University of Liège
Aerospace & Mechanical Engineering

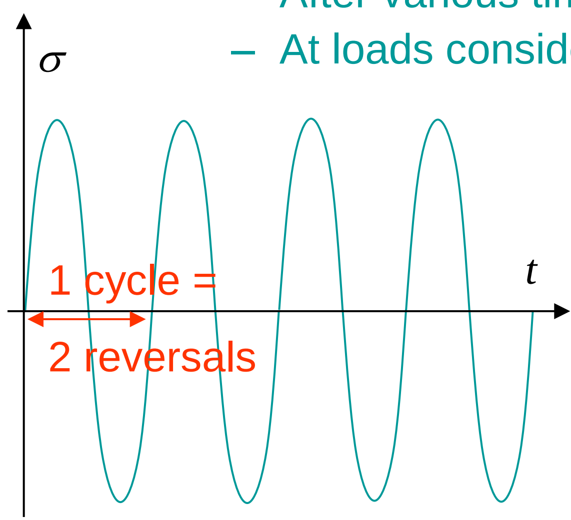
Aircraft Design
Introduction to Structure Life

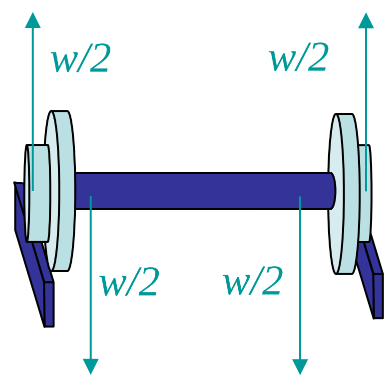
Ludovic Noels

Computational & Multiscale Mechanics of Materials – CM3
http://www.ltas-cm3.ulg.ac.be/
Chemin des Chevreuils 1, B4000 Liège
L.Noels@ulg.ac.be

Total life design

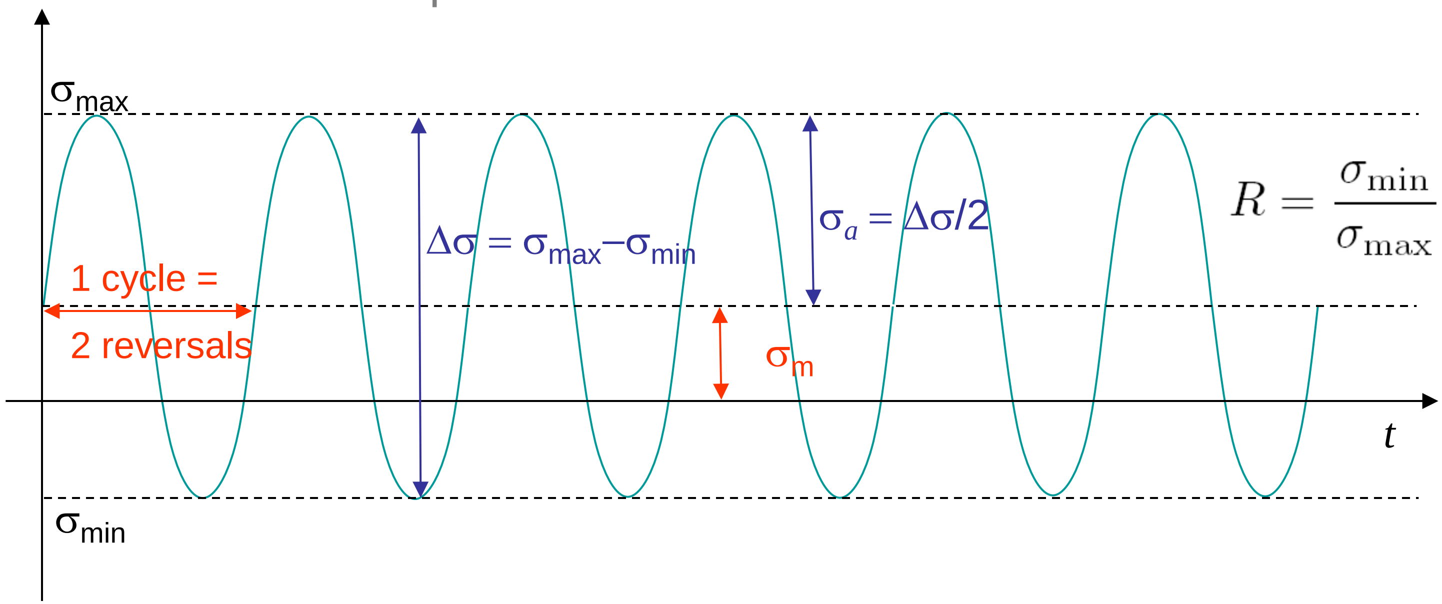
- Design with stresses lower than
  - Elastic limit (σ_p^0) or
  - Tensile strength (σ_{TS})
- ~1860, Wöhler
  - Technologist in the German railroad system
  - Studied the failure of railcar axles
    - Failure occurred
      - After various times in service
      - At loads considerably lower than expected





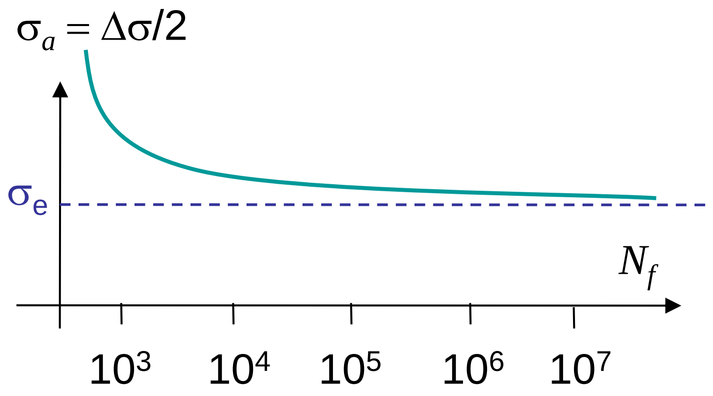
    - Failure due to cyclic loading/unloading
  - « Total life » approach
    - **Empirical** approach of fatigue

- Empirical approach: Total life
  - Life of a structure depends on



- Minimal & maximal stresses: σ_{min} & σ_{max}
- Mean stress: $\sigma_m=(\sigma_{max} + \sigma_{min})/2$
- Amplitude: $\sigma_a = \Delta\sigma/2 = (\sigma_{max} - \sigma_{min})/2$
- Load Ratio: $R = \sigma_{min} / \sigma_{max}$
- Under particular environmental conditions (humidity, high temperature, …):
  - Frequency of cycles
  - Shape of cycles (sine, step, …)

- First kind of total life approach: « stress life » approach
  - For structures experiencing (essentially) elastic deformations
  - For $\sigma_m = 0$ & N_f identical cycles before failure
    - For $\sigma_a < \sigma_e$ (endurance limit): infinite life (>10^7 cycles)
    - For $\sigma_a > \sigma_e$, finite life
    - With $\sigma_e \sim [\ 0.35\ ;\ 0.5\ ]\ \sigma_{TS}$
    - 1910, Basquin Law



$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f\,(2N_f)^b$$

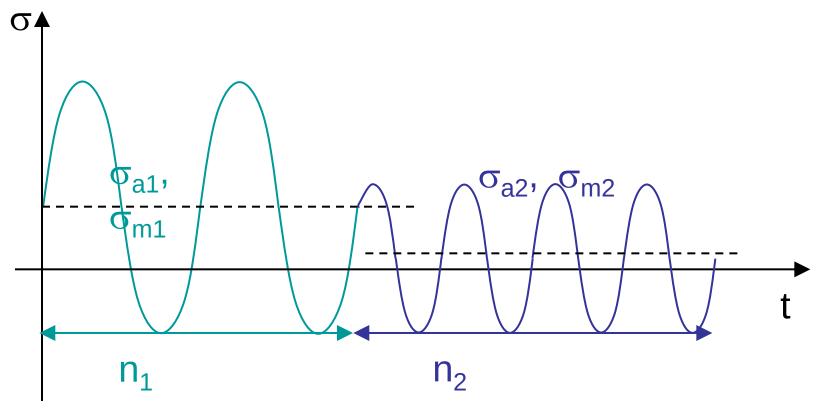
– σ_f' fatigue coefficient (mild steel T_{amb} : ~ [1; 3] GPa)

– b fatigue exponent (mild steel T_{amb} : ~ [-0.1; -0.06])

