Aircraft Structures
Design Example

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Goal

- From
  - Specifications
  - Preliminary design

- Design the rear fuselage of a two-seat trainer/semi-aerobatic aircraft
  - Stringers and Frames
  - Skin thickness
Goal (2)

• Specifications and preliminary design
  – Permit to calculate forces acting on the airplane
  – Fuselage has to resist the induced loading

• Step 1: Forces acting on the airplane are
  – Aerodynamic forces (→ flight envelope)
  – Thrust
  – Self-weight

• Step 2: The rear fuselage consists into
  – Stringers
  – Frames
  – Skin
  – Rivets

• All these elements must be calculated to resist the stresses induced by the forces (bending moments, torques and shearing)
Step 1 – Loading acting on the rear fuselage

Aerodynamic forces and self-weight
Specifications: flight envelope

- Required flight envelope
  - $n_1 = 6.28$
  - $V_D = 183.8 \text{ m/s}$
  - $V_C = 0.8V_D = 147.0 \text{ m/s}$
  - $n_2 = 0.75 \times n_1 = 4.71$
  - $n_3 = 0.5 \times n_1 = 3.14$

- Representative values for an aerobatic aircraft?

- Further requirements
  - Additional pitching acceleration allowed at any point of envelope
    - $\left(20 + \frac{475}{W}\right) \frac{n}{V}$ $[\text{rad/s}^2]$ Weight $W$ in $[\text{kN}]$
  - For asymmetric flight: angle of yaw allowed at any point of envelope
    - $\psi = 0.7n_1 + \frac{457.2}{V_D}$ $[\text{degrees}]$
    - The angle of yaw increases the overall pitching moment coefficient of the aircraft by $-0.0015 / \text{degree of yaw}$
Data - Aircraft

• Fully loaded aircraft
  – Weight $W = 37.43$ kN
  – Moment of inertia about center of gravity $G$: $I_\theta = 22,235$ kg·m²
  – Positions of $G$ and of the body drag centers known (see figure)

• Body drag coefficients
  – $C_{D,B}$ (engine on) = 0.01583
  – $C_{D,B}$ (engine off) = 0.0576

• Engine
  – Maximum horse power: 905 hp
  – Propeller efficiency: 90 %

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Aircraft Structures: Design Example 6
Data - Wing

- **Geometry/Aerodynamics**
  - Span $b = 14.07$ m
  - Gross area $S = 29.64$ m$^2$
  - Aerodynamic mean chord $c = 2.82$ m
  - Lift and drag coefficients
    - See picture

- **Pitching moment**
  - $C_M = -0.238 \, C_L$
  - Angle of incidence
    - Between fuselage axis and root chord
    - $-1.5^\circ$
    - Additional pitching moment coefficient $-0.036$
Data - Tailplane

- **Geometry/Aerodynamics**
  - Span $b_t = 6.55\,\text{m}$
  - Gross area $S_t = 8.59\,\text{m}^2$
  - Aerodynamic centre $P$

- **Yaw** → asymmetry of the slipstream → asymmetric load on the tailplane
  - Resulting torque $\frac{0.00125}{\sqrt{1 - M^2}} \rho V^2 S_t b_t \psi$
    - $M$ is the mach number
    - $\psi$ is in degree

Section CC

Loading induced by yaw on the tailplane
Data - Fin

- **Geometry/Aerodynamics**
  - Height $h_F = 1.65\, \text{m}$
  - Area $S_F = 1.80\, \text{m}^2$
  - Aspect-ratio $A_F = h_F^2/S_F = 1.5$
  - Lift-curve slope $a_1$
    - $a_1 = \frac{5.5A}{A + 2}$
    - With $A$ the aspect ratio of an equivalent wing: $A = (2h_F)^2/2S_F = 2 \times h_F^2/S_F = 2A_F = 3$
    - $a_1 = 3.3$

- **In yawed flight of angle $\psi$**
  - Fin has also an incidence of $\psi$
  - Aerodynamic loading
    - $F_{\text{fin}} = \frac{1}{2} \rho V^2 S_F a_1 \psi$
    - Where $\psi$ is in rad

- **Centre of pressure of the fin**
  - 1.13 m above the axis of the fuselage
  - 3.7 m aft section AA
Initial calculation – Flight envelope

- **Values**
  - \( n_1 = 6.28 \)
  - \( V_D = 183.8 \text{ m/s} \)
  - \( V_C = 0.8V_D = 147.0 \text{ m/s} \)
  - \( n_2 = 0.75 \times n_1 = 4.71 \)
  - \( n_3 = 0.5 \times n_1 = 3.14 \)

- **\( V_s^A \) (stalling speed on point A)?**
  - Assumption: Lift from wing only
  - From \( C_L = \frac{nW}{\frac{1}{2}\rho V^2S} \)
    - \( V_s = \left( \frac{2nW}{\rho SC_{L, \text{max}}} \right)^{1/2} = \sqrt{n} \left( \frac{2 \times 37.43 \times 10^3}{1.226 \times 29.64 \times 1.38} \right)^{1/2} = 38.6\sqrt{n} \)
    - \( V_s^A = 38.6\sqrt{n_1} = 38.6\sqrt{6.28} = 96.7 \text{ m/s} \)
Balancing out calculations

