Aircraft Design

Lecture 4:

Aircraft Performance

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Design for Performance

- The most important requirement for a new aircraft design is that it fulfills its mission
- This is assured through performance calculations at the design stage
- As these calculations are carried out, important aircraft parameters are chosen:
  - Size of wing
  - Engine

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Flight points

- Performance calculations are crucial at several flight points. The most important are:
  - Cruise
  - Take off
  - Climb
  - Landing
- The first performance design analyses for an airliner are carried out for cruise

Weight and drag

- In order to choose parameters such as engine and wing size, the aircraft's weight and drag must be known.
- Then the amount of lift and thrust required can be determined
- Of course, the weight and drag must be calculated at several important points in the flight envelope.

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Methodology

- It is impossible to know the weight and drag of an aircraft before it has even been designed
- There are two possibilities:
  - Carry out detailed simulations at the conceptual design stage; very costly
  - Use previous experience, statistical data, carry out industrial espionage etc

Statistics

There is an enormous wealth of data on civil aircraft. There are hundreds of types that are very similar. They can be used to extract some very useful statistics.

Weight guesstimates

- The first important weight to calculate is the take off weight, W_{to}.
- It is usually expressed as:

$$W_{to} = \frac{W_p + W_{fix}}{1 - \frac{W_{var}}{W_{to}} - \frac{W_f}{W_{to}}}$$

- Where W_p is the payload weight, W_f is the fuel weight, W_{fix} is the fixed empty weight (e.g. engine) and W_{var} is the variable empty weight.
- Notice that in this expression, $W_e = W_{fix} + W_{var}$ is the total empty weight of the aircraft

Light aircraft (W_{to}<5670Kg)

- By evaluating data from 100 different types of light aircraft, the following data was found:

$$\frac{W_{var}}{W_{to}} = \begin{cases} 0.45 \text{ - for normal category with fixed gear} \\ 0.47 \text{ - for normal category with retractable gear} \\ 0.50 \text{ - for utility category} \\ 0.55 \text{ - for acrobatic category} \end{cases}$$

$$\frac{W_f}{W_{to}} = 0.17\frac{R}{1000} r_{uc} AR^{-0.5} + 0.35$$

- Where R is the aircraft's range, AR is the main wing's aspect ratio and r_{uc}=1.00-1.35 is the undercarriage drag correction factor.
- The fixed weight is the engine weight. It is either known or can be approximated as 5%-6% of the take off weight

Undercarriage drag correction factor

- The undercarriage drag correction factor is used both in the calculation of fuel weight and in the calculation of the zero-lift drag (see later)
- For a fully retractable landing gear that disappears inside the aircraft lines, $r_{uc}=1$.
- Otherwise:

$$r_{uc} = \begin{cases} 1.35 \text{ - for fixed gear without streamlined wheel fairings} \\ 1.25 \text{ - for fixed gear with streamlined wheel fairings} \\ 1.08 \text{ - main gear retracted in streamlined fairings on the fuselage} \\ 1.03 \text{ - main gear retracted in engine nacelles} \end{cases}$$

Wheel Fairings

Cessna 180 with wheel fairing

Cessna 172 without wheel fairing

Fairings/Engine Nacelles

Antonov 225: fuselage fairings

Bombardier Dash 8: engine nacelles

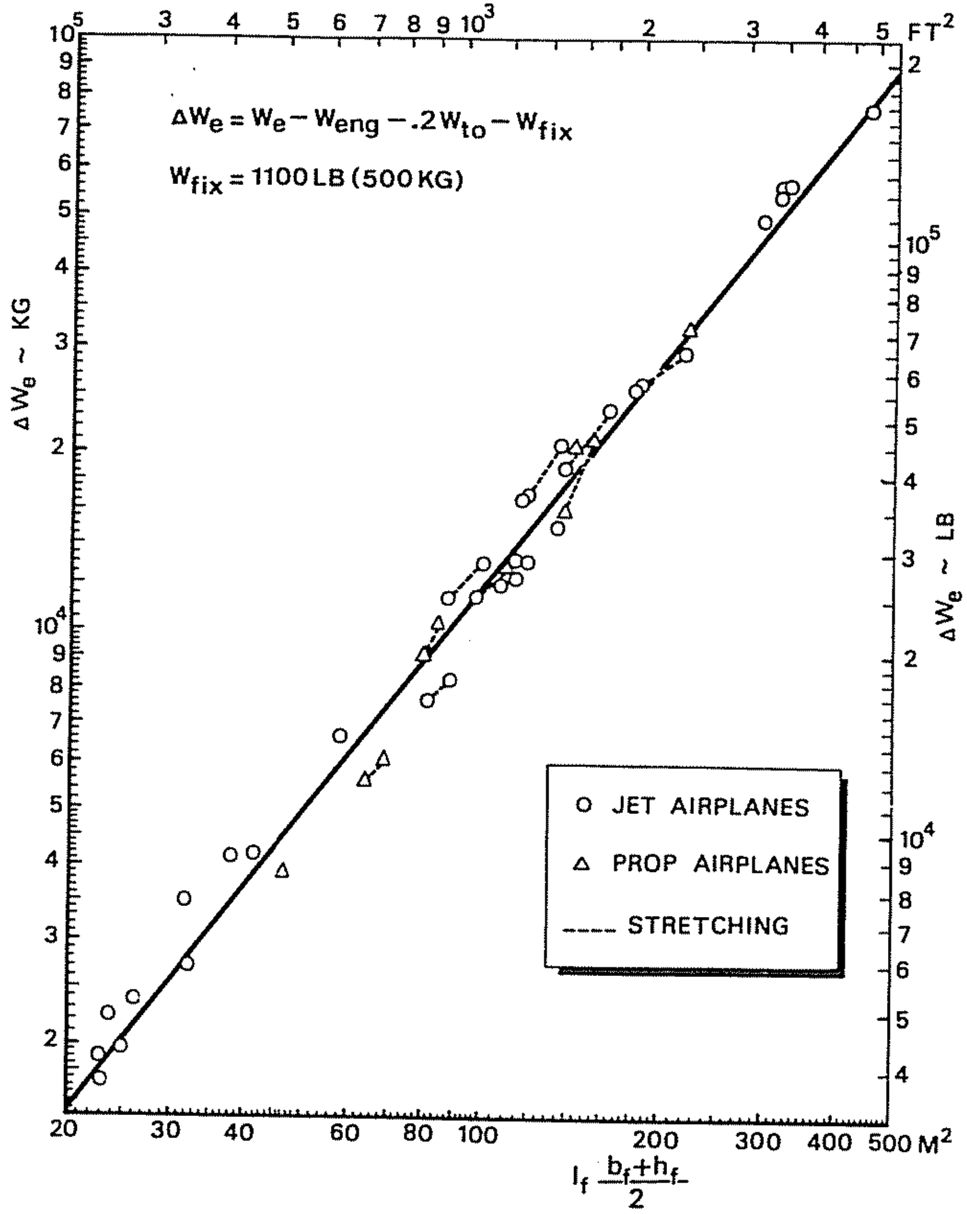
Transport Aircraft (W_{to}>5670Kg)

- Again, statistical studies show that

$$\frac{W_{var}}{W_{to}} = 0.2$$

$$W_{fix} = W_{eng} + 500 + \Delta W_e$$

- Where W_{eng} is the engine weight and ΔW_e is calculated from a statistical graph. All weights are in Kg.
- The fuel weight is also calculated from a statistical graph