– Parameters resulting from experimental tests

- First kind of total life approach: « stress life » approach (2)
  - For $\sigma_m \neq 0$ and N_f identical cycles, the maximal amplitude is corrected
    - Soderberg $\sigma_a = \sigma_a|_{\sigma_m=0}\left(1-\frac{\sigma_m}{\sigma_p}\right)$ $\Longrightarrow$ conservative
    - Goodman $\sigma_a = \sigma_a|_{\sigma_m=0}\left(1-\frac{\sigma_m}{\sigma_{\mathrm{TS}}}\right)$
    - Gerber $\sigma_a = \sigma_a|_{\sigma_m=0}\left[1-\left(\frac{\sigma_m}{\sigma_{\mathrm{TS}}}\right)^2\right]$ $\Longrightarrow$ only for alloys under tension
  - Varying amplitude loads
    - n_i cycles of constant amplitude

      lead to a damage $d_i = \frac{n_i}{N_{fi}}$
    - 1945, Miner-Palmgren law $\sum_i d_i = 1$



  - Does not account for the sequence in which the cycles are applied

- Second kind of total life approach: « strain life » approach
  - For structures experiencing (essentially)
    - Large plastic deformations
    - Stress concentration
    - High temperatures
  - For N_f identical cycles before failure
    - 1954, Manson-Coffin $$\frac{\Delta\bar{\varepsilon}^p}{2} = \varepsilon'_f \left(2N_f\right)^c$$
      - ε_f' : fatigue ductility coefficient ~ true fracture ductility (metals)
      - c : fatigue ductility coefficient exponent ~ [-0.7 , -0.5] (metals)
      - $\Delta\bar{\varepsilon}^p$ plastic strain increment during the loading cycles

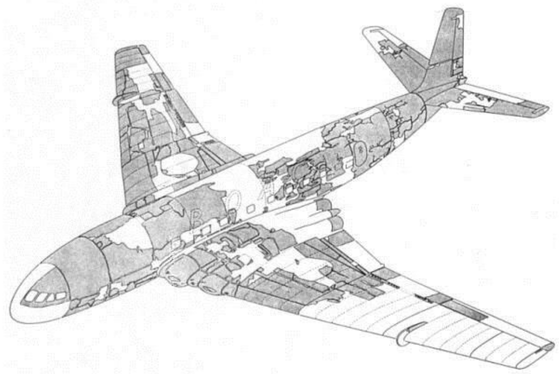
- 1952, De Havilland 106 Comet 1, UK (1)
  - First jetliner, 36 passengers, pressurized cabin (0.58 atm)
  - Wrong aerodynamics at high angle of attack (takeoff)
    - 1953, 2 crashes: lift loss due to swept wing and air intakes inefficient

  - The fuselage was designed using total life approach
    - 1952, a fuselage was tested against fatigue
      - Static loading at 1.12 atm, followed by
      - 10 000 cycles at 0.7 atm (> cabin pressurization at 0.58 atm)
  - Design issue
    - 1953, India, crash during storm
      - « Structural failure » of the stabilizer
      - The pilot does not “feel” the forces due to the fully powered controls (hydraulically assisted)
      - Fatigue due to overstress ?

Design using total life approach

- 1952, De Havilland 106 Comet 1, UK (2)
  - More design issues
    - 1954, January, flight BOAC 781 Rome-Heathrow
      - Plane G-ALYP disintegrated above the sea
      - After 1300 flights
      - Autopsies of passengers' lungs revealed explosive decompression
      - Bomb? Turbine failure ?

        ⟹ turbine rings with armor plates
    - 1954, April, flight SAA 201 Rome-Cairo
      - Plane G-ALYY disintegrated
    - 1954, April, reconstruction of plane ALYP from the recovered wreckages
      - Proof of fracture, but origin unknown
    - 1954, April, test of fuselage ALYU
      - Water tank for pressurization cycles
      - Rupture at port window after only 3057 pressurization cycles
  - Total life approach failed
    - Fuselages failed well before the design limit of 10000 cycles



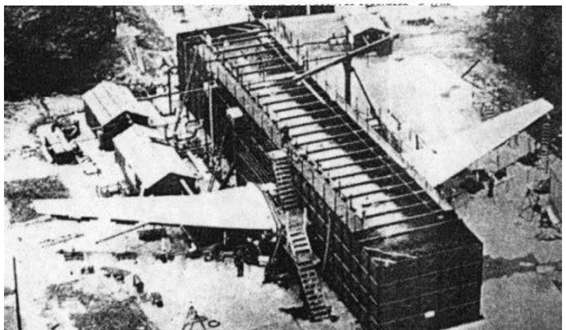
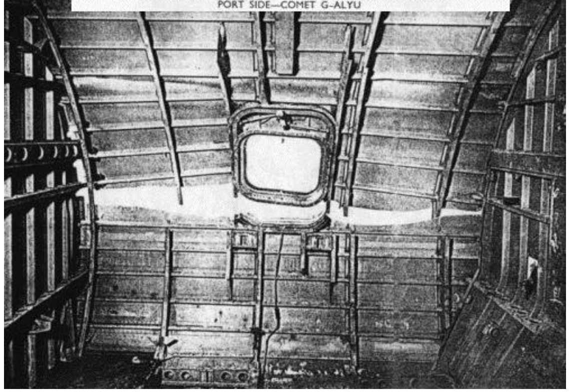


FIG. 6. AERIAL VIEW OF COMET G-ALYU IN TESTING TANK.

FIG. 7. VIEW FROM INSIDE OF FAILURE AT THE FORWARD ESCAPE HATCH ON THE PORT SIDE—COMET G-ALYU



Design using total life approach

- 1952, De Havilland 106 Comet 1, UK (3)
  - 1954, August, ALYP roof retrieved from sea
    - Origin of failure at the communication window
    - Use of square riveted windows
    - Punched riveting instead of drill riveting
    - ⟹ Existence of initial defects

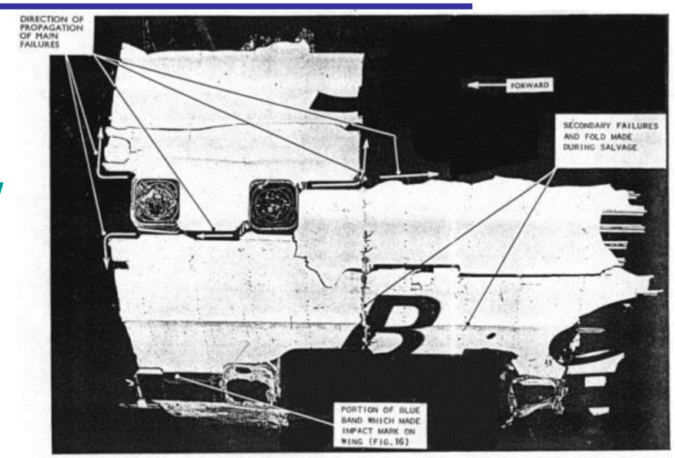
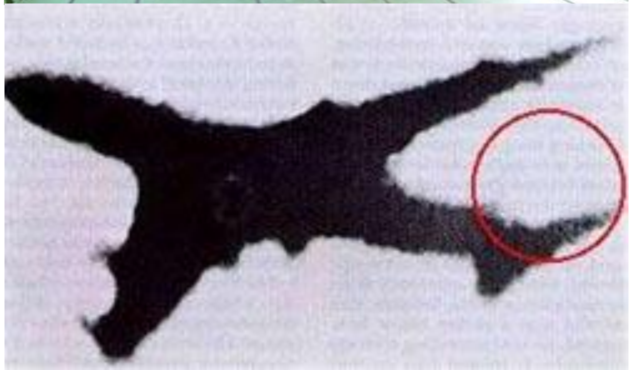


FIG. 12. PHOTOGRAPH OF WRECKAGE AROUND ADF AERIAL WINDOWS—G-ALYP.

- The total life approach
  - Accounts for crack initiation in smooth specimen
  - Does not account for inherent defects
    - Metal around initial defects could have hardened during the initial static test load of the fatigue tested fuselage
    - Production planes without this static test load …
- Life time can be improved by
  - “Shoot peening”: surface bombarded by small spherical media
    - Compression residual stresses in the surface layer
    - Prevents crack initiation
  - Surface polishing (to remove cracks)
- 1958, Comet 3 et 4
  - Round windows glued
  - Fuselage thicker

Other examples of fatigue failures

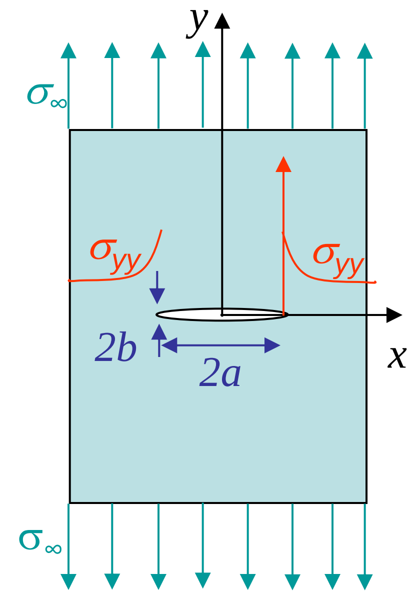
- 1985, B747 Japan Airline 123
  - 1978, tail touched the ground
  - Pressurization bulkhead repaired with a single row of rivets
  - To be safe-life 2 rows are required (Boeing repair manual)
- 1988, B737, Aloha Airlines 243
  - 2 fuselage panels not correctly glued
  - Salt water inbetween
  - Rust and dilatation
  - Fatigue of the rivets
- These structures perished by fatigue
  - At load lower than ultimate load
  - After a large number of cycle



Why fracture mechanics ?

