Aircraft Design

Lecture 9:

Stability and Control

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Stability and Control

- Aircraft stability deals with the ability to keep an aircraft in the air in the chosen flight attitude.
- Aircraft control deals with the ability to change the flight direction and attitude of an aircraft.
- Both these issues must be investigated during the preliminary design process.

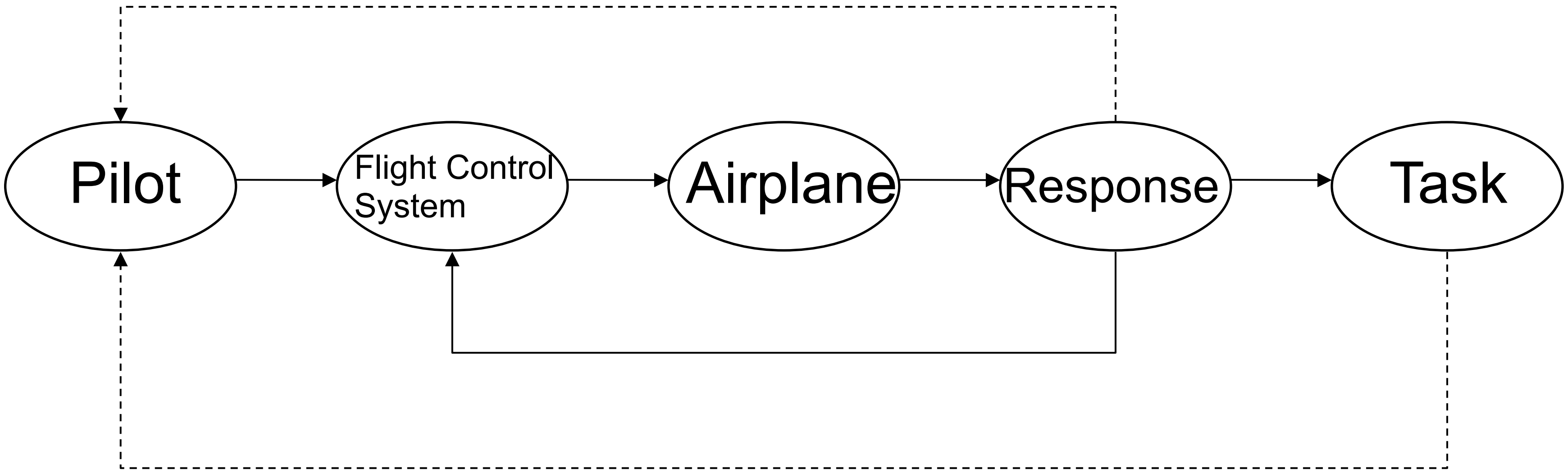
Design criteria?

- Stability and control are not design criteria
- In other words, civil aircraft are not designed specifically for stability and control
- They are designed for performance.
- Once a preliminary design that meets the performance criteria is created, then its stability is assessed and its control is designed.

Flight Mechanics

- Stability and control are collectively referred to as flight mechanics
- The study of the mechanics and dynamics of flight is the means by which :
  - We can design an airplane to accomplish *efficiently* a specific task
  - We can make the task of the pilot easier by ensuring good handling qualities
  - We can avoid unwanted or unexpected phenomena that can be encountered in flight

Aircraft description



The pilot has direct control only of the Flight Control System. However, he can tailor his inputs to the FCS by observing the airplane's response while always keeping an eye on the task at hand.

Control Surfaces

- Aircraft control is accomplished through control surfaces and power
  - Ailerons
  - Elevators
  - Rudder
  - Throttle
- Control deflections were first developed by the Wright brothers from watching birds

Modern control surfaces

Rudder

Elevator

Aileron

Elevon
(elevator+aileron)

Rudderon
(rudder+aileron)

Other devices

- Combinations of control surfaces and other devices: flaperons, spoilerons, decelerons (aileron and airbrake)
- Vectored thrust

Aircraft degrees of freedom

Six degrees of freedom:

3 displacements

x: horizontal motion

y: side motion

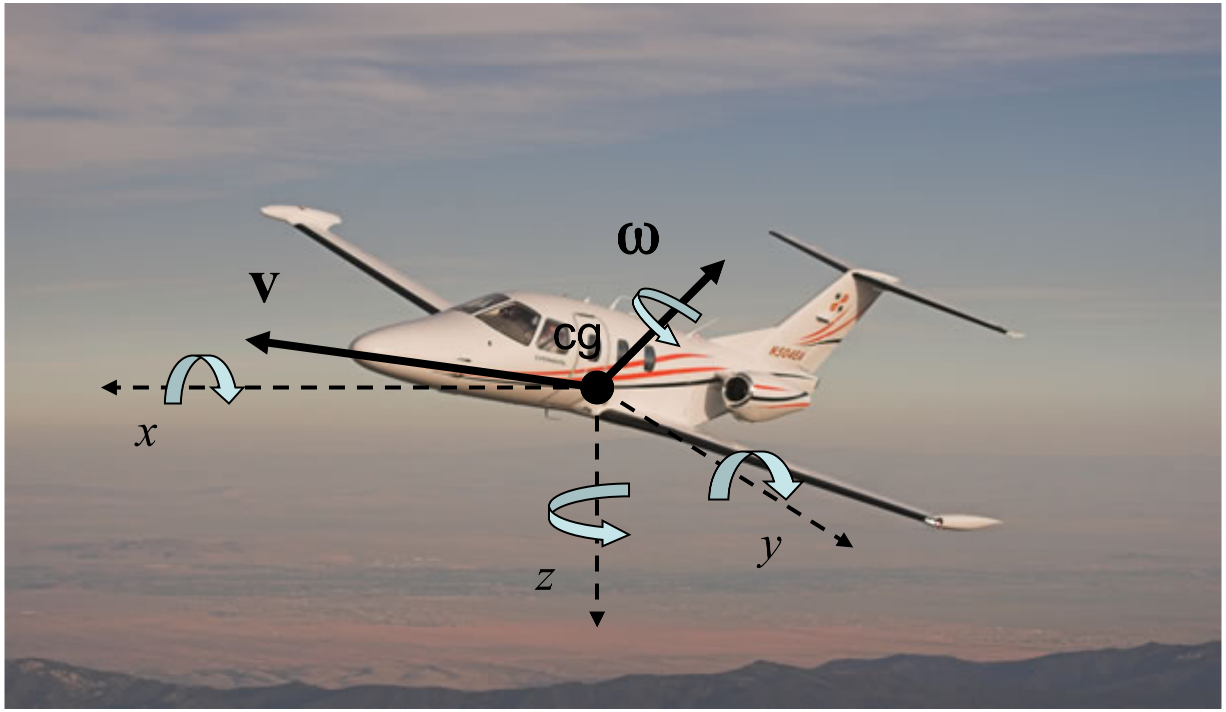
z: vertical motion

3 rotations

x: roll

y: pitch

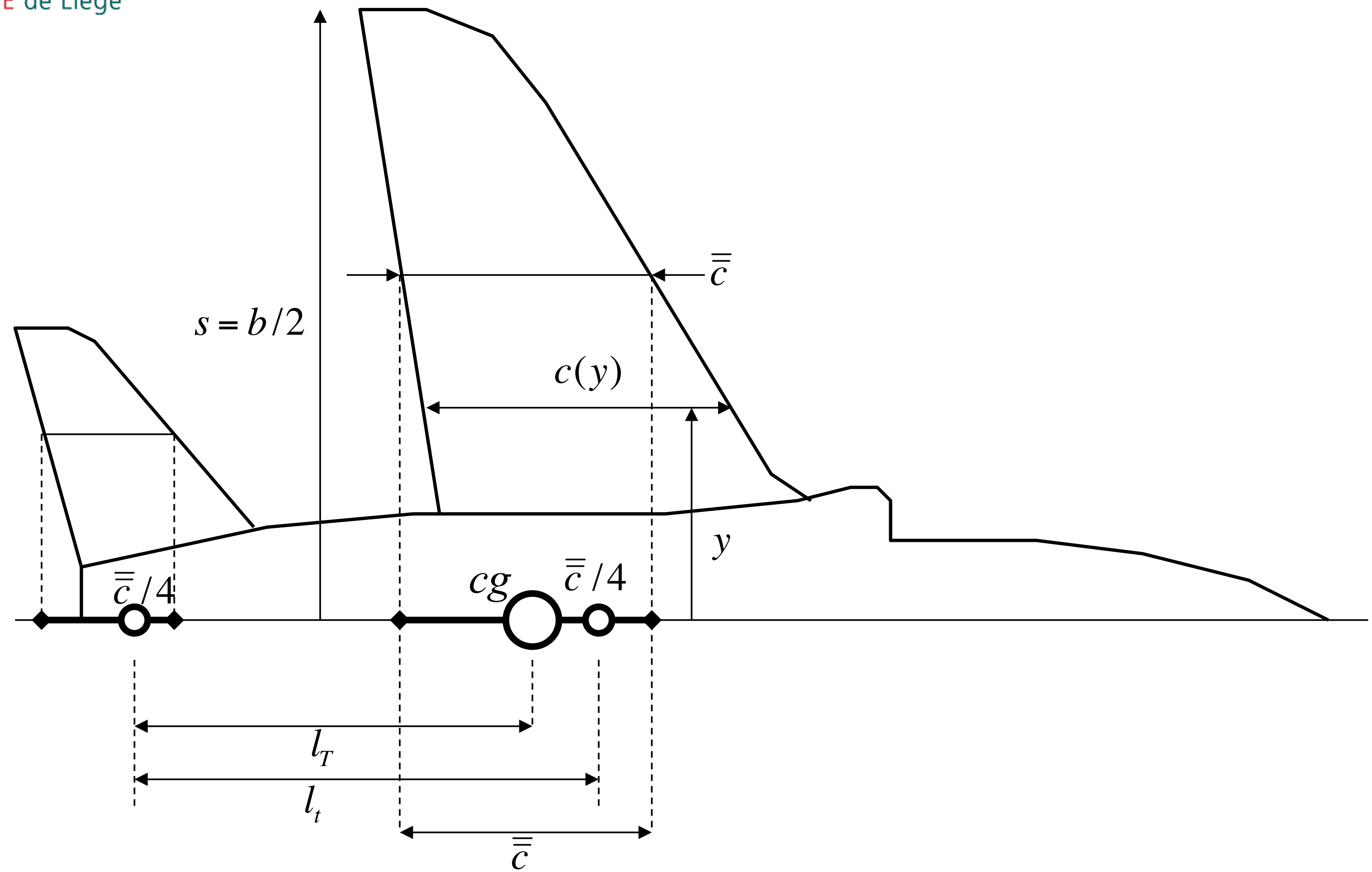
z: yaw



v: resultant linear velocity, cg: centre of gravity

ω: resultant angular velocity

Airplane geometry



Airplane references (1)

- Standard mean chord (smc)

$$\bar{c} = \int_{-s}^{s} c(y)\,\mathrm{d}y \Big/ \int_{-s}^{s} \mathrm{d}y$$

- Mean aerodynamic chord (mac)

$$\bar{\bar{c}} = \int_{-s}^{s} c^2(y)\,\mathrm{d}y \Big/ \int_{-s}^{s} c(y)\,\mathrm{d}y$$