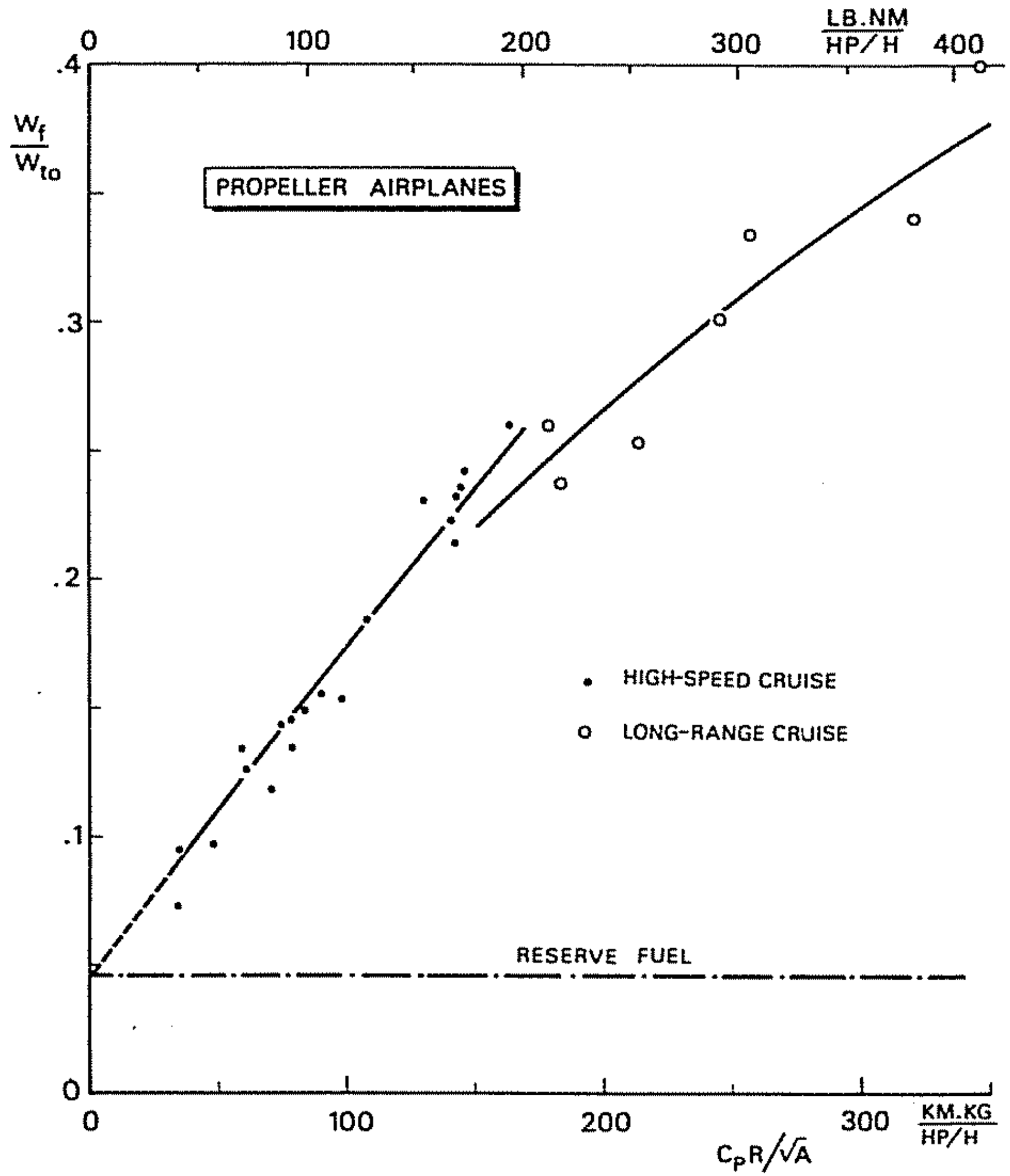


ΔW_e calculation

l_f = length of fuselage
b_f = width of fuselage
h_f = height of fuselage

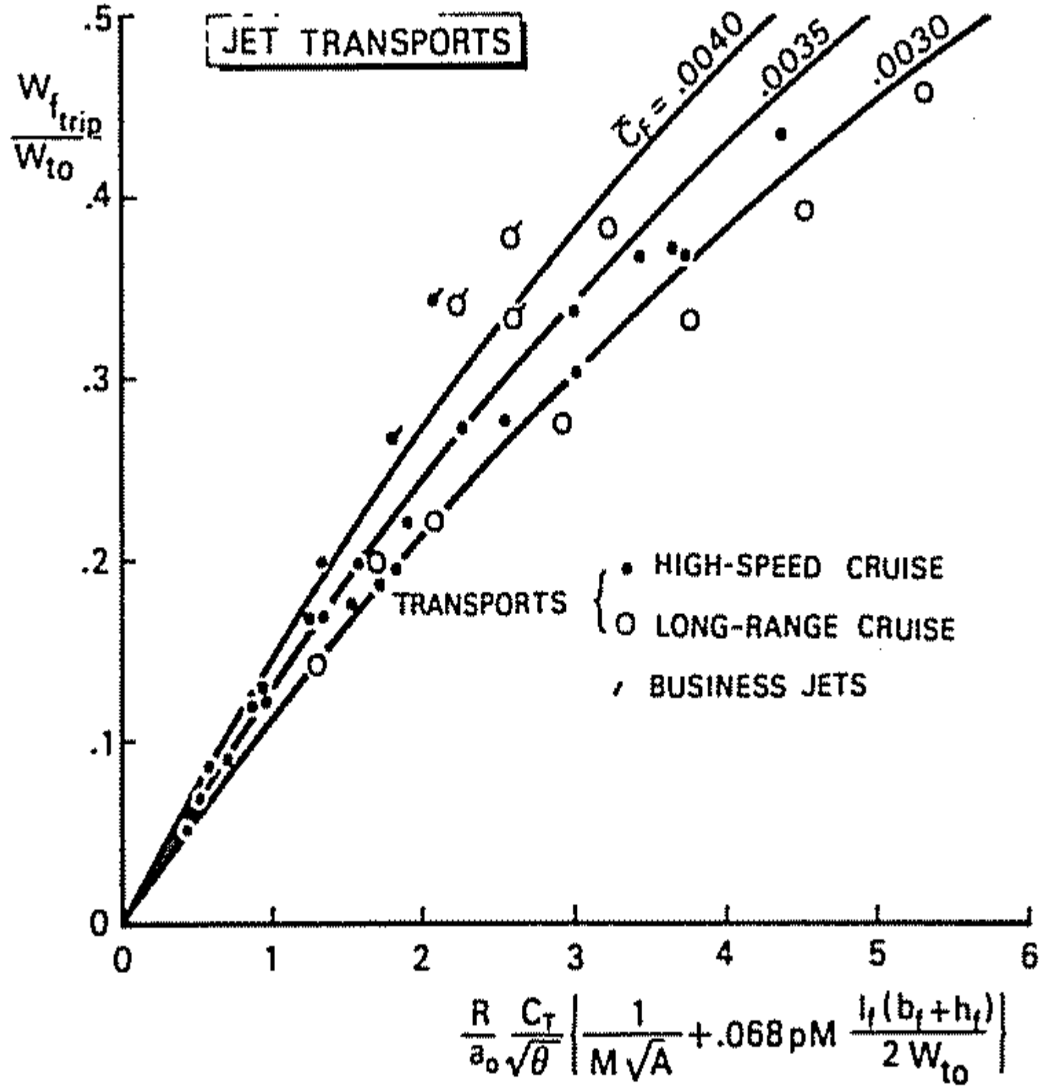
Use metric data

Fuel weight - Turboprops



C_p = Specific fuel consumption for propeller aircraft

Use metric data



Fuel weight - Turbojets

p : atmospheric pressure at cruise conditions

M : Mach number at cruise conditions

$\theta = T/T_0$, cruise temperature/ standard temperature

$C_T/\sqrt{\theta}$: Corrected specific fuel consumption at cruise conditions

a_0 : Speed of sound at sea level, International Standard Atmosphere

$\bar{C}_F$: Mean skin friction coefficient based on wetted area.

$$\bar{C}_F = \begin{cases} 0.003 & \text{- for large, long-range transports} \\ 0.0035 & \text{- for small, short-range transports} \\ 0.004 & \text{- for business and executive jets} \end{cases}$$

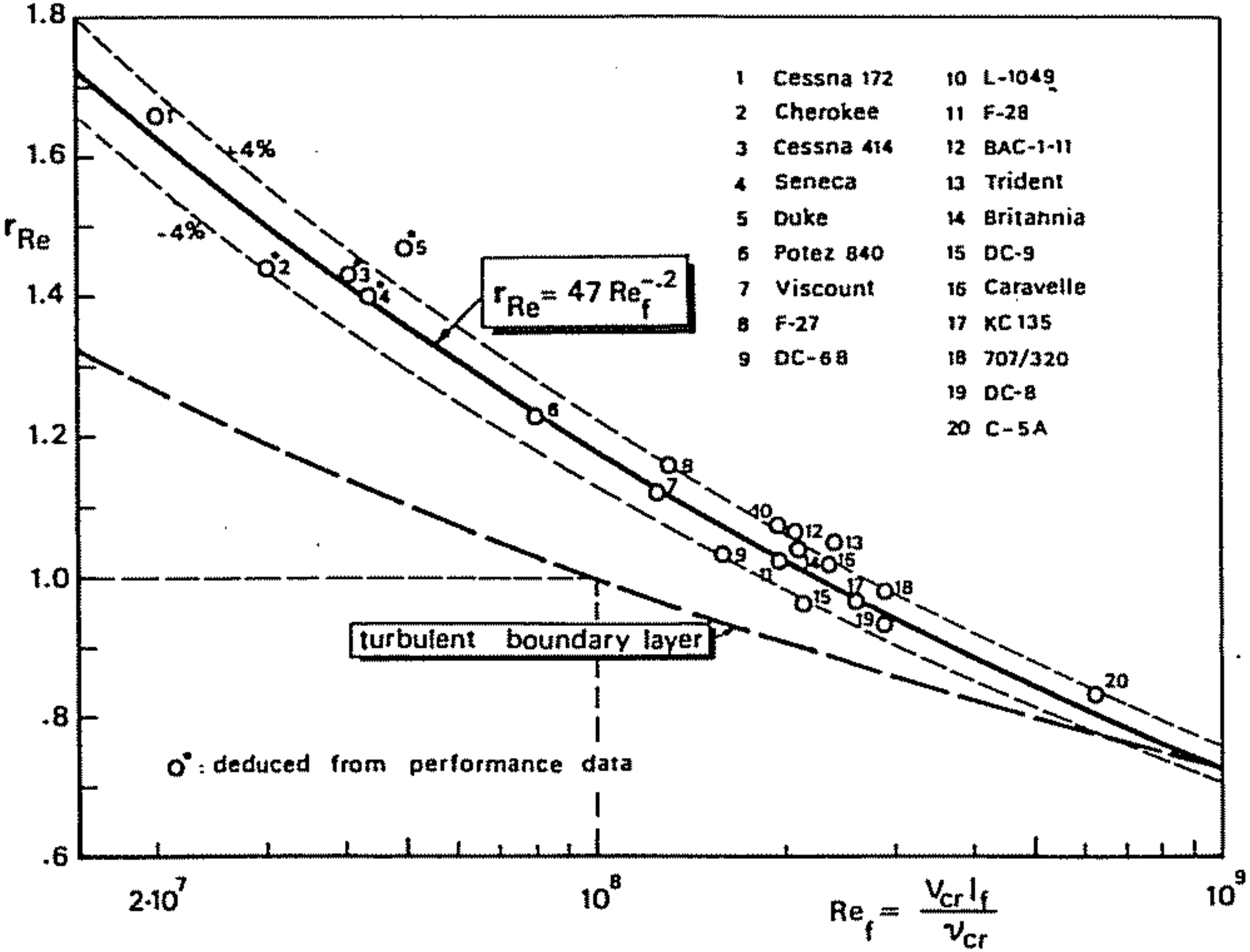
Skin friction coefficient

- An estimated of the drag force due to air friction over the full surface of the aircraft (wetted area).
- It can be estimated using Prandtl-Schlichting theory as

$$C_F = \frac{0.455}{\left(\log_{10}\left(Re_{cr}\right)\right)^{2.58}}$$

- Where Re_{cr} is the Reynolds number based on cruise flight conditions and the fuselage length

More skin friction



Skin friction coefficients for several aircraft types.

$$r_{Re} = C_F(\mathrm{Re})/C_F\left(10^8\right)$$

Caravelle type

Boeing 707 type

Boeing 727 type

Avro RJ85/100 (Bae 146)