- Limits of the total life approach
  - Does not account for inherent defects
    - What is happening when a defect is present ?
      - Example: Comet
    - Theoretical stress concentration
      - Infinite plane with an ellipsoidal void (1913, Inglis)

$$\sigma_{\max} = \sigma_{yy}(a, 0) = \sigma_{\infty}\left(1 + \frac{2a}{b}\right)$$

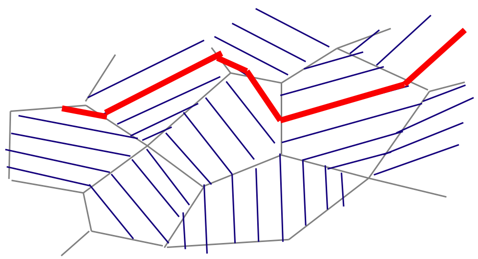


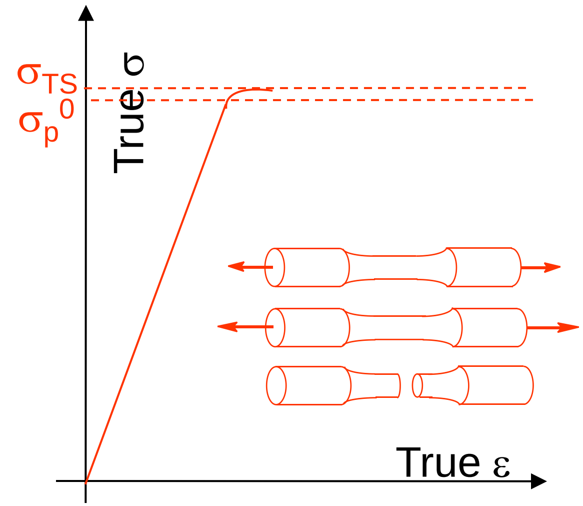
      - $b \rightarrow \upsilon$ ⟹ $\sigma_{\max} \rightarrow \infty$ ⟹ breaks for $\sigma_{\infty} \rightarrow 0$
      - In contradiction with Griffith and Irwin experiments
        - » Tensile strength depends on the crack size a and of surface energy γ_s

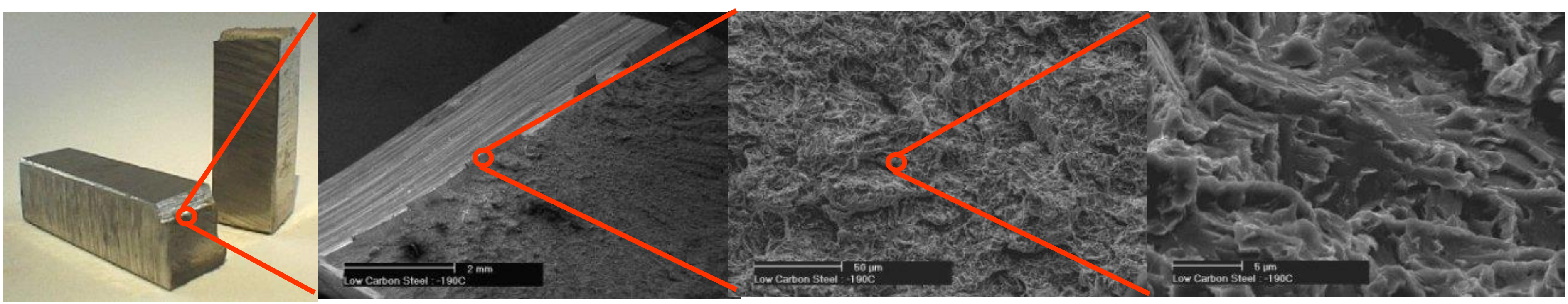
$$\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$$

- Development of the fracture mechanics field
  - How can we predict failure when a crack exists ?
  - Microscopic observations for cycling loading
    - Crack initiated at stress concentrations (nucleation)
    - Crack growth
    - Failure of the structure when the crack reaches a critical size
    - How can we model this?

- Mechanism of brittle failure
  - (Almost) no plastic deformations prior to the (macroscopic) failure
  - Cleavage: separation of crystallographic planes
    - In general inside the grains
    - Preferred directions: low bonding
    - Between the grains: corrosion, H_2, …
  - Rupture criterion
    - 1920, Griffith: $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$



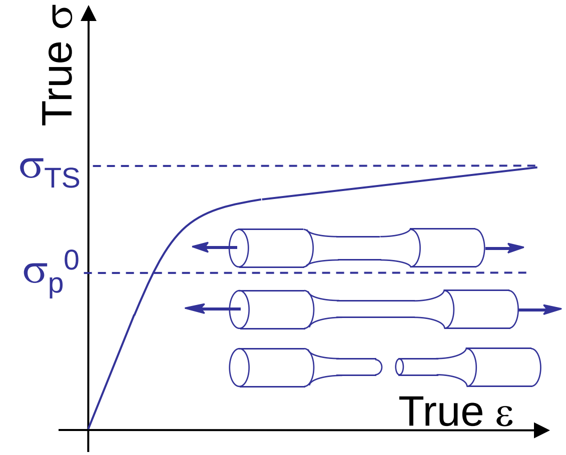


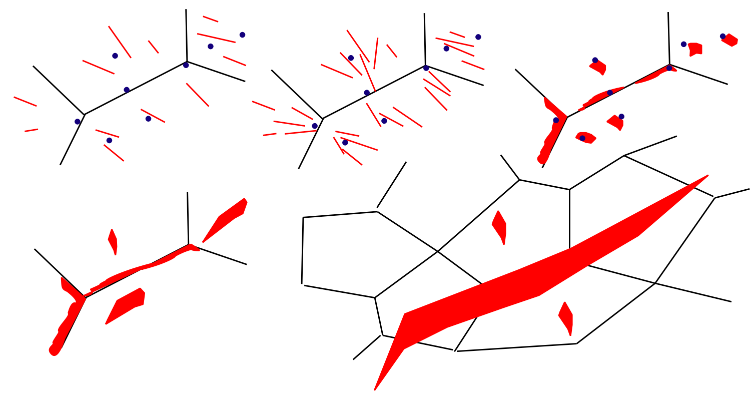


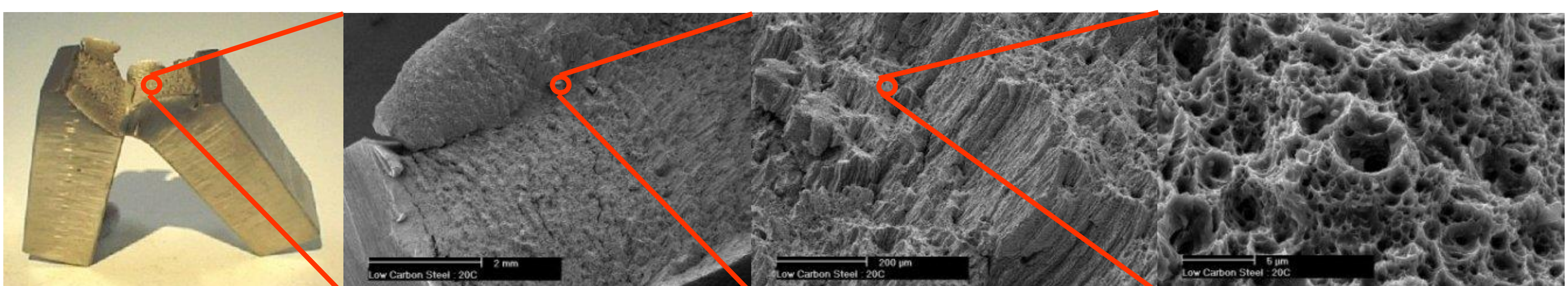
Brittle / ductile fracture

- Mechanism of ductile failure
  - Plastic deformations prior to (macroscopic) failure of the specimen
    - Dislocations motion ⟹ void nucleation around inclusions ⟹ micro cavity coalescence ⟹ crack growth
    - Is Griffith criterion $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$ still correct?
    - 1950, Irwin, the plastic work at the crack tip should be added to the surface energy:

$$\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\left(2\gamma_s + W_{\mathrm{pl}}\right)}$$