- Loads are calculated for various critical points of the flight envelope
  - Case A
    - Point A
    - Engine on
  - Case A*
    - Point A
    - Engine off
  - Case C
    - Point C
    - Engine off
  - Case D₁
    - Point D₁
    - Engine off
  - Case D₂
    - Point D₂
    - Engine off
Balancing out calculations – Case A – point A/engine on

- **Data (point A)**
  - \( n_1 = 6.28 \)
  - \( V = V_{sA} = 96.7 \text{ m/s} \)
  - \( C_{L,\text{max}} = 1.38 \)
  - \( \alpha_{L,\text{max}} = 18^\circ \)
  - \( C_{D,W} = 0.149 \)
Balancing out calculations – Case A – point A/engine on

- From measures on drawing
  - Or math, see next slide
Balancing out calculations – Case A – point A/engine on

- Math

\[ d = \sqrt{0.45^2 + 0.98^2} = 1.078 \text{ m} \]

\[ \alpha = \arctan \frac{0.45}{0.98} = 24.7^\circ \]

\[ l = d \cos (24.7^\circ - 18^\circ) = 1.07 \text{ m} \]
Balancing out calculations – Case A – point A/engine on

• Methodology
  – Forces that can be directly calculated
    • Trust: $T$ from engine
    • $nW$: $n$ & $W$ are known
    • Drag (body $B$, wings $W$) from $V$ and drag coefficients
    • Pitching moment $M$ from the pitching moment coefficients and the angle of yaw $\psi$
    • Pitching moment acceleration
  – Force equilibrium, and moment equilibrium
    • 2 Equations involving $P$, $L$
  – Find other forces acting on the rear fuselage
    • Tailplane torque, fin load, fin load torque, total torque
Balancing out calculations – Case A – point A/engine on

- **Trust of the engine**
  - **Data**
    - Maximum horse power 905
    - Propeller efficiency $\eta = 90\%$
    - $1 \text{[hp]} = 746 \text{[W]}$
  - **Thrust**
    - $T = \frac{\eta \cdot \text{hp} \cdot 746}{V} = \frac{0.9 \times 905 \times 746}{96.7} = 6284 \text{[N]}$
Balancing out calculations – Case A – point A/engine on

- **Weight**
  - **Data**
    - Fully loaded weight $W = 37.43 \text{ kN}$
  - **Loaded weight**
    - $nW = 6.28 \times 37.43 \times 10^3 = 235060 \text{ [N]}$
Balancing out calculations – Case A – point A/engine on

- **Drag**
  - **Data**
    - \( C_{D,W} = 0.149 \)
    - \( C_{D,B} = 0.01583 \)
    - \( V = 96.7 \) m/s
    - \( S = 29.64 \) m\(^2\)
  - **Wing drag**
    - \( D_W = \frac{1}{2} C_{D,W} \rho V^2 S = 26091 \) [N]
  - **Body drag**
    - \( D_B = \frac{1}{2} C_{D,B} \rho V^2 S = 2690 \) [N]
Balancing out calculations – Case A – point A/engine on

- **Pitching moment**
  - Data
    - \( n_1 = 6.28 \)
    - \( V_D = 183.8 \text{ m/s} \)
    - \( V = 96.7 \text{ m/s} \)
    - Fully loaded weight \( W = 37.43 \text{ kN} \)
  - Pitching moment coefficient
    - Maximum for the maximum yaw angle allowed during maneuver
    - Maximum angle of yaw allowed
      \[
      \psi = 0.7n_1 + \frac{457.2}{V_D} \quad [\text{degrees}]
      \]
      \[
      = 0.7 \times 6.28 + \frac{457.2}{183.8} = 6.9 \text{ [°]}
      \]
    - Pitching moment coefficient
      \[
      C_M = -0.238C_L - 0.036 - 0.0015\psi
      \]
      \[
      = -0.238 \times 1.38 - 0.036 - 0.0015 \times 6.9 = -0.375
      \]
      Wing
      Wing/fuselage incidence
      Pitching of aircraft due to yaw (in °)
  - Maximum pitching acceleration allowed
    - \( \ddot{\theta} = \left( 20 + \frac{475}{W} \right) \frac{n}{V} = \left( 20 + \frac{475}{37.43} \right) \frac{6.28}{96.7} = 2.12[\text{rad/s}^2] \)
Balancing out calculations – Case A – point A/engine on

- Pitching moment (2)
  - Data
    - \( V = 96.7 \text{ m/s} \)
    - \( S = 29.64 \text{ m}^2 \)
    - MAC: \( c = 2.82 \text{ m} \)
  - Pitching moment coefficient \( C_M = -0.375 \)
  - Pitching moment
    \[
    M = C_M \frac{1}{2} \rho V^2 S c
    \]
    \[
    = -0.375 \frac{1}{2} \times 1.226 \times 96.7^2 \times 29.64 \times 2.82 = -179669 \text{ [Nm]} 
    \]
Balancing out calculations – Case A – point A/engine on