Antonov An-72/74

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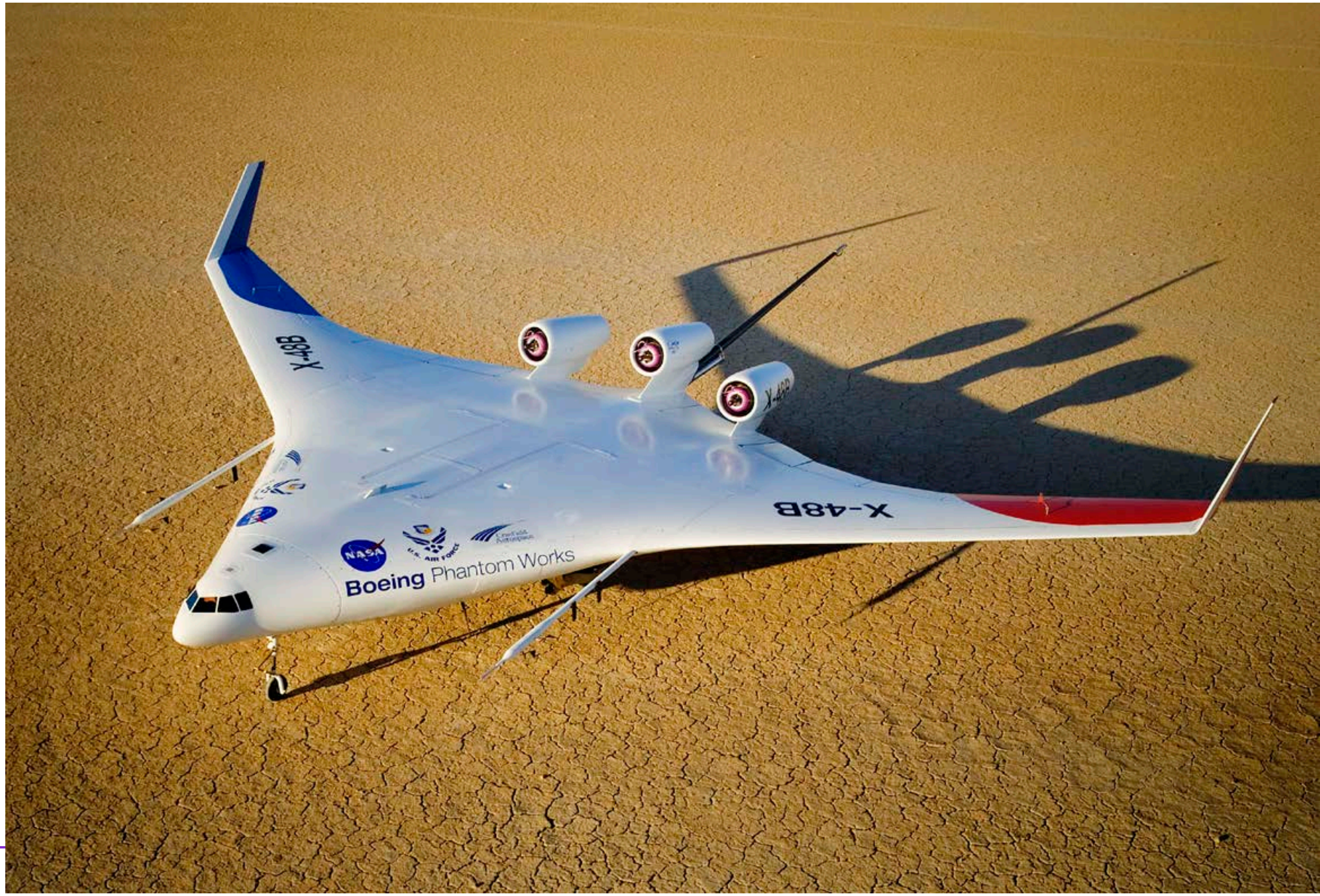
Turboprops

Blended Wing Body



Box wing

Circular wing

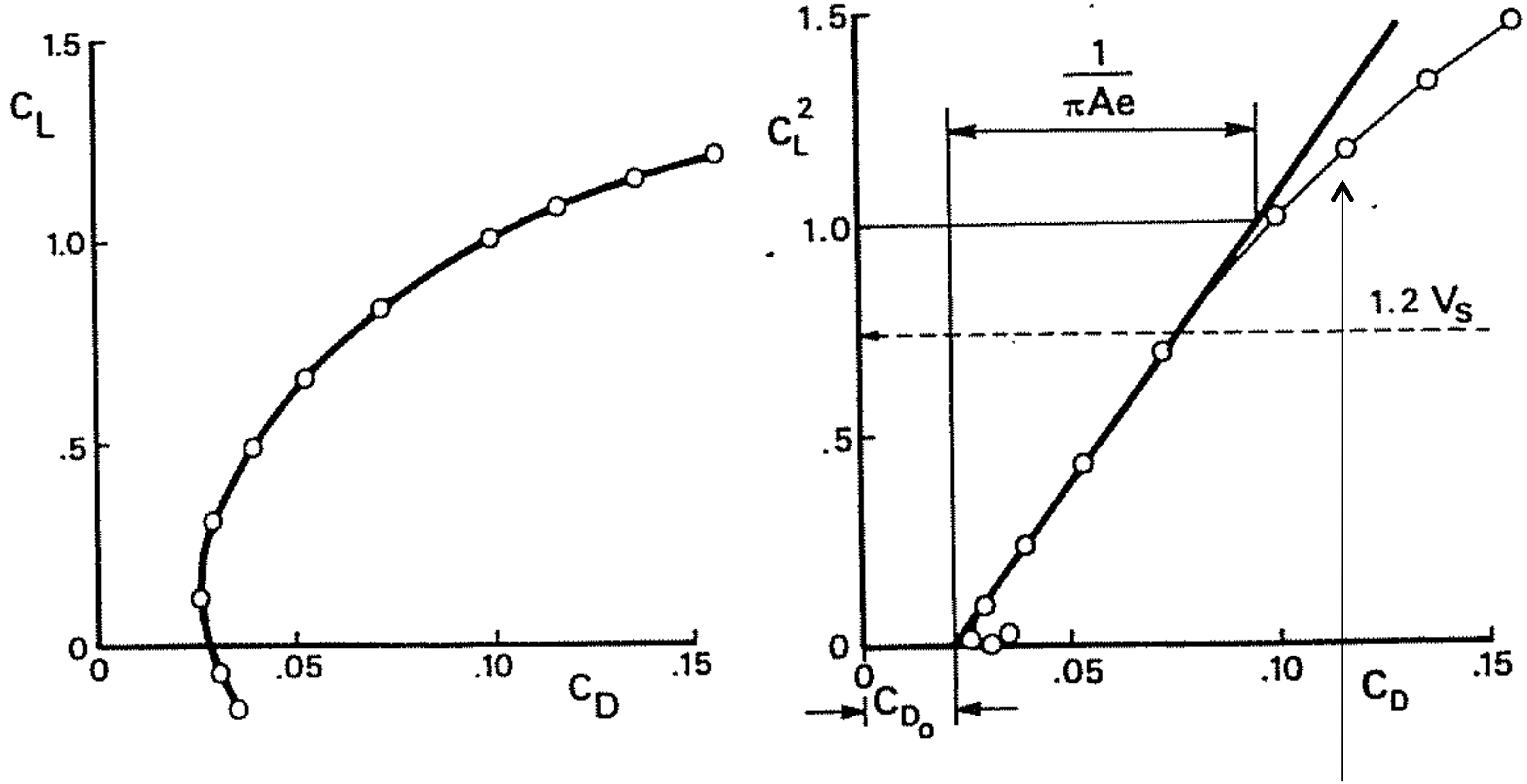
Drag Calculations

- There are many sources of aircraft drag
- They are usually summarized by the airplane drag polar:

$$C_D = C_{D_0} + \frac{C_L^2}{e\pi AR}$$

- Where C_{D0} is the drag that is independent of lift and e is the Oswald efficiency factor.

The drag polar



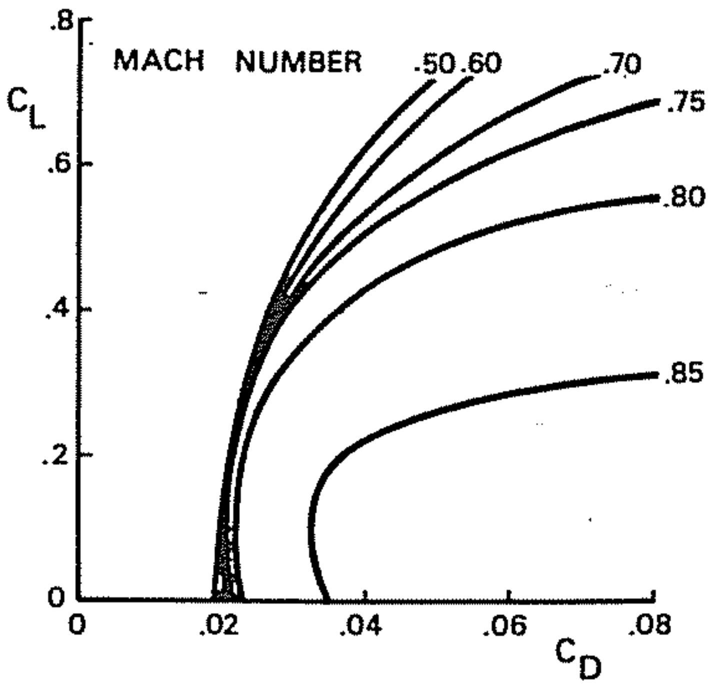
At high values of C_L the wing stalls

Drag figures for different aircraft

Aircraft Type	C_{D0}	e
High-subsonic jet	0.014-0.020	0.75-0.85
Large turboprop	0.018-0.024	0.80-0.85
Twin-engine piston aircraft	0.022-0.028	0.75-0.80
Single-engine piston aircraft with fixed gear	0.020-0.030	0.75-0.80
Single-engine piston aircraft with retractable gear	0.025-0.040	0.65-0.75
Agricultural aircraft without spray system	0.060	0.65-0.75
Agricultural aircraft with spray system	0.070-0.080	0.65-0.75

Compressibility drag

- Compressibility effects increase drag



A simple way of including compressibility effects in preliminary drag calculations is to add ΔC_D to C_{D0}, where ΔC_D=0.0005 for long-range cruise conditions ΔC_D=0.0020 for high-speed cruise conditions

Cruise

High speed performance

- At cruise, the flight speed is constant. Therefore, $T=D$. This can be written as

$$T = D = \frac{1}{2}\rho V^2 C_D S$$

- Where ρ is the cruise air density, V is the cruise airspeed and S is the wing area.
- The drag coefficient is obtained from the drag polar, using the fact that $L=W$, i.e.

$$W = L = \frac{1}{2}\rho V^2 C_L S$$

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$$

Thrust to weight

- The thrust to weight ratio is then

$$\frac{T}{W} = \frac{1}{2W}\rho V^2\left(C_{D_0} + \frac{C_L^2}{e\pi AR}\right)S = \frac{\rho V^2 C_{D_0}}{2W/S} + \frac{2W}{e\pi AR\rho V^2 S}$$

- The thrust here is the installed thrust, which is 4-8% lower than the un-installed thrust
- This equation can be used to choose an engine for the cruise condition

Minimum Thrust

- The thrust-to-weight ratio can be minimized as a function of wing loading W/S.
- Minimum thrust is required when

$$\frac{W}{S} = \frac{1}{2}\rho V^2 \sqrt{d_1 e \pi AR}$$

- Where d_1=0.008-0.010 for aircraft with retractable undercarriage. Therefore

$$\left(\frac{T}{W}\right)_{\min} = \frac{C_{D_0} + \sqrt{d_1}}{\sqrt{d_1 e \pi AR}}$$