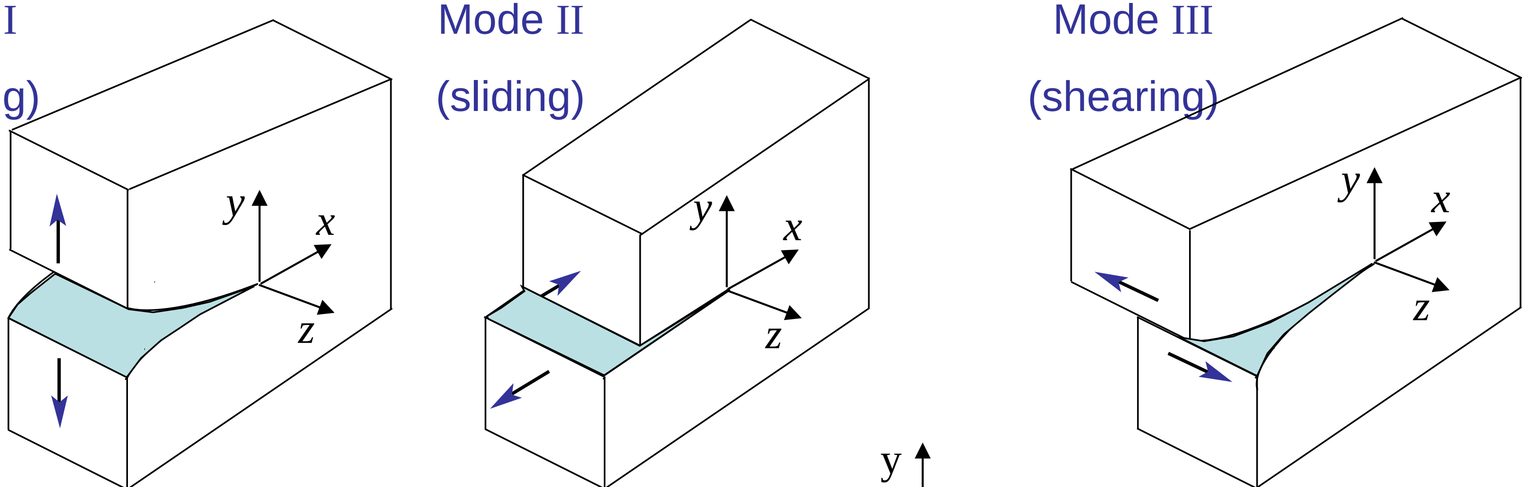




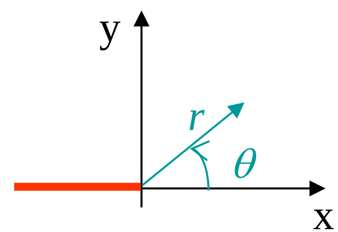


- Singularity at crack tip for linear and elastic materials
  - 1957, Irwin, 3 fracture modes

Mode I (opening) — Mode II (sliding) — Mode III (shearing)



- Boundary conditions



Mode I

$$\begin{cases} \boldsymbol{\sigma}_{zz} = 0 \text{ or } \boldsymbol{\varepsilon}_{zz} = 0 \\ \boldsymbol{\sigma}_{yy}(\theta = \pm\pi) = 0 \\ \boldsymbol{\sigma}_{xy}(\theta = \pm\pi) = 0 \\ \boldsymbol{u}_x(\theta > 0) = \boldsymbol{u}_x(\theta < 0) \\ \boldsymbol{u}_y(\theta > 0) = -\boldsymbol{u}_y(\theta < 0) \end{cases}$$

Mode II

$$\begin{cases} \boldsymbol{\sigma}_{zz} = 0 \text{ or } \boldsymbol{\varepsilon}_{zz} = 0 \\ \boldsymbol{\sigma}_{yy}(\theta = \pm\pi) = 0 \\ \boldsymbol{\sigma}_{xy}(\theta = \pm\pi) = 0 \\ \boldsymbol{u}_x(\theta > 0) = -\boldsymbol{u}_x(\theta < 0) \\ \boldsymbol{u}_y(\theta > 0) = \boldsymbol{u}_y(\theta < 0) \end{cases}$$

Mode III

$$\begin{cases} \boldsymbol{\sigma}_{xx} = \boldsymbol{\sigma}_{xy} = \boldsymbol{\sigma}_{yy} = \boldsymbol{\sigma}_{zz} = 0 \\ \boldsymbol{u}_y = \boldsymbol{u}_x = 0 \\ \boldsymbol{u}_z(\theta > 0) = -\boldsymbol{u}_z(\theta < 0) \end{cases}$$

Linear Elastic Fracture Mechanics (LEFM)

- Singularity at crack tip for linear and elastic materials (2)
  - Asymptotic solutions (Airy functions, see fracture mechanics classes)

Mode I

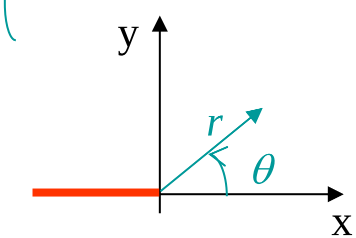
$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xx} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1-\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{yy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1+\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{\sigma}_{xy} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\sin\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
\boldsymbol{u}_{x} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\cos\left(\frac{\theta}{2}\right)\left[\kappa-1+2\sin^2\left(\frac{\theta}{2}\right)\right]\\
\boldsymbol{u}_{y} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)\left[\kappa+1-2\cos^2\left(\frac{\theta}{2}\right)\right]
\end{aligned}\right.$$

Mode II

$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xx} &= -\frac{C}{\sqrt{r}}\sin\frac{\theta}{2}\left[2+\cos\frac{3\theta}{2}\cos\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}\\
\boldsymbol{\sigma}_{yy} &= \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{xy} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}\left[1-\sin\frac{3\theta}{2}\sin\frac{\theta}{2}\right]+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{u}_{x} &= \frac{C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)\left[\kappa+1+2\cos^2\left(\frac{\theta}{2}\right)\right]\\
\boldsymbol{u}_{y} &= -\frac{C\left(1+\nu\right)}{E}\sqrt{r}\cos\left(\frac{\theta}{2}\right)\left[\kappa-1-2\sin^2\left(\frac{\theta}{2}\right)\right]
\end{aligned}\right.$$

Mode III

$$\left\{\begin{aligned}
\boldsymbol{\sigma}_{xz} &= -\frac{C}{\sqrt{r}}\sin\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}\\
&\boxed{\boldsymbol{\sigma}_{yz} = \frac{C}{\sqrt{r}}\cos\frac{\theta}{2}+\cancel{\mathcal{O}\left(r^0\right)}}\\
\boldsymbol{u}_{z} &= \frac{4C\left(1+\nu\right)}{E}\sqrt{r}\sin\left(\frac{\theta}{2}\right)
\end{aligned}\right.$$



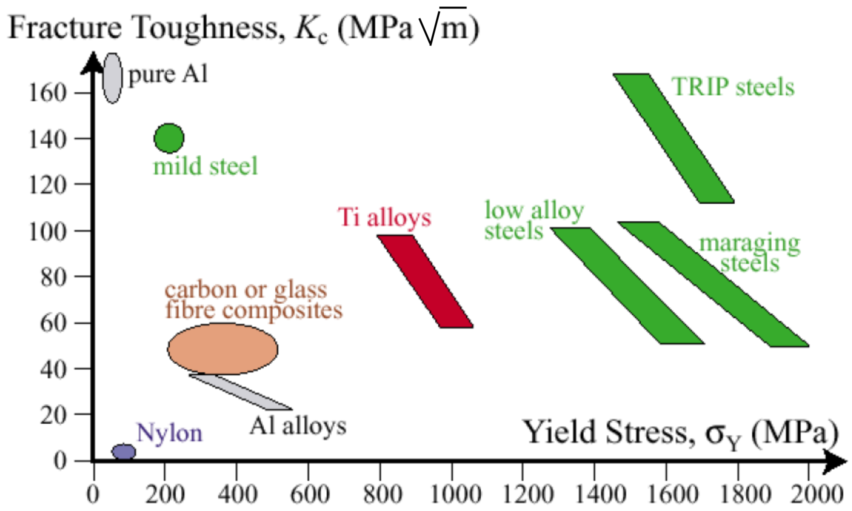
with for plane σ $\kappa = \dfrac{3-\nu}{1+\nu}$ & plane ε $\kappa = 3-4\nu$