- **Moments about G**

  \[
  1.07L - 0.18T + 0.04D_B - 0.12D_W - 6.28P + M = I_\theta \times \dot{\theta}
  \]

  \[
  1.07L - 0.18 \times 6284 + 0.04 \times 2690 - 0.12 \times 26901 - 6.28P - 179669 = 22235 \times 2.12
  \]

  \[
  5.86P = L - 216090
  \]

- **Vertical equilibrium**

  \[
  L + P = nW - T \sin(18^\circ - 1.5^\circ) = 235060 - 6284 \sin 16.5^\circ = 233275
  \]

  \[
  L + P = 233275
  \]
Balancing out calculations – Case A – point A/engine on

- 2 equations, 2 unknowns
  - \(5.86P = L - 216090\)
  - \(L + P = 233275\)

\[
\begin{align*}
P &= 2505 \text{ [N]} \\
L &= 230770 \text{ [N]}
\end{align*}
\]

- Remark: for other cases, \(\alpha\) is not known
  - Requires iterations on \(\alpha\) in order to determine \(C_L\)
  - Should also be done here as \(V_S\) was computed using \(C_L\) of wing (10% error)
Balancing out calculations – Case A – point A/engine on tail

• Tailplane torque
  – Due to asymmetric slipstream (yaw)
  – Data
    • \( b_t = 6.55 \text{ m} \)
    • \( S_t = 8.59 \text{ m}^2 \)
  – \( M_{\text{tail}} = \frac{0.00125 \rho V^2 S_t b_t \psi}{\sqrt{1 - Mach^2}} \)
  – \( = \frac{0.00125}{\sqrt{1 - (96.7/340.8)^2}} \times 1.226 \times 96.7^2 \times 8.59 \times 6.55 \times 6.9 \)
  – \( = 5802 \text{ [N \cdot m]} \)

• Fin load
  – Due to yaw
  – Data
    • \( S_F = 1.80 \text{ m}^2 \)
    • \( a_1 = 3.3 \)
  – \( F_{\text{fin}} = \frac{1}{2} \rho V^2 S_F a_1 \psi \)
  – \( = \frac{1}{2} \times 1.226 \times 96.7^2 \times 1.8 \times 3.3 \times \left( 6.9 \frac{\pi}{180} \right) = 4100 \text{ [N]} \)
Balancing out calculations – Case A – point A/engine on

- Total torque (rear fuselage)

\[ M_{\text{fus}} = M_{\text{tail}} + F_{\text{fin}} \times 1.13 = 5802 + 4100 \times 1.13 = 10435 \text{ [Nm]} \]
Balancing out calculations - End

- **Summary**
  - Other cases follow the same method

<table>
<thead>
<tr>
<th>Case</th>
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</table>
Fuselage loads – Preliminary choices

- Fuselage
  - Stringers
  - Frames
  - Skin
- Frames
  - Un-pressurized fuselage \(\leftrightarrow\) frames will not support significant loads
  - Frames are required to maintain the fuselage shape \(\leftrightarrow\) nominal in size
- Combination of stringer and skin will resist self-weight and aerodynamic loads
  - Shear forces
  - Bending moments
  - Torques
Fuselage loads – Preliminary choices

• Geometrical data

• Circular cross-section
  – Simple to fabricate
  – Simple to design
  – Will meet the design requirements

• Possible arrangement
  – 24 stringers arranged symmetrically spaced at
    • Section AA: ~168 mm
    • Section BB: ~96 mm
  – All stringers have the same cross-sectional area
Fuselage loads – Data

• Material: Aluminum alloy
  – Stingers & skin
  – 0.1 % proof stress: 232.5 [N/mm²] = 232.5 [MPa]
  – Shear strength: 145.5 [N/mm²] = 145.5 [MPa]

• Self-weight: Assumptions
  – Fuselage weight is from 4.8 to 8.0 % of the total weight
  – Tailplane/fin assembly is from 1.2 to 2.5 % of the total weight
  – Half of the fuselage weight is aft of the section AA

\[
W_{\text{rear fuselage}} = \frac{1}{2} \times 37.43 \times 10^3 \times 6.4\% = 1198 \text{ [N]}
\]

\[
W_{\text{tailplane+fin}} = 37.43 \times 10^3 \times 1.8\% = 674 \text{ [N]}
\]

– The weight distribution varies proportionally to the skin surface area
• **Assumptions on geometry**
  
  - Rear fuselage is uniformly tapered
  
  ![Diagram of fuselage cross-section](image)

  - CC is a section midway AA and BB.
  - The center of gravity of the tailplane/fin assembly has been estimated to be 4.06 m from the section AA on a line parallel to the fuselage centre line.

  $$\text{Area}_{skin} = \frac{1}{2} \pi (D_{max} + D_{min}) L$$

  $$= \frac{1}{2} \pi (1.28 + 0.1) 4.57 = 9.91 [m^2]$$
Fuselage loads – Data

- **Data**
  - Weight of rear fuselage: 1198 [N]
  - Skin area of rear fuselage: 9.91 [m$^2$]