Engine thrust

- The thrust of an engine at the cruise condition can be determined from:
  - Manufacturer's data
  - Approximate relationship to the take off thrust:

$$\frac{T}{T_{\mathrm{to}}} = 1 - \frac{0.454(1+\lambda)}{\sqrt{1+0.75\lambda}} M + \left(0.6 + \frac{0.13\lambda}{G}\right) M^2$$

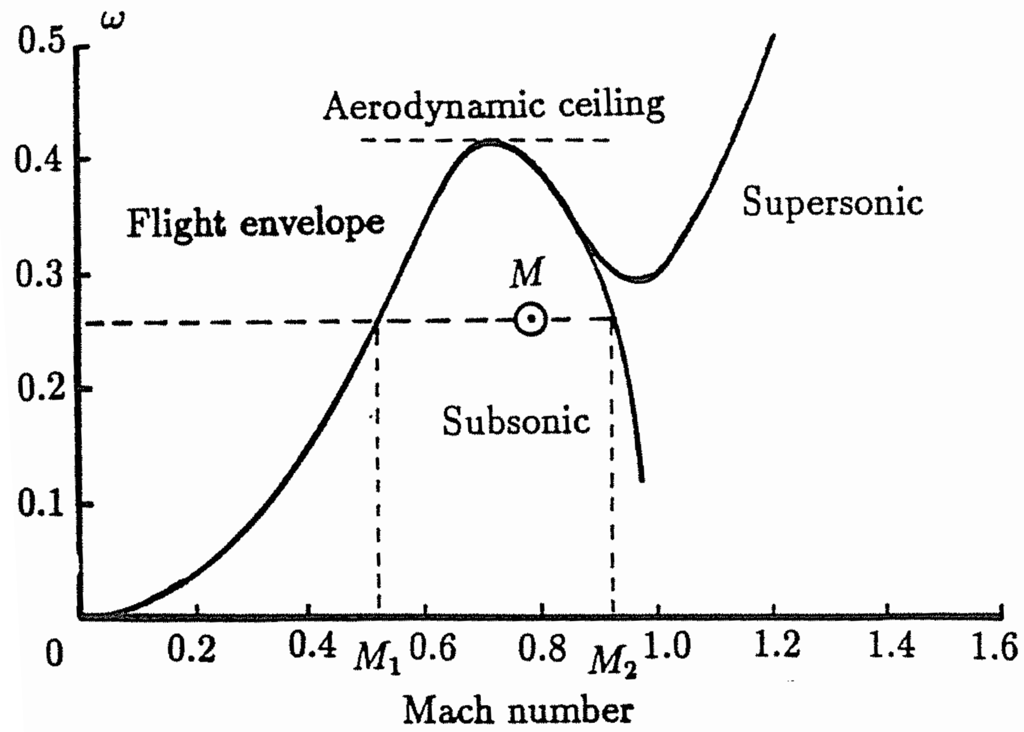
where λ is the bypass ratio, M is the cruise Mach number and G=0.9 for low bypass engines and G=1.1 for high bypass

Wing loading and Mach number

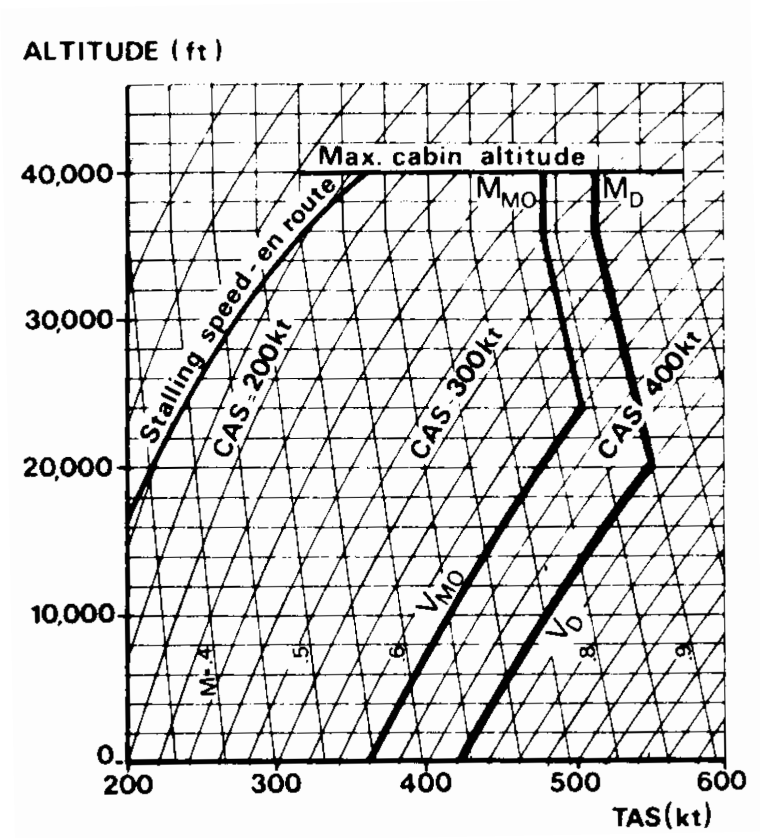
The maximum wing loading depends uniquely on $C_{L\max}$.

For an airliner, $C_{L\max}$ drops to neary zero as M approaches 1.

For supersonic aircraft, $C_{L\max}$ drops in the transonic region but not too much. It then assumes a near constant value for moderate supersonic speeds



The flight envelope



M_{MO}, V_{MO}=maximum operating Mach number, airspeed

M_C, V_C=design cruising Mach number, airspeed

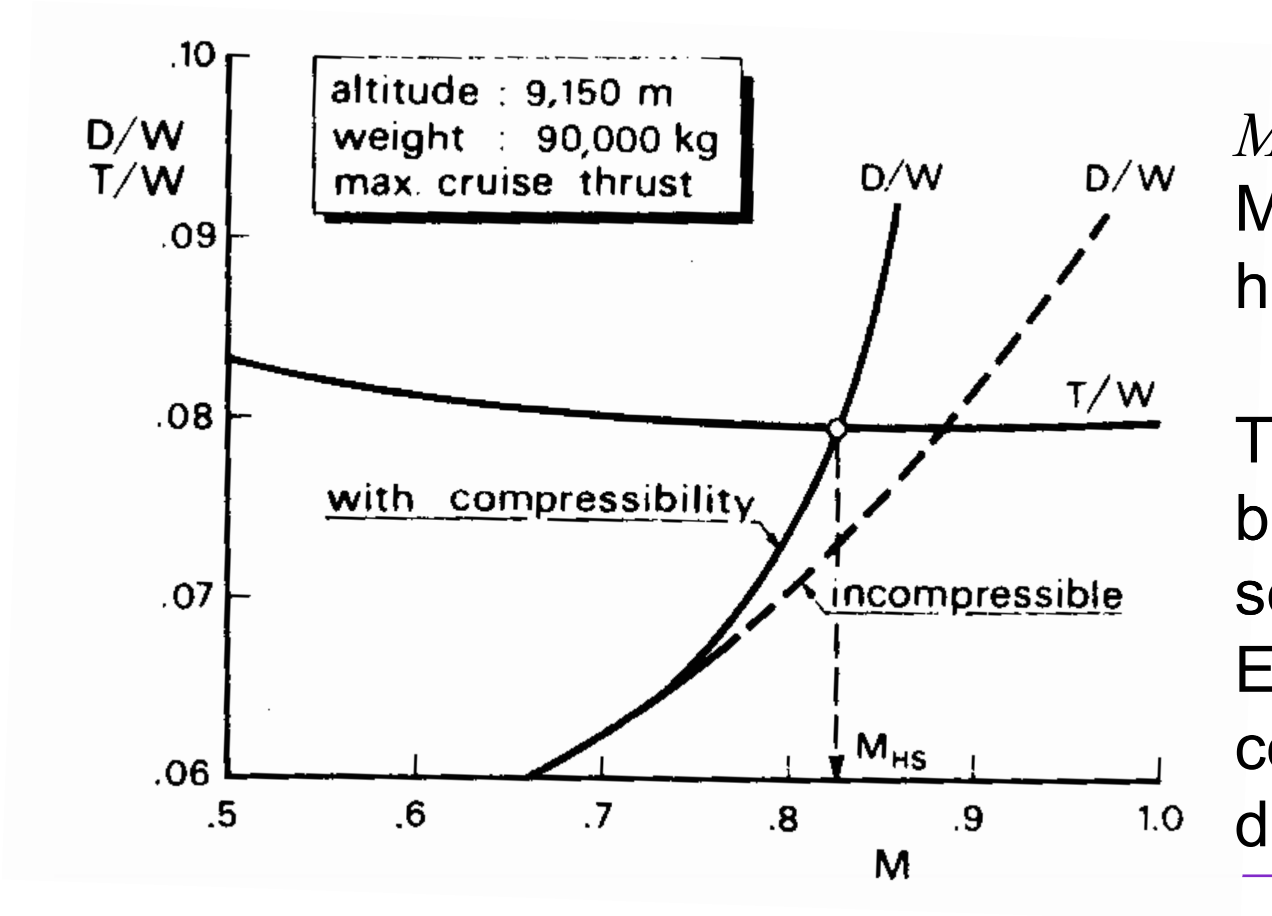
M_D,V_D=design diving Mach number, airspeed

V_S=stalling airspeed

CAS=calibrated airspeed (airspeed displayed on flight instruments)

TAS=true airspeed

Determination of cruise Mach number



M_{HS}=high-speed Mach number (for high-speed cruise)

This calculation must be repeated at several altitudes. Each altitude corresponds to a different cruise Mach.

Range performance

- The range of an aircraft can be estimated from the Bréguet range equation:

$$R = \frac{V}{C_T}\frac{L}{D}\ln\left(\frac{W_{\mathrm{i}}}{W_{\mathrm{i}} - W_{\mathrm{f}}}\right)$$

- This equation is applicable to cruise conditions only, i.e. L/D is the cruise lift-to-drag ratio, V is the cruise airspeed, W_{i} is the weight of the aircraft at the start of cruise and W_{f} is the cruise fuel weight

Maximizing Range

- The range equation can also be written as

$$\frac{R}{a_0} = \frac{ML/D}{C_T/\sqrt{\theta}} \ln\left(\frac{W_i}{W_i - W_f}\right)$$

- Where M is the cruise Mach number and a_0 is the speed of sound at sea level.
- The range can be maximized either by maximizing L/D or by maximizing ML/D.