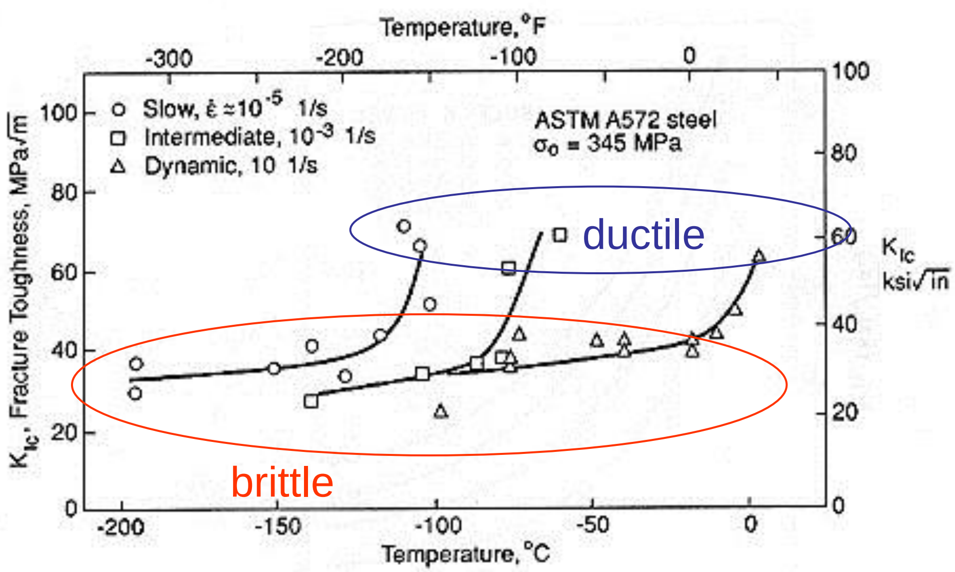
- Introduction of the Stress Intensity Factors - SIF (Pa m$^{1/2}$)

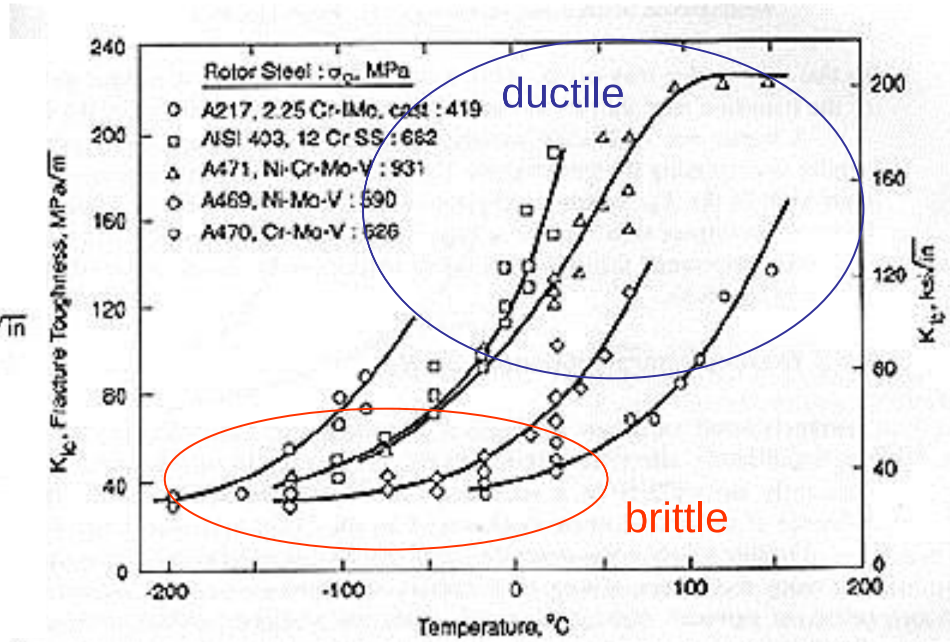
$$\left\{\begin{aligned}
K_I &= \lim_{r\to 0}\left(\sqrt{2\pi r}\boldsymbol{\sigma}_{yy}^{\text{mode I}}\big|_{\theta=0}\right) = C\sqrt{2\pi}\\
K_{II} &= \lim_{r\to 0}\left(\sqrt{2\pi r}\boldsymbol{\sigma}_{xy}^{\text{mode II}}\big|_{\theta=0}\right) = C\sqrt{2\pi}\\
K_{III} &= \lim_{r\to 0}\left(\sqrt{2\pi r}\boldsymbol{\sigma}_{yz}^{\text{mode III}}\big|_{\theta=0}\right) = C\sqrt{2\pi}
\end{aligned}\right. \Rightarrow \left\{\begin{aligned}
&\boxed{\boldsymbol{\sigma}^{\text{mode i}} = \frac{K_i}{\sqrt{2\pi r}}\mathbf{f}^{\text{mode i}}(\theta)}\\
&\boxed{\boldsymbol{u}^{\text{mode i}} = K_i\sqrt{\frac{r}{2\pi}}\boldsymbol{g}^{\text{mode i}}(\theta)}
\end{aligned}\right.$$

- ***K_i* are dependent on both the loading and the geometry**

- New failure criterion
  - 1957, Irwin, crack propagation
    - $\sigma_{max} \rightarrow \infty \Longrightarrow \sigma$ is irrelevant
    - If $K_i = K_{iC} \Longrightarrow$ crack growth
  - Toughness (ténacité) K_{Ic}
    - Steel, Al, … : see figures
    - Concrete: 0.2 - 1.4 MPa $m^{1/2}$ (brittle failure)







- Stress Intensity Factor (SIF)
  - Computation of the SIFs K_i
    - Analytical (crack $2a$ in an infinite plane)

$$\Rightarrow \begin{cases} K_I = \sigma_\infty \sqrt{\pi a} \\ K_{II} = \tau_\infty \sqrt{\pi a} \\ K_{III} = \tau_\infty \sqrt{\pi a} \end{cases}$$

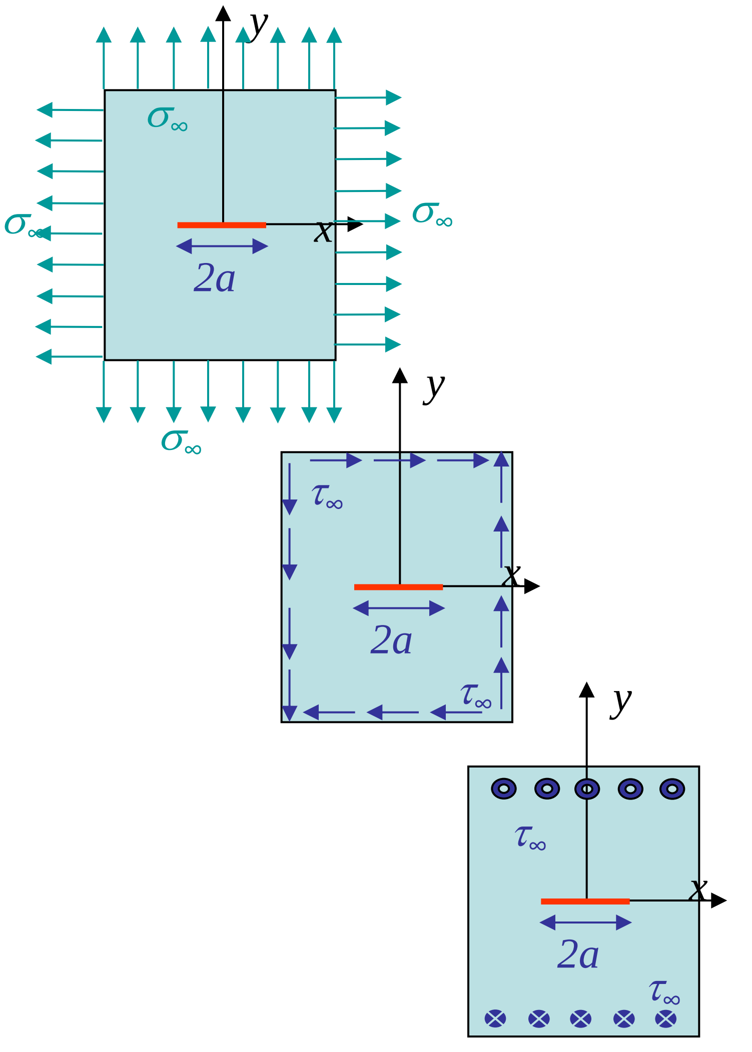
    - For other geometries or loadings

$$\Rightarrow \begin{cases} K_I = \beta_I \sigma_\infty \sqrt{\pi a} \\ K_{II} = \beta_{II} \tau_\infty \sqrt{\pi a} \\ K_{III} = \beta_{III} \tau_\infty \sqrt{\pi a} \end{cases}$$

    - β_i obtained by
      - Superposition
      - FEM
      - Energy approach
        - » Related to Griffith's work