- Wing area

$$S = b\bar{c}$$

- Aspect Ratio

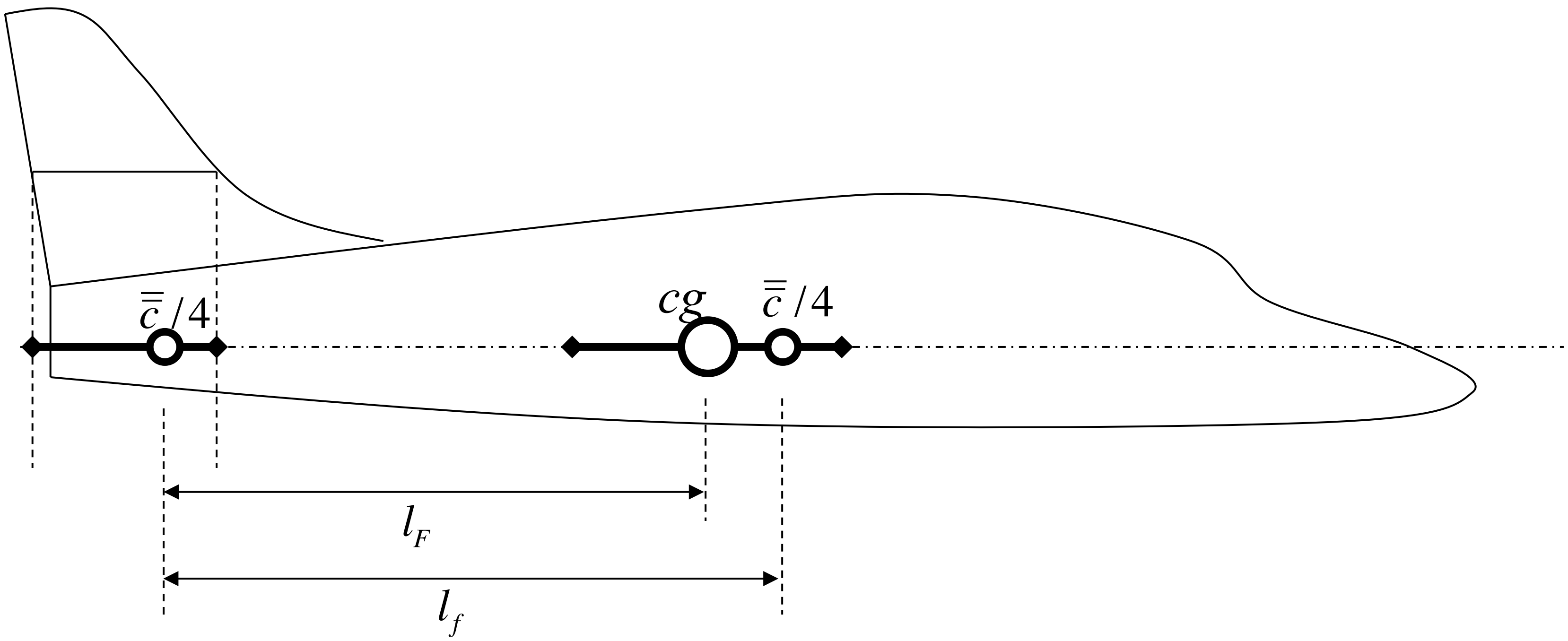
$$AR = b^2 / S$$

Airplane references (2)

- Centre of gravity (cg)
- Tailplane area (S_T)
- Tail moment arm (l_T)
- Tail volume ratio: A measure of the aerodynamic effectiveness of the tailplane

$$\overline{V}_T = \frac{S_T l_T}{S\overline{\overline{c}}}$$

Airplane references (3)



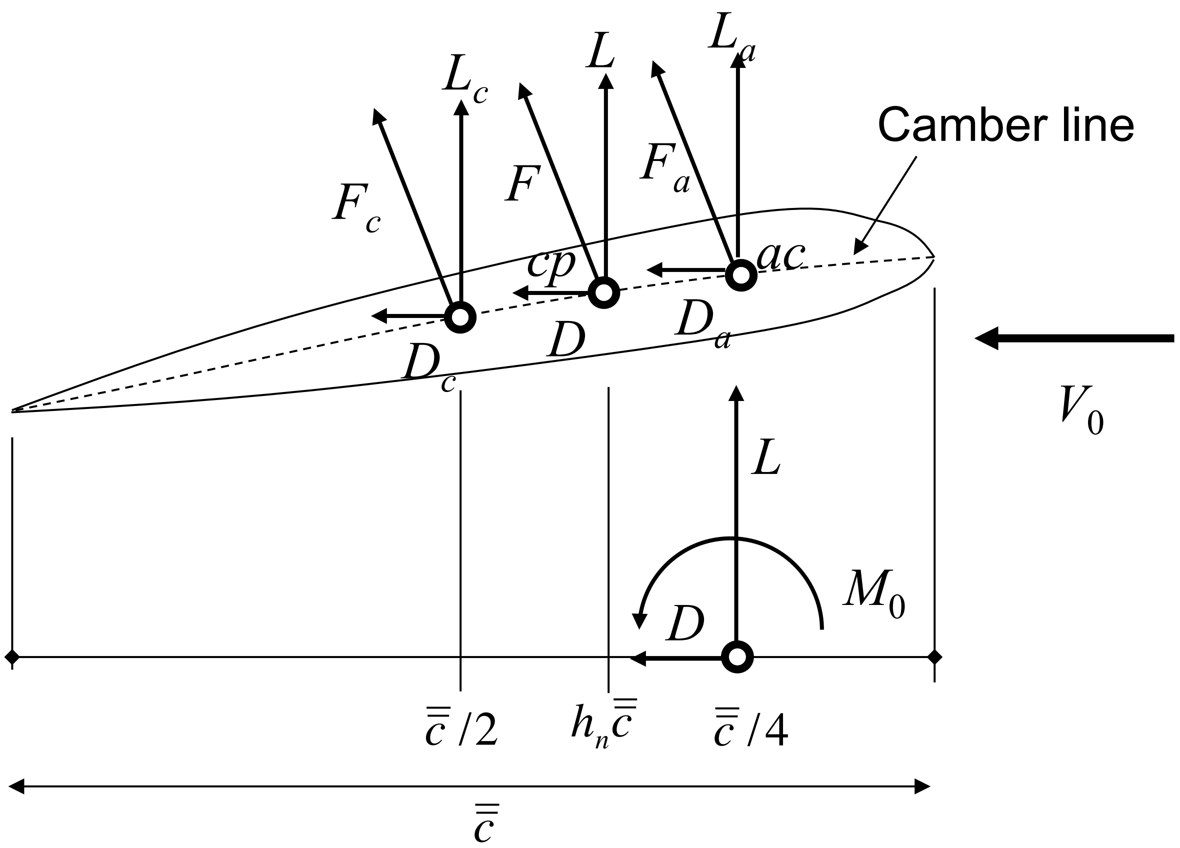
- Fin moment arm (l_F)
- Fin volume ratio $\bar{V}_F = \dfrac{S_F l_F}{S\bar{\bar{c}}}$

Aerodynamic Reference Centres

- Centre of pressure (cp): The point at which the resultant aerodynamic force F acts. There is no aerodynamic moment around the cp.
- Half-chord: The point at which the aerodynamic force due to camber, F_c, acts
- Quarter-chord (or aerodynamic centre): The point at which the aerodynamic force due to angle of attack, F_a, acts. The aerodynamic moment around the quarter-chord, M_0, is constant with angle of attack

Airfoil with centres

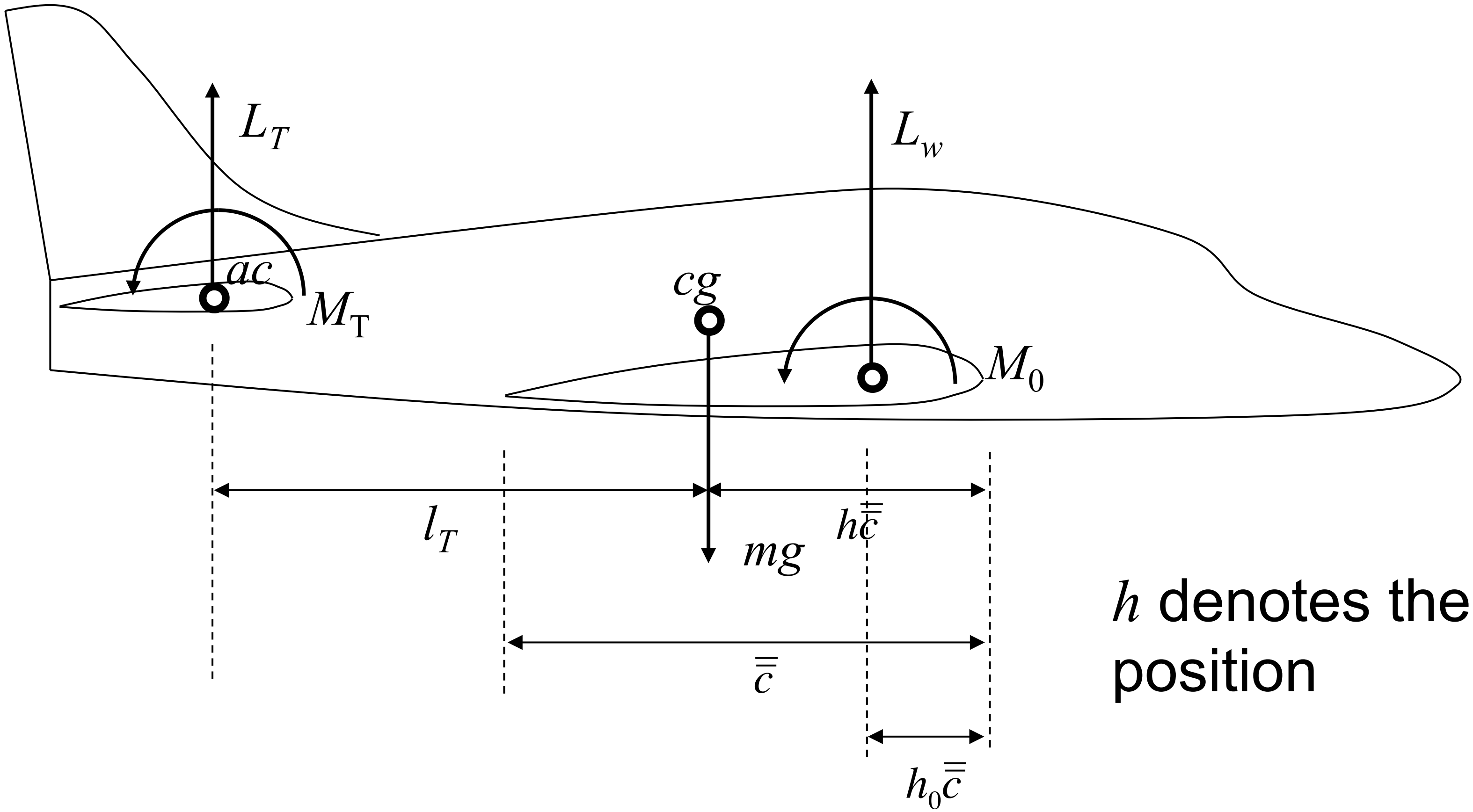
By placing all of the lift and drag on the aerodynamic centre we move the lift and drag due to camber from the half-chord to the quarter chord. This is balanced by the moment M_0



Static Stability

- Most aircraft (apart from high performance fighters) are statically stable
- Static stability implies:
  - All the forces and moments around the aircraft's cg at a fixed flight condition and attitude are balanced
  - After any small perturbation in flight attitude the aircraft returns to its equilibrium position
- The equilibrium position is usually called the trim position and is adjusted using the trim tabs

Pitching moment equation



h denotes the cg position

Equilibrium equations

- Steady level flight is assumed
- Thrust balances drag and they both pass by the cg
- Force equilibrium:

$$L_w + L_T - mg = 0$$

- Pitching moment around cg equilibrium:

$$M = M_0 + L_w(h - h_0)\bar{\bar{c}} - L_T l_T + M_T = 0$$

(nose up moment is taken to be positive)

Stable or Unstable?