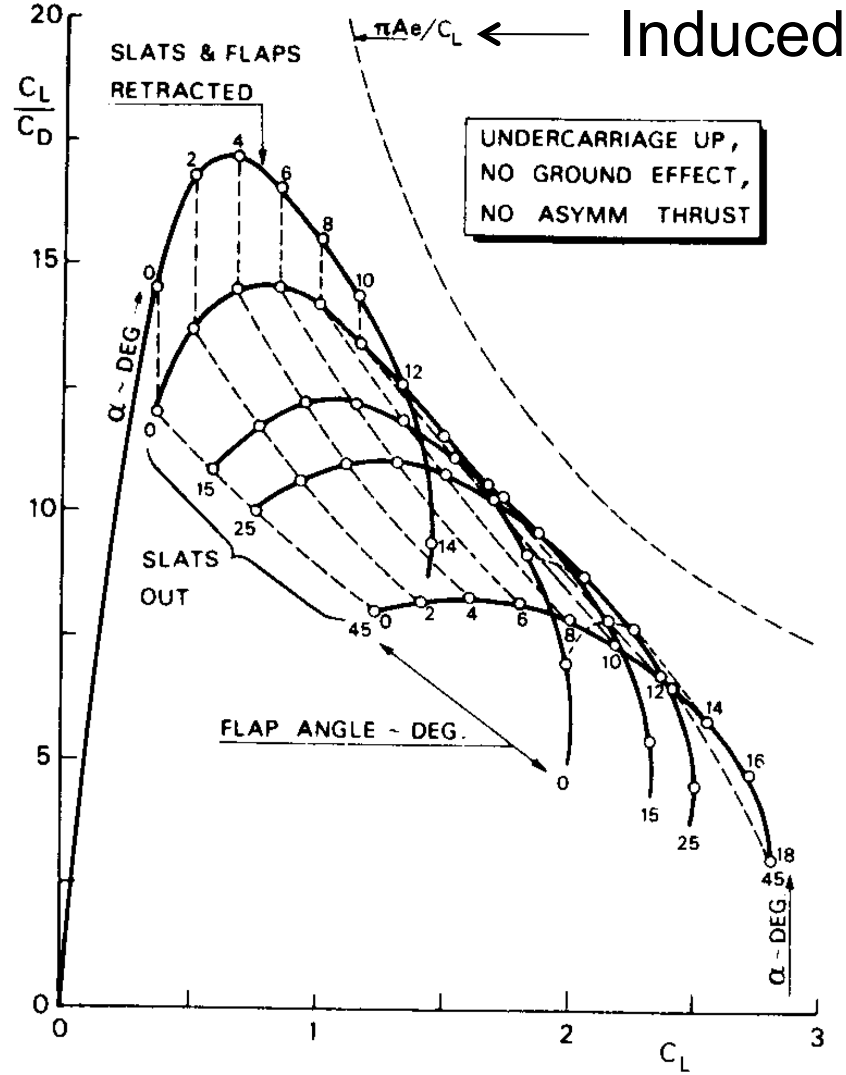
To maximize L/D:

$$C_L = \sqrt{C_{D_0} e\pi AR}$$

To maximize ML/D:

$$C_L = \sqrt{\frac{1}{3} C_{D_0} e\pi AR}$$

Lift to drag ratio



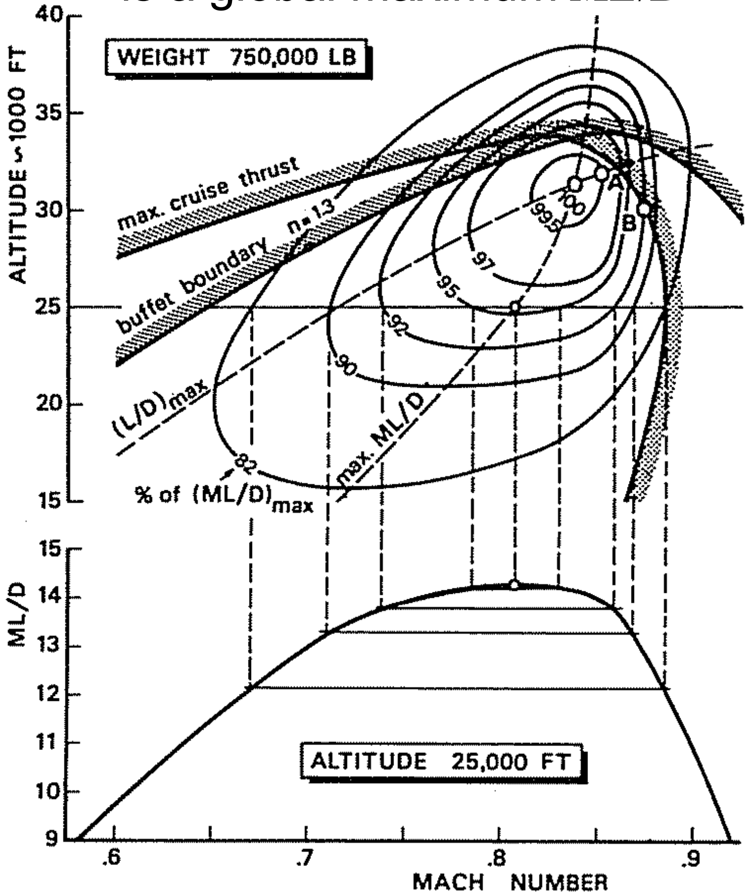
Induced drag only

Example of lift to drag ratio variation with lift, angle of attack and the deployment of slats and flaps

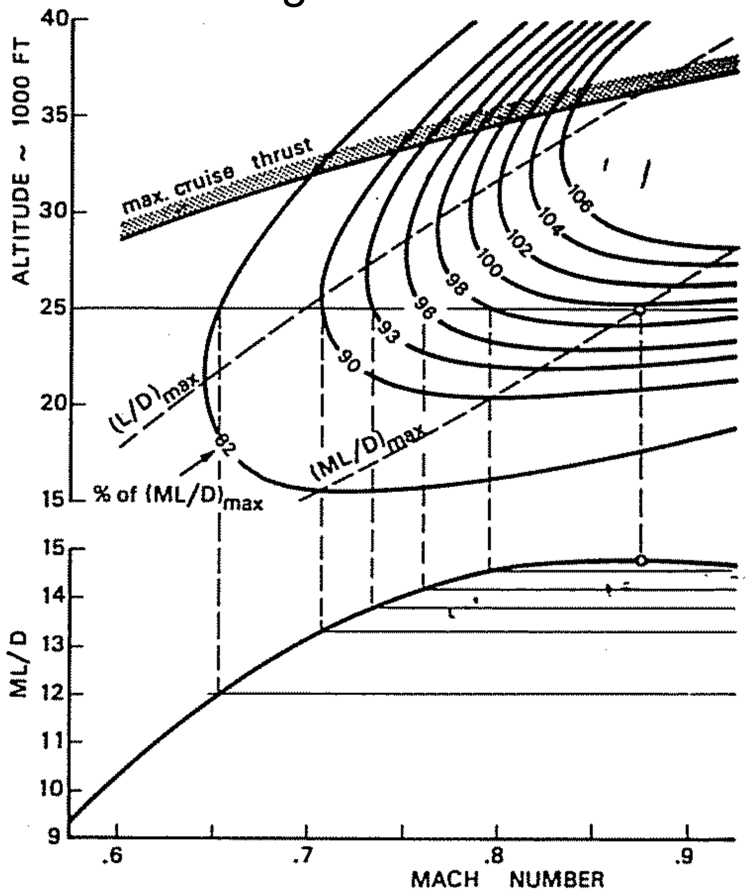
The best L/D is obtained for the clean configuration

Compressibility effect on range

With compressibility – there is a global maximum ML/D



Without compressibility – there is no global maximum



Range design

- The designer's problem is to choose a favorable combination of:
  - Speed
  - Altitude
  - Airplane geometry
  - Engine
- in order to achieve the best range performance or fuel efficiency

Some considerations

- For long-haul aircraft the most important consideration is fuel efficiency
- For short-haul aircraft the most important consideration is engine weight
- There are some complications though:
  - Cruise fuel is only part of the fuel weight
  - For short haul aircraft the engine thrust is frequently determined by take off field length
  - Air traffic controllers decide the allowable cruise altitudes
  - An aircraft can have more than one engine

Reserve Fuel

- One definition of reserve fuel for international airline operations (ATA 67):
- The airliner must carry enough reserve fuel to:
  - Continue flight for time equal to 10% of basic flight time at normal cruise conditions
  - Execute missed approach and climbout at destination airport
  - Fly to alternate airport 370km distant
  - Hold at alternate airport for 30 minutes at 457m above the ground
  - Descend and land at alternate airport
- One approximate calculation:

$$W_{f_{res}} / W_{to} = 0.18 C_T / \sqrt{\theta AR}$$

Range for propeller aircraft

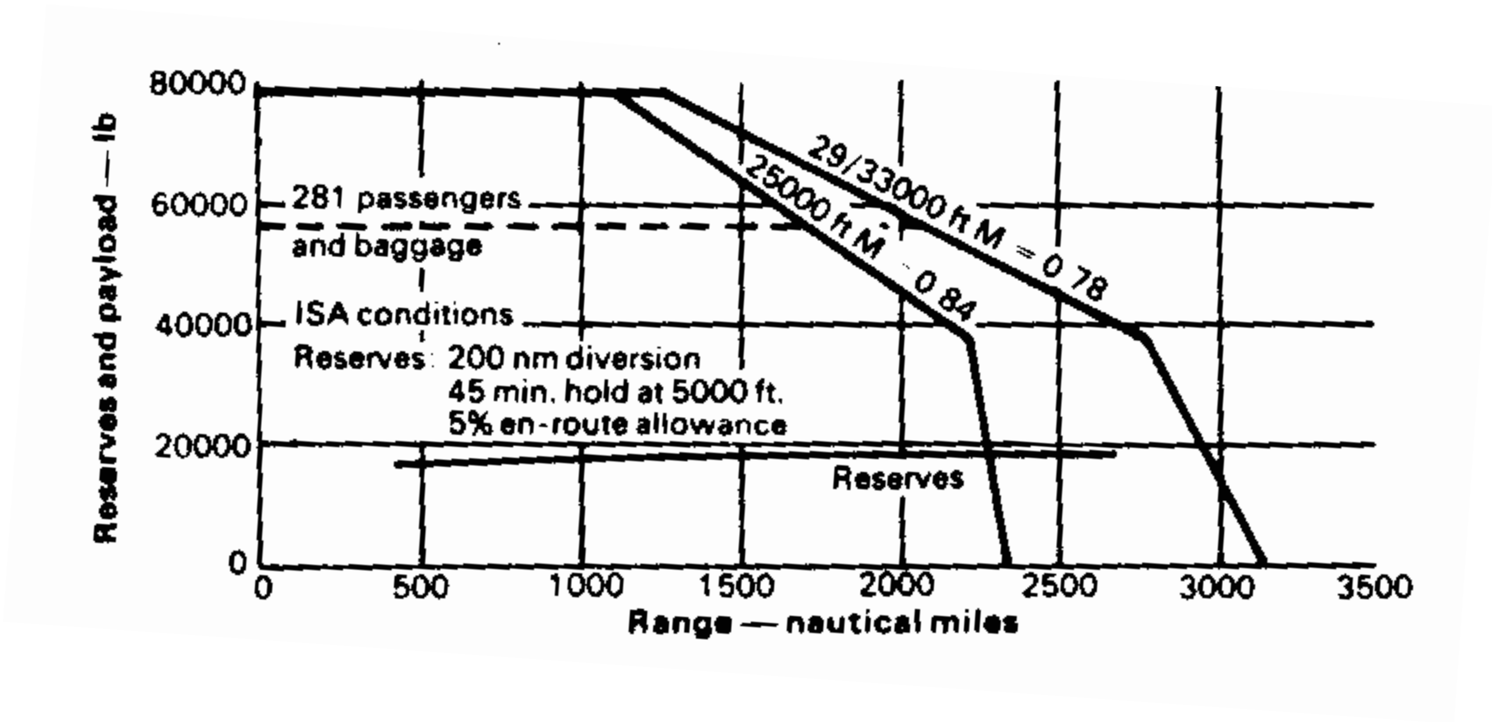
- The Bréguet range equation for propeller aircraft is