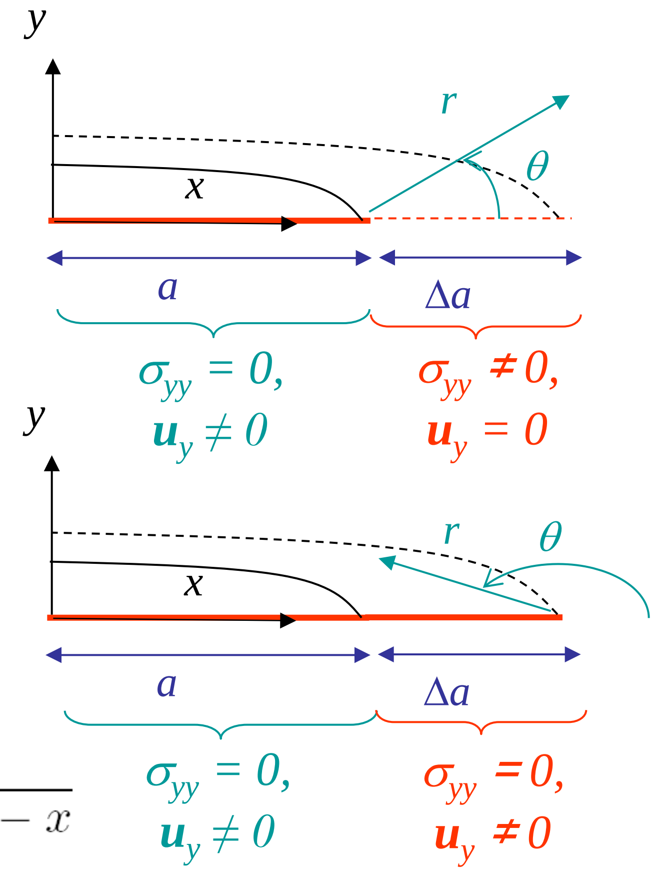
$$\sigma_{\mathrm{TS}} \sqrt{a} \div \sqrt{E\left(2\gamma_s + W_{\mathrm{pl}}\right)}$$

        - » See next slides
  - For 2 loadings a & b: $K_I = K_I^a + K_I^b$
  - **BUT** for 2 modes $K \neq K_I + K_{II}$



- Energy approach
  - Mode I
    - Initial crack $2a$

$$\begin{cases} \boldsymbol{\sigma}_{yy}^{0}\left(\theta=0,\ r=x-a\right)=\dfrac{\sigma_{\infty}\sqrt{a}}{\sqrt{2\left(x-a\right)}} \\ \boldsymbol{u}_{y}^{0}\left(\theta=0,\ r=x-a\right)=0 \end{cases}$$



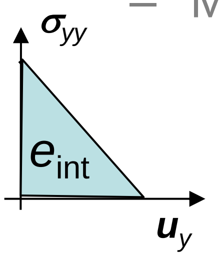
    - Crack grows to $2(a+\Delta a)$

$$\begin{cases} \boldsymbol{\sigma}_{yy}^{1}\left(\theta=\pi,\ r=a+\Delta a-x\right)=0 \\ \boldsymbol{u}_{y}^{1}\left(\theta=\pm\pi,\ r=a+\Delta a-x\right)= \\ \qquad \pm\dfrac{\sigma_{\infty}\left(1+\nu\right)\left(\kappa+1\right)}{E\sqrt{2}}\sqrt{a+\Delta a}\sqrt{a+\Delta a-x} \end{cases}$$

    - Energy is needed for crack to grow by $2\Delta a$ as there is a work done by σ_{yy}

$$\Delta E_{\text{int}}=-4dz\int_{a}^{a+\Delta a}\int_{\boldsymbol{u}_y^0}^{\boldsymbol{u}_y^1}\boldsymbol{\sigma}_{yy}d\boldsymbol{u}_y dx$$ (x4 as it is for $x>0$, $x<0$ & for $y<0$, $y>0$)

- Energy approach (2)
  - Mode I



    - Energy needed for crack to grow by Δa $\Delta E_{\text{int}} = -4dz \int_a^{a+\Delta a} \int_{\boldsymbol{u}_y^0}^{\boldsymbol{u}_y^1} \boldsymbol{\sigma}_{yy} d\boldsymbol{u}_y dx$
    - Assumption: $\boldsymbol{\sigma}_{yy}$ linear in terms of $\boldsymbol{u}_y$ $\Longrightarrow$ $\Delta E_{\text{int}} = -2dz \int_a^{a+\Delta a} \boldsymbol{\sigma}_{yy}^0 \boldsymbol{u}_y^1 dx$
    - Change of variable $x = a + \Delta a \cos^2\theta$

$$\Longrightarrow \Delta E_{\text{int}} = -\frac{dz\sigma_\infty^2 \sqrt{a\left(a+\Delta a\right)}\left(1+\nu\right)\left(1+\kappa\right)}{2E}\pi\Delta a$$

    - G : energy release rate **for a straight ahead growth**

$$\Longrightarrow \boxed{G = -\frac{dE_{\text{int}}}{dA} = -\lim_{\Delta a \to 0}\frac{\Delta E_{\text{int}}}{2\Delta a dz} = \frac{\pi a \sigma_\infty^2}{4E}\left(1+\nu\right)\left(\kappa+1\right) = \frac{\pi a \sigma_\infty^2}{E'} = \frac{K_I^2}{E'}}$$

$$E' = \frac{4E}{\left(1+\nu\right)\left(\kappa+1\right)} = \begin{cases} E & \text{P}\sigma \\ \frac{E}{1-\nu^2} & \text{P}\varepsilon \end{cases}$$

  - The crack has been assumed lying in an infinite plane. But

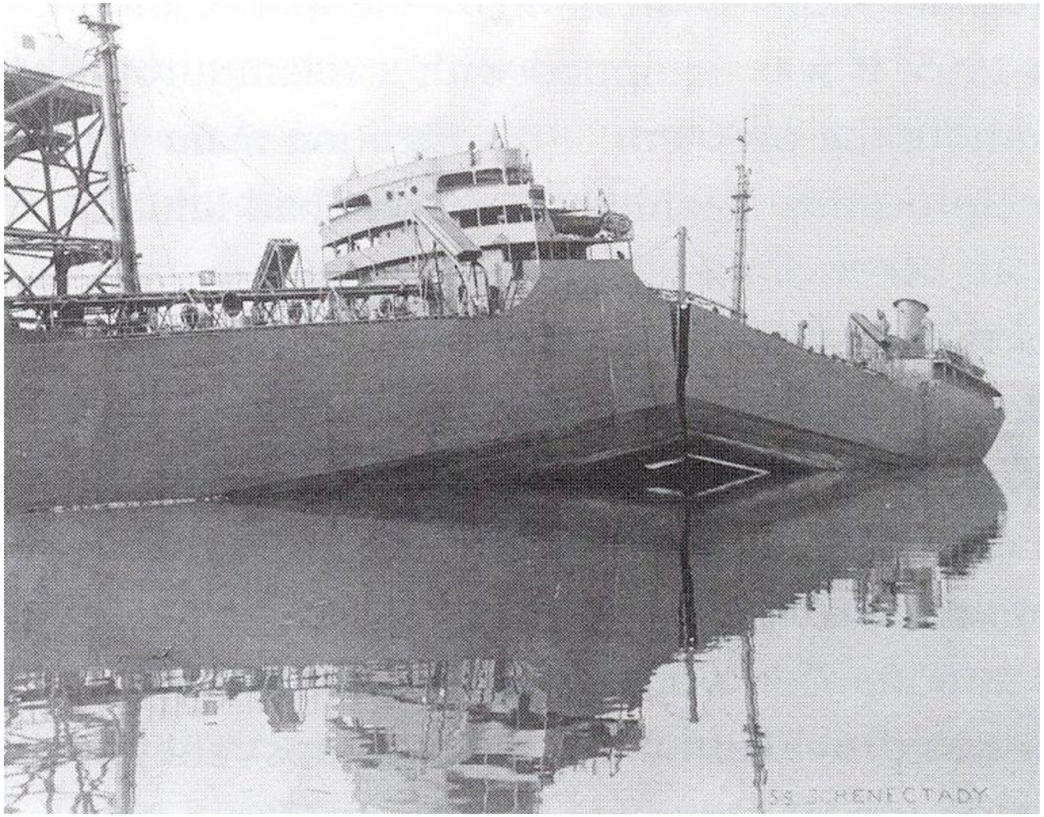
$G = \frac{K_I^2}{E'}$ still holds for other expressions of K_I (see fracture mechanics)

