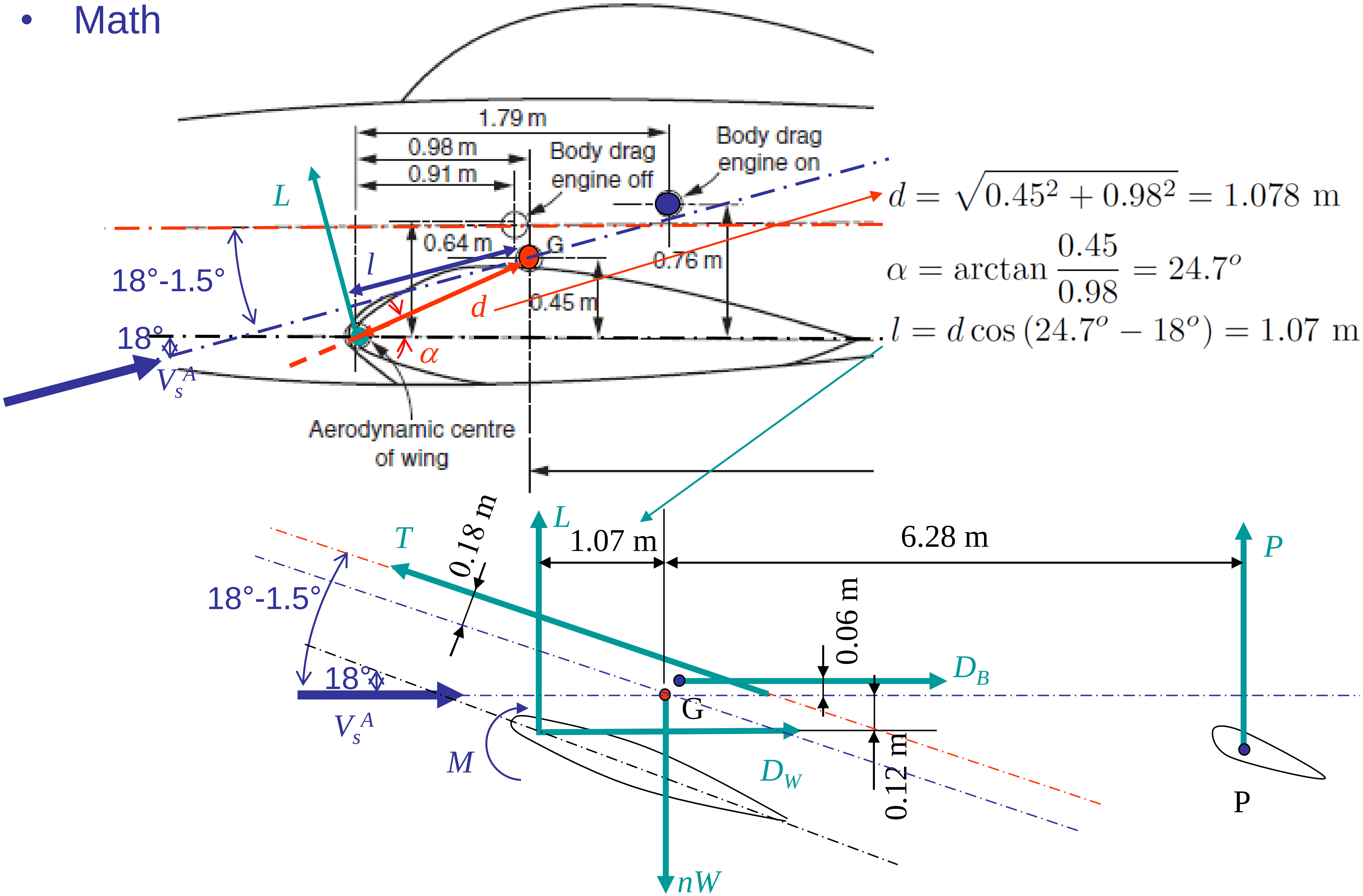


- From measures on drawing
  - Or math, see next slide



Balancing out calculations – Case A – point A/engine on

- Math



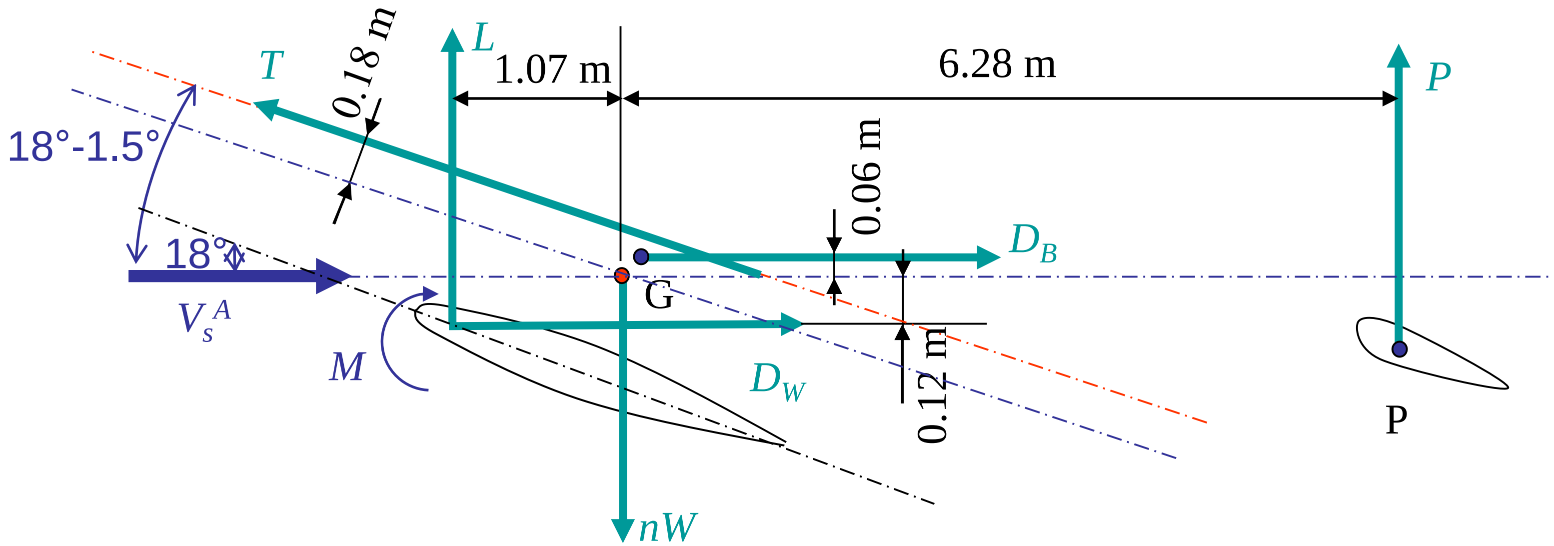
$$d = \sqrt{0.45^2 + 0.98^2} = 1.078 \text{ m}$$

$$\alpha = \arctan \frac{0.45}{0.98} = 24.7^o$$

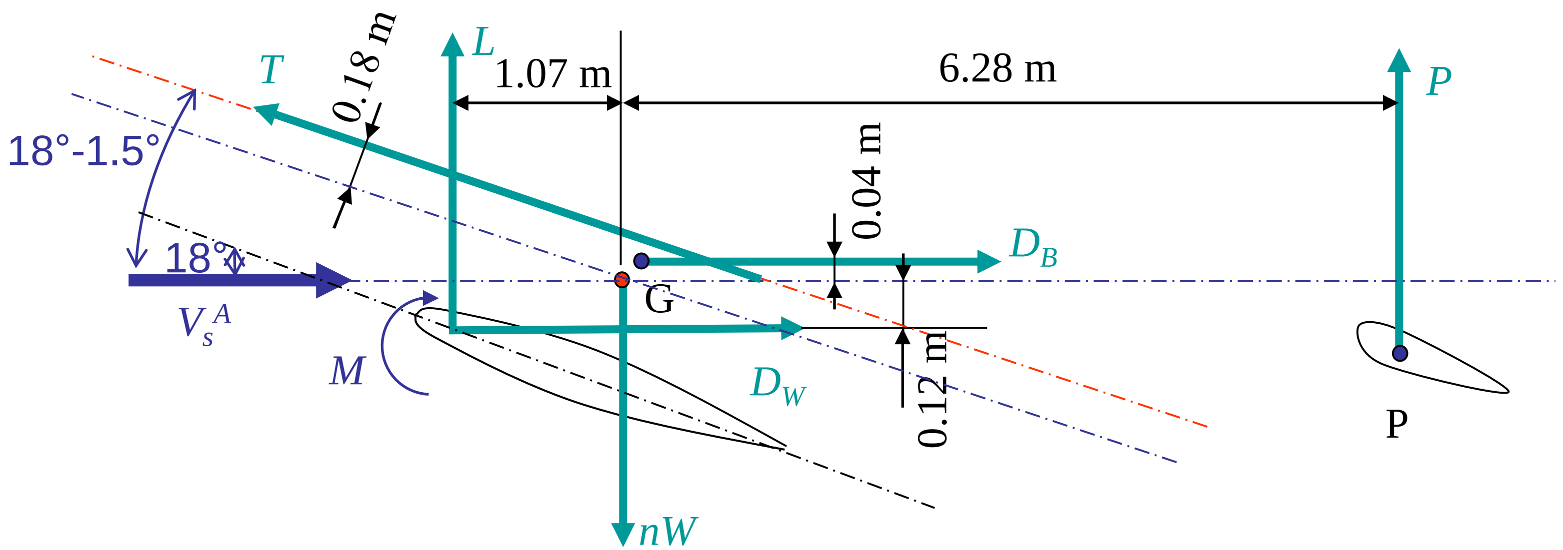
$$l = d \cos\left(24.7^o - 18^o\right) = 1.07 \text{ m}$$

Balancing out calculations – Case A – point A/engine on

- Methodology
  - Forces that can be directly calculated
    - Trust: T from engine
    - nW: n & W are known
    - Drag (body B, wings W) from V and drag coefficients
    - Pitching moment M from the pitching moment coefficients and the angle of yaw ψ
    - Pitching moment acceleration
  - Force equilibrium, and moment equilibrium
    - 2 Equations involving P, L
  - Find other forces acting on the rear fuselage
    - Tailplane torque, fin load, fin load torque, total torque

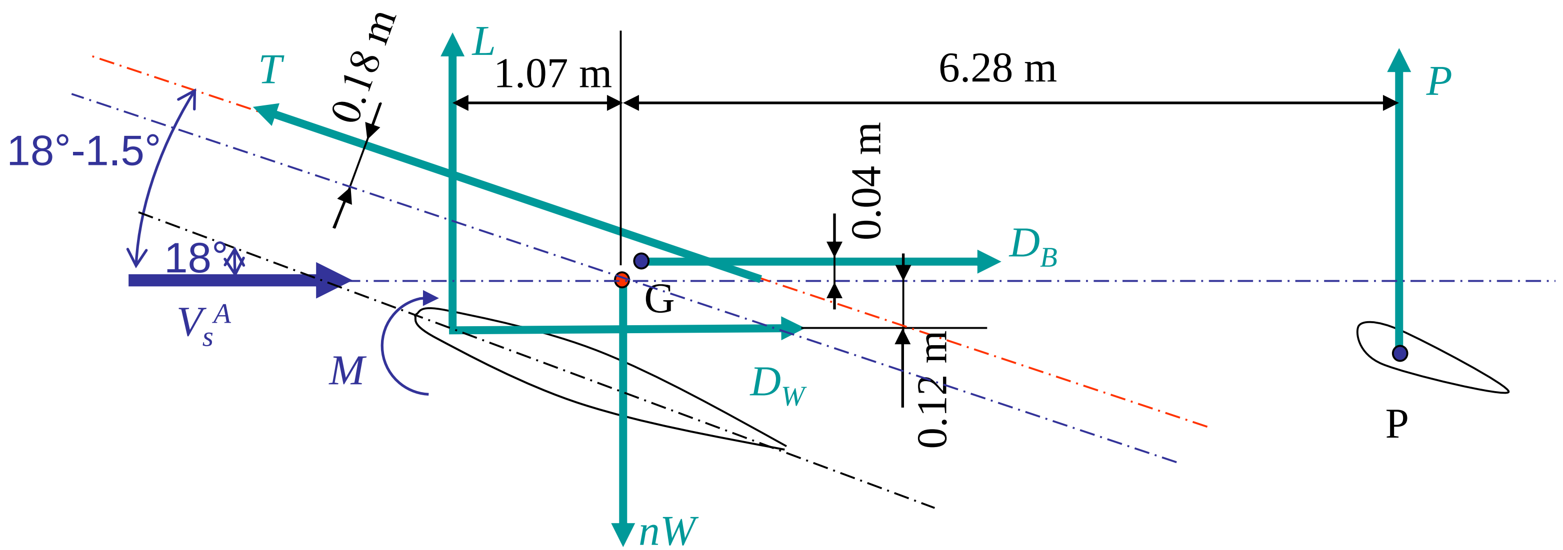


- Trust of the engine
  - Data
    - Maximum horse power 905
    - Propeller efficiency η = 90 %
    - 1 [hp] = 746 [W]
  - Thrust
    - $T = \frac{\eta . hp . 746}{V} = \frac{0.9 \times 905 \times 746}{96.7} = 6284\,[N]$

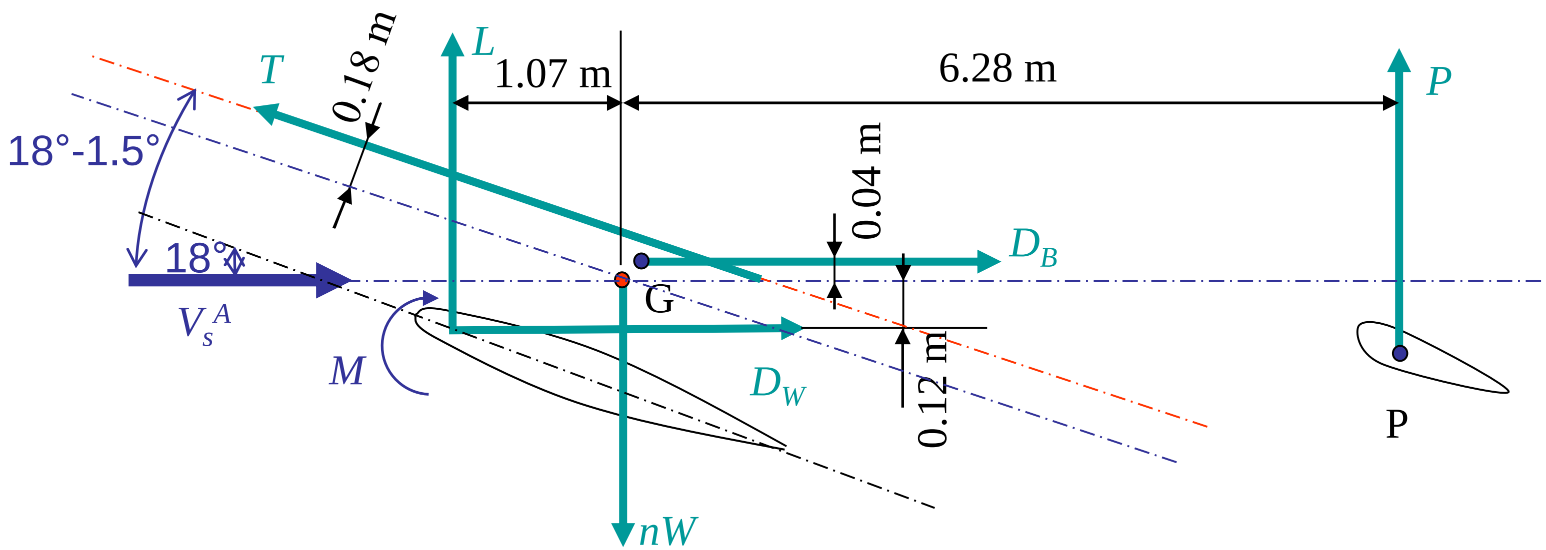


Balancing out calculations – Case A – point A/engine on

- Weight
  - Data
    - Fully loaded weight W = 37.43 kN
  - Loaded weight
    - $nW = 6.28 \times 37.43\ 10^3 = 235\,060\ [N]$



- Drag
  - Data
    - $C_{D,W}$ = 0.149
    - $C_{D,B}$ = 0.01583
    - V = 96.7 m/s
    - S = 29.64 m^2
  - Wing drag
    - $D_W = \frac{1}{2} 0.149 \times 1.226 \times 96.7^2 \times 29.64 = 26091\ [N]$
  - Body drag
    - $D_B = \frac{1}{2} C_{D,B} \rho V^2 S = 2690\ [N]$



- Pitching moment
  - Data
    - n_1 = 6.28
    - V_D = 183.8 m/s
    - V = 96.7 m/s
    - Fully loaded weight W = 37.43 kN
  - Pitching moment coefficient
    - Maximum for the maximum yaw angle allowed during maneuver
    - Maximum angle of yaw allowed $$\begin{aligned}\psi &= 0.7n_1 + \frac{457.2}{V_D} \quad [degrees] \\ &= 0.7 \times 6.28 + \frac{457.2}{183.8} = 6.9\,[^\circ]\end{aligned}$$
    - Pitching moment coefficient

$$\begin{aligned}C_M &= -0.238\,C_L - 0.036 - 0.0015\psi \\ &= -0.238 \times 1.38 - 0.036 - 0.0015 \times 6.9 = -0.375\end{aligned}$$

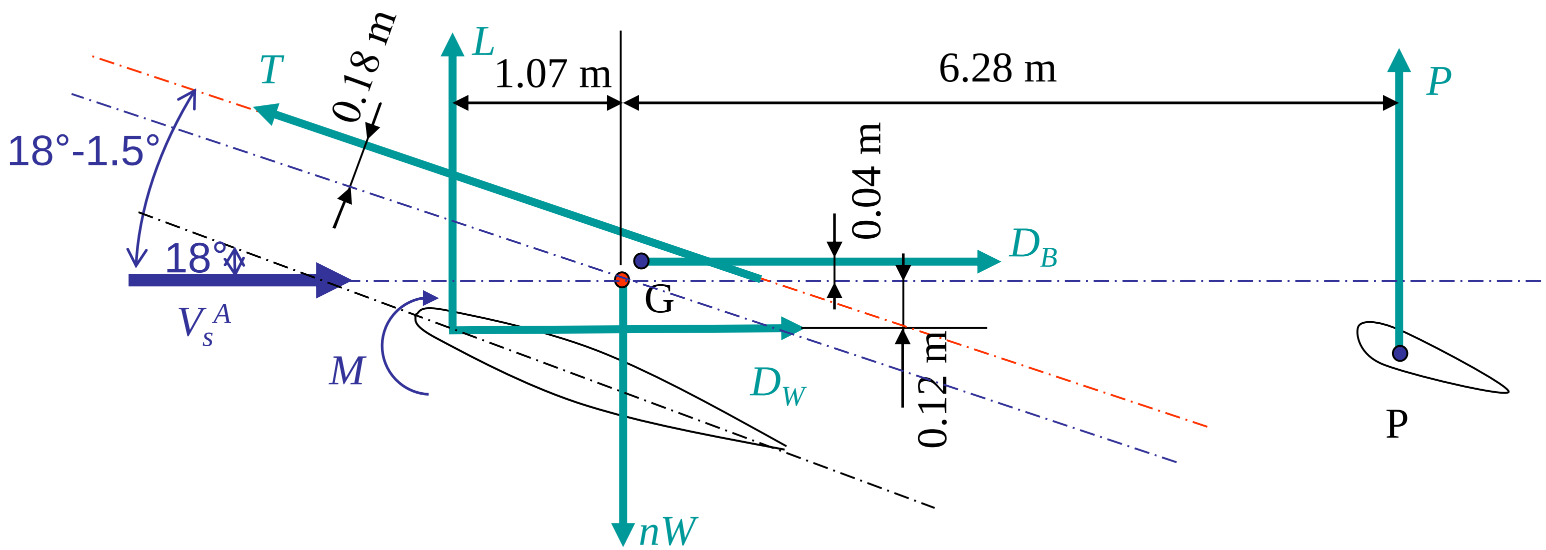
Wing

Wing/fuselage incidence

Pitching of aircraft due to yaw (in °)