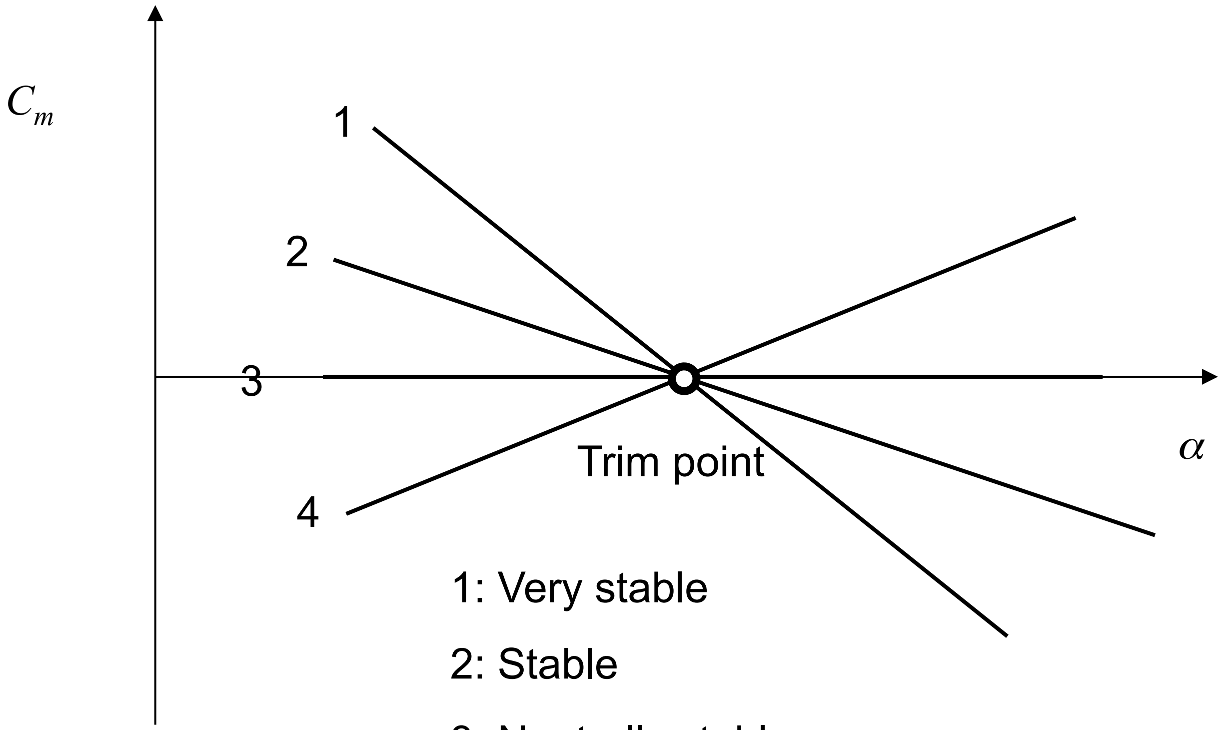
- An equilibrium point can be stable, unstable or neutrally stable
- A stable equilibrium point is characterized by

$$M = 0 \quad \text{and} \quad \frac{\mathrm{d}M}{\mathrm{d}\alpha} < 0$$

- A more general condition (takes into account compressibility effects) is

$$M = 0 \quad \text{and} \quad \frac{\mathrm{d}M}{\mathrm{d}L} < 0 \qquad \text{or} \qquad C_m = 0 \quad \text{and} \quad \frac{\mathrm{dC_m}}{\mathrm{d}C_\mathrm{L}} < 0$$

Degree of stability



1: Very stable

2: Stable

3: Neutrally stable

4: Unstable

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Pitching moment stability (1)

- The pitching moment equation can be written as

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - C_{L_T}\overline{V}_T = 0$$

- Where

$$C_m = \frac{M}{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}}, \quad C_{L_w} = \frac{L_w}{\frac{1}{2}\rho V_0^2 S}, \quad C_{L_T} = \frac{L_T}{\frac{1}{2}\rho V_0^2 S_T}$$

- And the tailplane is assumed to be symmetric so that $M_T=0$

Pitching moment stability (2)

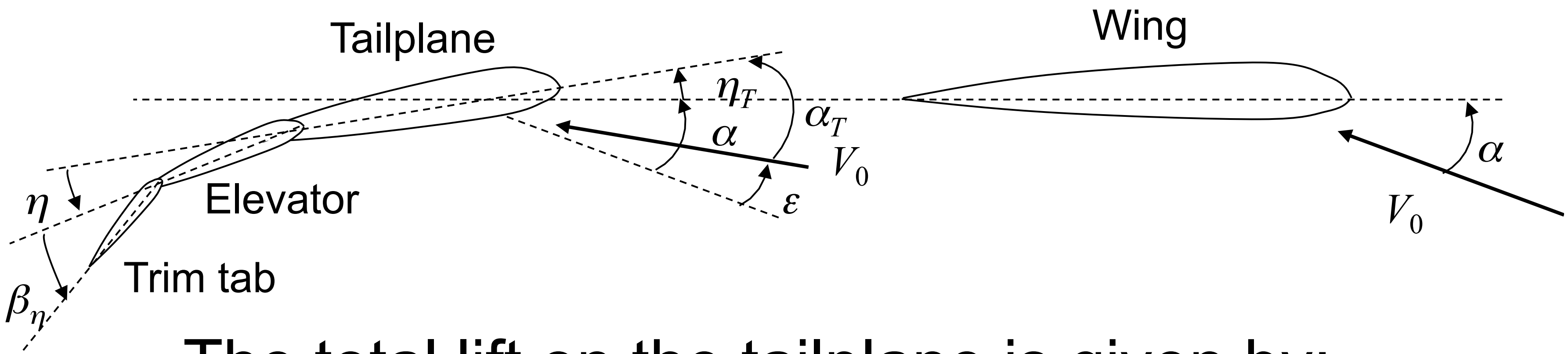
- For static stability $\mathrm{dC}_m / \mathrm{d}C_L < 0$ or, approximately, $\mathrm{dC}_m / \mathrm{d}C_{Lw} < 0$
- Then

$$\mathrm{d}C_m / \mathrm{d}C_{L_w} = (h - h_0) - \overline{V}_T \mathrm{d}C_{L_T} / \mathrm{d}C_{L_w}$$

- Since M_0 is a constant.
- Unfortunately, the derivative of the tail lift with respect to the wing lift is unknown

Wing-tail flow geometry

- The downwash effect of the wing deflects the free stream flow seen by the tailplane by an angle ε.
- Total angle of attack of tail: $\alpha_T = \alpha - \varepsilon + \eta_T$



- The total lift on the tailplane is given by:

$$C_{L_T} = \alpha_0 + a_1 \alpha_T + a_2 \eta + a_3 \beta_\eta$$

Pitching moment stability (3)

- For small disturbances the downwash angle is a linear function of wing incidence α:

$$\varepsilon = \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\alpha$$

- Wing lift is also a linear function of α:

$$C_{L_w} = a\alpha \ \text{ or } \ \alpha = C_{L_w}/a$$

- So that

$$\alpha_T = \frac{C_{L_w}}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \eta_T$$

Pitching moment stability (4)

- The tail lift coefficient can then be written as

$$C_{L_T} = C_{L_w} \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta$$

- And the derivative of the pitching moment coefficient becomes

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_{L_w}} = (h - h_0) - \bar{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + a_2\frac{\mathrm{d}\eta}{\mathrm{d}C_{L_w}} + a_3\frac{\mathrm{d}\beta_\eta}{\mathrm{d}C_{L_w}}\right)$$

- since η_T is a constant

Controls fixed stability

- Assume that the aircraft has reached trim position and the controls are locked
- What will happen if there is a small perturbation to the aircraft's position (due to a gust, say)?
- The pitching moment equation becomes

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_{L_w}} = (h - h_0) - \overline{V}_T \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)$$

Stability margin

- Define the controls fixed stability margin as

$$K_n = -\frac{\mathrm{d}C_m}{\mathrm{d}C_{L_w}}$$

- And the controls fixed neutral point as

$$K_n = h_n - h, \text{ so that } h_n = h_0 + \overline{V}_T \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)$$

- A stable aircraft has positive stability margin. The more positive, the more stable.
- If the cg position (h) is ahead of the neutral point (h_n) the aircraft will by definition be stable
- Too much stability can be a bad thing!

Stability margin

- Certification authorities specify that

$$K_n \geq 0.05$$

at all times

- Of course, the stability margin can change:
  - If fuel is used up
  - If payload is released:
    - Bombs
    - Missiles
    - External fuel tanks
    - Paratroopers
    - Anything else you can dump from a plane

Controls Free Stability

- Pilots don't want to hold the controls throughout the flight.
- The trim tab can be adjusted such that, if the elevator is allowed to float freely, it will at an angle corresponding to the desired trim condition.
- This is sometimes called a hands-off trim condition.
- Therefore the pilot can take his hands off the elevator control and the aircraft will remain in trim.

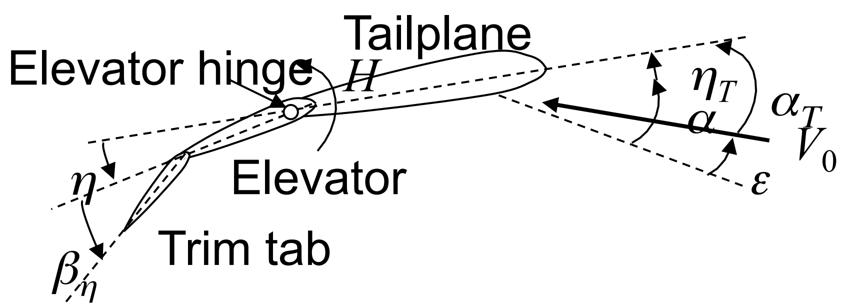
Controls Free Stability Margin

- This is an expression for η that can be substituted into the pitching moment equation.
- Differentiating the latter with respect to wing lift coefficient gives

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_{L_w}} = \left(h - h_0\right) - \overline{V}_T \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

- Where b_1, b_2 and b_3 define the elevator hinge moment coefficient

$$C_H = b_1\alpha_T + b_2\eta + b_3\beta_\eta$$



Controls Free Neutral Point

- Define the Controls Free Stability Margin, K'_n, such that

$$K'_n = -\frac{\mathrm{d}C_m}{\mathrm{d}C_{L_w}} = h'_n - h$$

- The controls free neutral point is then

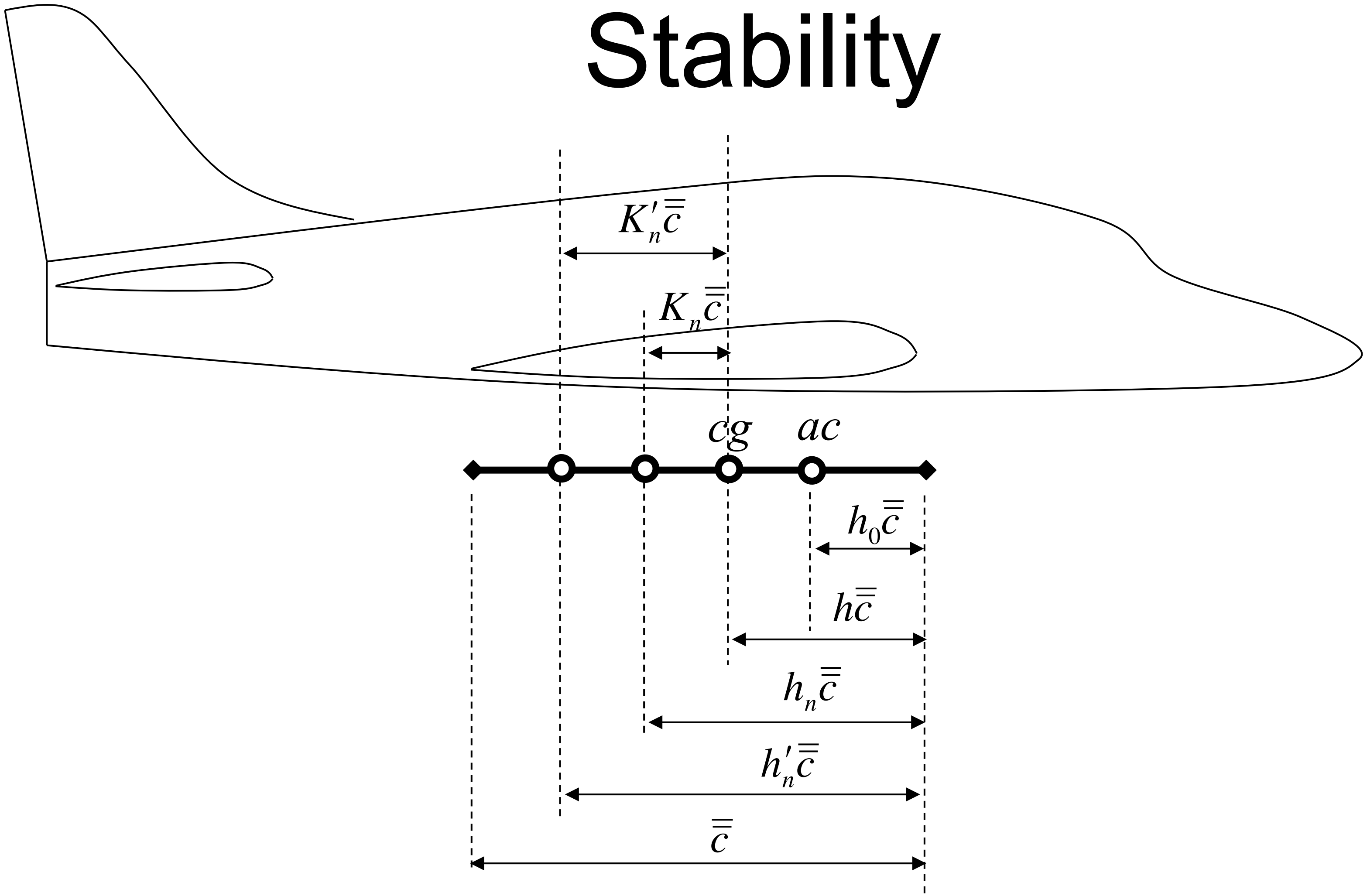
$$h'_n = h_0 + \bar{V}_T \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