- **Self-weight / m of the fuselage**

\[
\text{weight/m} = \frac{W_{\text{rear fuselage}} \times \pi \times D}{\text{Area}_{\text{skin}}}
\]

\[
\begin{align*}
\text{weight/m}_{AA} &= \frac{1}{9.91} \times 1198 \times \pi \times 1.28 = 486.1 \text{ [N/m]} \\
\text{weight/m}_{CC} &= \frac{1}{9.91} \times 1198 \times \pi \times 1.01 = 383.6 \text{ [N/m]} \\
\text{weight/m}_{BB} &= \frac{1}{9.91} \times 1198 \times \pi \times 0.73 = 277.2 \text{ [N/m]} \\
\text{weight/m}_{DD} &= \frac{1}{9.91} \times 1198 \times \pi \times 0.1 = 38.0 \text{ [N/m]}
\end{align*}
\]
Fuselage loads – Shear force and bending moment due to self-weight

- **Self-weight induces**
  - Shear forces: SF
  - Bending moments: BM

- **SF and BM are calculated by equilibrium (‘MNT’)**

![Diagram showing shear force and bending moment calculations](image)

- Shear forces: SF
- Bending moments: BM

**Resultant** = \( W_{\text{rear fuselage}} = 1198 \text{ [N]} \)

**Calculations**:
- \( W_{\text{tailplane+fin}} = 674 \text{ [N]} \)
- \( 4.06 \text{ m} \)
- \( 4.57 \text{ m} \)
- \( 2.13 \text{ m} \)
- \( 2.44 \text{ m} \)
- \( 1.065 \text{ m} \)

**Dimensions**:
-\( 98.0 \text{ [N/m]} \)
-\( 486.1 \text{ [N/m]} \)
-\( 383.6 \text{ [N/m]} \)
-\( 277.2 \text{ [N/m]} \)
-\( 38.0 \text{ [N/m]} \)

**Equilibrium Conditions**:
- Shear forces
- Bending moments
Fuselage loads – Shear force and bending moment due to self-weight

- Transform self-weight into
  - A triangular repartition: $q_1(x)$
  - And a constant linear force: $q_2$

\[
q_1(x) = \begin{cases} 
448.1 [\text{N/m}] & \text{for } 0 \leq x < 4.57 \text{ m} \\
0 & \text{for } 4.57 \text{ m} \leq x \leq 4.06 \text{ m}
\end{cases}
\]

\[
q_2 = 38 \text{ [N/m]}
\]

\[
W_{\text{tailplane+fin}} = 674 \text{ [N]}
\]
Fuselage loads – Shear force and bending moment due to self-weight

- Effect of load factor

  - The self weight is multiplied by the load factor $n$
    - Forces are not applied in the cross section plane
    - Forces are $(\alpha-1.5^\circ)$ – inclined with this section
      - Will be multiplied by $\cos(\alpha-1.5^\circ)$ later
    - Rigorously, an axial loading should also be considered
  - Distance along fuselage axis is multiplied by $\cos(\alpha-1.5^\circ)$
    - When computing bending moment
  - We do not compute the fuselage compression
    - Should be done and risk of buckling avoided
Fuselage loads – Shear force and bending moment due to self-weight

- Reactions at section AA

\[ SF_A = n \times (Q_1 + Q_2 + W_{\text{tailplane+fin}}) \]
\[ = n \times \left( \frac{1}{2} \times 448.1 \times 4.57 + 38.0 \times 4.57 + 674 \right) = 1872n \quad [N] \]

\[ W_{\text{rear fuselage}} = 1198 \quad [N] \]

\[ BM_A = n \cos (\alpha - 1.5^\circ) \times \left[ 4.06 \times W_{\text{tailplane+fin}} + \frac{1}{3} \times 4.57 \times Q_1 + \frac{1}{2} \times 4.57 \times Q_2 \right] \]
\[ = 4693n \cos (\alpha - 1.5^\circ) \quad [Nm] \]
Fuselage loads – Shear force and bending moment due to self-weight

- Reactions at section CC

\[ SF_C = n \times (Q_1 + Q_2 + W_{\text{tailplane+fin}}) \]
\[ = n \times \left( \frac{1}{2} 345.6 \times 3.51 + 38.0 \times 3.51 + 674 \right) = 1409n \quad [N] \]

\[ BM_C = n \cos (\alpha - 1.5^\circ) \times \left[ (4.06 - 1.065) \times W_{\text{tailplane+fin}} + \frac{1}{3} 3.51 \times Q_1 + \frac{1}{2} 3.51 \times Q_2 \right] \]
\[ = 2959n \cos (\alpha - 1.5^\circ) \quad [Nm] \]
Fuselage loads – Shear force and bending moment due to self-weight

- Reactions at section BB

\[ SF_B = n \times (Q_1 + Q_2 + W_{\text{tailplane+fin}}) \]
\[ = n \times \left( \frac{1}{2} \times 239.2 \times 2.44 + 38.0 \times 2.44 + 674 \right) = 1059n \quad [N] \]

\[ BM_B = n \cos(\alpha - 1.5^\circ) \times \left[ (4.06 - 2.13) \times W_{\text{tailplane+fin}} + \frac{1}{3} \times 2.44 \times Q_1 + \frac{1}{2} \times 2.44 \times Q_2 \right] \]
\[ = 1651n \cos(\alpha - 1.5^\circ) \quad [Nm] \]
Total shear forces, bending moments and torque