- Energy approach (3)
  - 1920, Griffith, energy conservation: $E = E_{\text{int}} + \Gamma$
    - Total energy E is the sum of the internal (elastic) energy E_{int} with the energy Γ needed to create surfaces A $\Longrightarrow$ $\dfrac{\partial E}{\partial A} = \dfrac{\partial E_{\text{int}}}{\partial A} + \dfrac{\partial \Gamma}{\partial A} = 0$
    - If γ_s is the surface energy (material property of brittle material)

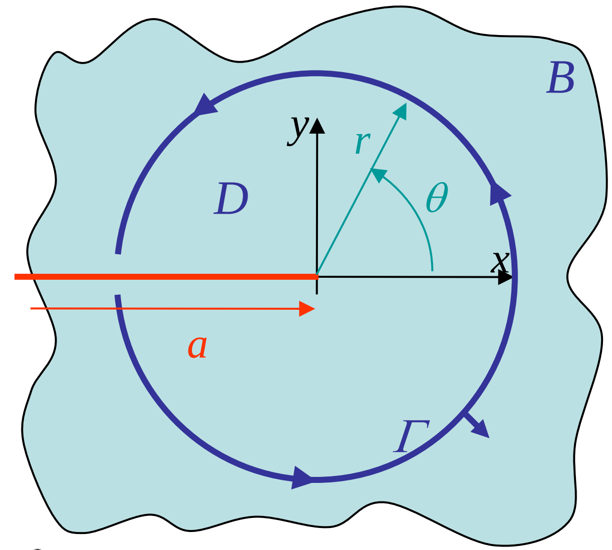
      $\Longrightarrow$ crack growth if $G = G_c = -\dfrac{dE_{\text{int}}}{dA} = \dfrac{d\Gamma}{dA} = 2\gamma_s$ → A crack creates 2 surfaces A
  - Mode I, infinite plane
    - Strength $G = \dfrac{\pi a \sigma_{\infty}{}^2}{E'} = \dfrac{K_I^2}{E'}$ $\Longrightarrow$ $\sigma_{\text{TS}} = \sqrt{\dfrac{2\gamma_s E}{\pi a}}$ → Depend on the crack size
    - Glass: $G_c = 2\gamma_s \sim 2\ \text{Jm}^{-2}$, $E = 60$ GPa $\Longrightarrow$ $\sigma_{\text{TS}}\sqrt{a} = 195\text{kPa m}^{\frac{1}{2}}$
    - Steel: G_c = plast. dissipation ~ 200 kJm^{-2}, $E = 210$ GPa $\Longrightarrow$ $\sigma_{\text{TS}}\sqrt{a} = 115\text{MPa m}^{\frac{1}{2}}$
  - Straight ahead propagation for general loading
    - Proceeding as for mode I: $G = \dfrac{K_I^2}{E'} + \dfrac{K_{II}^2}{E'} + \dfrac{(1+\nu)\,K_{III}^2}{E}$

      $\Longrightarrow$ crack growth if $G = G_c$

- Example: Liberty ships (WWII)
  - Steel at low $T°$: brittle
    - $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\,2\gamma_s}$ with γ_s ~ 3400 J m^{-2}
  - Steel at room $T°$: ductile
    - $\sigma_{\mathrm{TS}}\sqrt{a} \div \sqrt{E\left(2\gamma_s + W_{\mathrm{pl}}\right)}$ with $2\gamma_s + W_{pl}$ ~ 200 kJ m^{-2}
  - Use of low-grade steel
    - In cold weather:
      DBTT ~ water temperature
    - When put in water existing cracks lead to failure
    - 30% of the liberty ships suffered from fracture



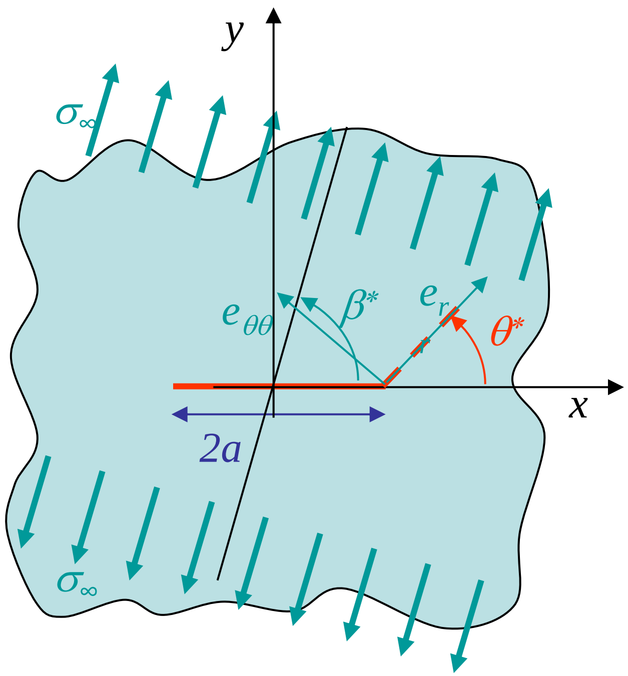
- Energy approach: J-integral
  - Energy release rate
    - Straight ahead propagation for linear elasticity $G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{(1+\nu)\,K_{III}^2}{E}$
  - Should be related to the energy flowing toward the crack tip
    - J-integral $J = \int_\Gamma \left[U(\boldsymbol{\varepsilon})\, \boldsymbol{n}_x - \boldsymbol{u}_{,x} \cdot \boldsymbol{T} \right] dl$

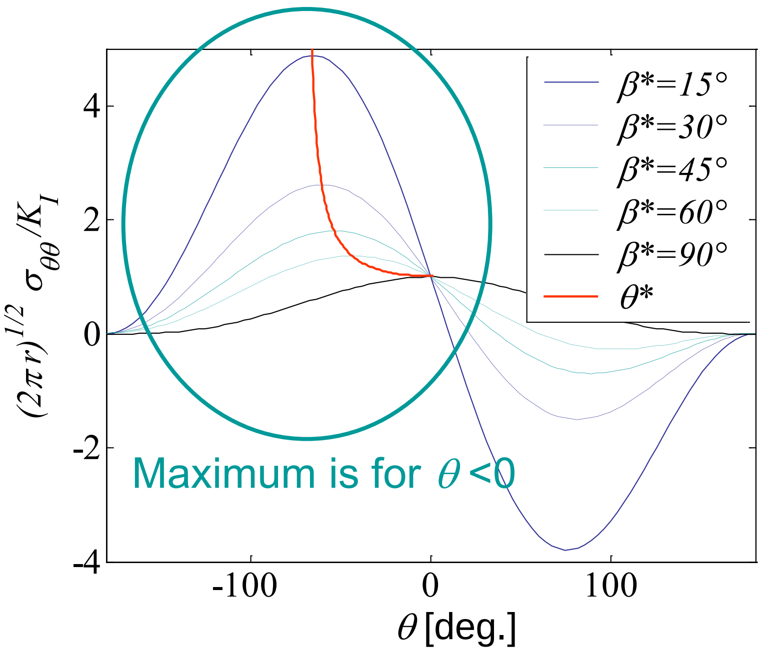


      - Defined even for non-linear materials
      - Is path independent if the contour Γ embeds a straight crack tip
      - BUT no assumption on subsequent growth direction
      - If crack grows straight ahead $\Longrightarrow G=J$
      - If linear elasticity $\Longrightarrow J = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$
      - Can be extended to plasticity if no unloading (see fracture mechanics)
    - Advantages
      - Efficient numerical computation of the SIFs
      - Useful for non perfectly brittle materials

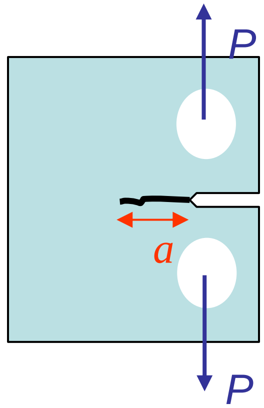
- Direction of crack grow
  - Assumptions: the crack will grow in the direction where the SIF related to mode I in the new frame is maximal
    - Crack growth if $\left(\sqrt{2\pi r}\boldsymbol{\sigma}_{\theta\theta}\left(r,\,\theta*\right)\right) \geq K_C$ with $\partial_\theta \boldsymbol{\sigma}_{\theta\theta}|_{\theta*} = 0$
  - From direction of loading, one can compute the propagation direction