$$\text{or} \quad h'_n = h_n - \bar{V}_T \frac{a_2 b_1}{a b_2}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)$$

Discussion

- As with the controls fixed stability margin, the controls free stability margin is positive when the aircraft is stable.
- Similarly, the centre of gravity position must be ahead of the controls free neutral point if the aircraft is to be stable.
- Usually, the constants of the elevator and tab are such that $h'_n > h_n$.
- An aircraft that is stable controls fixed will usually be also stable controls free

Summary of Longitudinal Stability



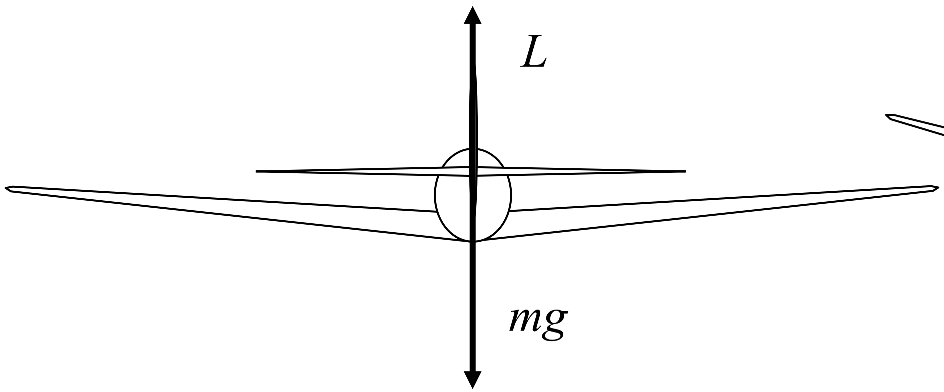
Lateral stability

- There are two types of lateral motion for an aircraft:
  - Roll
  - Yaw
- The aircraft must be stable in both of these directions of motion

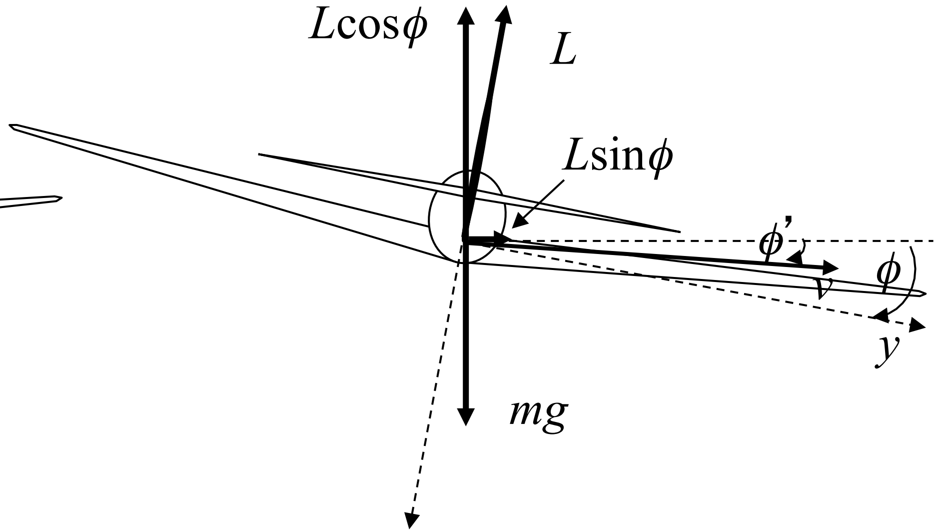
Roll Stability Mechanism

- There is no active stabilizing mechanism for lateral stability (e.g. tail for longitudinal stability, rudder for yaw stability)
- Wing dihedral, Γ, is the only stabilizing mechanism
- The higher the dihedral angle, the more stable the aircraft
- As usual, too much stability can be a bad thing

Roll Motion



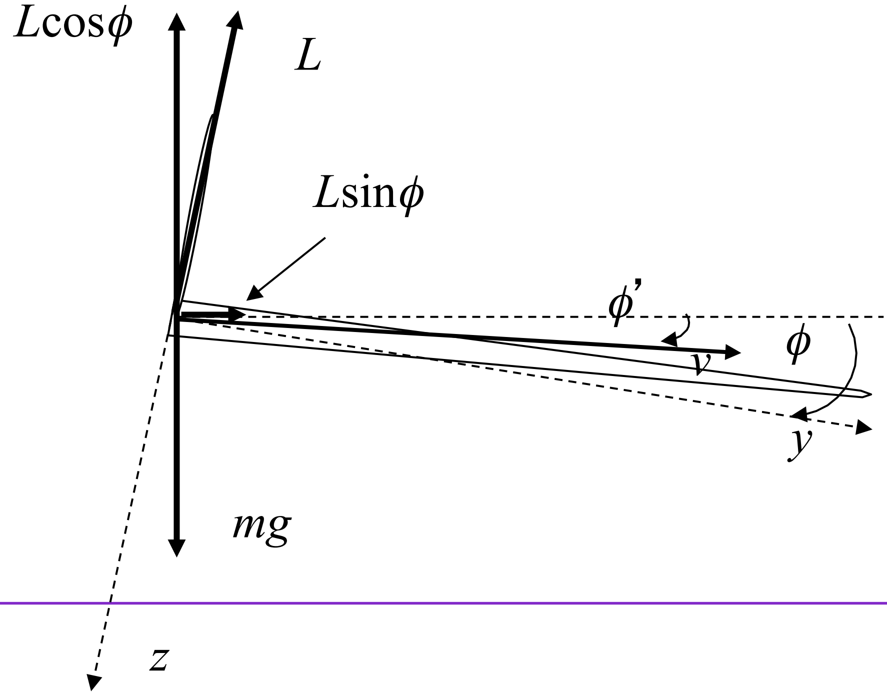
Steady level flight: $L=mg$



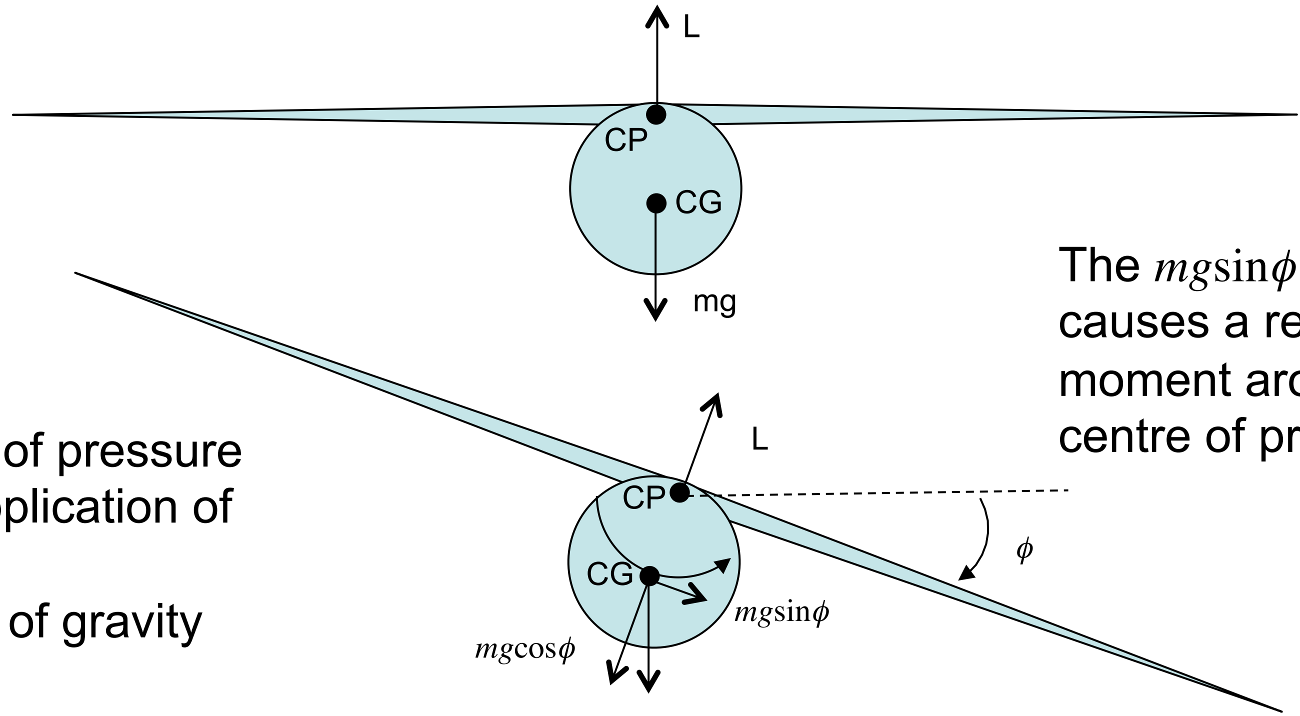
When at a roll angle ϕ, the lift L is still equal to mg. However, $L\cos\phi < mg$ and there is a sideslip force $L\sin\phi$. Therefore the aircraft is moving to the side and down, with speed v.

Sideslip angle

- Using trigonometry it can be shown that, the angle ϕ' between the sideslip vector v and the horizontal is always $\phi/2$.