$$R = \frac{\eta_p}{C_P}\frac{L}{D}\ln\left(\frac{W_i}{W_i - W_f}\right)$$

- Where η_p is the propeller efficiency and C_p is the specific fuel consumption.
- The range can be maximized by:
  – Minimizing airplane drag
  – Minimizing engine power

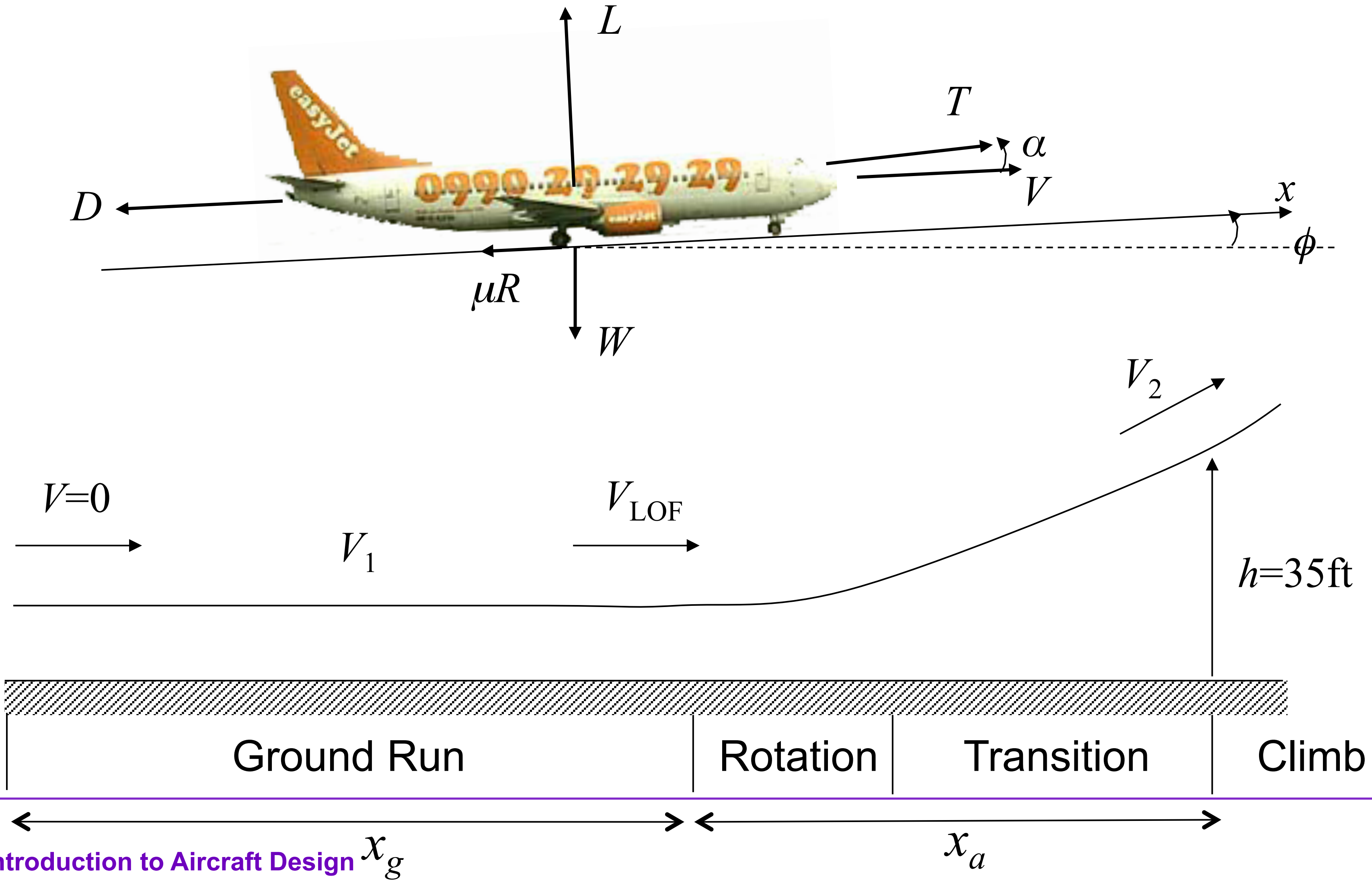
Payload-range diagram

Two cases: high-speed cruise, long-range cruise. Example: A-300B



Take off

Take Off Performance



Take Off Description

- Take off starts at time t_0, with airspeed V_0 and the runway may have an angle to horizontal of ϕ.
- Lift off occurs at time t_g, after a distance of x_g, usually at speed V_{LOF}.
- Take off is completed when the aircraft has reached sufficient height to clear an obstacle 35ft high (50ft for military aircraft)
- Finally, the climb out phase takes the aircraft to 500ft at the climb throttle setting.

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Ground Run

- The ground run can either start at $V_0=0$.
- The angle of attack is defined with respect to the thrust line.
  - For aircraft with a tricycle landing gear the angle of attack is nearly zero. At a sufficient speed the nose is lifted up a little bit to achieve an optimum angle of attack.
  - For aircraft with a tailwheel the angle of attack is very high. At an airspeed where the control surfaces are effective the angle of attack is decreased to decrease the drag and increase acceleration.

Lift off

- In principle an aircraft can lift off once it has reached its stall speed, V_{stall}.
- At this airspeed it can increase its angle of attack α_{max}, with a resulting $C_{L\mathrm{max}}$. This lift coefficient will generally satisfy $L>W$.
- However, for safety reasons, the lift off speed is defined as $V_{\mathrm{LOF}}=k_1 V_{\mathrm{stall}}$ where
  - $k_1=1.10$ is an indicative value
  - In general k_1 varies with type of aircraft

Rotation and Transition

- After lift off the aircraft rotates to deflect the velocity from nearly horizontal to a few degrees upward.
- The rotation lasts usually for a couple of seconds.
- The transition stage follows, at the end of which the aircraft is required to clear a hypothetical obstacle.
- The speed at the clearance of the obstacle is usually $V_2 = k_2 V_{stall}$ where $k_2 = 1.2$.

Take off details

- V_1 is called the decision speed for multi-engined aircraft.
  - Below this speed, if one engine fails, the take off is aborted.
  - Above this speed, if one engine fails, the take off is continued.
- Take off field length: Distance from rest to obstacle clearance
- The climb stage follows the transition stage.
- The take off phase is usually considered to be complete at around 500ft once the flaps have been retracted.

Equations of motion

- The total force applied to the aircraft parallel to the runway is:

$$F = T - \mu R - D = ma$$

- The acceleration is equal to $a = \mathrm{d}V/\mathrm{d}t$.
- The velocity is equal to $V = \mathrm{d}x/\mathrm{d}t$.
- Then

$$\frac{V}{a} = \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}V} = \frac{\mathrm{d}x}{\mathrm{d}V} \quad \text{or} \quad \frac{\mathrm{d}x}{\mathrm{d}V} = \frac{V}{a}$$

- To obtain x, we need to integrate with respect to V:

$$x_g = \int_0^{V_{\mathrm{LOF}}} \frac{V}{a}\mathrm{d}V$$

Friction force

- In order to have a friction force, it must be that $R>0$, i.e.

$$R = W - L = W - \frac{1}{2}\rho V^2 S C_L$$

- Where it is assumed that the runway's inclination is small. The total force can be written as

$$ma = T - \mu W - \frac{1}{2}\rho V^2 S\left(C_D - \mu C_L\right)$$

Friction Coefficients

Runway type	μ
Concrete, asphalt	0.02
Hard turf	0.04
Field with short grass	0.05
Field with long grass	0.10
Soft field, sand	0.10-0.30

Approximate solution (1)

- An approximate solution for the ground run can be obtained as

$$x_g \approx \frac{V_{\text{LOF}}^2 / 2g}{\dfrac{\overline{T}}{W_{\text{to}}} - \mu'}$$

where

$$\overline{T} = \text{thrust at } V_{\text{LOF}} / \sqrt{2} \approx 0.75 \frac{5+\lambda}{4+\lambda} T_{\text{to}}$$

$$C_L = \mu e \pi AR$$

$$\mu' = \mu + 0.72 \frac{C_{D_0}}{C_{L_{\max}}}$$

Approximate solution (2)

- An approximate solution for the air run can be obtained as

$$x_a \approx \frac{V_{\mathrm{LOF}}^2}{g\sqrt{2}} + \frac{h}{\gamma_{\mathrm{LOF}}} \quad \text{where}$$

$$\gamma_{\mathrm{LOF}} = \left(\frac{T-D}{W}\right)_{\mathrm{LOF}} \approx 0.9\frac{\bar{T}}{W_{\mathrm{to}}} - \frac{0.3}{\sqrt{AR}}$$

- The airspeed at the takeoff obstacle is

$$V_2 = V_{\mathrm{LOF}}\sqrt{1+\gamma_{\mathrm{LOF}}\sqrt{2}}$$

Discussion

- The ground run distance increases with
  - Aircraft weight
  - Altitude
  - Temperature
  - Rolling friction coefficient, positive runway slope
- And it decreases with
  - Thrust
  - High lift devices
  - Additional thrust, e.g. booster jets