  - Maximum pitching acceleration allowed
    - $$\ddot{\theta} = \left(20 + \frac{475}{W}\right)\frac{n}{V} = \left(20 + \frac{475}{37.43}\right)\frac{6.28}{96.7} = 2.12[rad/s^2]$$

- Pitching moment (2)
  - Data
    - V = 96.7 m/s
    - S = 29.64 m^2
    - MAC: c = 2.82 m
  - Pitching moment coefficient C_M = -0.375
  - Pitching moment
    - $$\begin{aligned} M &= C_M \frac{1}{2} \rho V^2 S c \\ &= -0.375 \frac{1}{2} \times 1.226 \times 96.7^2 \times 29.64 \times 2.82 = -179\,669\ [Nm] \end{aligned}$$



- Moments about G

$$1.07L - 0.18T + 0.04D_B - 0.12D_W - 6.28P + M \quad = \quad I_\theta \times \ddot{\theta}$$

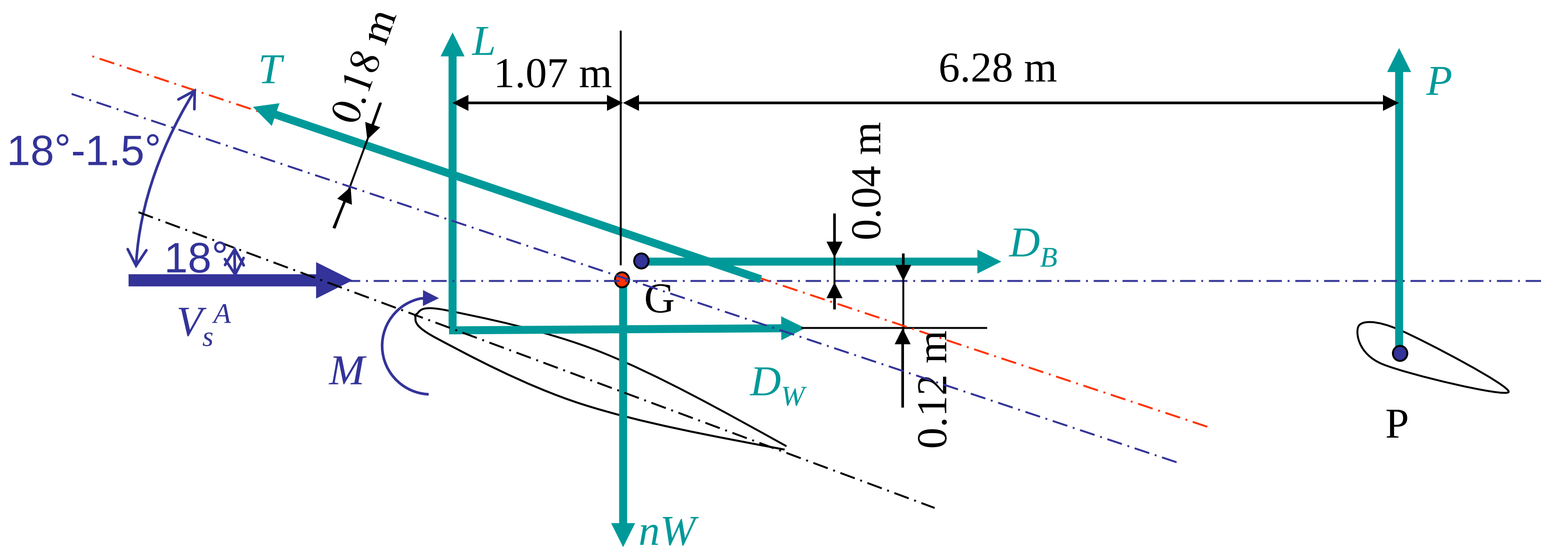
$$\Longrightarrow 1.07L - 0.18 \times 6284 + 0.04 \times 2690 - 0.12 \times 26901 - 6.28P - 179669 \quad = \quad 22235 \times 2.12$$

$$\Longrightarrow \quad 5.86P = L - 216090$$

- Vertical equilibrium

$$L + P = nW - T\sin(18^\circ - 1.5^\circ) = 235\,060 - 6284\sin 16.5^\circ = 233275$$

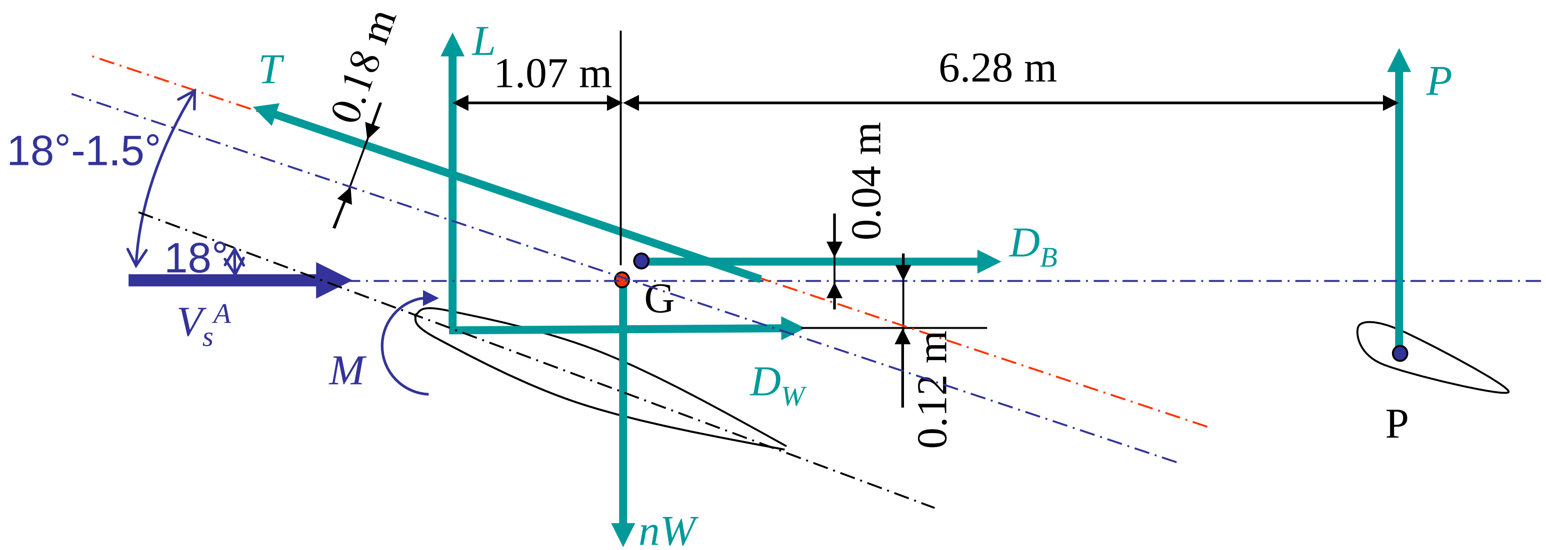
$$\Longrightarrow \quad L + P = 233\,275$$



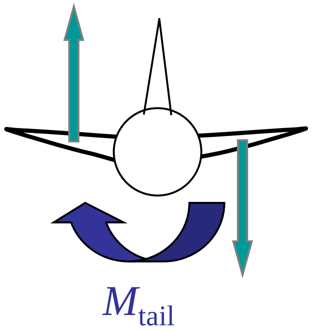
- 2 equations, 2 unknowns
  - $5.86P = L - 216090$
  - $L + P = 233\,275$

$$\Rightarrow \begin{cases} P = 2\,505 \text{ [N]} \\ L = 230\,770 \text{ [N]} \end{cases}$$

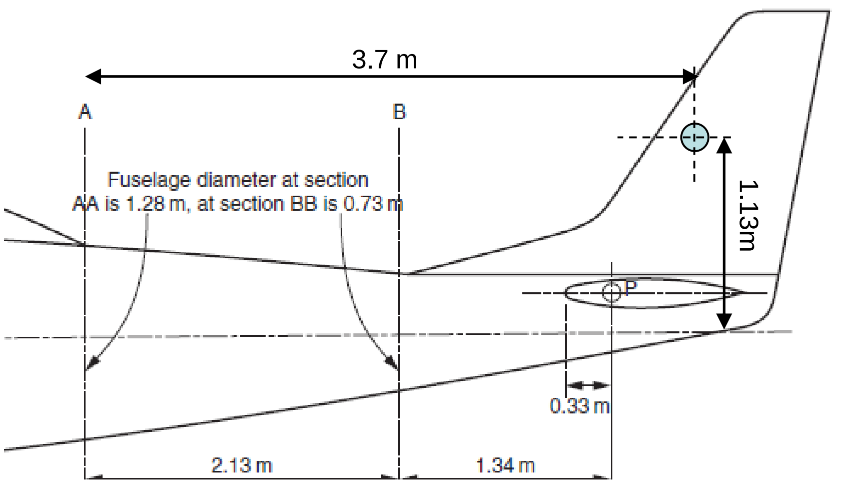
- Remark: for other cases, α is not known
  - Requires iterations on α in order to determine C_L
  - Should also be done here as V_S was computed using C_L of wing (10% error)



Balancing out calculations – Case A – point A/engine on

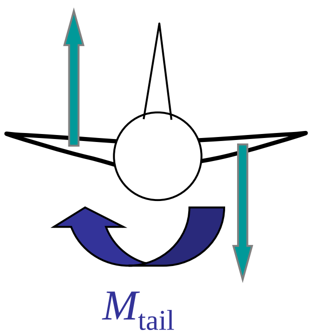


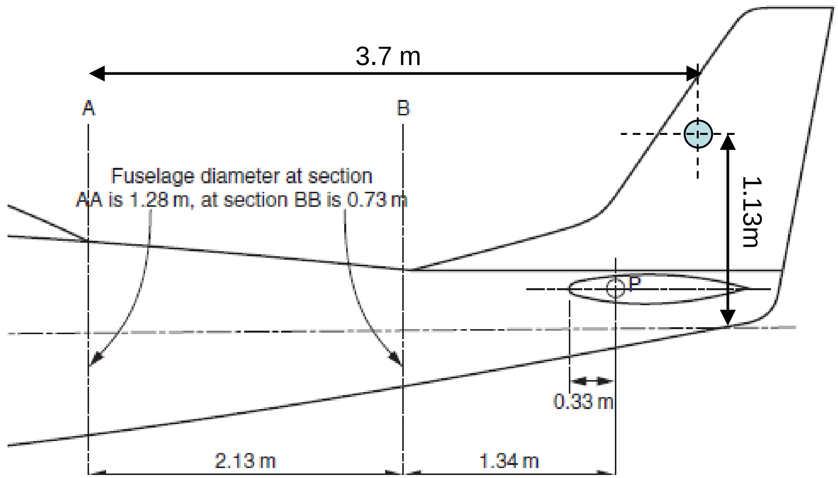
- Tailplane torque
  - Due to asymmetric slipstream (yaw)
  - Data
    - b_t = 6.55 m
    - S_t = 8.59 m^2
  - $$\begin{aligned} M_{\text{tail}} &= \frac{0.00125}{\sqrt{1 - Mach^2}} \rho V^2 S_t b_t \psi \\ &= \frac{0.00125}{\sqrt{1-(96.7/340.8)^2}} \times 1.226 \times 96.7^2 \times 8.59 \times 6.55 \times 6.9 \\ &= 5802\ [N \cdot m] \end{aligned}$$

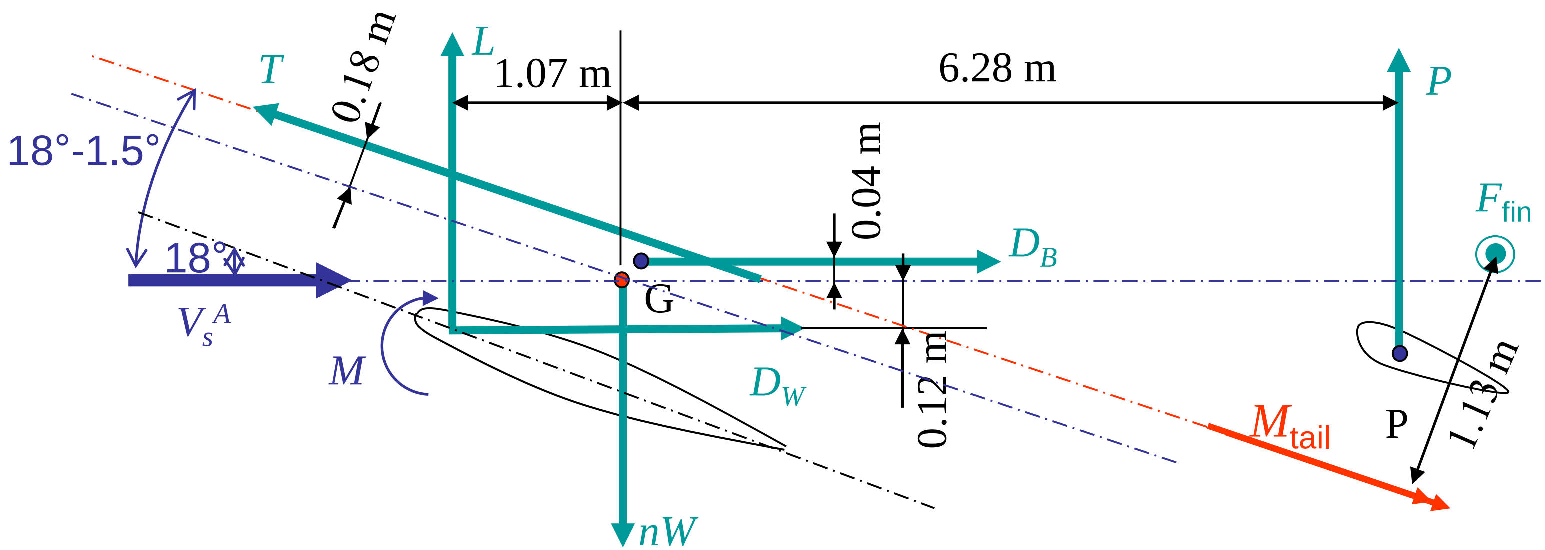


- Fin load
  - Due to yaw
  - Data
    - S_F = 1.80 m^2
    - a_1 = 3.3
  - $$\begin{aligned} F_{\text{fin}} &= \frac{1}{2} \rho V^2 S_F a_1 \psi \\ &= \frac{1}{2} \times 1.226 \times 96.7^2 \times 1.8 \times 3.3 \times (6.9 \frac{\pi}{180}) = 4100\ [N] \end{aligned}$$

- Total torque (rear fuselage)
  - $M_{\text{fus}} = M_{\text{tail}} + F_{\text{fin}} 1.13 = 5802 + 4100 \times 1.13 = 10\,435\ [Nm]$



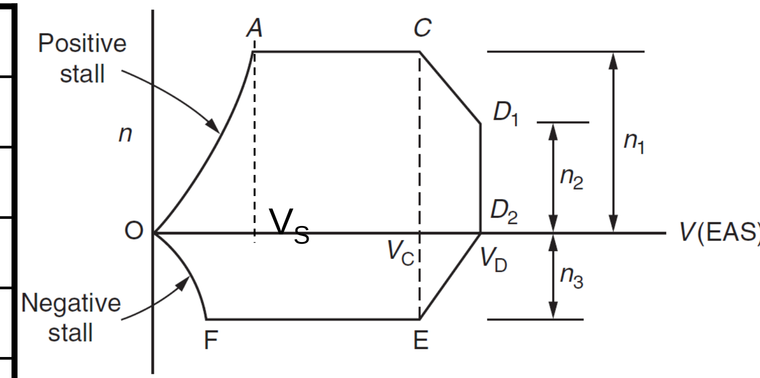


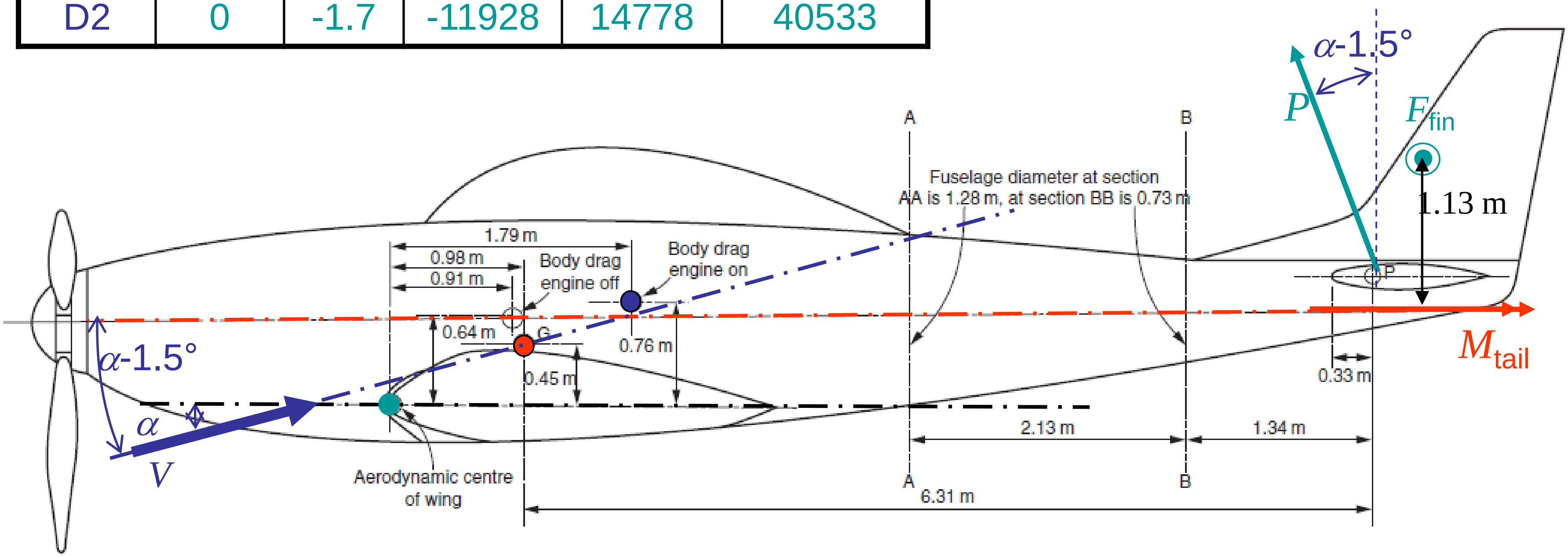


Balancing out calculations - End

- Summary
  - Other cases follow the same method

Case	n [-]	α [°]	P [N]	F_{fin} [N]	M_{fus} [Nm]
A	6.28	18	2505	4100	10439
A'	6.28	18	3174	4100	10439
C	6.28	6.7	137	9453	24906
D1	4.71	2.3	-5849	14778	40533
D2	0	-1.7	-11928	14778	40533