High-wing aircraft

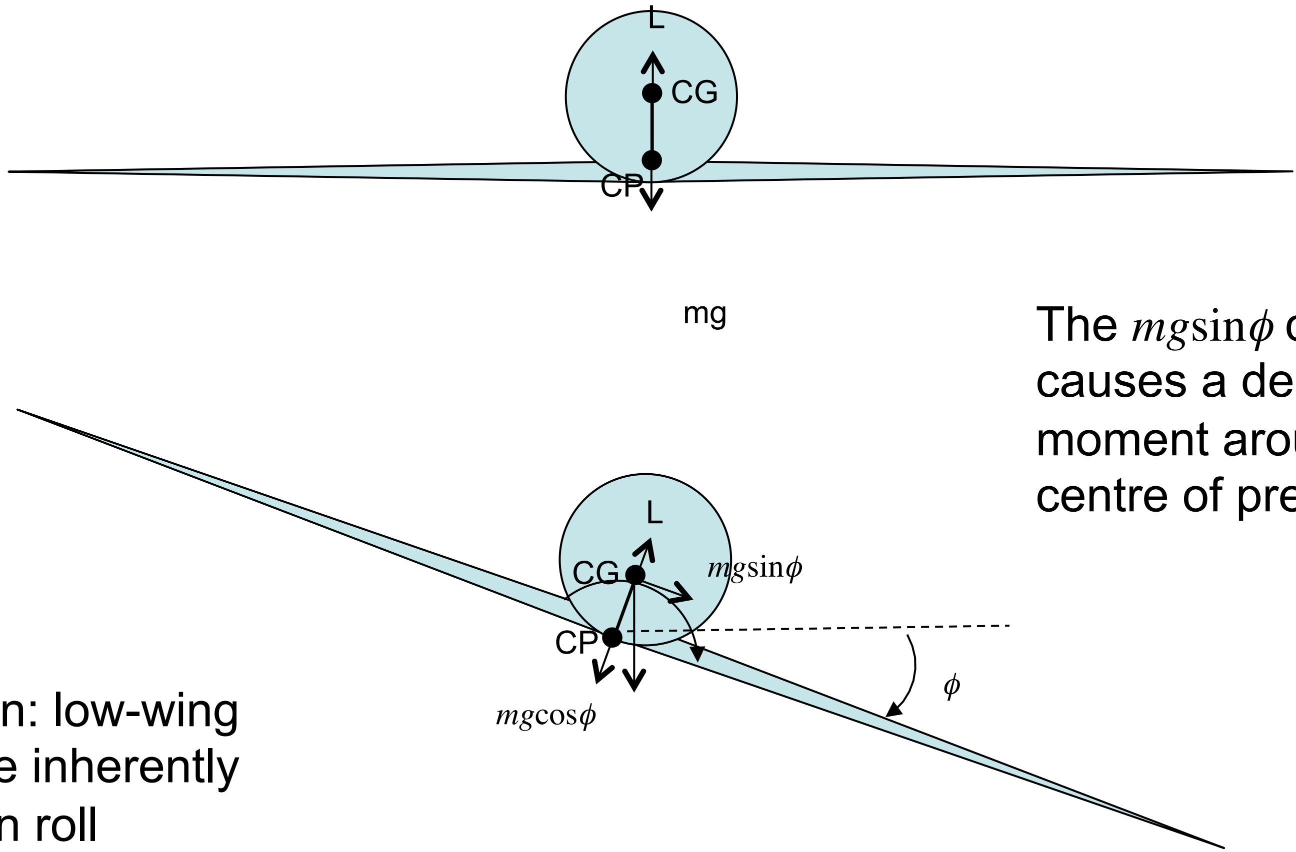


CP: centre of pressure (point of application of lift vector)
CG: centre of gravity

The $mg\sin\phi$ component causes a restoring moment around the centre of pressure

Conclusion: high-wing aircraft are inherently stable in roll

Low-wing aircraft

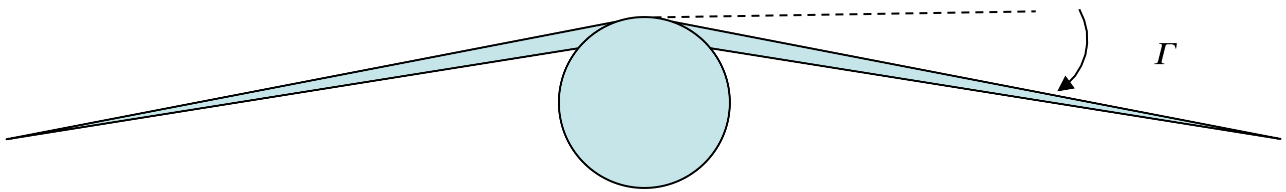


The $mg\sin\phi$ component causes a destabilising moment around the centre of pressure

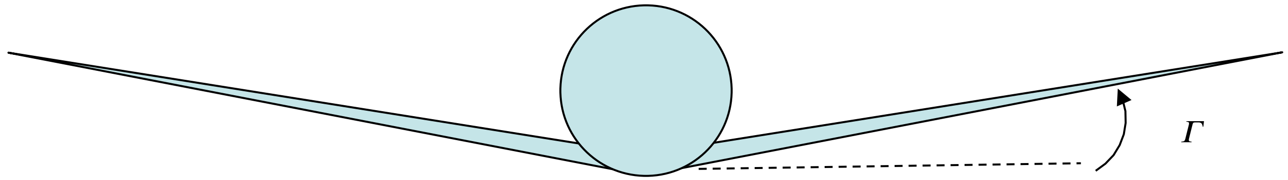
Conclusion: low-wing aircraft are inherently unstable in roll

Dihedral

High-wing aircraft: Dihedral reduces stability

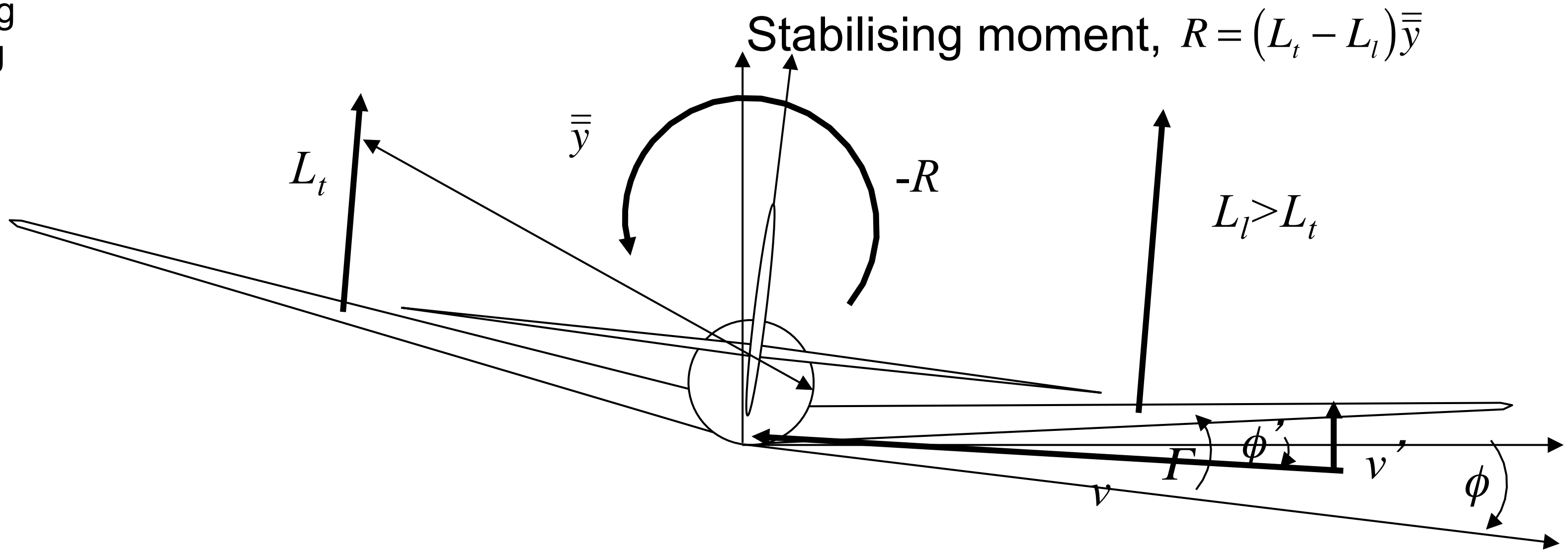


Low-wing aircraft: Dihedral increases stability



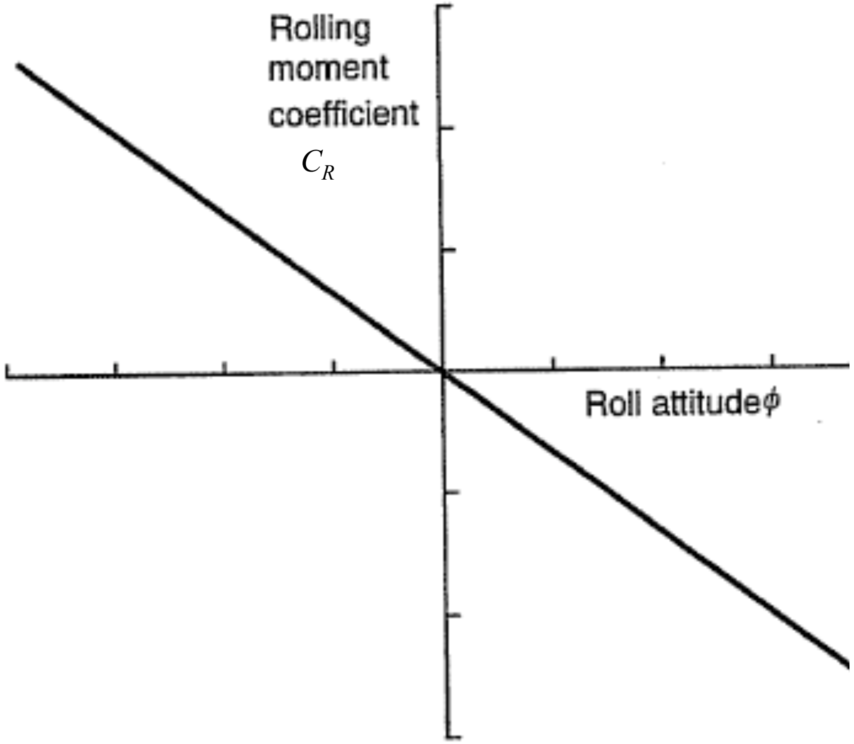
Restoring moment

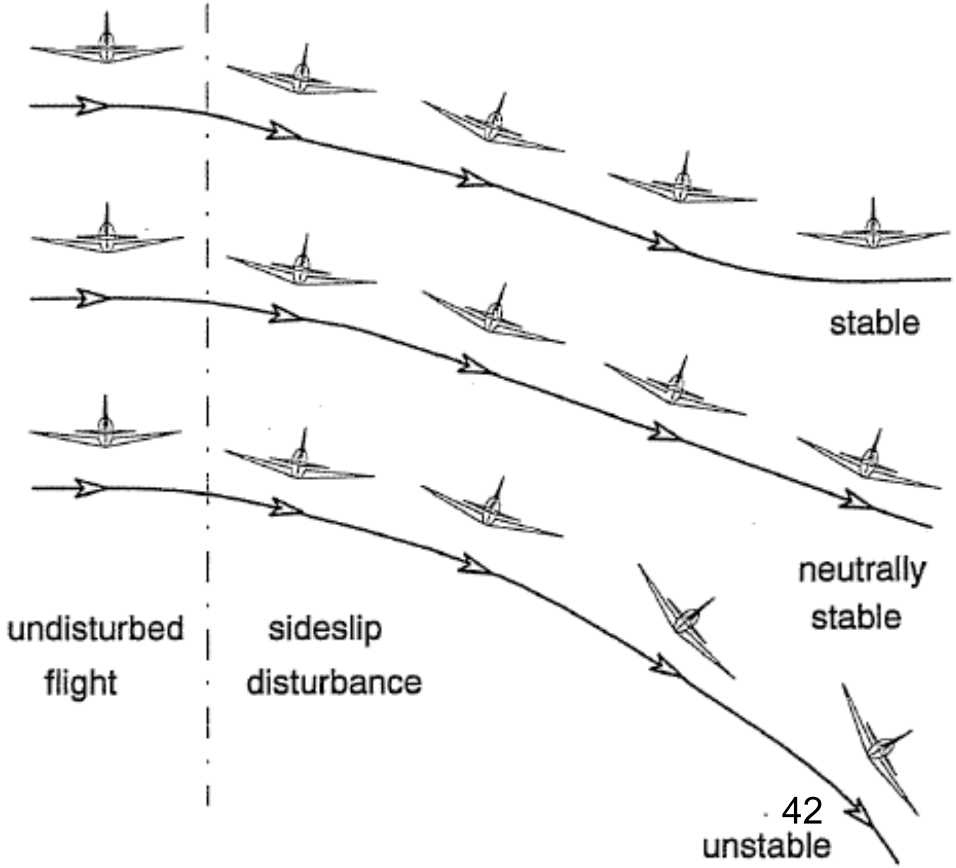
$_l$=leading
$_t$=trailing



Roll stability

- For a stable aircraft in roll, $\mathrm{d}C_R/\mathrm{d}\phi<0$.