- Resultant forces in each section
  - Example section AA

\[
\begin{align*}
T_{y}^{AA} &= -F_{\text{fin}} \\
T_{z}^{AA} &= (SF_{A} - P) \cos (\alpha - 1.5^\circ) \\
M_{y}^{AA} &= BM_{A} - P \times 3.47 \text{ m} \cos (\alpha - 1.5^\circ) \\
M_{z}^{AA} &= F_{\text{Fin}} \times 3.7 \text{ m} \\
M_{x}^{\text{section}} &= -M_{\text{tail}} - 1.13F_{\text{fin}} = -M_{\text{fus}}
\end{align*}
\]
Total shear forces, bending moments and torque

- Resultant forces in each section (2)

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- Example case A, section AA
  - $SF_A = 1872 \, n$ [N] & $BM_A = 4693 \, n \cos(\alpha-1.5°)$ [Nm]

\[
\begin{align*}
T_y^{AA} &= -F_{\text{fin}} = -4100 \, \text{N} \\
T_z^{AA} &= (SF_A - P) \cos (\alpha - 1.5°) = (1872 \times 6.28 - 2505) \cos 16.5° = 8580 \, \text{N} \\
M_y^{AA} &= BM_A - P \times 3.47 \, \text{m} \cos (\alpha - 1.5°) \\
&= (4693 \times 6.28 - 2505 \times 3.47) \cos 16.5° = 20774 \, \text{Nm} \\
M_z^{AA} &= F_{\text{Fin}} \times 3.7 \, \text{m} = 4100 \times 3.7 = 15174 \, \text{Nm} \\
M_x^{\text{section}} &= -M_{\text{tail}} - 1.13F_{\text{fin}} = -M_{\text{fus}} = -10439 \, \text{Nm}
\end{align*}
\]
Total shear forces, bending moments and torque

- **Table of sections loading**
  - $SF_A = 1872 \, n \, [N]$ & $BM_A = 4693 \, n \, \cos(\alpha-1.5^\circ) \, [Nm]$  
  - $SF_C = 1409 \, n \, [N]$ & $BM_C = 2959 \, n \, \cos(\alpha-1.5^\circ) \, [Nm]$  
  - $SF_B = 1059 \, n \, [N]$ & $BM_B = 1651 \, n \, \cos(\alpha-1.5^\circ) \, [Nm]$  

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Step 2 – Design of the rear fuselage

Stringers, frames, skin and rivets
Fuselage design calculation – Material and approach

• 2 types of design are possible
  – Elastic design
    • Allowed stress = 0.1% proof stress / \( s \)
    • \( s \) : safety factor = 1.5
  – Ultimate load design
    • Ultimate stresses
    • Actual load multiplied by an ultimate load factor

• For linear systems : both designs give the same results

• We use the elastic design as 0.1 proof stress is known

\[
\sigma_{max} = \frac{\sigma_{0.1%}}{s} = \frac{232.5}{1.5} = 155 \text{ MPa}
\]
\[
\tau_{max} = \frac{\tau_{strength}}{s} = \frac{145.5}{1.5} = 97 \text{ MPa}
\]
Stringers section: Data

- **Frames**
  - Un-pressurized fuselage frames will not support significant loads.
  - Frames are required to maintain the fuselage shape nominal in size.

- **Circular cross-section**
  - 24 stringers arranged symmetrically and spaced at around
    - Section AA – \(l\) ~168 mm
    - Section BB – \(l\) ~96 mm
  - All stringers have the same cross-sectional area.
Stringers section

- **Direct stress**
  - Induced by $M_z$ and $M_y$
  - Obtained from $\sigma_{xx} = \frac{M_y}{I_{yy}} z - \frac{M_z}{I_{zz}} y$ (as $I_{yz} = 0$)

- **Unknown**
  - $B$: the area of the stringers in a section
  - $B$ should be chosen such that

  $$\sigma_{xx} \leq \sigma_{max} = 155 \text{ MPa}$$

- **Second moments of area**

  $$I_{zz} = I_{yy} = \sum_{i=1}^{24} B \times z_i^2$$

  $$= 4BD^2 \left( 0.1294^2 + 0.25^2 + 0.353^2 + 0.433^2 + 0.483^2 + \frac{0.5^2}{2} \right)$$

  $$= 3BD^2$$
Stringers section

- **Values of \( M_x \) and \( M_y \)**
  - Stress
    \[
    \sigma_{xx} = \frac{M_y}{I_{yy}} z - \frac{M_z}{I_{zz}} y
    \]
  - Worst case
    - Case D1 (dive)
    - \( M_z \) and \( M_y \) have same sign
    - \( y \) and \( z \) of opposite sign