Climb

Climb performance

- Operational requirements, e.g.
  - Rate of climb at sea level
  - Service ceiling altitude for a maximum rate of climb of 0.5m/s
- Airworthiness requirements
  - Minimum climb gradient in takeoff, en route, landing
  - Rate if climb at a specified altitude with one engine inoperative

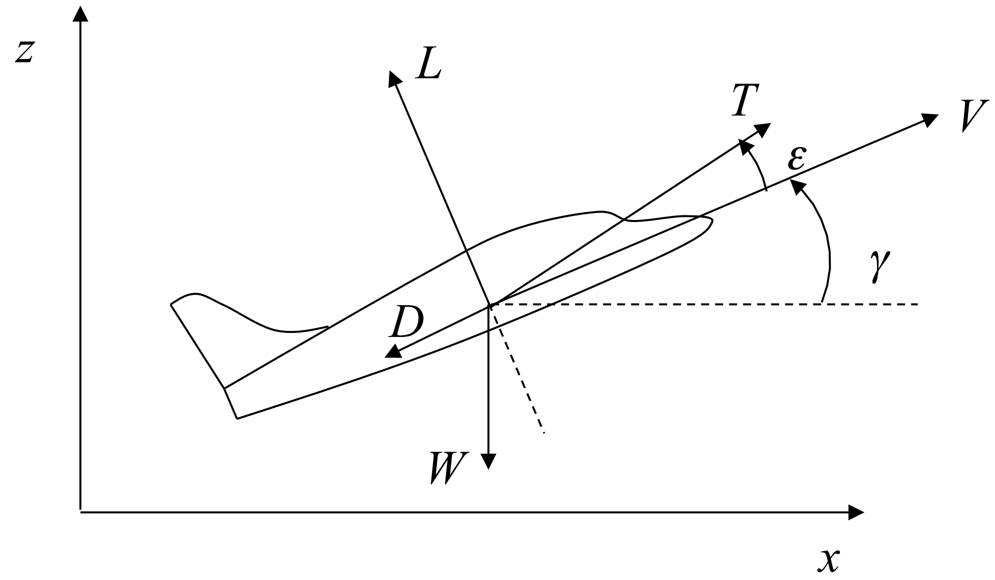
Climb description

- Climb follows immediately after take off and its objective is to bring the aircraft to cruising height
- In general, climb is performed in a vertical plane, i.e. no turning during climb.
- Climb takes up a relatively short percentage of flight time so it can be assumed that the aircraft's weight remains constant
- Variations in speed and flight path are considered small, i.e. constant speed climb

Climb diagram

Notice that the thrust line is not necessarily parallel to the flight path

Also note that, for constant speed V, the aircraft cannot be accelerating



Thrust and Lift equations

- Assume that $\varepsilon=0$. Then, write the equilibrium equations on the body axes:

$$T = \frac{1}{2}\rho S V^2 C_D + W \sin\gamma$$

$$W \cos\gamma = \frac{1}{2}\rho S V^2 C_L$$

- As mentioned before, the drag can be expressed in terms of lift using the drag polar

Thrust and Power

- It is customary to express the thrust equation in terms of power by multiplying it by V:

$$TV = \frac{1}{2}\rho S V^3 C_D + WV \sin\gamma$$

- Define

$$P_a = TV,\ P_r = \frac{1}{2}\rho S V^3 C_D,\ V_z = V \sin\gamma$$

- Where P_a is the power available for climb, P_r is the power required for level flight at the same airspeed and V_z is the rate of climb.
- Then $V_z = \dfrac{P_a - P_r}{W}$ and $\sin\gamma = \dfrac{P_a - P_r}{VW}$

Climb of a jet aircraft

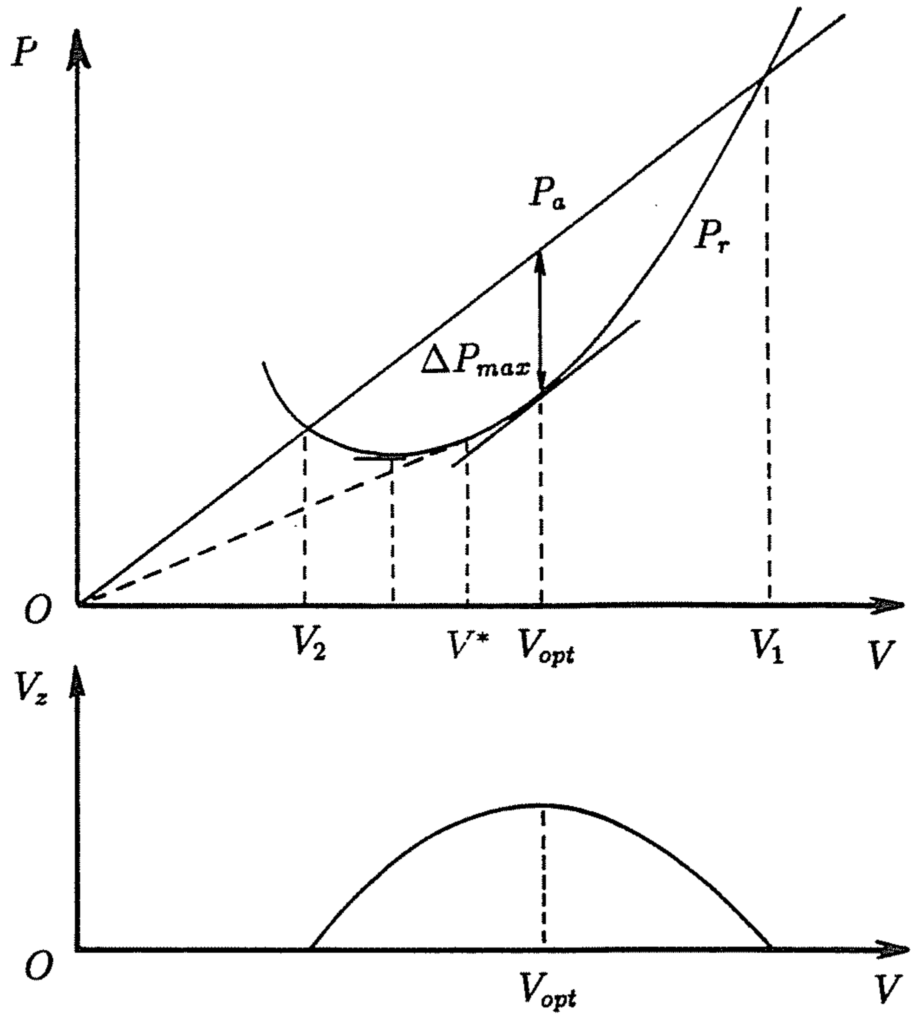
- For jet aircraft the amount of power available for climb varies linearly with airspeed.
- The power required for level flight varies non-linearly with airspeed.
- There is an optimum airspeed for maximum rate of climb which maximizes P_a-P_r.
- This airspeed is not necessarily at the airspeed yielding the minimum value of P_r.

Optimum airspeed

The optimum climb rate is obtained when the power difference ΔP is maximized.

It occurs at the point where the tangent to curve P_r is parallel to P_a.

At airspeeds below V_2 and above V_1 the aircraft cannot climb; the power required for level flight is higher than the power available for climb.



Rate of climb

- The equation for the rate of climb can be written as

$$V_z = \frac{P_a - P_r}{W} = \frac{1}{W}\left(TV - \frac{1}{2}\rho S V^3 C_{D_0} - \frac{2KW^2}{\rho S V}\right)$$

- Where a parabolic drag polar was substituted for C_D and the rate of climb was assumed small.
- The maximum climb rate is obtained when $\partial V_z/\partial \mathrm{d}V=0$, which gives:

$$\frac{3\rho S C_{D_0}}{W}V^4 - \frac{2T}{W}V^2 - \frac{4KW}{\rho S} = 0$$

Maximum climb rate

- Solving the quartic equatio we get

$$\frac{V_{z_{\max}}}{V} = \frac{1}{3E_{\max}}\left(\tau + \sqrt{\tau^2 + 3}\right) - \frac{3}{E_{\max}\left(\tau + \sqrt{\tau^2 + 3}\right)}$$

- Where $\tau = E_{\max} T/W$.
- Therefore, the maximum climb rate depends on:
  - Thrust available
  - Weight
  - Altitude
  - Wing surface

Climb rate requirements

PHASE OF FLIGHT		AIRPLANE CONFIGURATION: flap setting	u.c.	engine	thrust (power)	speed	altitude	MINIMUM CLIMB GRADIENT %: $N_e=2$	$N_e=3$	$N_e=4$
TAKEOFF CLIMB POTENTIAL ("first segment")		t.o.	↓	one engine out	t.o.	V_{LOF}	$0 \rightarrow h_{uu}$ 1)	0	.3	.5
TAKEOFF FLIGHT PATH	"second segment"	t.o.	↑	one engine out	t.o.	V_2 2)	$h_{uu} \rightarrow 400$ ft	2.4	2.7	3.0
TAKEOFF FLIGHT PATH	final takeoff ("third segment")	en route	↑	one engine out	max. cont.	$V \geq 1.25 V_S$	$400 \rightarrow 1{,}500$ ft	1.2	1.5	1.7
APPROACH CLIMB POTENTIAL		approach 3)	↑	one engine out	t.o.	$V \leq 1.5 V_S$	0 1)	2.1	2.4	2.7
LANDING CLIMB POTENTIAL		landing	↓	all engines	takeoff 4)	$V \leq 1.3 V_S$	0 1)	3.2	3.2	3.2

Nomenclature:

- V_{LOF} - liftoff speed
- V_2 - takeoff safety speed
- V_R - rotation speed
- V_S - stalling speed
- u.c. - undercarriage position
- h_{uu} - height at which u.c. retraction is completed
- N_e - number of engines per a/c

1) out of ground effect
2) defined in Section 2 of Appendix K
3) flap setting such that $V_S \leq 1.10\ V_S$ for landing
4) more precisely: the engine power (thrust) available 8 seconds after throttle opening to takeoff rating
5) takeoff requirements are at actual weight, other requirements at landing (touchdown) weight