Aircraft with negative dihedral


F104 Starfighter

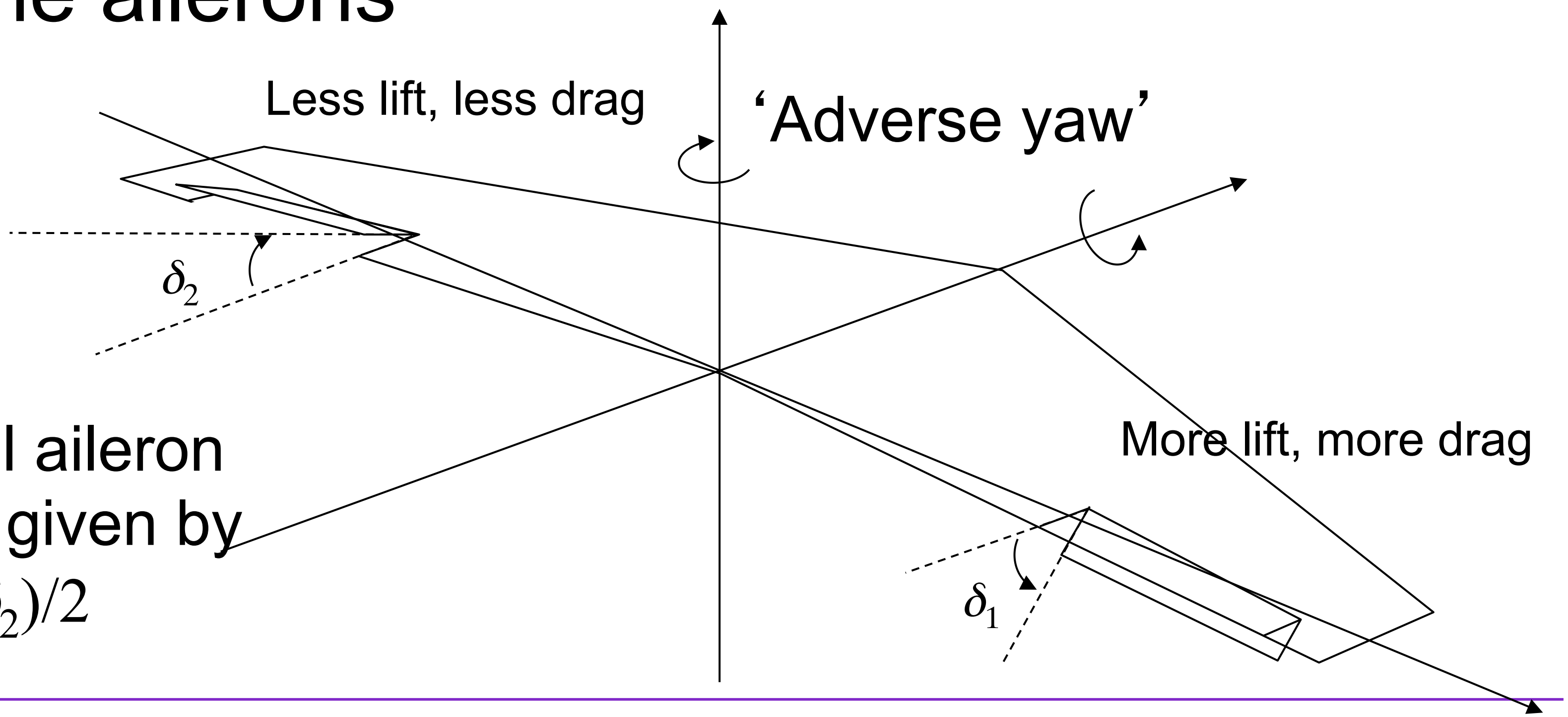

Alpha Jet


C5 Galaxy


Antonov 225

Roll Control

- Roll control can be accomplished using the ailerons



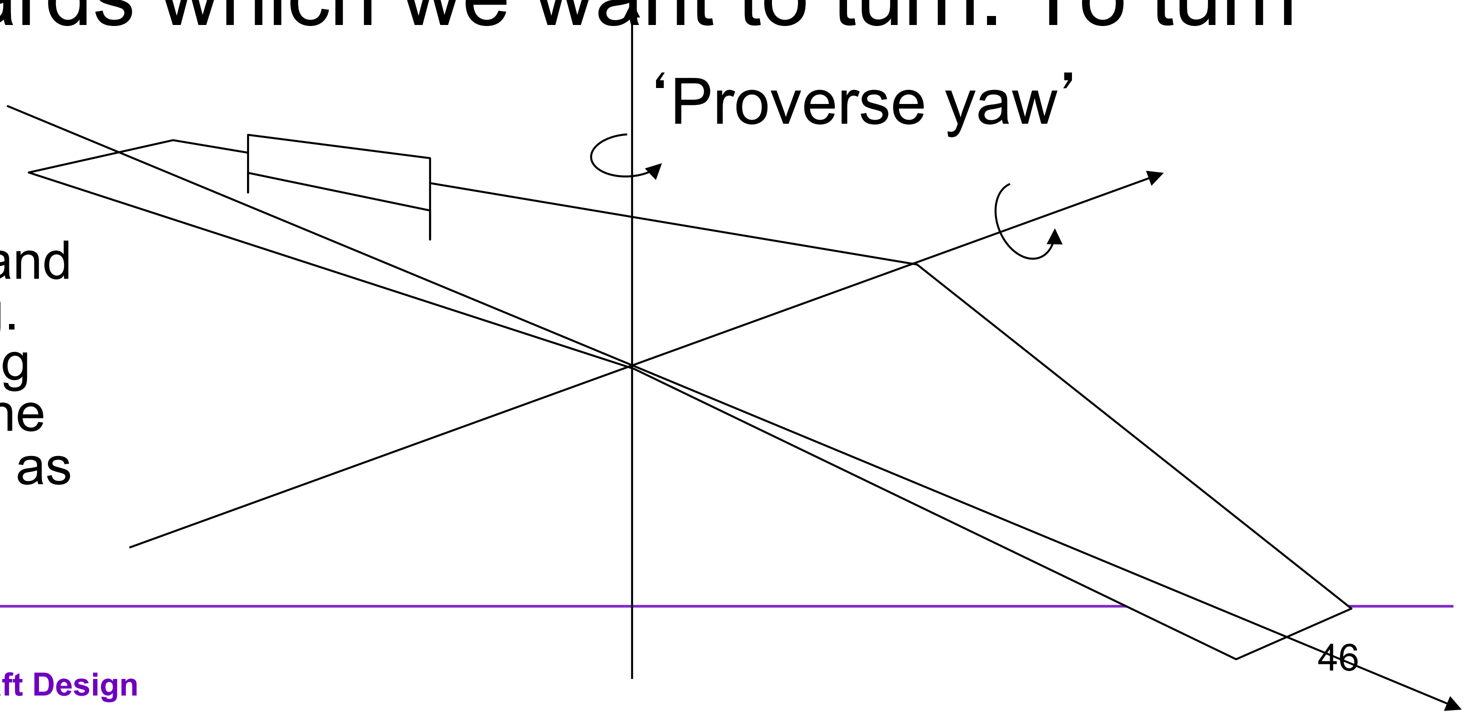
The total aileron angle is given by $\delta_a=(\delta_1+\delta_2)/2$

Aileron adverse yaw

- Increasing the lift also increases the drag and vice versa.
- When deflecting ailerons, there is a net yawing moment in an opposite direction to the rolling moment.
  - When rolling left (in order to turn left), there is a yawing moment to the right
  - This can make turning very difficult, especially for high aspect ratio wings

Roll control by spoilers

- Another way of performing roll control is by deforming a spoiler on the wing towards which we want to turn. To turn left:

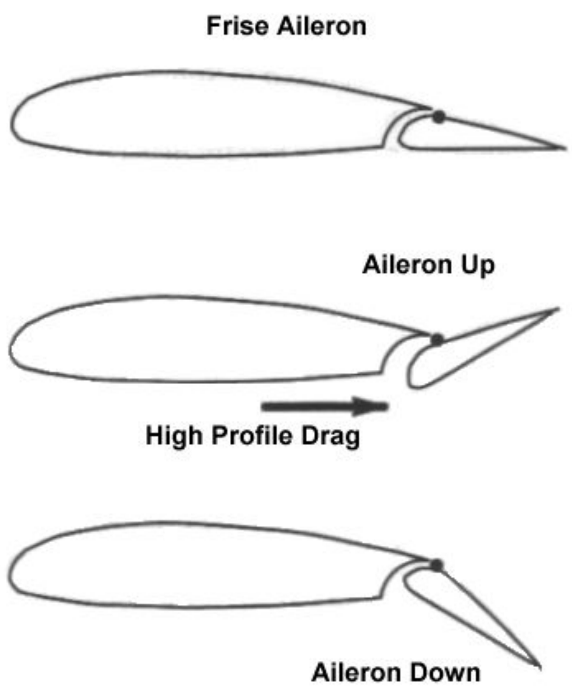


The spoiler decreases lift and increases drag. Now the yawing moment is in the same direction as the roll.

Frise Ailerons

The idea is to counteract the higher lift induced drag of the down wing with higher profile drag on the up wing.

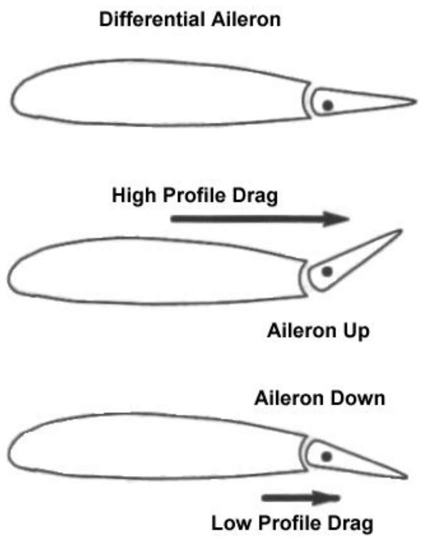
Frise ailerons are especially designed to create very high profile drag when deflected upwards. When deflected downwards the profile drag is kept low. Thus, they alleviate or, even, eliminate adverse yaw