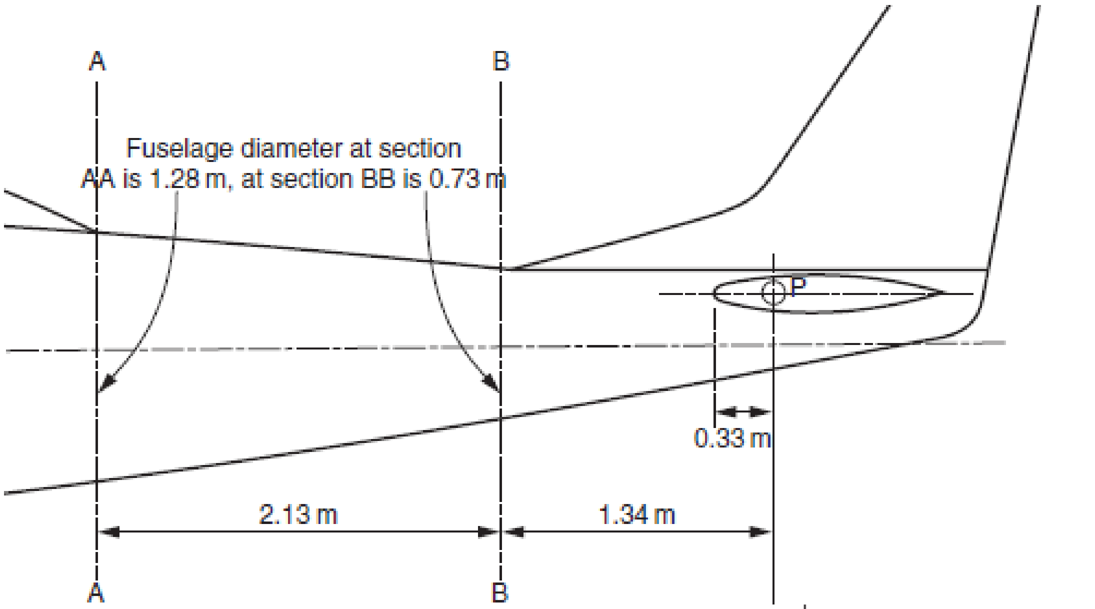


Fuselage loads – Preliminary choices

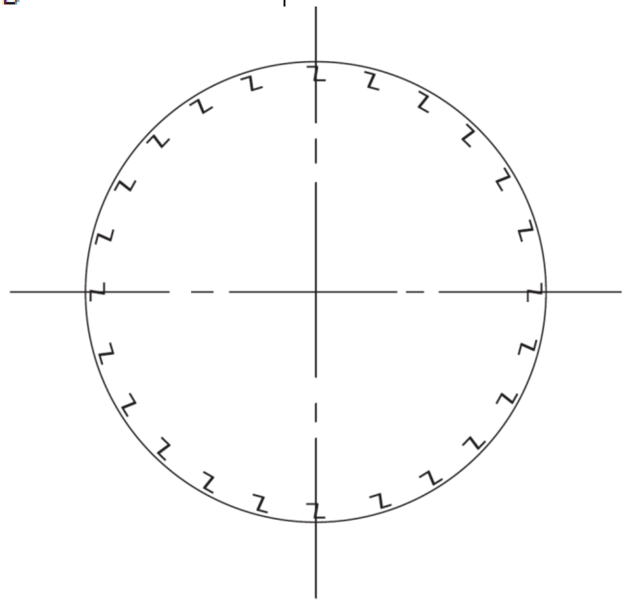
- Fuselage
  - Stringers
  - Frames
  - Skin
- Frames
  - Un-pressurized fuselage ←→ frames will not support significant loads
  - Frames are required to maintain the fuselage shape ← → nominal in size
- Combination of stringer and skin will resist self-weight and aerodynamic loads
  - Shear forces
  - Bending moments
  - Torques

Fuselage loads – Preliminary choices

- Geometrical data



- Circular cross-section
  - Simple to fabricate
  - Simple to design
  - Will meet the design requirements
- Possible arrangement
  - 24 stringers arranged symmetrically spaced at
    - Section AA: ~168 mm
    - Section BB: ~96 mm
  - All stringers have the same cross-sectional area



Fuselage loads – Data

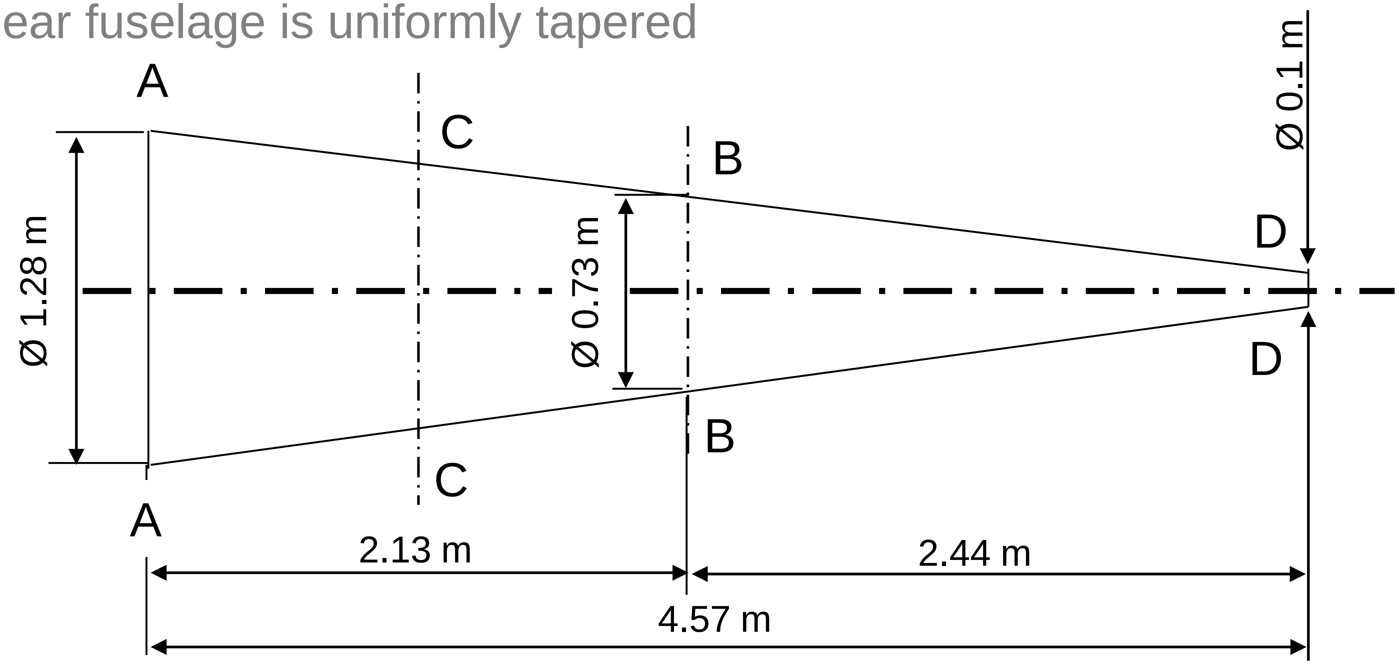
- Material: Aluminum alloy
  - Stingers & skin
  - 0.1 % proof stress: 232.5 [N/mm^2] = 232.5 [MPa]
  - Shear strength: 145.5 [N/mm^2] = 145.5 [MPa]

- Self-weight: Assumptions
  - Fuselage weight is from 4.8 to 8.0 % of the total weight
  - Tailplane/fin assembly is from 1.2 to 2.5 % of the total weight
  - Half of the fuselage weight is aft of the section AA

$$
\begin{aligned}
W_{rear\ fuselage} &= \frac{1}{2} \times 37.43\ 10^3 \times 6.4\% = 1198\,[N] \\
W_{tailplane+fin} &= 37.43\ 10^3 \times 1.8\% = 674\,[N]
\end{aligned}
$$

  - The weight distribution varies proportionally to the skin surface area

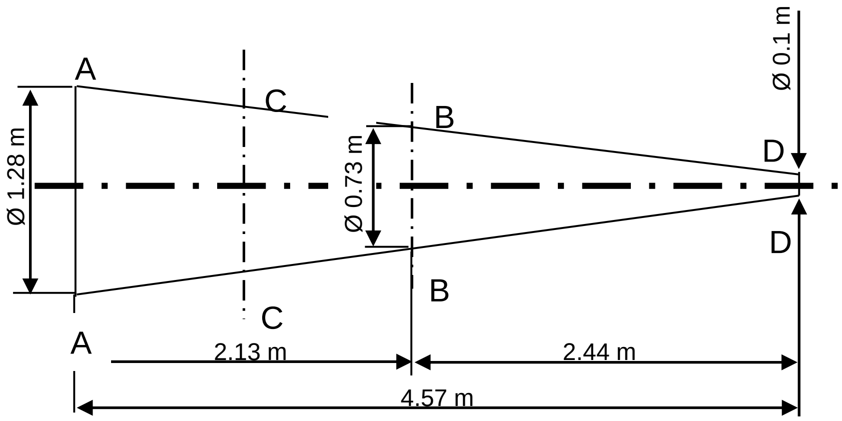
- Assumptions on geometry
  - Rear fuselage is uniformly tapered



$$\Rightarrow \quad \begin{aligned} Area_{skin} &= \frac{1}{2}\pi\left(D_{max}+D_{min}\right)L \\ &= \frac{1}{2}\pi\left(1.28+0.1\right)4.57 = 9.91\ [m^2] \end{aligned}$$

  - CC is a section midway AA and BB.
  - The center of gravity of the tailplane/fin assembly has been estimated to be 4.06 m from the section AA on a line parallel to the fuselage centre line

- Data
  - Weight of rear fuselage: 1198 [N]
  - Skin area of rear fuselage: 9.91 [m^2]

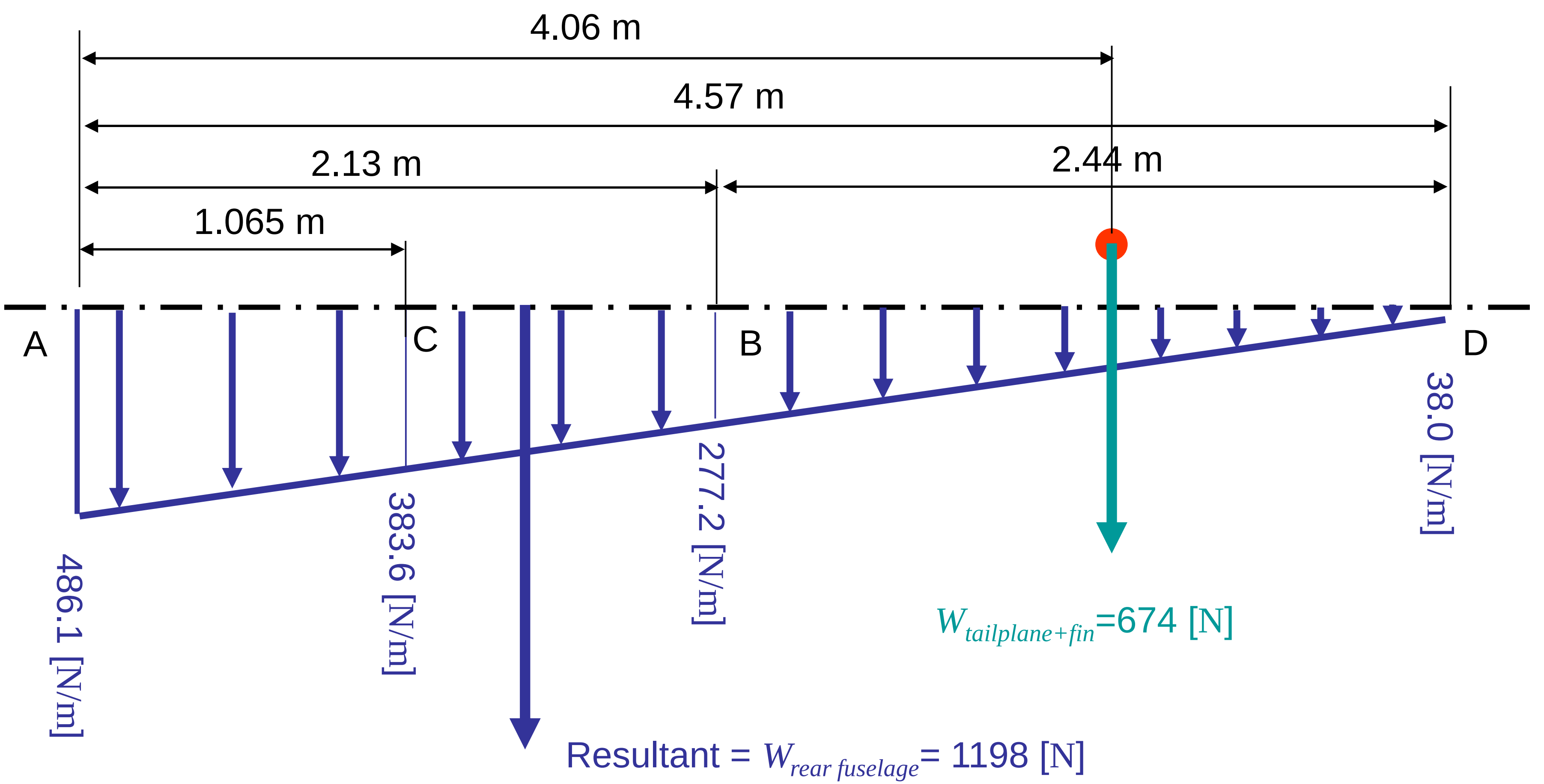


- Self-weight / m of the fuselage
  - $$weight/m = \frac{W_{rear\ fuselage} \times \pi \times D}{Area_{skin}}$$

$$\Rightarrow \left\{ \begin{array}{lcl} weight/m_{AA} = \frac{1}{9.91} \times 1198 \times \pi \times 1.28 & = & 486.1\,[N/m] \\ weight/m_{CC} = \frac{1}{9.91} \times 1198 \times \pi \times 1.01 & = & 383.6\,[N/m] \\ weight/m_{BB} = \frac{1}{9.91} \times 1198 \times \pi \times 0.73 & = & 277.2\,[N/m] \\ weight/m_{DD} = \frac{1}{9.91} \times 1198 \times \pi \times 0.1 & = & 38.0\,[N/m] \end{array} \right.$$

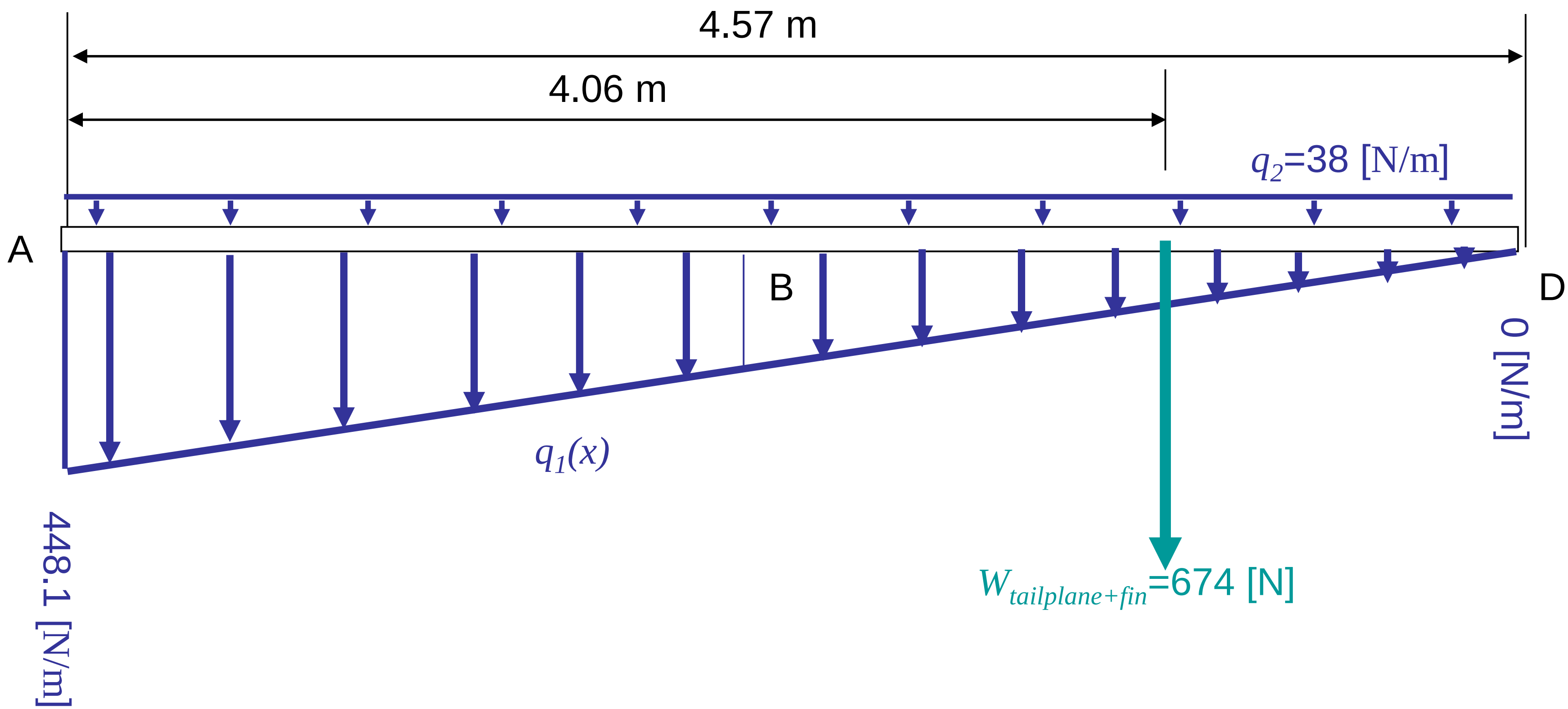
Fuselage loads – Shear force and bending moment due to self-weight

- Self-weight induces
  - Shear forces : SF
  - Bending moments : BM
- SF and BM are calculated by equilibrium (‘MNT’)



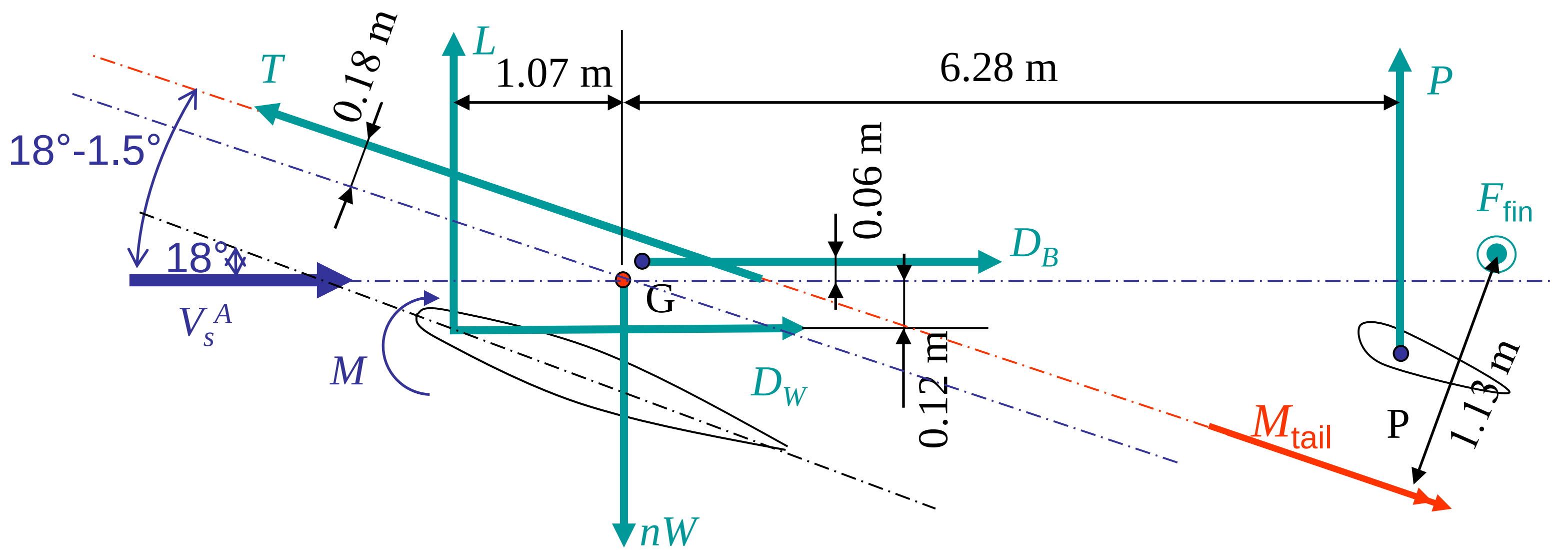
Fuselage loads – Shear force and bending moment due to self-weight