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Stringers section

- Calculate $B\sigma_{xx}$

$$\sigma_{xx} = \frac{M_y}{3BD^2} \phi - \frac{M_z}{3D^2} \psi$$

$$B\sigma_{xx} = \frac{M_y}{3D^2} \phi - \frac{M_z}{3D^2} \psi$$

- For each
  - Section
    - $M$ & $D$ change
  - Stringer
    - $y$ & $z$ change
  - Determine minimal value of stringers’ area $B$ such that

$$\sigma_{xx} \leq \sigma_{max} = 155 \text{ MPa}$$

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Max. $B\sigma_{xx}$ [kN] | 8.94 | 7.97 | 6.41
Min. $B$ [mm²] | 57.7 | 51.4 | 41.4
Stringers and frames

- From calculation $B_{\text{min}}(AA) > B_{\text{min}}(CC) > B_{\text{min}}(CC)$
  - Lighter stringer between CC and BB

![Diagram of stringers and frames]

- **Type A stringers**: $B > 57.7 \, \text{mm}^2$
- **Type B stringers**: $B > 51.4 \, \text{mm}^2$
Stringers and frames

- **Stringer type “Z-section”**

  Type A stringer: $B = 58.1 > 57.7 \text{ mm}^2$

  Type B stringer: $B = 51.9 > 51.4 \text{ mm}^2$
Frames

- Non load bearing
- Must be of sufficient size to be connected with stringers (AA/BB/CC sections)
  - Use of brackets
- Must be of sufficient size to allow to be cut
  - So stringers can pass through them
Skin thickness

- Skin must resist shear flow due to
  - Shear loads $T_y$, $T_z$ &
  - Torque $M_x$

- Calculate the shear flow
  - At each boom there is a discontinuity

\[
q^{i+1} - q^i = -\frac{T_z}{I_{yy}} B_i z_i - \frac{T_y}{I_{zz}} B_i y_i
\]

\[
= -\frac{T_z}{3D^2} z_i - \frac{T_y}{3D^2} y_i
\]

As stringers have constant area in one section and as $I_{yy} = I_{zz} = 3BD^2$
Shear flow due to $T_Z$

- Following equations
  \[ q^{i+1} - q^i = - \frac{T_z}{3D^2} z_i \]

\[
\begin{align*}
q^{12} &= q^{241} - \frac{T_z}{3D^2} \times 0.1294D = q^{241} - 0.043 \frac{T_z}{D} \\
q^{23} &= q^{12} - \frac{T_z}{3D^2} \times 0.25D = q^{241} - 0.126 \frac{T_z}{D} \\
q^{34} &= q^{23} - \frac{T_z}{3D^2} \times 0.353D = q^{241} - 0.244 \frac{T_z}{D} \\
q^{45} &= q^{34} - \frac{T_z}{3D^2} \times 0.433D = q^{241} - 0.388 \frac{T_z}{D} \\
q^{56} &= q^{45} - \frac{T_z}{3D^2} \times 0.483D = q^{241} - 0.549 \frac{T_z}{D} \\
q^{67} &= q^{56} - \frac{T_z}{3D^2} \times 0.5D = q^{241} - 0.716 \frac{T_z}{D}
\end{align*}
\]

- By symmetry (no torque)
  \[ q^{67} = - q^{56} \]

\[ q^{241} - 0.716 \frac{T_z}{D} = -q^{241} + 0.549 \frac{T_z}{D} \iff q^{241} = \frac{0.633 \times T_z}{D} \]
Skin thickness – Shear flow

- Shear flow due to $T_z$ (2)
  - Final form of $qD/T_z$
Skin thickness – Shear flow

- **Shear flow due to** $T_y$
  - Following equations
    \[ q^{i+1} - q^i = -\frac{T_y}{3D^2} y_i \]

\[
\begin{align*}
q^{1\ 2} &= q^{24\ 1} - \frac{T_y}{3D^2} \times 0.483D = q^{24\ 1} - 0.161 \frac{T_y}{D} \\
q^{2\ 3} &= q^{1\ 2} - \frac{T_y}{3D^2} \times 0.433D = q^{24\ 1} - 0.305 \frac{T_y}{D} \\
q^{3\ 4} &= q^{2\ 3} - \frac{T_y}{3D^2} \times 0.353D = q^{24\ 1} - 0.4232 \frac{T_y}{D} \\
q^{4\ 5} &= q^{3\ 4} - \frac{T_y}{3D^2} \times 0.25D = q^{24\ 1} - 0.507 \frac{T_y}{D} \\
q^{5\ 6} &= q^{4\ 5} - \frac{T_y}{3D^2} \times 0.1294D = q^{24\ 1} - 0.55 \frac{T_y}{D}
\end{align*}
\]

- But we also have
  \[ q^{24\ 1} = q^{23\ 24} - \frac{T_y}{3D^2} \times 0.5D = q^{23\ 24} - 0.1667 \frac{T_y}{D} \]

- By symmetry (no torque)
  \[ q^{23\ 24} = - q^{24\ 1} \quad \Rightarrow \quad 2q^{24\ 1} = -0.1667 \frac{T_y}{D} \iff q^{24\ 1} = -\frac{0.0833 \times T_y}{D} \]
• Shear flow due to $T_y$ (2)
  – Final form of $qD/T_y$
Skin thickness – Shear flow caused by torque