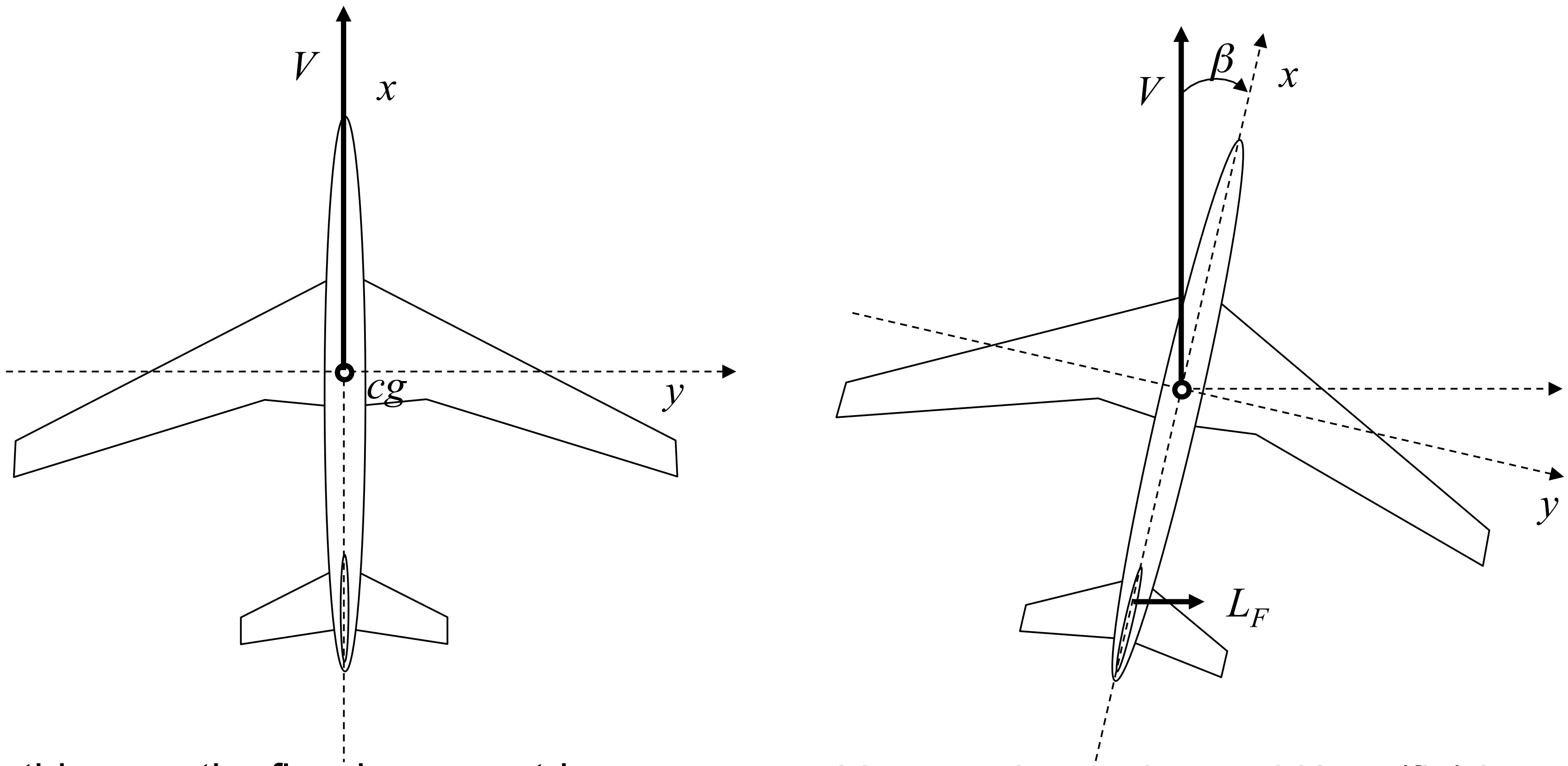
Differential Aileron Deflection

- The roll rate of the aircraft depends on the mean aileron deflection angle. The individual deflections δ_1 and δ_2 do not have to be equal.

Differential deflection means that the up aileron is deflected by a lot while the down aileron is deflected by a little.



Yaw Stability



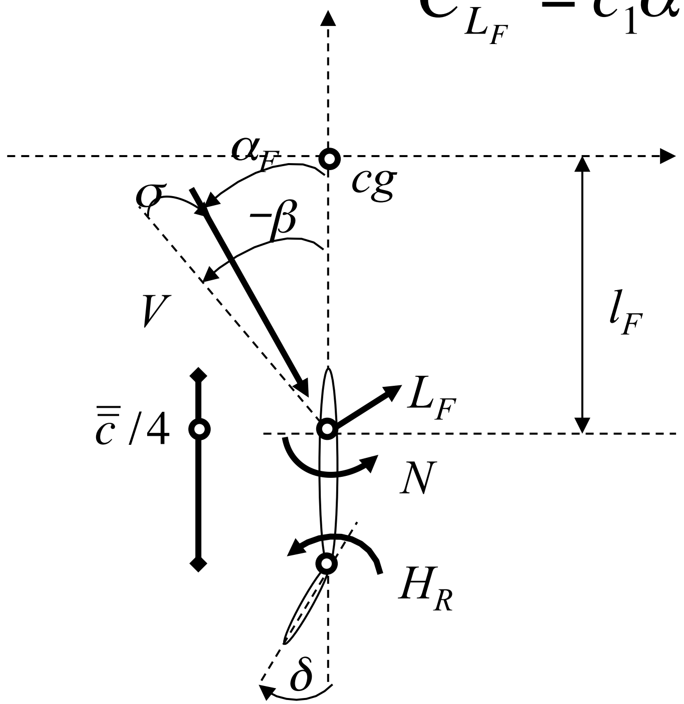
In this case the flow is symmetric;
There is no net moment around the z axis

In this case the vertical stabilizer (fin) is at a positive angle of attack. It produces a stabilizing moment around the z axis

Fin Moment

The lift coefficient of the fin can be expressed as

$$C_{L_F} = c_1\alpha_F + c_2\delta = c_1(-\beta+\sigma) + c_2\delta$$



Where σ is the sidewash velocity, a local windspeed component induced by the effect of the fuselage, wing and possibly propellers.

$$C_n = C_{L_F}\overline{V}_F, \text{ where } \overline{V}_F = \frac{S_F l_F}{S\bar{\bar{c}}}$$

Yaw stability

- The stability condition for yaw motion is then $$\frac{\mathrm{d}C_n}{\mathrm{d}\beta} < 0, \text{ or, } \bar{V}_F\left(-c_1 + c_1\frac{\mathrm{d}\sigma}{\mathrm{d}\beta} + c_2\frac{\mathrm{d}\delta}{\mathrm{d}\beta}\right) < 0$$
- Note that, in this case, it makes no sense to differentiate the yawing moment by the lift since the two are independent.
- The sidewash factor, $\mathrm{d}\sigma/\mathrm{d}\beta$, is very difficult to estimate

Sidewash factor

- There are three main contributions to sidewash:
  – Fuselage: It acts as a lifting body when at a yaw angle
  – Wing: The flow over the wing is asymmetric. The resulting sidewash is more pronounced for low aspect ratio sweptback wings.
  – Propeller: The flow behind the propeller is also yawed, causing additional asymmetry.
- Sidewash factors can be estimated most accurately by carefully designed wind tunnel experiments.

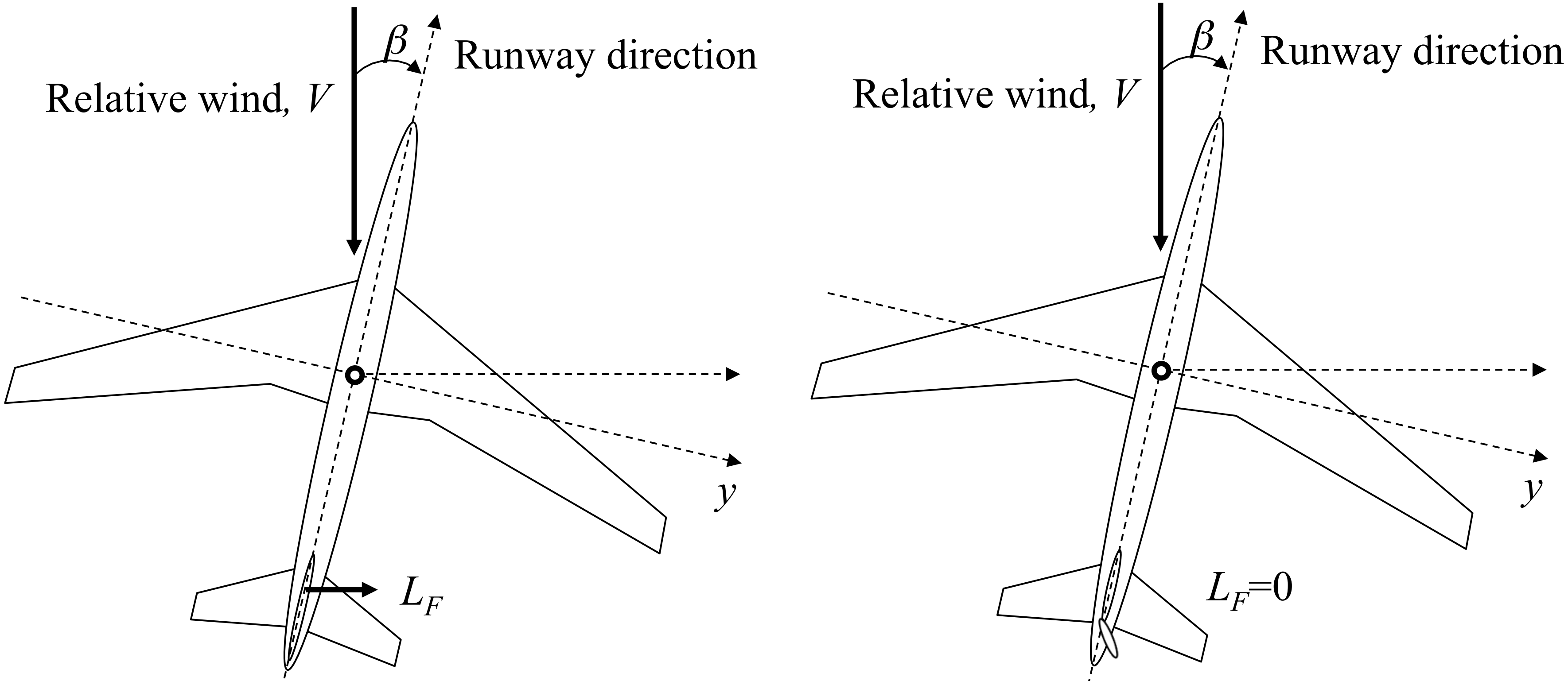
Yaw control

- During most flight conditions the yaw angle must be zero - this minimizes drag
- This is achieved through the deflection of the rudder.
- Rudder power: Rate of change of fin moment with rudder angle $$\frac{\mathrm{d}C_n}{\mathrm{d}\delta} = \overline{V}_F \frac{\mathrm{d}C_{L_F}}{\mathrm{d}\delta} = c_2 \overline{V}_F$$
- This quantity must be large enough to maintain zero yaw even at the most extreme flight conditions.

Take off and landing (1)

- During cruise, aircraft tend to turn towards the wind in order to minimize their drag. Therefore, the objective is to achieve 0^o yaw.
- At take-off and landing this is not possible. The aircraft must remain aligned with the runway, even in the presence of a very strong sidewind.
- Therefore, the rudder must be able to provide a moment that can keep the aircraft aligned with the runway.

Take off and landing (2)



Undeflected rudder: fin has a sizeable lift causing a moment around the c.g.

Deflected rudder: fin creates 0 lift and 0 moment around the c.g.

Roll-Yaw coupling

- Roll and yaw are always coupled.
- There are several reasons for the coupling:
  - Rolling produces sideslip
  - Ailerons cause adverse yaw
  - Dihedral cases additional coupling
  - Wingtip vortices cause additional coupling
  - Sweepback causes additional coupling
  - The fin causes additional coupling

Summary on Control surfaces

- **Elevators** contribute to pitch stability and control pitch angle.
- **Rudder** contributes to yaw stability and controls yaw angle.
- **Ailerons** do not contribute to stability. Furthermore, they control the *roll rate*, not the *roll angle*. There is no moment to balance the effect of the ailerons: They provide a constant moment that causes continuous roll rotation, whose rate also depends on the moment of inertia of the aircraft.

More control surfaces

- **Elevons** contribute to stability and control of both pitch and roll. They are ailerons that can also move up or down in unison, just like elevators.
- **Flaperons** contribute to stability and control of both pitch and roll. They are ailerons that can also move downwards only, just like flaps.
- **Spoilerons** contribute to stability and control of both pitch and roll. They are ailerons that can also move upwards only, just like spoilers.