- Transform self-weight into
  - A triangular repartition: $q_1(x)$
  - And a constant linear force: q_2



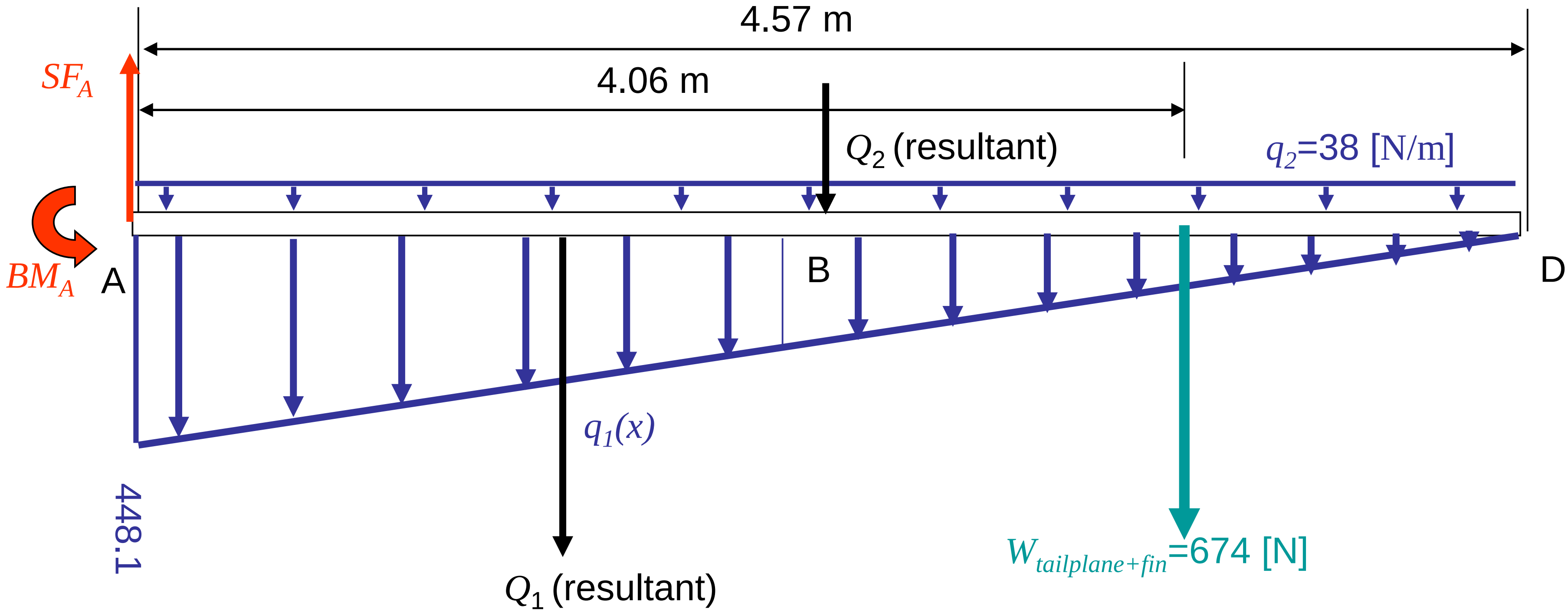
Fuselage loads – Shear force and bending moment due to self-weight

- Effect of load factor



  - The self weight is multiplied by the load factor n
    - Forces are not applied in the cross section plane
    - Forces are $(\alpha - 1.5°)$ –inclined with this section
      - Will be multiplied by cos $(\alpha - 1.5°)$ later
    - Rigorously, an axial loading should also be considered
  - Distance along fuselage axis is multiplied by cos $(\alpha - 1.5°)$
    - When computing bending moment
  - We do not compute the fuselage compression
    - Should be done and risk of buckling avoided

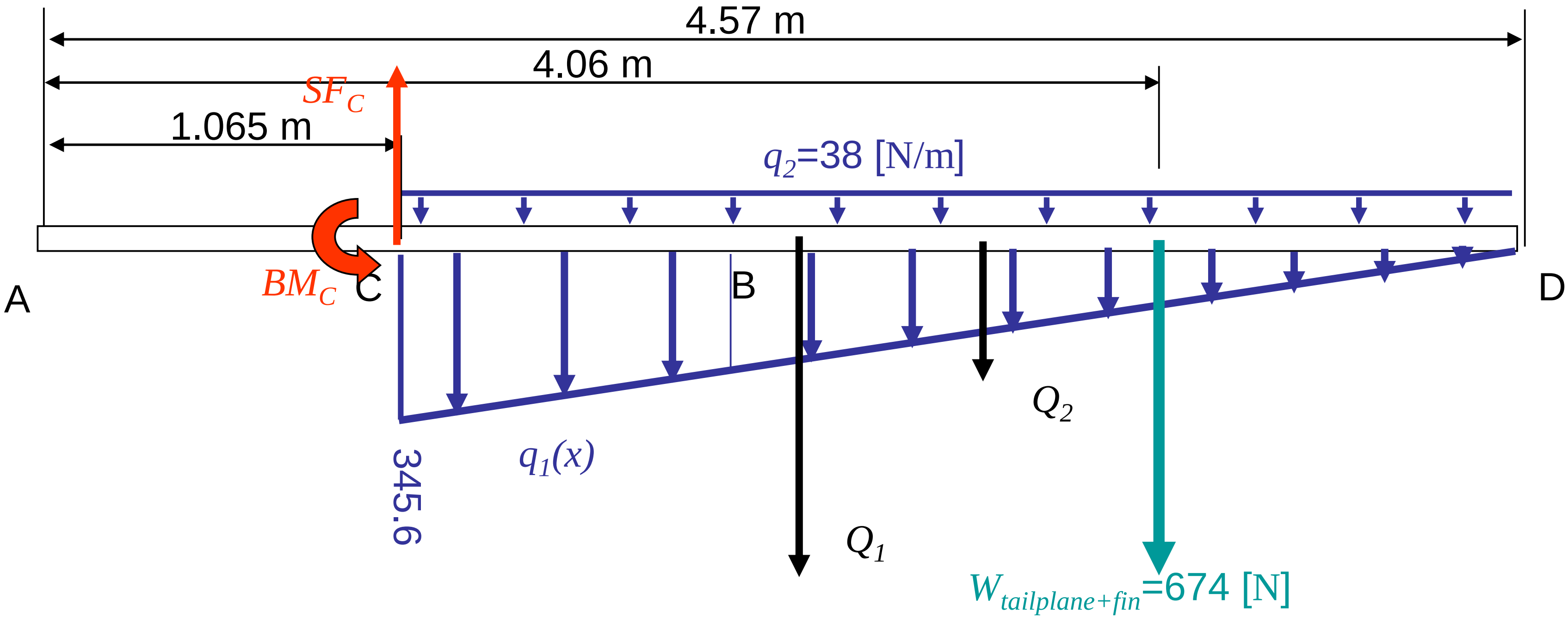
- Reactions at section AA



$$
\begin{aligned}
SF_A &= n \times (Q_1 + Q_2 + W_{tailplane+fin}) \\
&= n \times (\underbrace{\frac{1}{2} 448.1 \times 4.57 + 38.0 \times 4.57}_{1198 = W_{rear\ fuselage}} + 674) = 1872n \quad [N]
\end{aligned}
$$

$$
\begin{aligned}
BM_A &= n \cos(\alpha - 1.5^o) \times \left[4.06 \times W_{tailplane+fin} + \frac{1}{3} 4.57 \times Q_1 + \frac{1}{2} 4.57 \times Q_2 \right] \\
&= 4693 n \cos(\alpha - 1.5^o) \ [Nm]
\end{aligned}
$$

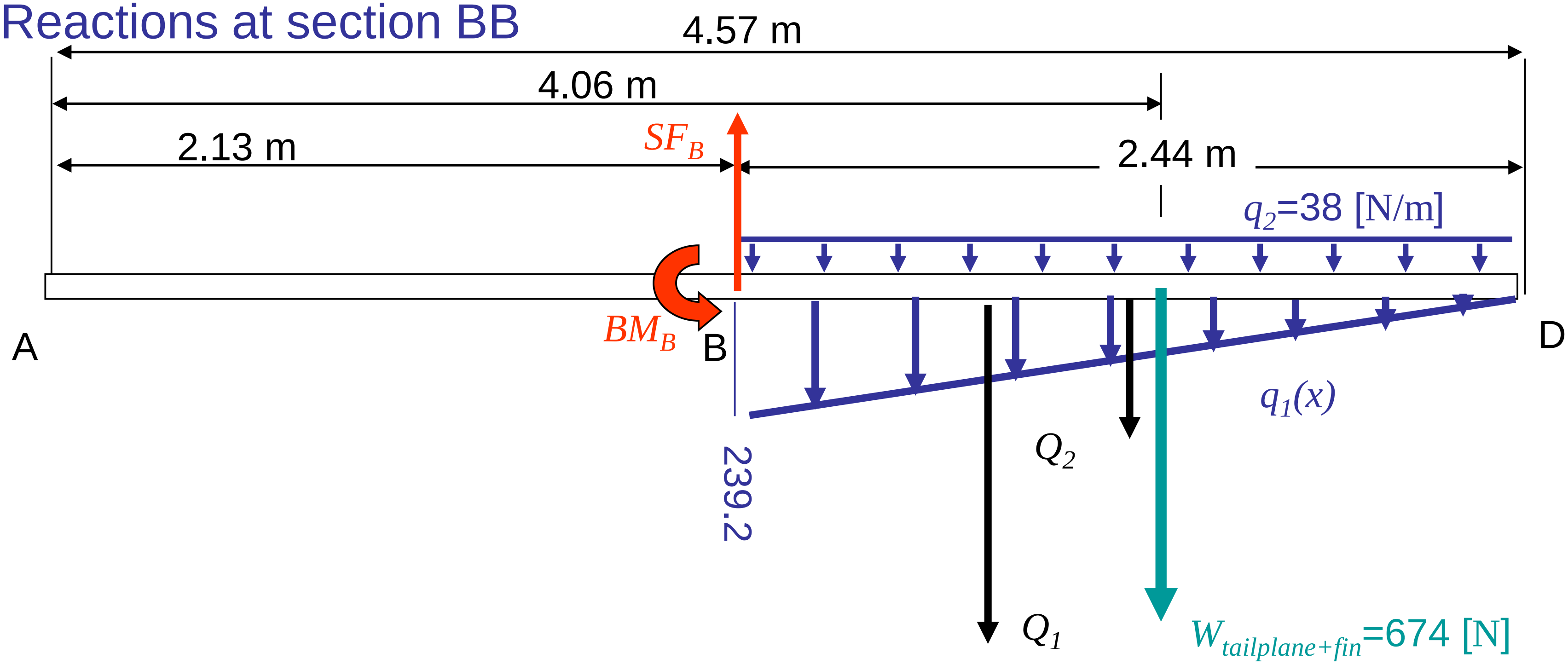
- Reactions at section CC



$$SF_C = n \times (Q_1 + Q_2 + W_{tailplane+fin})$$
$$= n \times (\frac{1}{2}345.6 \times 3.51 + 38.0 \times 3.51 + 674) = 1409n \quad [N]$$

$$BM_C = n \cos(\alpha - 1.5^o) \times \left[(4.06 - 1.065) \times W_{tailplane+fin} + \frac{1}{3}3.51 \times Q_1 + \frac{1}{2}3.51 \times Q_2\right]$$
$$= 2959n \cos(\alpha - 1.5^o)\,[Nm]$$

- Reactions at section BB



$$SF_B = n \times (Q_1 + Q_2 + W_{tailplane+fin})$$

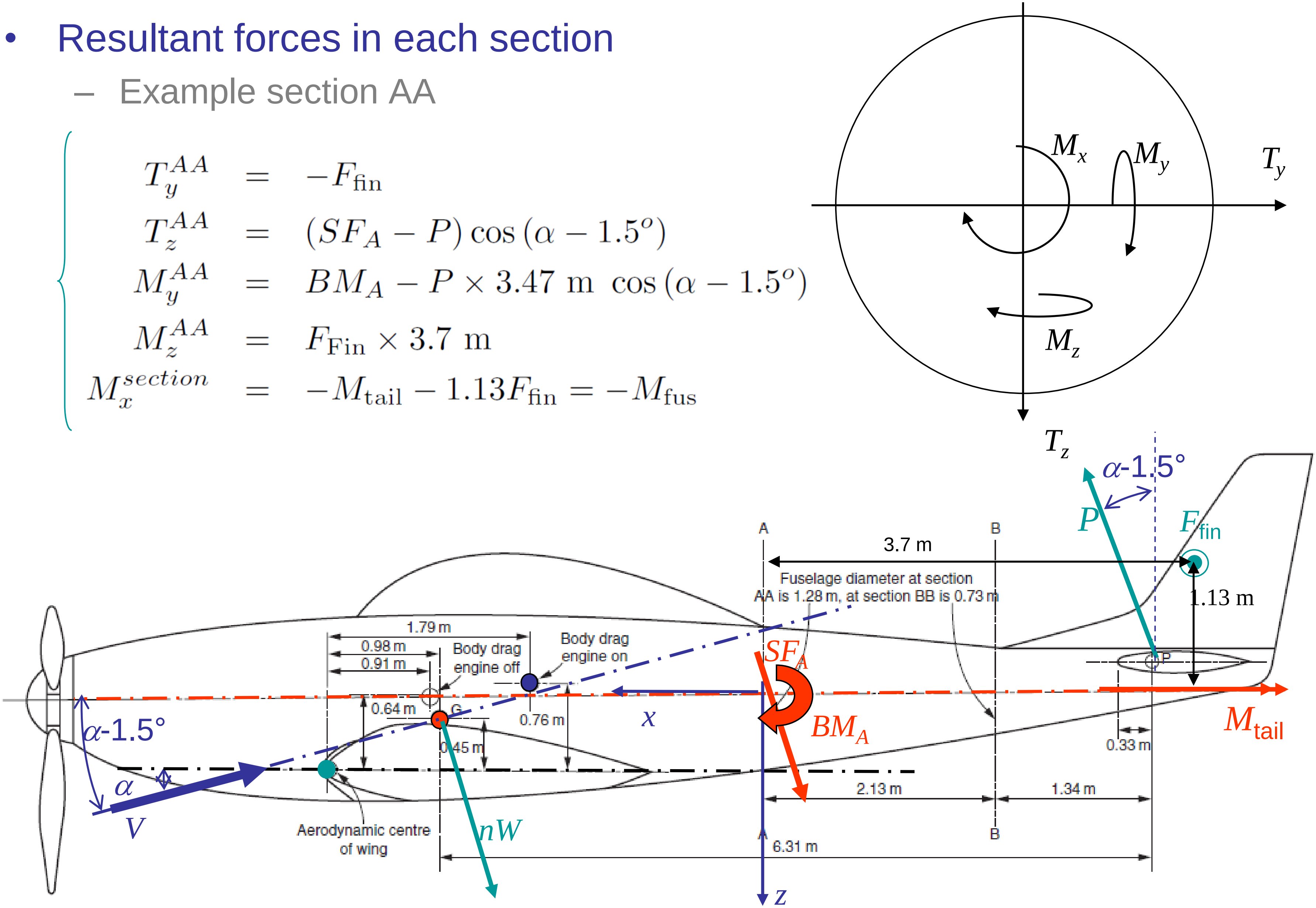
$$= n \times (\frac{1}{2}239.2 \times 2.44 + 38.0 \times 2.44 + 674) = 1059n \quad [N]$$

$$BM_B = n\cos(\alpha - 1.5^o) \times \left[(4.06 - 2.13) \times W_{tailplane+fin} + \frac{1}{3}2.44 \times Q_1 + \frac{1}{2}2.44 \times Q_2\right]$$

$$= 1651n\cos(\alpha - 1.5^o)\ [Nm]$$

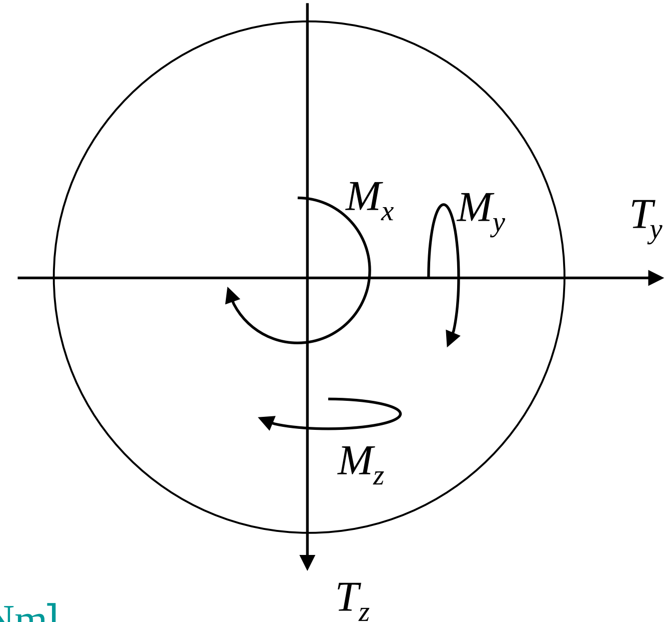
- Resultant forces in each section
  - Example section AA

$$\begin{cases} T_y^{AA} = -F_{\text{fin}} \\ T_z^{AA} = (SF_A - P)\cos(\alpha - 1.5^o) \\ M_y^{AA} = BM_A - P \times 3.47 \text{ m } \cos(\alpha - 1.5^o) \\ M_z^{AA} = F_{\text{Fin}} \times 3.7 \text{ m} \\ M_x^{section} = -M_{\text{tail}} - 1.13 F_{\text{fin}} = -M_{\text{fus}} \end{cases}$$



- Resultant forces in each section (2)

Case	n [-]	α [°]	P [N]	F_{fin} [N]	M_{fus} [Nm]
A	6.28	18	2505	4100	10439
A’	6.28	18	3174	4100	10439
C	6.28	6.7	137	9453	24906
D1	4.71	2.3	-5849	14778	40533
D2	0	-1.7	-11928	14778	40533