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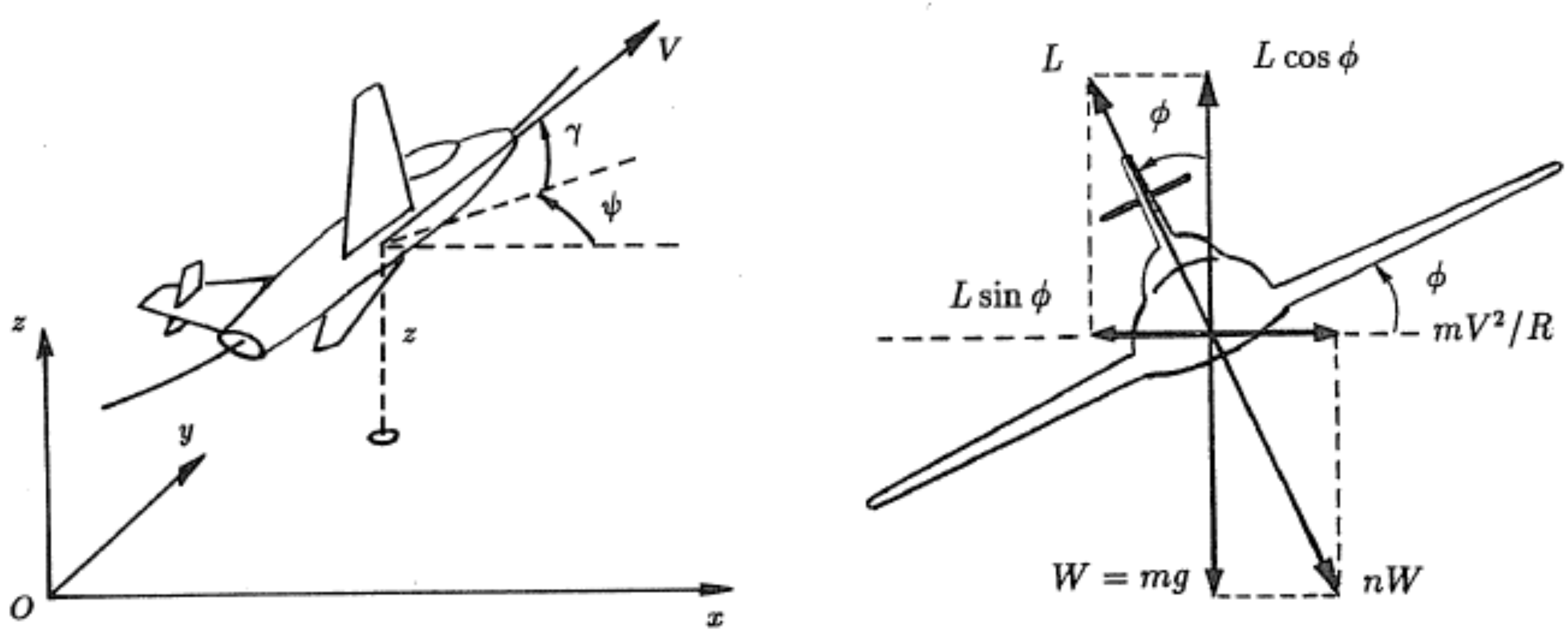
Turning

Turning

- The most general turn is a turn accompanied by a change in height.
- All the turn angles of the aircraft are involved, roll, pitch and yaw
- In this lecture we will only cover turns in a horizontal plane, i.e. no change of height

Turn diagrams



Constant speed horizontal turn

- For a horizontal turn, $\gamma=0$.
- For constant speed, $\mathrm{d}V/\mathrm{d}t=0$.
- Also assume that the thrust is aligned with the flight path.
- The equilibrium equations are simply:

$$L\cos\phi = W, \quad T = D$$

- For a perfectly circular turn of radius R, we add:

$$L\sin\phi = \frac{mV^2}{R}$$

Load factor

- The load factor is quite important for turns. If the load factor is too great the pilot can be harmed or the aircraft's structure can be damaged.
- The load factor is defined as $n=L/W$.
- From the equations of motion:

$$n = 1/\cos\phi$$

- During a turn, the lift must balance not only the weight but also the centrifugal force. This means that the loads acting on the aircraft are far more important.

Lift required for turning

- From the turning diagram, it is seen that:

$$nW = \sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2}$$

- Furthermore, $L=nW$.
- Hence, the required lift is simply:

$$nW = \frac{1}{2}\rho S V^2 C_L$$

Thrust required for turning

- The drag can be obtained, as usual, by a drag polar, i.e.

$$C_D = C_{D_0} + \frac{C_L^2}{e\pi AR} = C_{D_0} + \frac{1}{e\pi AR}\left(\frac{2nW}{\rho SV^2}\right)^2$$

- The required thrust is then given by

$$T = \frac{1}{2}\rho SV^2 C_D = \frac{1}{2}\rho SV^2\left(C_{D_0} + \frac{1}{e\pi AR}\left(\frac{2nW}{\rho SV^2}\right)^2\right)$$

Thrust required for turning (2)

- The thrust equations can be written directly as:

$$T = D = \frac{LD}{L} = nW\frac{C_D}{C_L}$$

- If it is assumed that the lift to drag ratio is constant, then the thrust required for turning is directly proportional to the load factor.

Maximum load factor

- Aircraft structures can only withstand a maximum load factor
- Furthermore, passengers can withstand very low load factors.
- It must be verified that the radius R will correspond to a load factor lower than n_{max}.

$$\frac{\sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2}}{W} < n_{max}$$

- For commercial transports n_{max} is usually 2.5.
- For aerobatic aircraft it can be 6 or higher.

Maximum turning rate

- The lift coefficient can never be higher than $C_{L\max}$.
- The load factor can never be higher than $n_{\max}$.
- Therefore the maximum turning rate is obtained by

$$\left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\right)_{\max} = g\sqrt{\frac{\rho S C_{L_{\max}}}{2W}\left(n_{\max} - \frac{1}{n_{\max}}\right)}$$

The turning radius

- For a given load factor n, the turning radius can be expressed as

$$R = \frac{V^2}{g \tan\phi} = \frac{V^2}{g\sqrt{n^2 - 1}}$$

- Expressing the airspeed in terms of the load factor and lift coefficient we get:

$$R = \left(\frac{2W}{g\rho S C_L}\right)\frac{n}{\sqrt{n^2 - 1}}$$

Landing

Landing

- The landing is performed in two phases:
  - Approach over a hypothetical obstacle to touch-down
  - Ground run to a full stop
- Before reaching the obstacle the aircraft makes an approach along the axis of the runway with a glide angle between -2.5 and -3.5 degrees.
- The approach speed is V_2, i.e. $1.2 V_{\text{stall}}$.

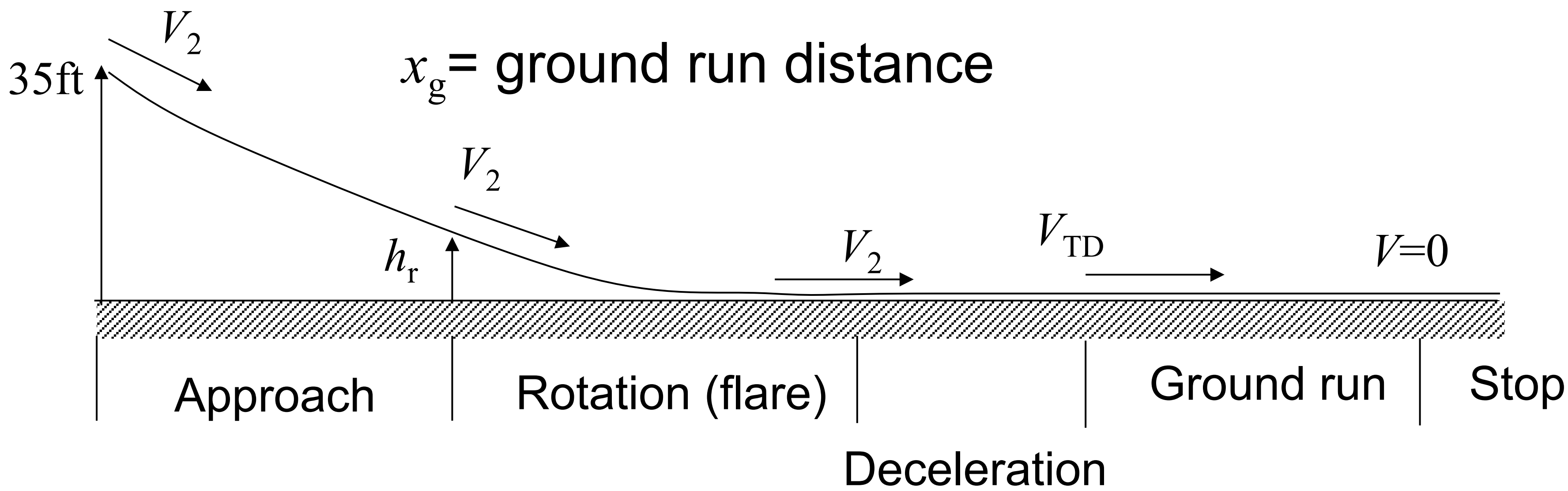
Landing diagram

x_3= approach distance

x_2= rotation distance

x_1= deceleration distance

x_g= ground run distance



Approach

- The aircraft is approaching the runway with a descent angle γ.
- If the height of the hypothetical object is h_{ob} and the rotation height is h_r, then the approach distance x_3 is given by

$$x_3 = \frac{h_{ob} - h_r}{\tan\gamma}$$

- And the approach time is

$$t_3 = \frac{x_3}{V_2 \cos\gamma}$$

Rotation

- Rotation is a circular arc with radius R.
- As in the take-off case, we have

$$R = \frac{V_2^2}{g(n-1)}$$

- Where n is the load factor.
- The distance traveled during rotation is

$$x_2 = R\sin\gamma$$

- And the time taken is

$$t_2 = \frac{\gamma V_2}{g(n-1)}$$

Ground run

- Once the aircraft has touched down the airspeed drops from V_{TD} to 0.
- The distance of the ground run can be approximated by $x_g = V_{TD}^2 / 2\bar{a}$
- Where $\bar{a}$ is the mean deceleration:

$$\bar{a} = \begin{cases} 0.30 - 0.35 \text{ for light aircraft with simple brakes} \\ 0.35 - 0.45 \text{ for turboprop aircraft without reverse propeller thrust} \\ 0.40 - 0.50 \text{ for jets with spoilers, anti-skid devices, speed brakes} \\ 0.50 - 0.60 \text{ as above, with nosewheel breaks} \end{cases}$$

Parametric design

- Design for performance is above all a process of optimization.
- The aim is to satisfy or exceed all performance requirements by finding the optimal combination of parameters
- The parameters are:
  - Poweplant: Takeoff thrust, number of engines, engine type of configuration
  - Wing: Wing area, Aspect Ratio, high-lift devices

Flow diagram