Modes of vibration of aircraft

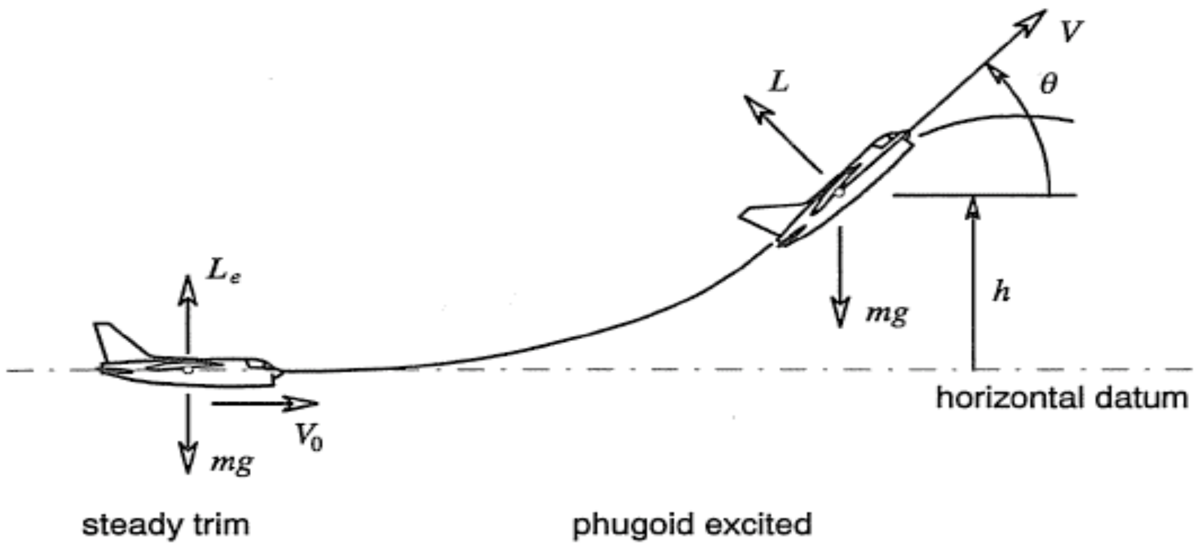
- Stability must also be investigated in a dynamic sense
- Aircraft have several modes of vibration:
  - Longitudinal modes:
    - Short Period Oscillation
    - Phugoid
  - Lateral modes:
    - Spiral mode
    - Roll subsidence
    - Dutch roll

Phugoid oscillations

- Phugoids are long period oscillations that occur only in the longitudinal direction.
- The angle of attack is constant; the aircraft climbs and descends in an oscillatory manner.
- Phugoids are also very lightly damped.
- Phugoid periods:
  - Microlight aircraft: 15-25s
  - Light aircraft: over 30s
  - Jet aircraft: minutes
- Phugoids are neutralized by re-trimming the aircraft in the new flight condition.

Phugoid Videos

Phugoid approximation



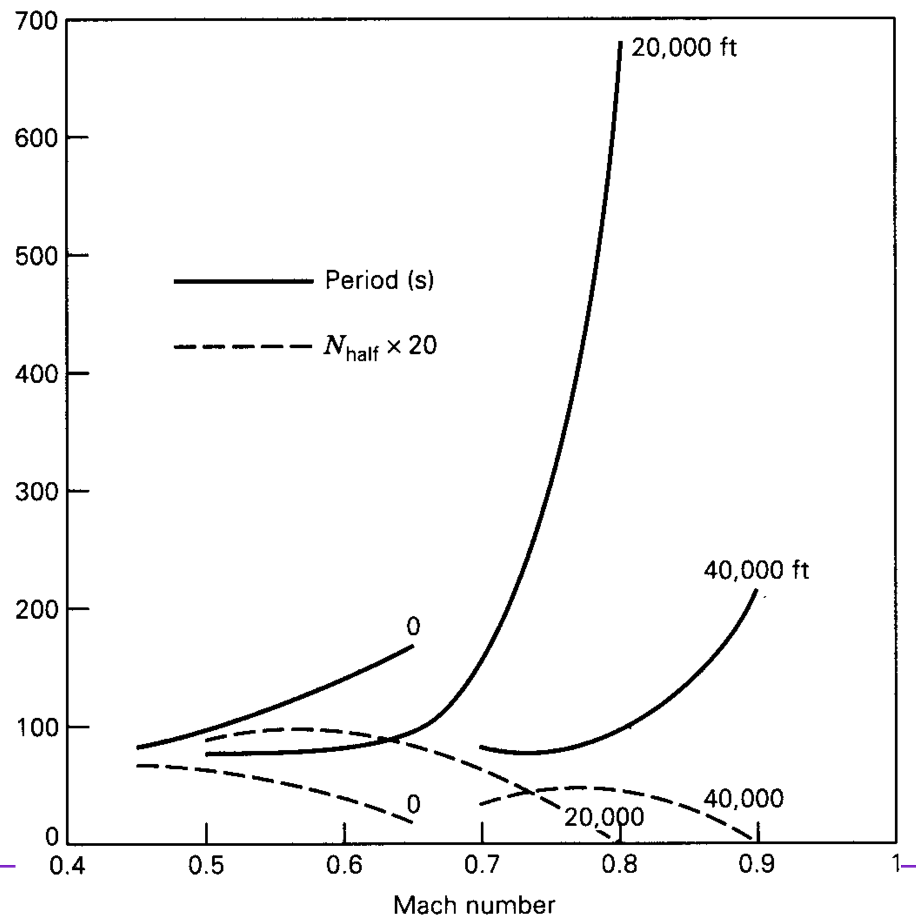
- The Lanchester approximation states that the phugoid damping ratio and frequency are given by:

$$\zeta_p \omega_p = \frac{g C_D}{C_L V_0}, \quad \omega_p = \frac{g\sqrt{2}}{V_0}$$

More about Phugoids

- Phugoid period increases with airspeed. Phugoid damping increases with airspeed.
- Compressibility effects
- Period and damping for a Boeing 747 at several altitudes and Mach numbers

N_{half}= number of periods until the amplitude is halved

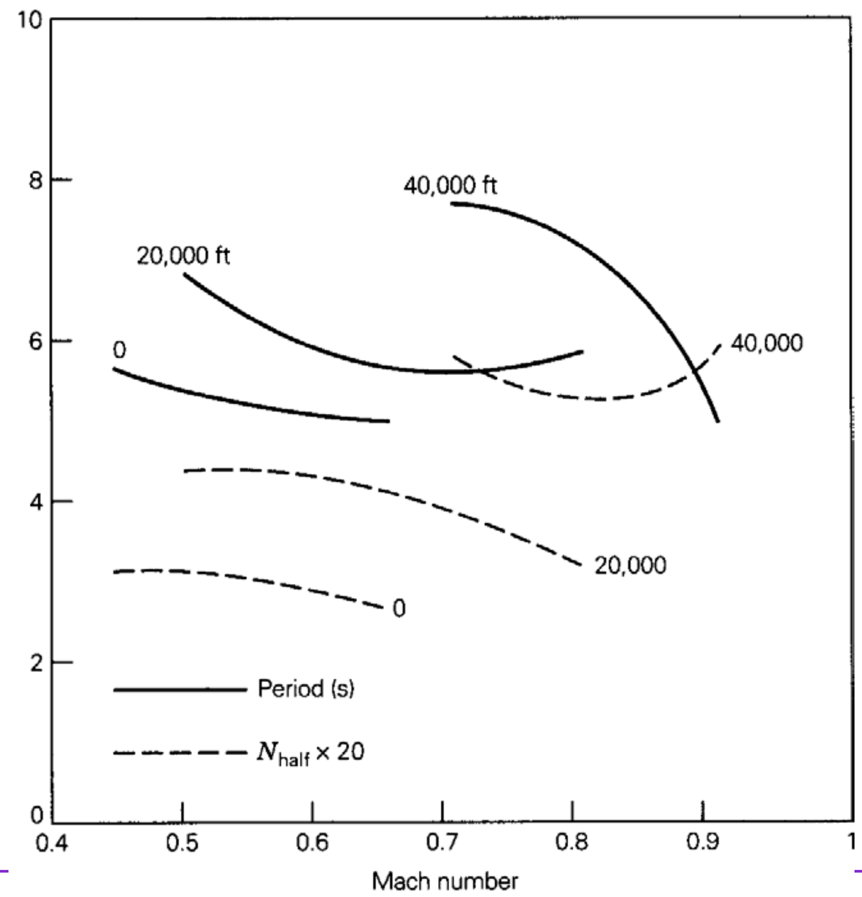


Short period oscillations

- Short period oscillations have a much higher frequency than Phugoids.
- They are driven by the angle of attack (in french they are called oscillations d'incidence).
- Speed changes are negligible
- They occur after abrupt input changes. Slower input changes do not cause significant short period oscillations

Short Period Dependencies

- The period generally decreases with airspeed. The damping can either decrease or increase
- Compressibility effects
- Period and damping for a Boeing 747 at several altitudes and Mach numbers



Spiral mode

- This mode is quite visible in the impulse response of the lateral equations
- It is the non-oscillatory mode with large time constant
- It is mainly a yaw movement with a little roll
- This mode can be stable or unstable. It is unstable quite often but that is not a problem because of its large time constant
- The typical half-life of a spiral mode is of the order of a minute.
- The spiral movement is usually stopped by a corrective control input

Spiral Mode Video

Roll subsidence

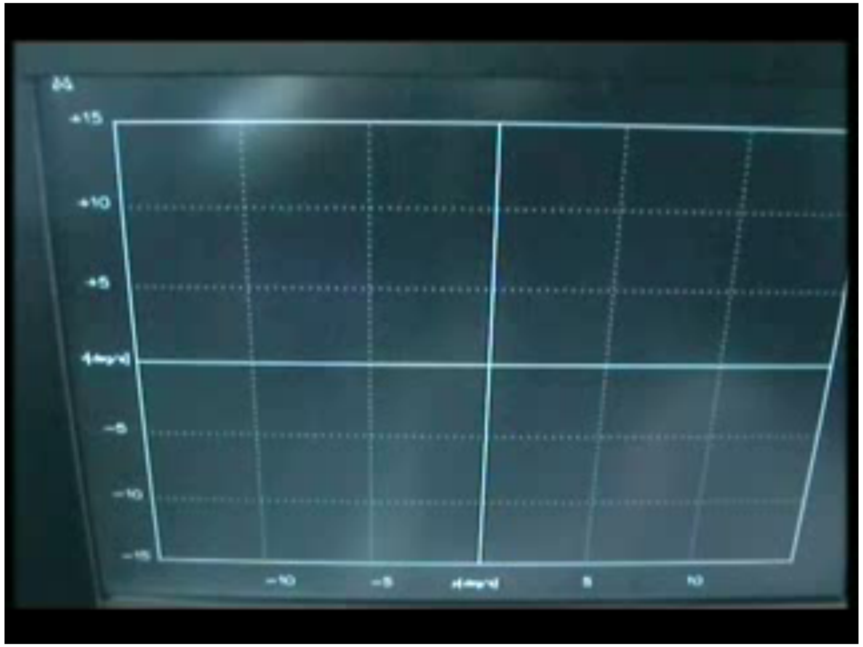
- An impulse aileron input will start the aircraft rolling.
- In general, the aircraft will stop rolling with time (i.e. the roll rate becomes zero after sufficient time)
- The aircraft will find itself at a roll angle which depends on how fast the roll rate tends to zero.
- This phenomenon is called roll subsidence.

Roll Subsidence Video

Dutch Roll

- The name Dutch Roll is due to the fact that the phenomenon resembles an ice skating figure called Dutch Roll.
- The centre of gravity remains on a straight trajectory while the roll and yaw angles oscillate.
- The roll velocity also oscillates but the yaw velocity is very low.
- The Dutch roll damping increases with airspeed while its period first increases and then decreases with airspeed.
- The typical period of a Dutch roll is in the order of 5 to 10 seconds.

Dutch Roll Videos



Lateral Modes of Boeing 747

Altitude	Mach	Spiral	Dutch Roll	
		Half-life	Period	N_{half}
0	0.45	35.7	5.98	0.87
0	0.65	34.1	4.54	0.71
20,000	0.5	76.7	7.3	1.58
20,000	0.65	64.2	5.89	1.33
20,000	0.8	67.3	4.82	1.12
40,000	0.7	-296	7.99	1.93
40,000	0.8	94.9	6.64	3.15
40,000	0.9	-89.2	6.19	1.18