  – Example case A, section AA
    - $SF_A = 1872\, n$ [N] & $BM_A = 4693\, n \cos(\alpha\text{-}1.5°)$ [Nm]

$$
\left\{
\begin{array}{rcl}
T_y^{AA} & = & -F_{\text{fin}} = -4100 \text{ N} \\
T_z^{AA} & = & (SF_A - P)\cos(\alpha - 1.5^o) = (1872 \times 6.28 - 2505)\cos 16.5^o = 8580 \text{ N} \\
M_y^{AA} & = & BM_A - P \times 3.47 \text{ m } \cos(\alpha - 1.5^o) \\
 & = & (4693 \times 6.28 - 2505 \times 3.47)\cos 16.5^o = 20774 \text{ Nm} \\
M_z^{AA} & = & F_{\text{Fin}} \times 3.7 \text{ m } = 4100 \times 3.7 = 15174 \text{ Nm} \\
M_x^{section} & = & -M_{\text{tail}} - 1.13 F_{\text{fin}} = -M_{\text{fus}} = -10439 \text{ Nm}
\end{array}
\right.
$$

Total shear forces, bending moments and torque

- Table of sections loading
  - $SF_A = 1872\ n$ [N] & $BM_A = 4693\ n \cos(\alpha\text{-}1.5°)$ [Nm]
  - $SF_C = 1409\ n$ [N] & $BM_C = 2959\ n \cos(\alpha\text{-}1.5°)$ [Nm]
  - $SF_B = 1059\ n$ [N] & $BM_B = 1651\ n \cos(\alpha\text{-}1.5°)$ [Nm]

Sect	Case	T_y [N]	T_z [N]	M_y [Nm]	M_z [Nm]	M_x [Nm]
AA	A	-4100	9249	20774	15174	-10439
	A'	-4100	8580	18454	15174	-10439
	C	-9453	11615	28987	34976	-24906
	D1	-14778	14662	42387	54680	-40533
	D2	-14778	11924	41376	54680	-40533
CC	A	-4100	6342	12555	10806	-10439
	A'	-4100	5673	10946	10806	-10439
	C	-9453	8708	18247	24909	-24906
	D1	-14778	12482	27995	38941	-40533
	D2	-14778	11924	28677	38941	-40533
BB	A	-4100	4144	7010	6439	-10439
	A'	-4100	3476	6114	6439	-10439
	C	-9453	6511	10181	14841	-24906
	D1	-14778	10834	15609	23202	-40533
	D2	-14778	11924	15978	23202	-40533

Step 2 – Design of the rear fuselage

Stringers, frames, skin and rivets

Fuselage design calculation – Material and approach

- 2 types of design are possible
  - Elastic design
    - Allowed stress = 0.1% proof stress / s
    - s : safety factor = 1.5
  - Ultimate load design
    - Ultimate stresses
    - Actual load multiplied by an ultimate load factor
- For linear systems : both designs give the same results

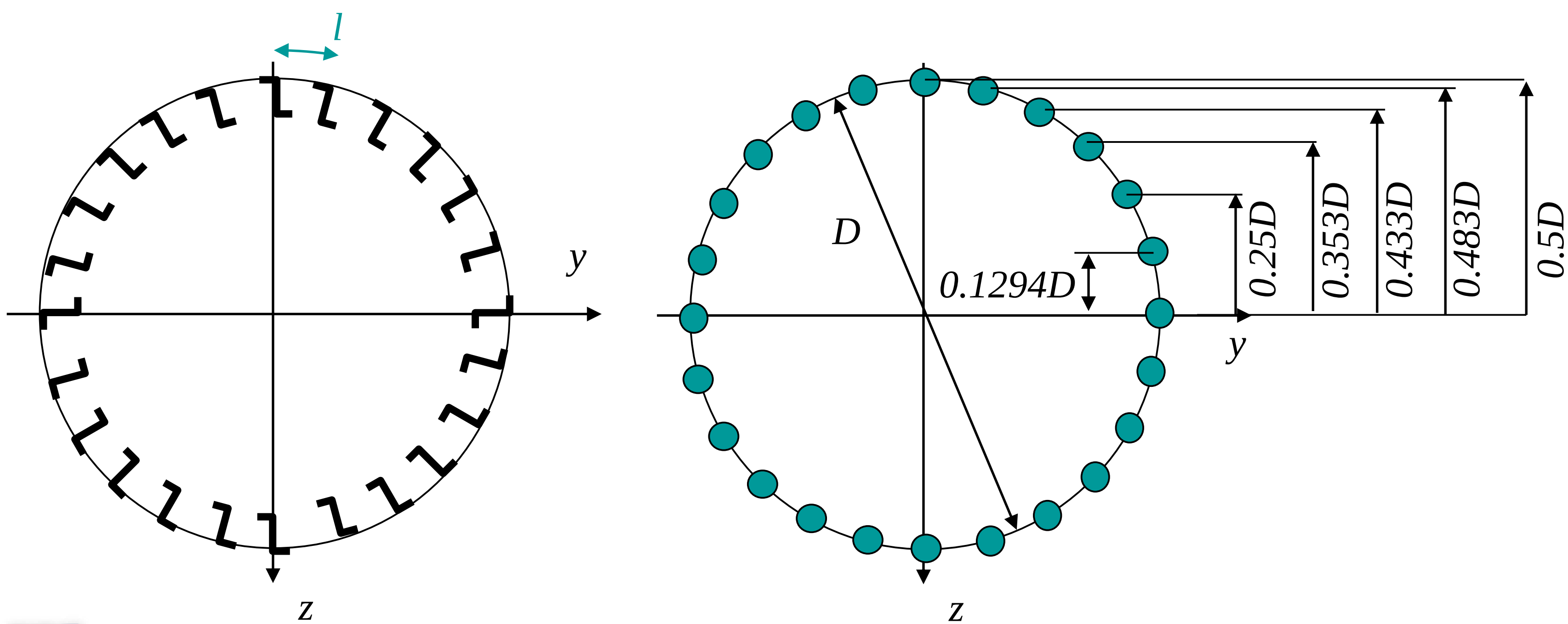
- We use the elastic design as 0.1 proof stress is known

$$\sigma_{max} = \frac{\sigma_{0.1\%}}{s} = \frac{232.5}{1.5} = 155\,MPa$$

$$\tau_{max} = \frac{\tau_{strength}}{s} = \frac{145.5}{1.5} = 97\,MPa$$

Stringers section: Data

- Frames
  - Un-pressurized fuselage ←→ frames will not support significant loads
  - Frames are required to maintain the fuselage shape ← → nominal in size
- Circular cross-section
  - 24 stringers arranged symmetrically and spaced at around
    - Section AA – l ~168 mm
    - Section BB – l ~96 mm
  - All stringers have the same cross-sectional area

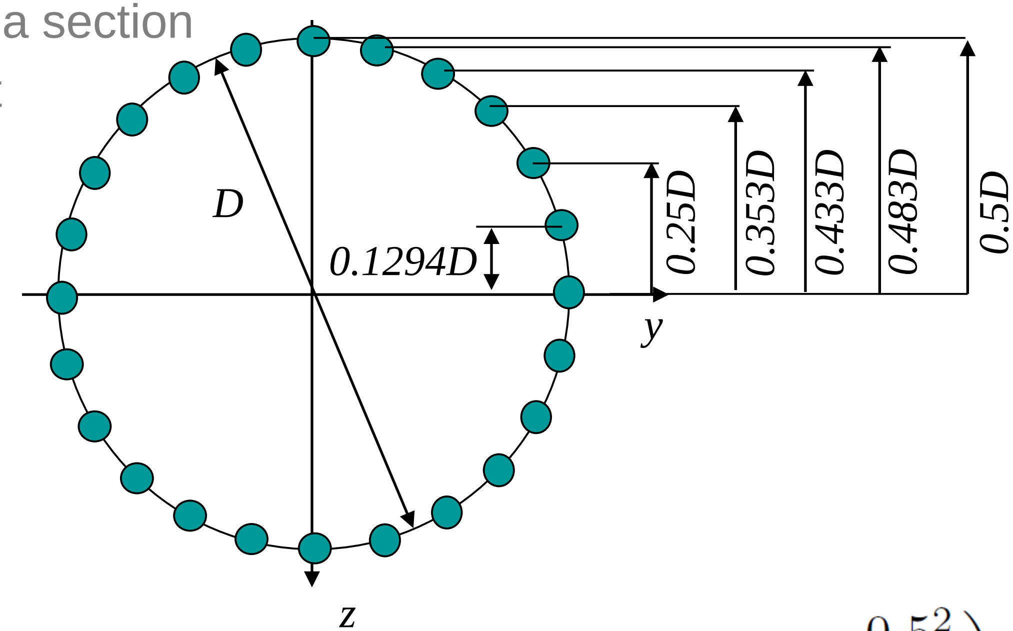


Stringers section

- Direct stress
  - Induced by M_z and M_y
  - Obtained from $\boldsymbol{\sigma}_{xx} = \frac{M_y}{I_{yy}} z - \frac{M_z}{I_{zz}} y$ (as I_{yz} = 0)

- Unknown
  - B: the area of the stringers in a section
  - B should be chosen such that

$$\boldsymbol{\sigma}_{xx} \leq \sigma_{max} = 155\, MPa$$



- Second moments of area

$$I_{zz} = I_{yy} = \sum_{i=1}^{24} B \times z_i^2$$

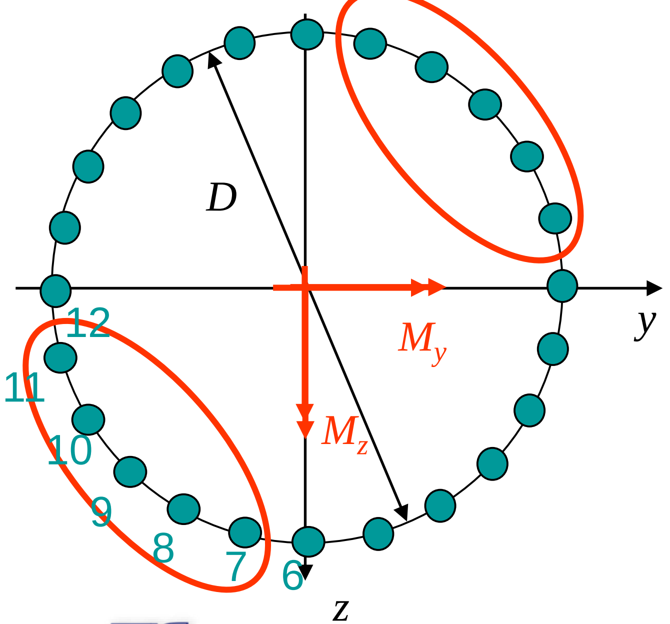
$$= 4BD^2 \left(0.1294^2 + 0.25^2 + 0.353^2 + 0.433^2 + 0.483^2 + \frac{0.5^2}{2}\right)$$

$$= 3BD^2$$

- Values of M_x and M_y
  - Stress

$$\sigma_{xx} = \frac{M_y}{I_{yy}} z - \frac{M_z}{I_{zz}} y$$

  - Worst case
    - Case D1 (dive)
    - M_z and M_y have same sign
    - y and z of opposite sign



Sect	Case	T_y [N]	T_z [N]	M_y [Nm]	M_z [Nm]	M_x [Nm]
AA	A	-4100	9249	20774	15174	-10439
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	D1	-14778	10834	15609	23202	-40533
	D2	-14778	11924	15978	23202	-40533

- Calculate $B\sigma_{xx}$
  - $$\boldsymbol{\sigma}_{xx} = \frac{M_y}{3BD^2} z - \frac{M_z}{3BD^2} y$$

$$\Rightarrow \quad B\boldsymbol{\sigma}_{xx} = \frac{M_y}{3D^2} z - \frac{M_z}{3D^2} y$$

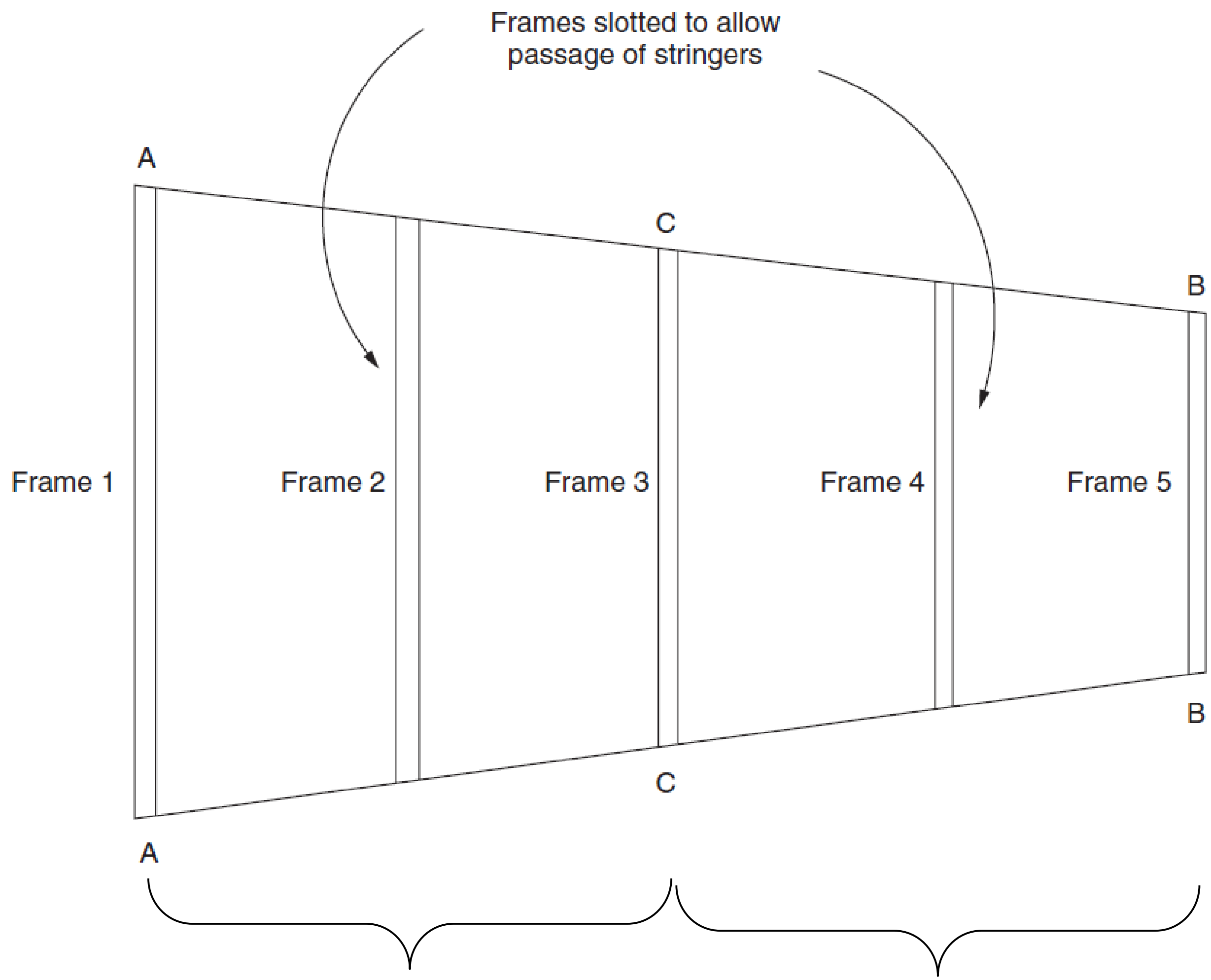
  - For each
    - Section
      - M & D change
    - Stringer
      - y & z change
  - Determine minimal value of stringers' area B such that

$$\boldsymbol{\sigma}_{xx} \leq \sigma_{max} = 155\, MPa$$

Sect.		AA			CC			BB		
Data		D	1.28 m		D	1.01 m		D	0.73 m	
		M_y	42 kN·m		M_y	29 kN·m		M_y	16 kN·m	
		M_z	55 kN·m		M_z	39 kN·m		M_z	23 kN·m	
Stringers	n°	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]
	6	0	0.64	5.52	0	0.50	4.76	0	0.37	3.65
	7	0.16	0.62	7.17	0.13	0.49	6.27	0.09	0.35	4.89
	8	0.32	0.55	8.34	0.25	044	7.34	0.18	0.32	5.81
	9	0.45	0.45	**8.94**	0.36	0.36	7.93	0.26	0.26	6.33
	10	0.55	0.32	8.92	0.44	0.25	**7.97**	0.32	0.18	**6.41**
	11	0.62	0.16	8.30	0.49	0.13	7.47	0.35	0.09	6.06
	12	0.64	0	7.12	0.50	0	6.46	0.37	0	5.30
Max. $B\sigma_{xx}$ [kN]				8.94			7.97			6.41
Min. B [mm^2]				57.7			51.4			41.4

Stringers and frames

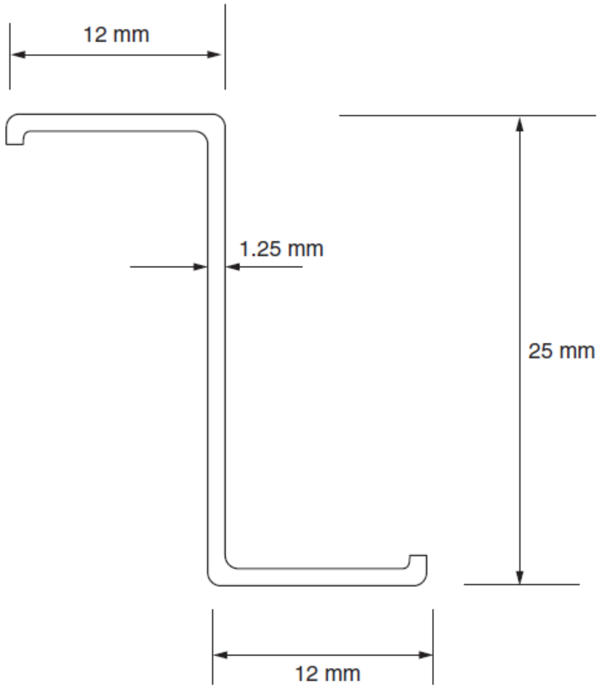
- From calculation $B_{\mathrm{min}}(\mathrm{AA}) > B_{\mathrm{min}}(\mathrm{CC}) > B_{\mathrm{min}}(\mathrm{CC})$
  - Lighter stringer between CC and BB



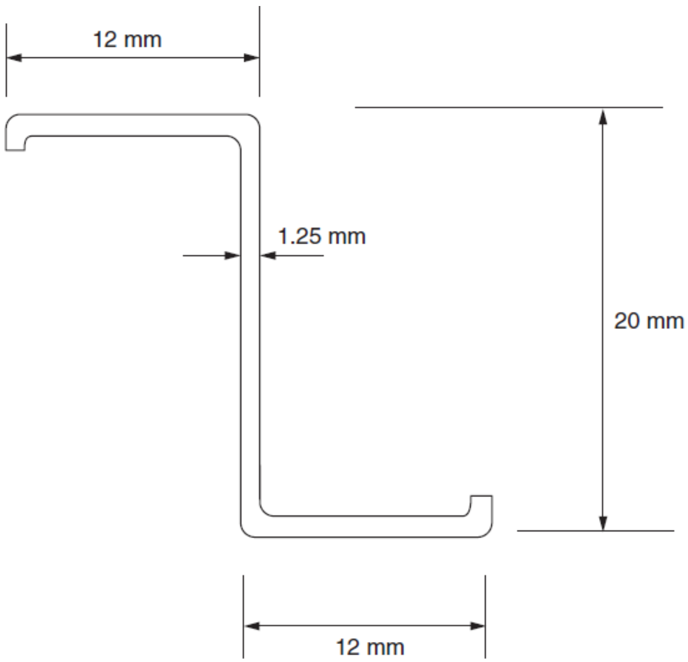
type A stringers : B > 57.7 mm²   type B stringers : B > 51.4 mm²

- Stringer type "Z-section"