- Shear flow due to torque

\[
q_T = \frac{M_x}{2A} = \frac{M_x}{2(\pi D^2/4)} = \frac{0.637 \times M_x}{D^2}
\]
Skin thickness – Shear flow caused by torque

- **Maximum shear flow**
  - As $T_z > 0$, $T_y < 0$ & $M_x < 0$
  - Maximum shear flow is in a skin panel between stringers 12 and 18

- Example: $q_{1213} = -0.633 \frac{T_z}{D} + 0.083 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2}$
Skin Thickness – Maximum shear flow

- Maximum shear flow (2)
  - Equations

\[
\begin{align*}
q^{12\ 13} &= -0.633 \frac{T_z}{D} + 0.083 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q^{13\ 14} &= -0.590 \frac{T_z}{D} + 0.244 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q^{14\ 15} &= -0.507 \frac{T_z}{D} + 0.389 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q^{15\ 16} &= -0.389 \frac{T_z}{D} + 0.507 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q^{16\ 17} &= -0.244 \frac{T_z}{D} + 0.590 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2} \\
q^{17\ 18} &= -0.083 \frac{T_z}{D} + 0.633 \frac{T_y}{D} + 0.637 \frac{M_x}{D^2}
\end{align*}
\]
**Skin Thickness – Maximum shear flow**

- **Maximum shear flow (3)**
  - Critical case: D1

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Skin Thickness – Maximum shear flow

- Computation
  - See Table

- Minimum skin thickness
  - From
    \[
    \frac{q_{\text{max}}}{t} \leq \tau_{\text{max}}
    \]
    \[
    \frac{q_{\text{max}}}{\tau_{\text{max}}} \leq t
    \]
    \[
    t = \frac{65}{97} = 0.67 \text{ mm}
    \]
  - But must support rivets
    - 1mm skin thickness is chosen

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<tr>
<th>Sect.</th>
<th>AA</th>
<th>CC</th>
<th>BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>D</td>
<td>1.28 m</td>
<td>1.01 m</td>
<td>0.73 m</td>
</tr>
<tr>
<td>(T_y)</td>
<td>-15 kN</td>
<td>-15 kN</td>
<td>-15 kN</td>
</tr>
<tr>
<td>(T_z)</td>
<td>15 kN</td>
<td>12 kN</td>
<td>12 kN</td>
</tr>
<tr>
<td>(M_x)</td>
<td>-41 kN(\cdot)m</td>
<td>-40 kN(\cdot)m</td>
<td>-40 kN(\cdot)m</td>
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<table>
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<tr>
<th>n°</th>
<th>q [kN(\cdot)m(^{-1})]</th>
<th>q [kN(\cdot)m(^{-1})]</th>
<th>q [kN(\cdot)m(^{-1})]</th>
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<tbody>
<tr>
<td>12-13</td>
<td>-22.04</td>
<td>-32.18</td>
<td>-57.07</td>
</tr>
<tr>
<td>13-14</td>
<td>-23.96</td>
<td>-34.64</td>
<td>-60.45</td>
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<td>14-15</td>
<td>-25.33</td>
<td>-36.47</td>
<td>-63.03</td>
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<tr>
<td>15-16</td>
<td>-26.04</td>
<td>-37.55</td>
<td>-64.56</td>
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<td>-64.35</td>
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<tr>
<td>Min. t [mm]</td>
<td>0.27</td>
<td>0.39</td>
<td>0.67</td>
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</table>
Rivets size – Skin/stringers

• What are the forces acting on the rivet?
  – At a stringer, we found for $T_z$
    • $q^{i+1} - q^i = -\frac{T_z}{3D^2}z_i$
    • This corresponds to
      – The shear flow balanced by
      – All the rivets linking the
        skin to the stringer
  – Therefore the shear load per
    unit stringer length acting on the
    rivets fixing the skin to the stringer $i$ is
    • $R^i = \left(-\frac{T_y}{3D^2}y_i - \frac{T_z}{3D^2}z_i\right)$
      • Maximum between stringers 6 and 12
        – $T_y < 0$ & $T_z > 0$ $\rightarrow$ $y < 0$ & $z > 0$
        – Critical case is still D1
        – Remark: the torque does not lead
          to a discontinuity in the shear flow
Rivets size – Skin/stringers

- **Results**
  - Maximal load:
    - 4.91 kN per unit stringer length

<table>
<thead>
<tr>
<th>Sect.</th>
<th>AA</th>
<th>CC</th>
<th>BB</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$D$ 1.01 m</td>
<td>$D$ 0.73 m</td>
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<tr>
<td>$T_y$</td>
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<td>$T_y$ -15 kN</td>
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<td>$T_z$</td>
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<td>$T_z$ 12 kN</td>
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<table>
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<tr>
<th>n°</th>
<th>-y [m]</th>
<th>z [m]</th>
<th>-R [kN/m]</th>
<th>-y [m]</th>
<th>z [m]</th>
<th>-R [kN/m]</th>
<th>-y [m]</th>
<th>z [m]</th>
<th>-R [kN/m]</th>
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<td>0.62</td>
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<td>0.35</td>
<td>3.50</td>
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<tr>
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<td>0.32</td>
<td>0.55</td>
<td>2.62</td>
<td>0.25</td>
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<td>0.36</td>
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<td>2.45</td>
<td>0.37</td>
<td>0</td>
<td>3.37</td>
</tr>
</tbody>
</table>