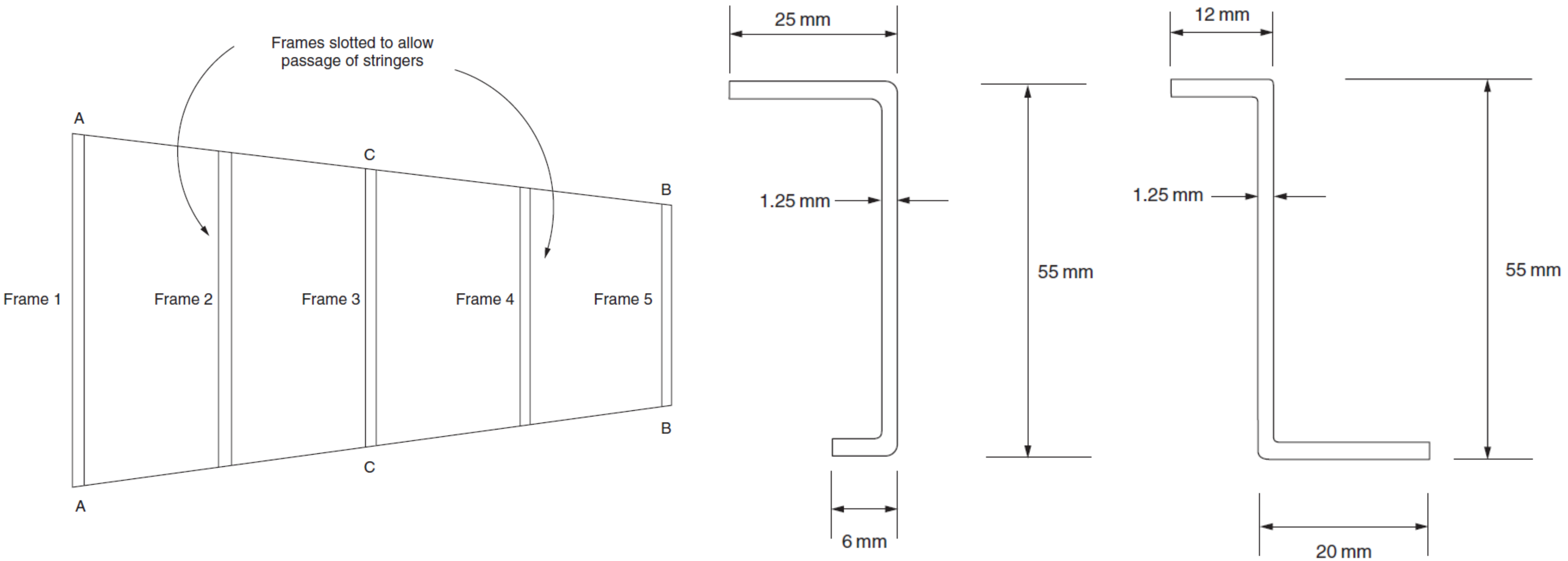
Type A stringer: B = 58.1 > 57.7 mm²



Type B stringer: B = 51.9 > 51.4 mm²



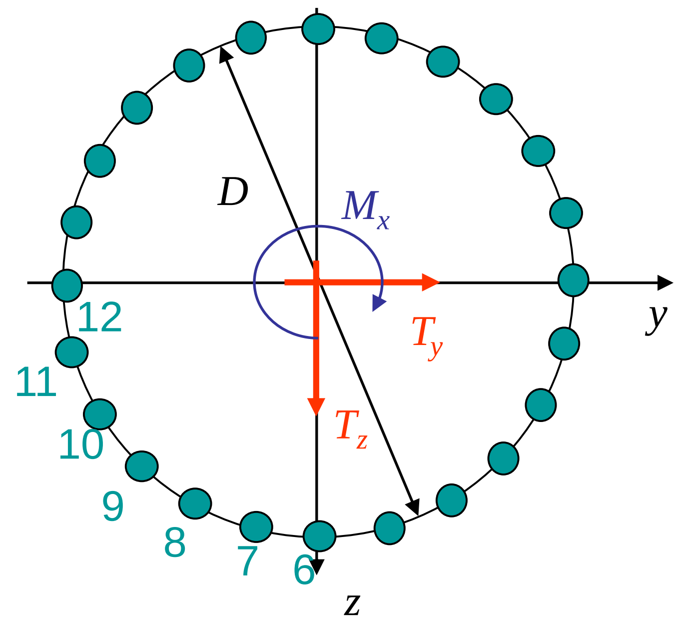
- Frames
  - Non load bearing
  - Must be of sufficient size to be connected with stringers (AA/BB/CC sections)
    - Use of brackets
  - Must be of sufficient size to allow to be cut
    - So stringers can pass trough them



(a) Frames 1 and 5

(b) Frames 2, 3 and 4

- Skin must resist shear flow due to
  - Shear loads T_y, T_z &
  - Torque M_x
- Calculate the shear flow
  - At each boom there is a discontinuity

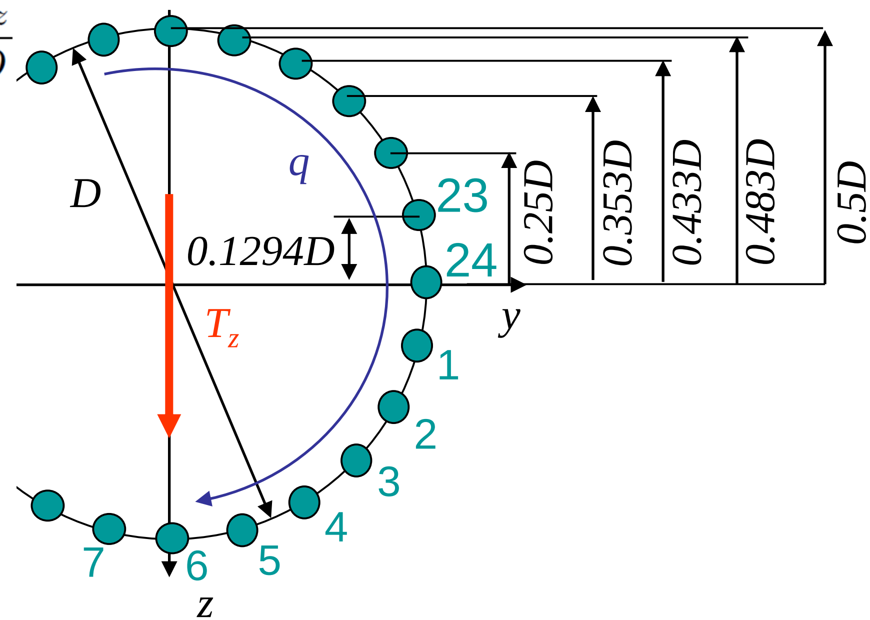


$$
\begin{aligned}
q^{i+1} - q^{i} &= -\frac{T_z}{I_{yy}} B_i z_i - \frac{T_y}{I_{zz}} B_i y_i \\
&= -\frac{T_z}{3D^2} z_i - \frac{T_y}{3D^2} y_i
\end{aligned}
$$

As stringers have constant area in one section and as $I_{yy} = I_{zz} = 3BD^2$

- Shear flow due to T_z
  - Following equations $q^{i+1} - q^{i} = -\frac{T_z}{3D^2} z_i$

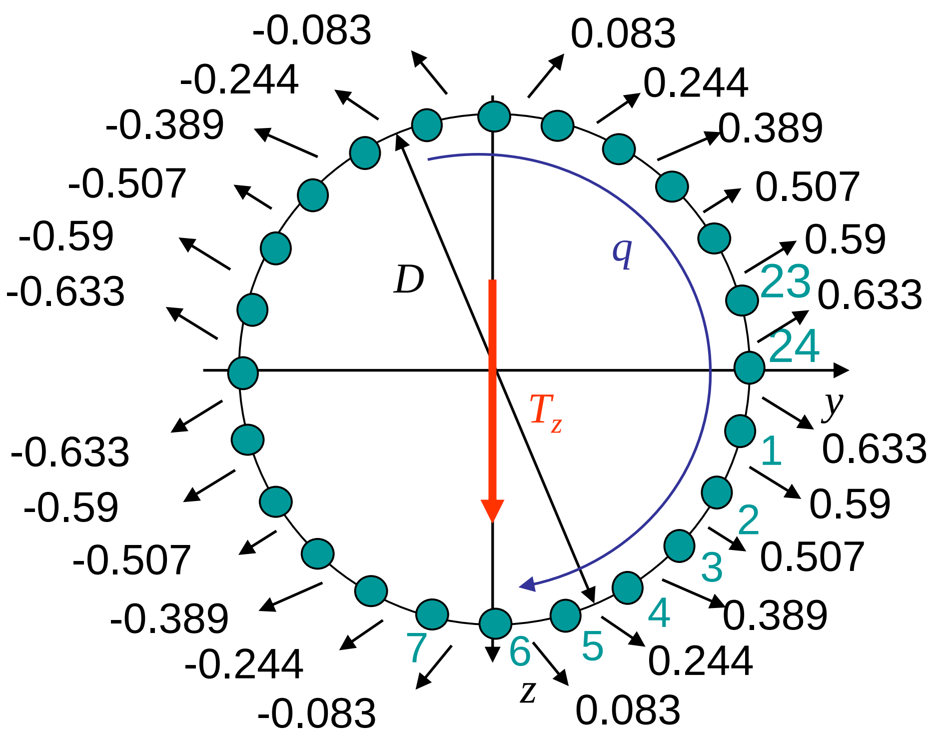
$$
\begin{cases}
q^{1\,2} = q^{24\,1} - \frac{T_z}{3D^2} \times 0.1294D = q^{24\,1} - 0.043\frac{T_z}{D} \\
q^{2\,3} = q^{1\,2} - \frac{T_z}{3D^2} \times 0.25D = q^{24\,1} - 0.126\frac{T_z}{D} \\
q^{3\,4} = q^{2\,3} - \frac{T_z}{3D^2} \times 0.353D = q^{24\,1} - 0.244\frac{T_z}{D} \\
q^{4\,5} = q^{3\,4} - \frac{T_z}{3D^2} \times 0.433D = q^{24\,1} - 0.388\frac{T_z}{D} \\
q^{5\,6} = q^{4\,5} - \frac{T_z}{3D^2} \times 0.483D = q^{24\,1} - 0.549\frac{T_z}{D} \\
q^{6\,7} = q^{5\,6} - \frac{T_z}{3D^2} \times 0.5D = q^{24\,1} - 0.716\frac{T_z}{D}
\end{cases}
$$



  - By symmetry (no torque)
    - $q^{6\,7} = - q^{56}$

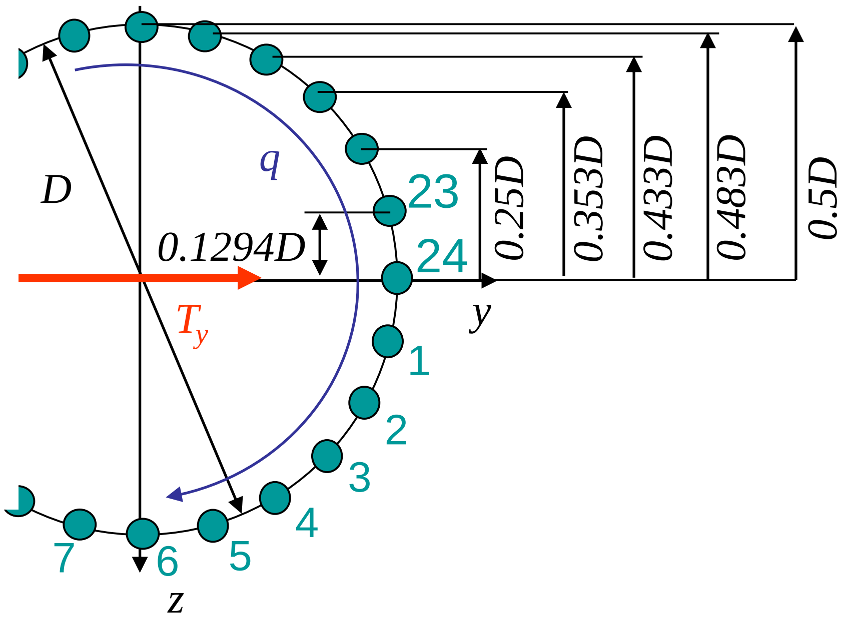
$$\Longrightarrow \quad q^{24\,1} - 0.716\frac{T_z}{D} = -q^{24\,1} + 0.549\frac{T_z}{D} \Leftrightarrow q^{24\,1} = \frac{0.633 \times T_z}{D}$$

- Shear flow due to T_z (2)
  - Final form of qD/T_z



- Shear flow due to T_y
  - Following equations $q^{i+1} - q^{i} = -\frac{T_y}{3D^2} y_i$

$$\begin{cases} q^{1\,2} = q^{24\,1} - \frac{T_y}{3D^2} \times 0.483D = q^{24\,1} - 0.161\frac{T_y}{D} \\ q^{2\,3} = q^{1\,2} - \frac{T_y}{3D^2} \times 0.433D = q^{24\,1} - 0.305\frac{T_y}{D} \\ q^{3\,4} = q^{2\,3} - \frac{T_y}{3D^2} \times 0.353D = q^{24\,1} - 0.4232\frac{T_y}{D} \\ q^{4\,5} = q^{3\,4} - \frac{T_y}{3D^2} \times 0.25D = q^{24\,1} - 0.507\frac{T_y}{D} \\ q^{5\,6} = q^{4\,5} - \frac{T_y}{3D^2} \times 0.1294D = q^{24\,1} - 0.55\frac{T_y}{D} \end{cases}$$

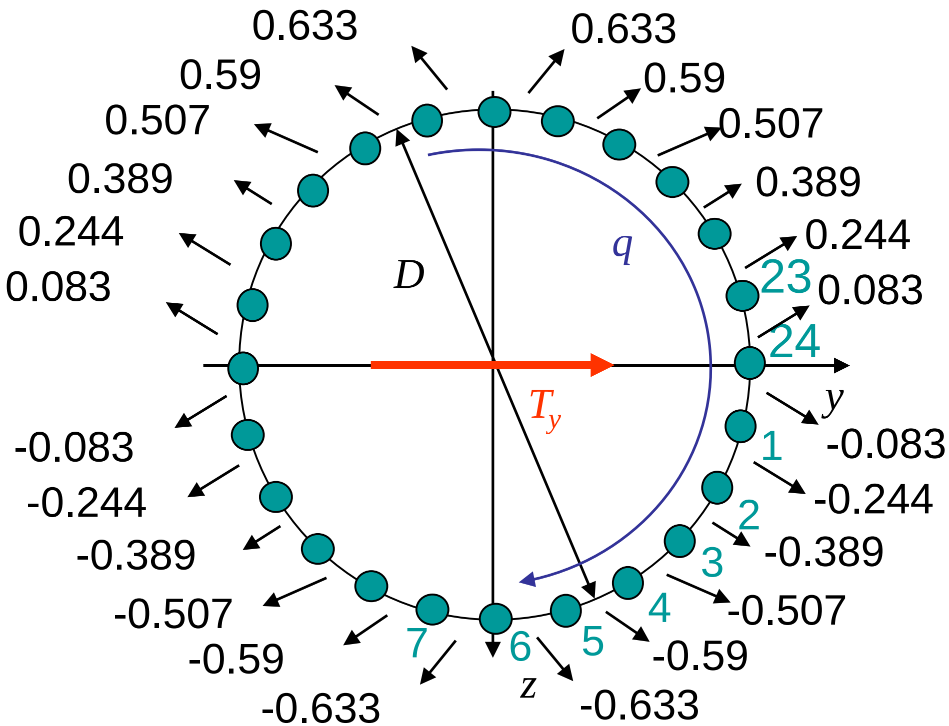


  - But we also have

$$q^{24\,1} = q^{23\,24} - \frac{T_y}{3D^2} \times 0.5D = q^{23\,24} - 0.1667\frac{T_y}{D}$$

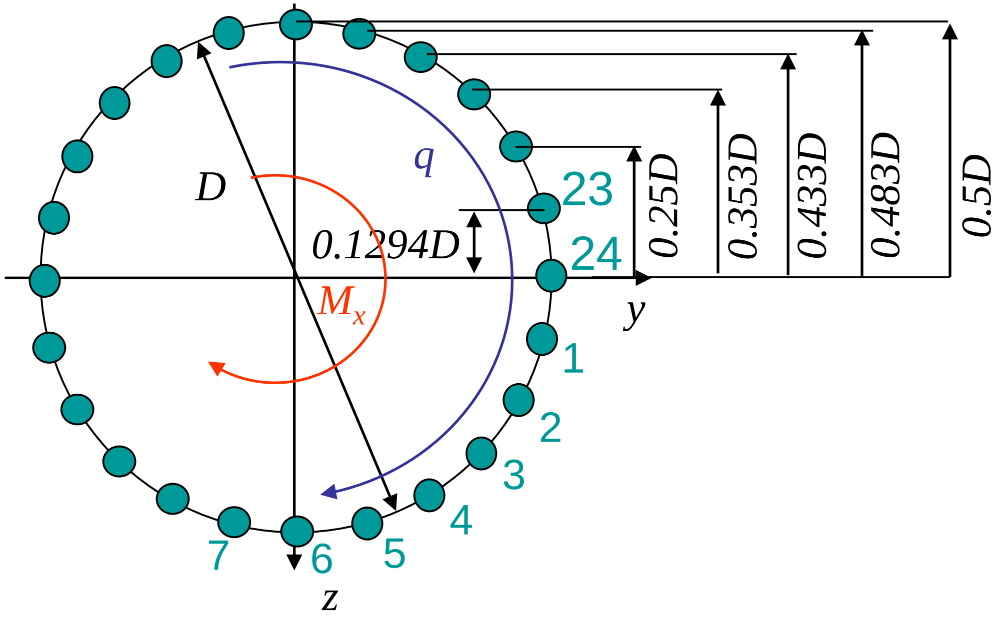
  - By symmetry (no torque)
    - $q^{23\,24} = - q^{24\,1}$ $\Longrightarrow$ $2q^{24\,1} = -0.1667\frac{T_y}{D} \Leftrightarrow q^{24\,1} = -\frac{0.0833 \times T_y}{D}$

- Shear flow due to T_y (2)
  - Final form of qD/T_y



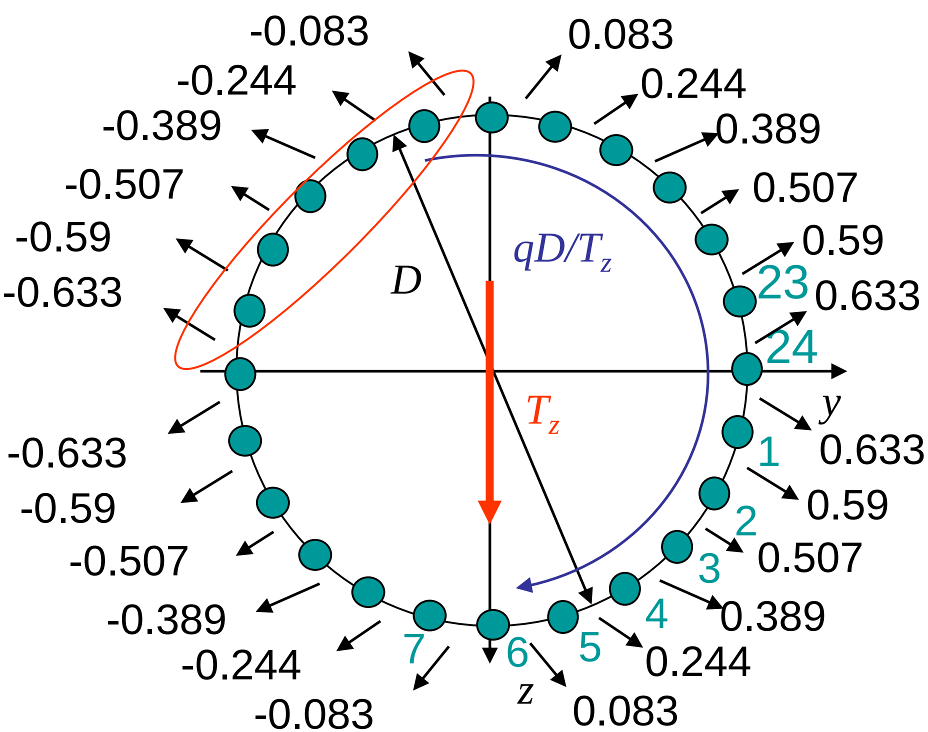
- Shear flow due to torque

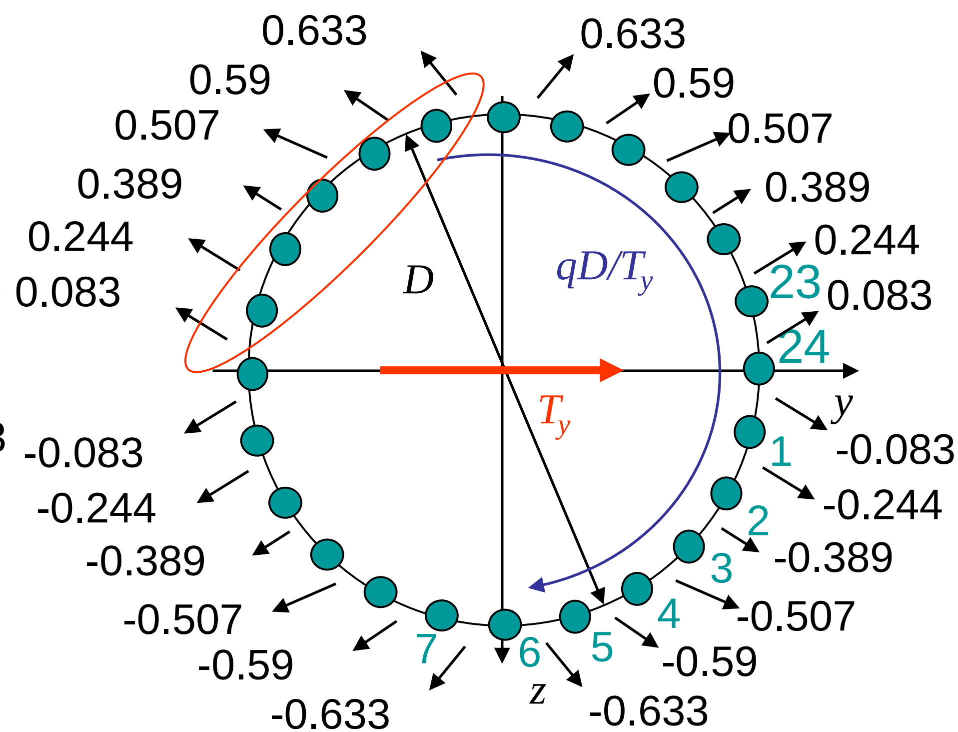
$$
\begin{aligned}
q_T &= \frac{M_x}{2A} \\
&= \frac{M_x}{2(\pi D^2/4)} \\
&= \frac{0.637 \times M_x}{D^2}
\end{aligned}
$$



Skin thickness – Shear flow caused by torque

- Maximum shear flow
  - As T_z >0, T_y < 0 & M_x < 0
  - Maximum shear flow is in a skin panel between stringers 12 and 18
    - Example: $q^{12\,13} \quad = \quad -0.633\frac{T_z}{D} + 0.083\frac{T_y}{D} + 0.637\frac{M_x}{D^2}$





- Maximum shear flow (2)
  - Equations

$$\left\{\begin{array}{lcl} q^{12\,13} & = & -0.633\dfrac{T_z}{D}+0.083\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \\ q^{13\,14} & = & -0.590\dfrac{T_z}{D}+0.244\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \\ q^{14\,15} & = & -0.507\dfrac{T_z}{D}+0.389\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \\ q^{15\,16} & = & -0.389\dfrac{T_z}{D}+0.507\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \\ q^{16\,17} & = & -0.244\dfrac{T_z}{D}+0.590\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \\ q^{17\,18} & = & -0.083\dfrac{T_z}{D}+0.633\dfrac{T_y}{D}+0.637\dfrac{M_x}{D^2} \end{array}\right.$$

Skin Thickness – Maximum shear flow

- Maximum shear flow (3)
  - Critical case: D1

Sect	Case	T_y [N]	T_z [N]	M_y [Nm]	M_z [Nm]	M_x [Nm]
AA	A	-4100	9249	20774	15174	-10439
	A'	-4100	8580	18454	15174	-10439
	C	-9453	11615	28987	34976	-24906
	D1	-14778	14662	42387	54680	-40533
	D2	-14778	11924	41376	54680	-40533
CC	A	-4100	6342	12555	10806	-10439
	A'	-4100	5673	10946	10806	-10439
	C	-9453	8708	18247	24909	-24906
	D1	-14778	12482	27995	38941	-40533
	D2	-14778	11924	28677	38941	-40533
BB	A	-4100	4144	7010	6439	-10439
	A'	-4100	3476	6114	6439	-10439
	C	-9453	6511	10181	14841	-24906
	D1	-14778	10834	15609	23202	-40533
	D2	-14778	11924	15978	23202	-40533

Skin Thickness – Maximum shear flow

- Computation
  - See Table
- Minimum skin thickness
  - From

$$\frac{q_{max}}{t} \leq \tau_{max}$$

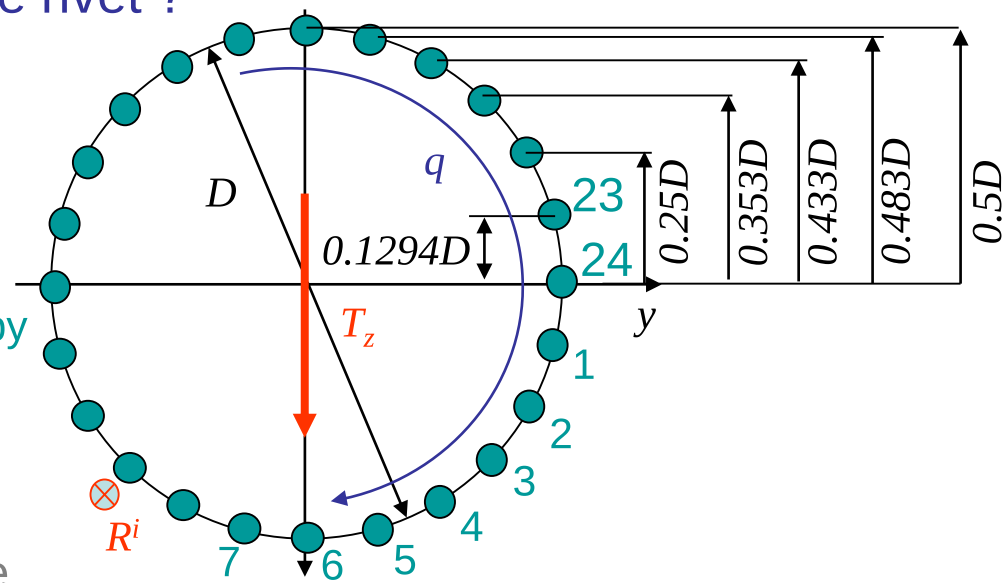
$$\frac{q_{max}}{\tau_{max}} \leq t$$

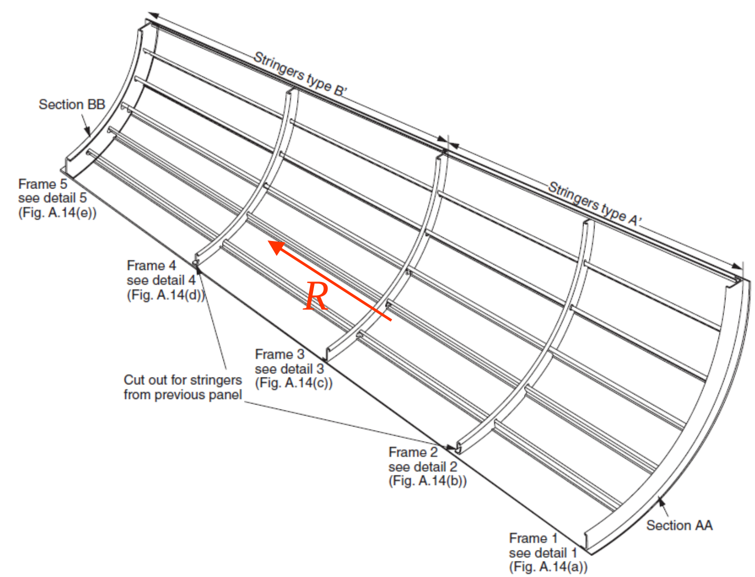
$$t = \frac{65}{97} = 0.67\,mm$$

  - But must support rivets
    - 1mm skin thickness is chosen

Sect.		AA		CC		BB	
Data		D	1.28 m	D	1.01 m	D	0.73 m
		T_y	-15 kN	T_y	-15 kN	T_y	-15 kN
		T_z	15 kN	T_z	12 kN	T_z	12 kN
		M_x	-41 kN·m	M_x	-40 kN·m	M_x	-40 kN·m
Panels	n°	q [kN·m^{-1}]		q [kN·m^{-1}]		q [kN·m^{-1}]	
	12-13	-22.04		-32.18		-57.07	
	13-14	-23.96		-34.64		-60.45	
	14-15	-25.33		-36.47		-63.03	
	15-16	-26.04		-37.55		-64.56	
	16-17	**-26.05**		**-37.82**		**-65.02**	
	17-18	-25.36		-37.26		-64.35	
Max. q [kN·m^{-1}]		-26.1		-37.8		-65.0	
Min. t [mm]		0.27		0.39		**0.67**	

- What are the forces acting on the rivet ?
  - At a stringer, we found for T_z
    - $$q^{i+1} - q^{i} = -\frac{T_z}{3D^2} z_i$$
    - This corresponds to
      - The shear flow balanced by
      - All the rivets linking the skin to the stringer
  - Therefore the shear load per unit stringer length acting on the rivets fixing the skin to the stringer i is
    - $$R^{i} = \left(-\frac{T_y}{3D^2} y_i - \frac{T_z}{3D^2} z_i \right)$$
    - Maximum between stringers 6 and 12
      - T_y <0 & T_z >0 $\Longrightarrow$ y <0 & z >0
      - Critical case is still D1
  - Remark: the torque does not lead to a discontinuity in the shear flow





Rivets size – Skin/stringers

- Results
  - Maximal load:
    - 4.91 kN per unit stringer length

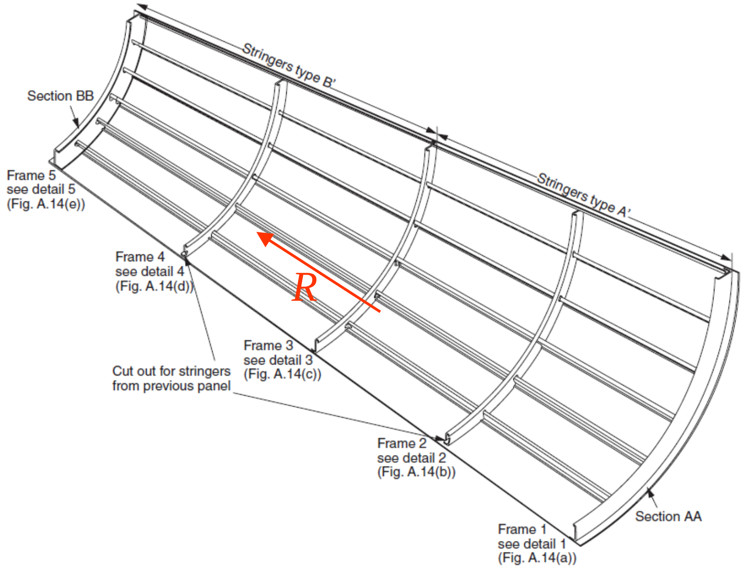
Sect.	AA			CC			BB		
Data	D	1.28 m		D	1.01 m		D	0.73 m	
	T_y	-15 kN		T_y	-15 kN		T_y	-15 kN	
	T_z	15 kN		T_z	12 kN		T_z	12 kN	
Stringers n°	$-y$ [m]	z [m]	$-R$ [kN/m]	$-y$ [m]	z [m]	$-R$ [kN/m]	$-y$ [m]	z [m]	$-R$ [kN/m]
6	0	0.64	1.91	0	0.50	2.07	0	0.37	2.72
7	0.16	0.62	2.34	0.13	0.49	2.63	0.09	0.35	3.50
8	0.32	0.55	2.62	0.25	044	3.02	0.18	0.32	4.04
9	0.45	0.45	**2.71**	0.36	0.36	**3.20**	0.26	0.26	**4.31**
10	0.55	0.32	2.62	0.44	0.25	3.16	0.32	0.18	4.28
11	0.62	0.16	2.35	0.49	0.13	2.90	0.35	0.09	3.96
12	0.64	0	1.92	0.50	0	2.45	0.37	0	3.37
Max. $\vert R\vert$ [kN/m]			2.71			3.20			**4.91**

Rivets size – Skin/stringers

- Rivets
  - Maximal load:
    - 4.91 kN per unit stringer length
- Rivets
  - For 2.5 mm diameter countersunk rivets with skin thickness 1.0 mm
    - Allowable load in shear: 668 N
    - The number of rivets/m is given by

$$n = \frac{4910}{668} = 7.35 \text{ or } 8 \text{ rivets/m}$$

    - This corresponds to a rivet pitch of 125 mm
    - Too large: does not ensure structural rigidity
  - We choose 25 mm rivet pitch: 40 rivets/m



Rivets size – Frames/skin

- What are the forces acting between frames & stringers ?
  - These forces correspond to the direct stresses in the stringers
  - Already calculated
  - Maximal forces
    - Section AA: 9 kN
    - Section CC: 8 kN
    - Section BB: 6.4 kN

Sect.		AA			CC			BB		
Data		D	1.28 m		D	1.01 m		D	0.73 m	
		M_y	42 kN·m		M_y	29 kN·m		M_y	16 kN·m	
		M_z	55 kN·m		M_z	39 kN·m		M_z	23 kN·m	
Stringers	n°	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]	$-y$ [m]	z [m]	$B\sigma_{xx}$ [kN]
	6	0	0.64	5.52	0	0.50	4.76	0	0.37	3.65
	7	0.16	0.62	7.17	0.13	0.49	6.27	0.09	0.35	4.89
	8	0.32	0.55	8.34	0.25	044	7.34	0.18	0.32	5.81
	9	0.45	0.45	**8.94**	0.36	0.36	7.93	0.26	0.26	6.33
	10	0.55	0.32	8.92	0.44	0.25	**7.97**	0.32	0.18	**6.41**
	11	0.62	0.16	8.30	0.49	0.13	7.47	0.35	0.09	6.06
	12	0.64	0	7.12	0.50	0	6.46	0.37	0	5.30
Max. $B\sigma_{xx}$ [kN]				8.94			7.97			6.41
Min. B [mm^2]				57.7			51.4			41.4

- Number of rivets between the skin and frames
  - Section AA
    - Maximal forces: 9 kN
    - This force is resisted by the rivets between the skin and frame
    - Distance available for one stringer l = 0.168 m
    - Resistance of one rivet: 668 N

$$\text{pitch} = \frac{0.168}{\frac{8940}{668}} = 0.0126\ \mathrm{m}$$

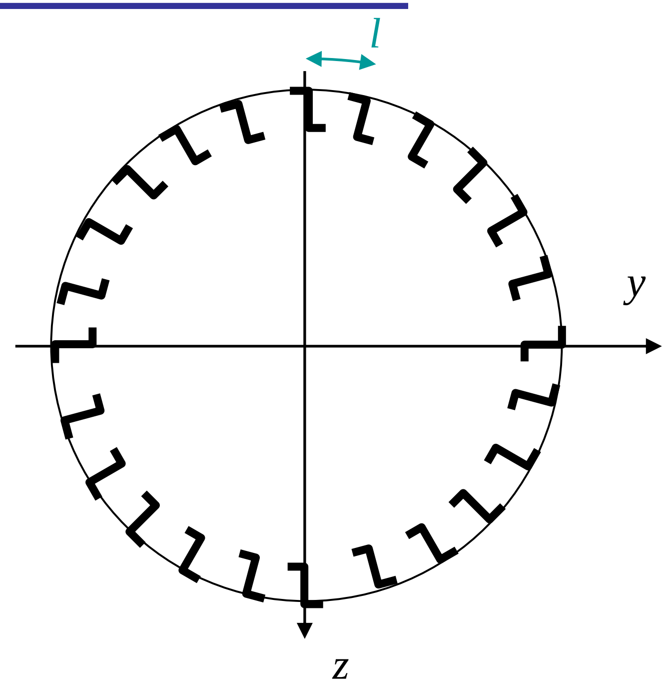
  - Section CC

$$\text{pitch} = \frac{0.132}{\frac{7970}{668}} = 0.011\ \mathrm{m}$$

  - Section BB

$$\text{pitch} = \frac{0.096}{\frac{6410}{668}} = 0.01\ \mathrm{m}$$

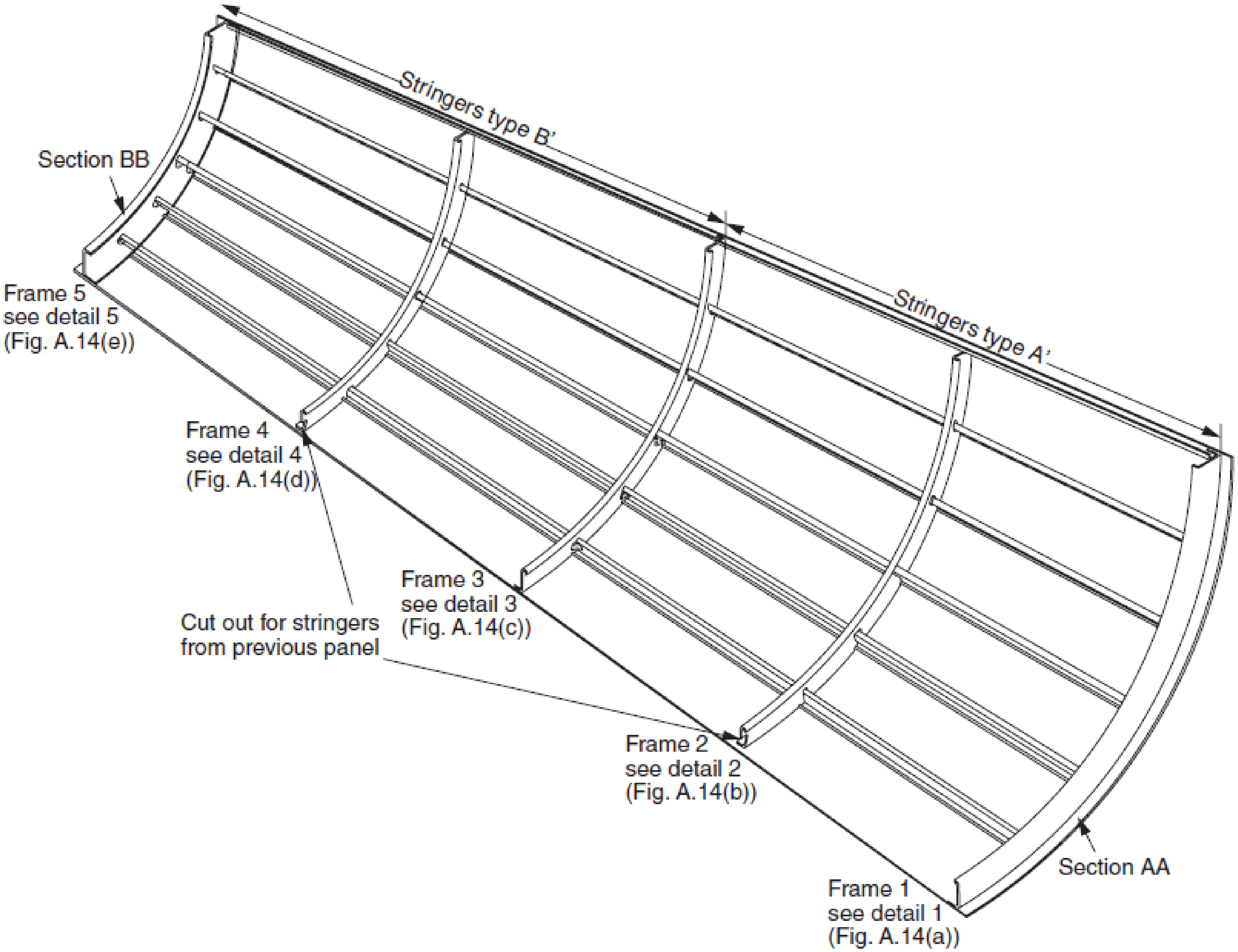
  - Distance between the rivets: 10 mm for all frames