Max. $|R|$ [kN/m] | 2.71 | 3.20 | **4.91** |
Rivets size – Skin/stringers

- Rivets
  - Maximal load:
    - 4.91 kN per unit stringer length

- Rivets
  - For 2.5 mm diameter countersunk rivets with skin thickness 1.0 mm
    - Allowable load in shear: 668 N
    - The number of rivets/m is given by
      \[
      n = \frac{4910}{668} = 7.35 \text{ or } 8 \text{ rivets/m}
      \]
    - This corresponds to a rivet pitch of 125 mm
    - Too large: does not ensure structural rigidity
  - We choose 25 mm rivet pitch: 40 rivets/m
• What are the forces acting between frames & stringers?
  – These forces correspond to the direct stresses in the stringers
  – Already calculated
  – Maximal forces
    • Section AA: 9 kN
    • Section CC: 8 kN
    • Section BB: 6.4 kN

<table>
<thead>
<tr>
<th>Sect.</th>
<th>Data</th>
<th>AA</th>
<th>CC</th>
<th>BB</th>
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<tbody>
<tr>
<td></td>
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<td>$D$ [m]</td>
<td>$D$ [m]</td>
<td>$D$ [m]</td>
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<tr>
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<td>1.28</td>
<td>1.01</td>
<td>0.73</td>
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<td></td>
<td>$M_y$ [kN·m]</td>
<td>42</td>
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<td>16</td>
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<tr>
<td></td>
<td>$M_z$ [kN·m]</td>
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<table>
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<tr>
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<th>z [m]</th>
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<th>-y [m]</th>
<th>z [m]</th>
<th>$B\sigma_{xx}$ [kN]</th>
<th>-y [m]</th>
<th>z [m]</th>
<th>$B\sigma_{xx}$ [kN]</th>
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<td>0.37</td>
<td>0</td>
<td>5.30</td>
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<tr>
<td></td>
<td>Max. $B\sigma_{xx}$ [kN]</td>
<td>8.94</td>
<td>7.97</td>
<td>6.41</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>Min. $B$ [mm$^2$]</td>
<td>57.7</td>
<td>51.4</td>
<td>41.4</td>
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</tbody>
</table>
• Number of rivets between the skin and frames
  – Section AA
    • Maximal forces: 9 kN
    • This force is resisted by the rivets between the skin and frame
    • Distance available for one stringer \( l = 0.168 \text{ m} \)
    • Resistance of one rivet: 668 N
    \[
    \text{pitch} = \frac{0.168}{\frac{668}{668}} = 0.0126 \text{ m}
    \]
  – Section CC
    \[
    \text{pitch} = \frac{0.132}{\frac{7970}{668}} = 0.011 \text{ m}
    \]
  – Section BB
    \[
    \text{pitch} = \frac{0.096}{\frac{6410}{668}} = 0.01 \text{ m}
    \]
  – Distance between the rivets: 10 mm for all frames
Fuselage design - End

Plans and Figures
Design of the rear fuselage

Section BB

Frame 5
see detail 5
(Fig. A.14(e))

Section AA

Frame 1
see detail 1
(Fig. A.14(a))

Frame 2
see detail 2
(Fig. A.14(b))

Frame 3
see detail 3
(Fig. A.14(c))

Cut out for stringers from previous panel

Frame 4
see detail 4
(Fig. A.14(d))

Stringers type A'

Stringers type B'
For Details of Bracket
see Fig. A.14 (c)
Skins from Previous
Sections Overlap Where
Necessary.
All Rivets 2.5 mm Countersunk
Except for Bracket.
Rear fuselage: details (A14C)

10 mm

Stringer Type ‘A’

Stringer Type ‘B’

Frame 3

Section on AA

A Elevation From Port Side

All Rivets 2.5 mm Countersunk
Except for Bracket Use 2.5 mm
Mushroom. Skins from Previous
Sections Overlap Where Necessary

1.2 mm Thickness
Material
No. Of Holes 48
Rivets 2.5 mm
Mushroom

Bracket

6 mm
25 mm
8 mm
22 mm

6 mm
25 mm
8 mm

6 mm
25 mm
8 mm

Jogged
Rear fuselage: details (A14D)

Section on AA

Stringer Type ‘B’

Framo 4

Elevation on Port Side

10 mm

25 mm

All rivets 2.5 mm Countersunk. Skins from Previous Sections Overlap Where Necessary.

Lip Cut Away
Rear fuselage: details (A14E)

Elevation on Port Side

Frame 5

Section on ABCDEA

Brackets, As Shown
Material 1.2 mm thick
No. off 24
Rivets 2.5 mm Diameter Mushroom

Skins from Previous
Sections Overlap
Where Necessary
Rivets 2.5 mm Diameter Countersunk.