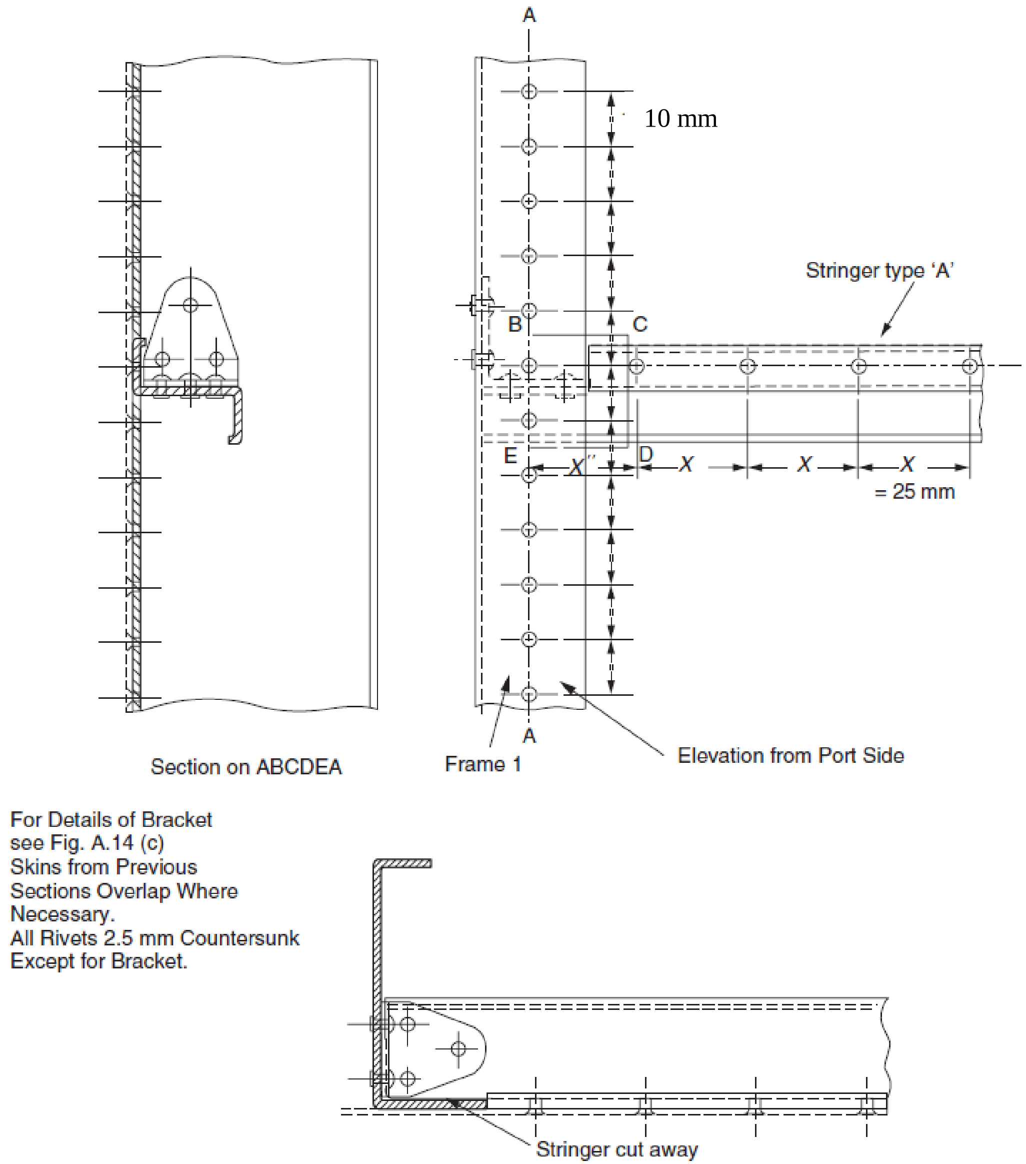
Fuselage design - End

Plans and Figures

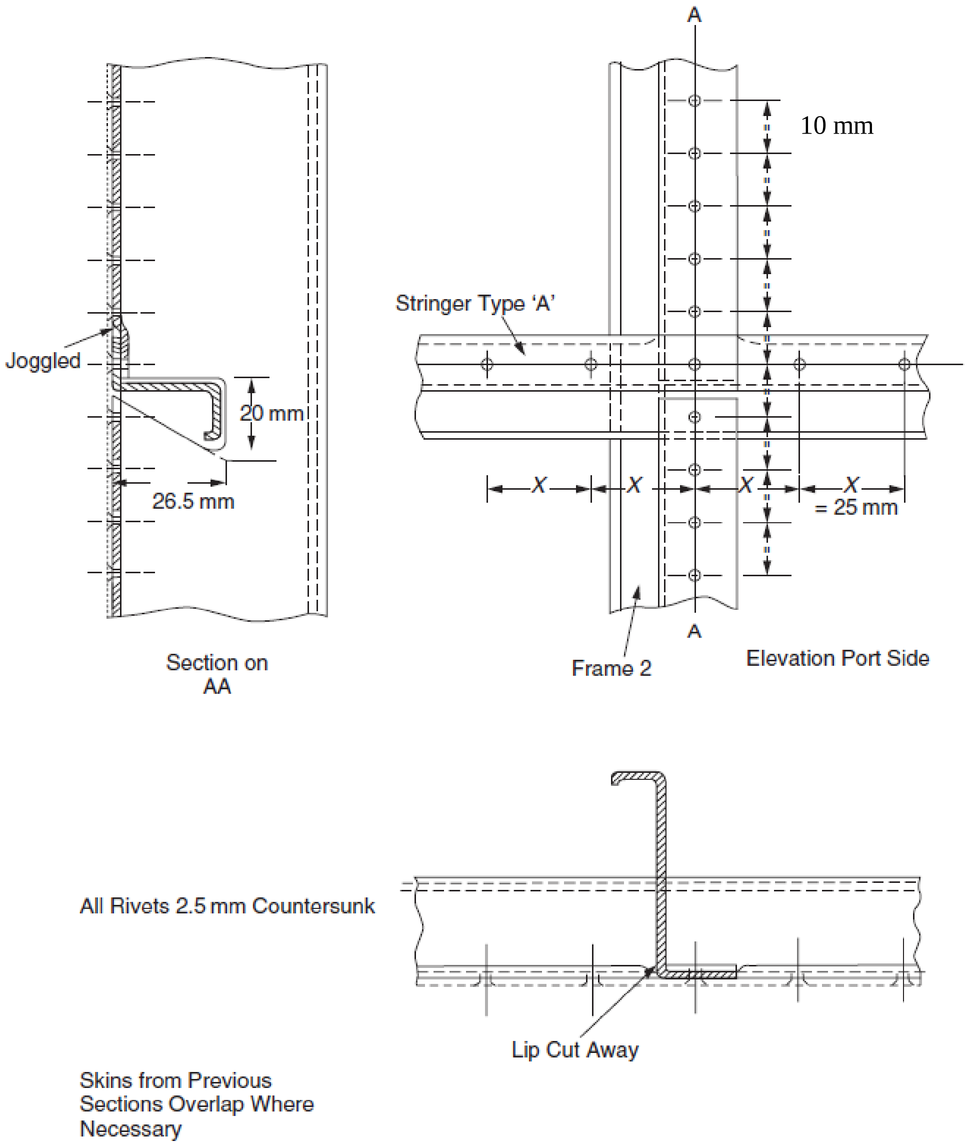
Design of the rear fuselage



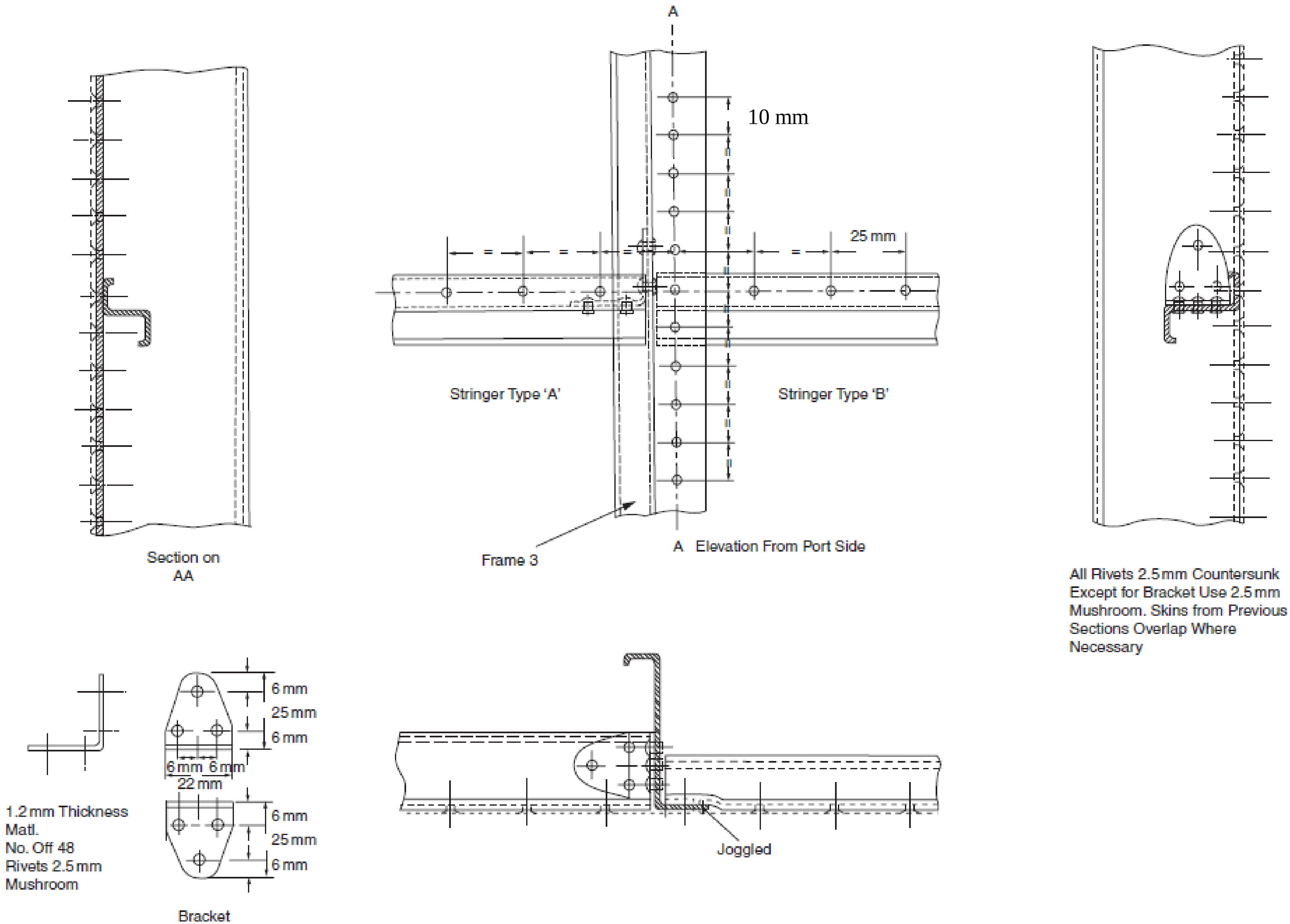
Rear fuselage: details (A14A)



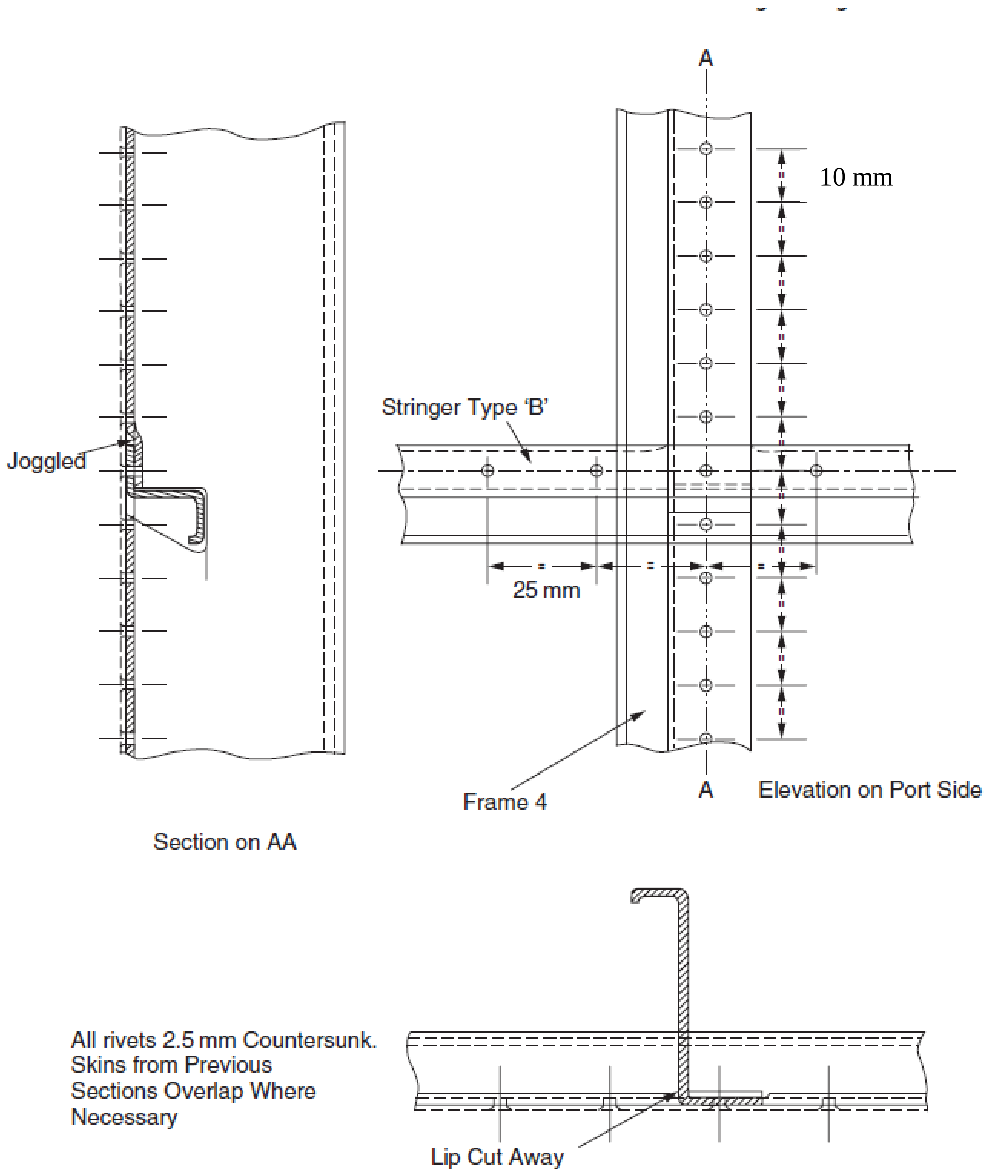
Rear fuselage: details (A14B)



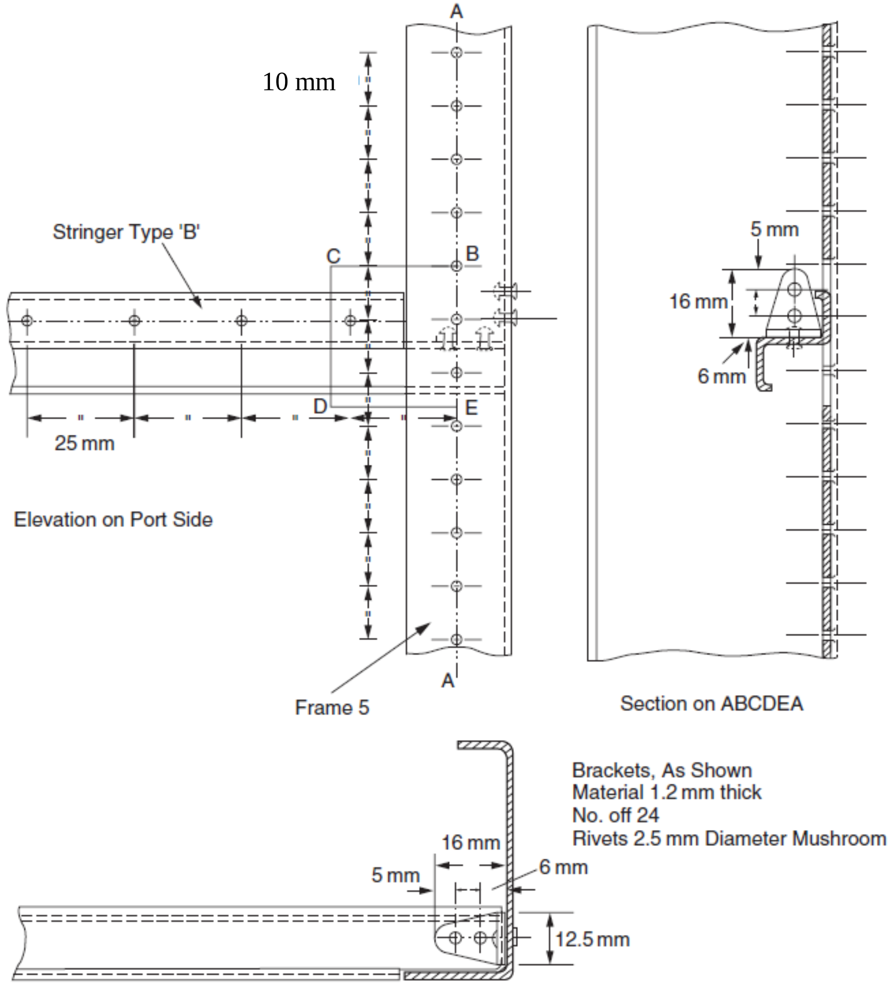
Rear fuselage: details (A14C)



Rear fuselage: details (A14D)



Rear fuselage: details (A14E)



Skins from Previous
Sections Overlap
Where Necessary
Rivets 2.5 mm Diameter Countersunk.