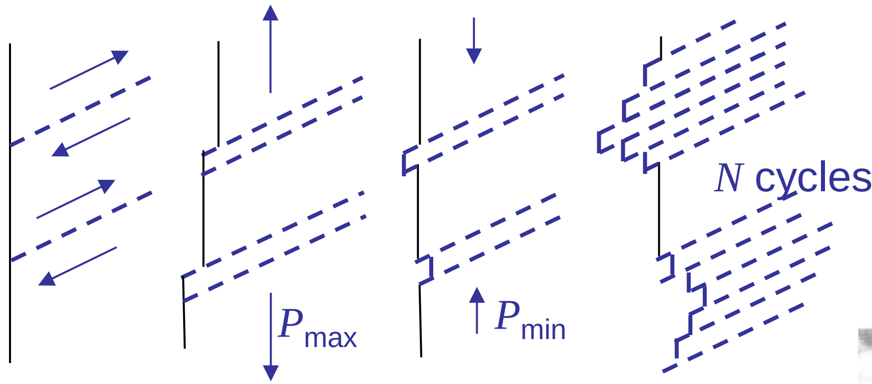
$$\cot\beta^* = \frac{K_{II}}{K_I} \Longrightarrow \boldsymbol{\sigma}_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\cos^3\frac{\theta}{2} - \frac{3\cot\beta^*}{2}\sin\theta\cos\frac{\theta}{2}\right]$$



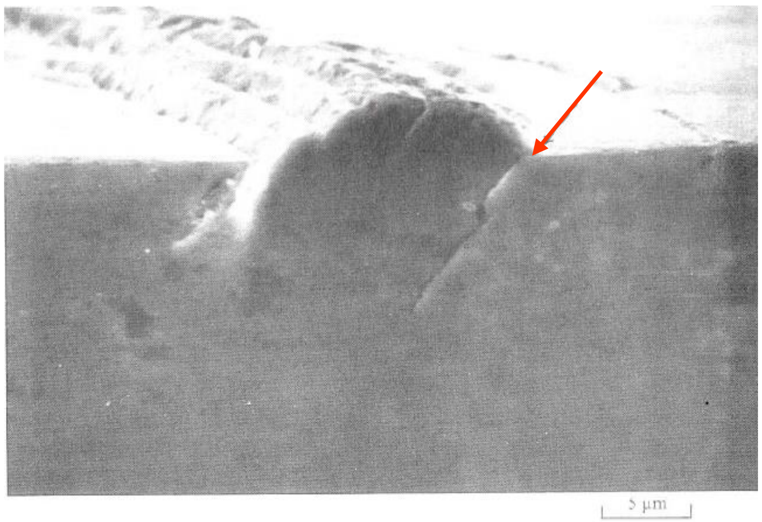


- Fatigue failure
  - Tests performed with different $\Delta P = P_{max} - P_{min}$
  - Nucleation: cracks initiated for $K < K_c$
    - Surface: deformations result from dislocations motion along slip planes

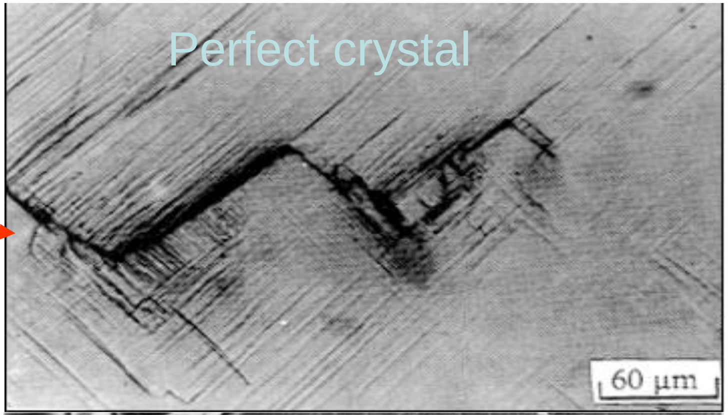


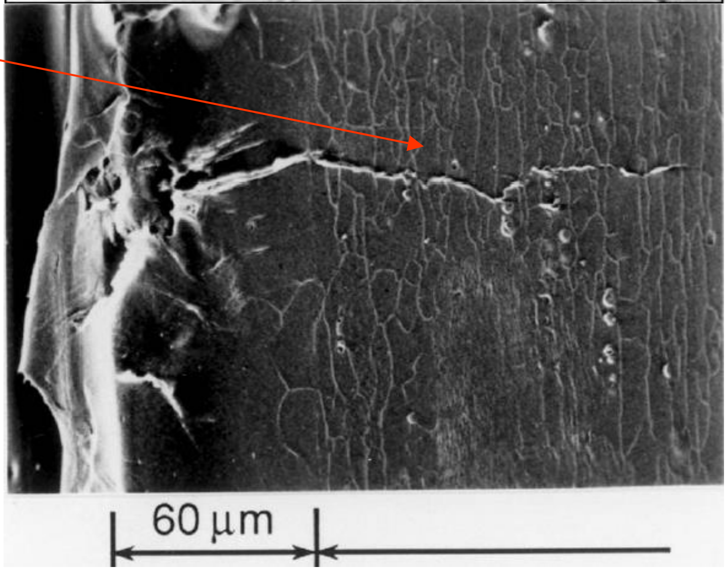


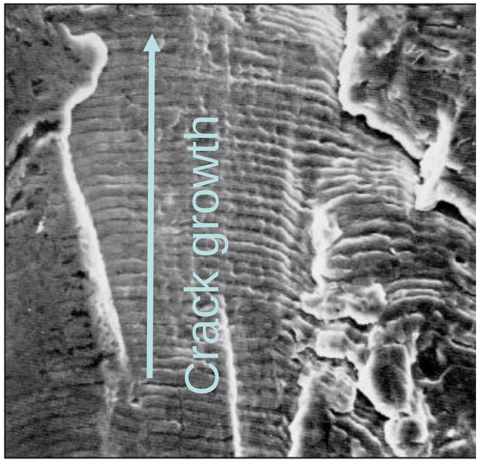
    - Can also happen around a bulk defect



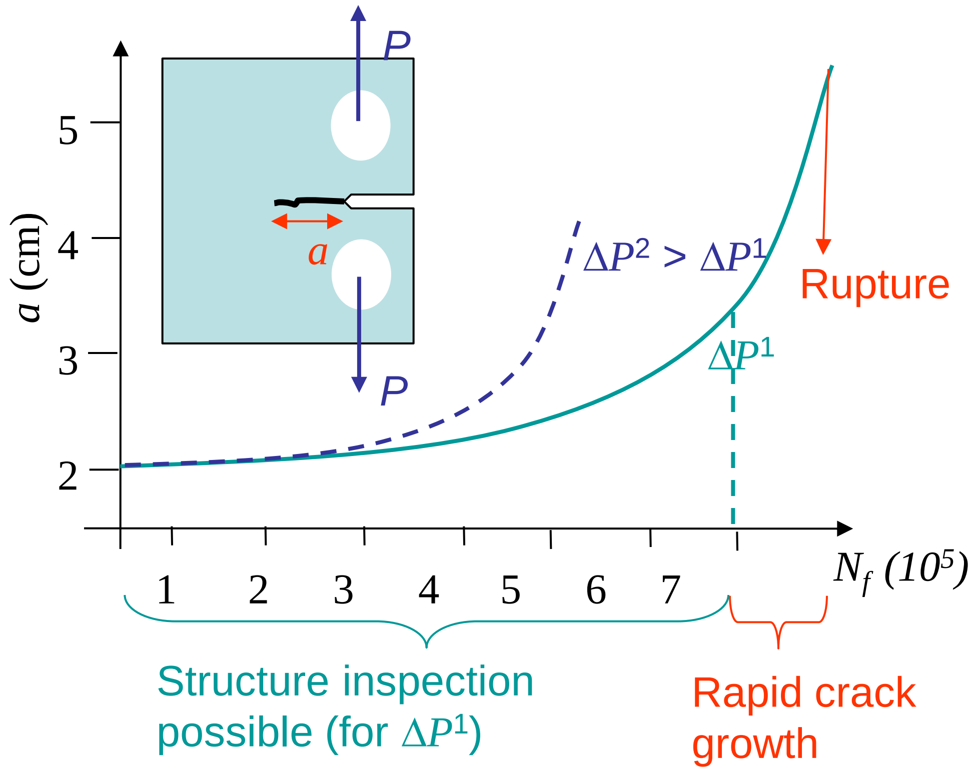
- Fatigue failure (2)
  - Stage I fatigue crack growth:
    - Along a slip plane
  - Stage II fatigue crack growth:
    - Across several grains
      - Along a slip plane in each grain
      - Straight ahead macroscopically
    - Striation of the failure surface: corresponds to the cycles







- Fatigue failure (3)
  - SSY assumption
    - Tests: conditioning parameters
      - ΔP &
      - $P_{\min}$ / $P_{\max}$
    - Therefore fatigue failure can be described by
      - $\Delta K = K_{\max} - K_{\min}$ &
      - $R = K_{\min}/K_{\max}$



$$\Rightarrow \boxed{\frac{da}{dN_f} = f(\Delta K, R)}$$

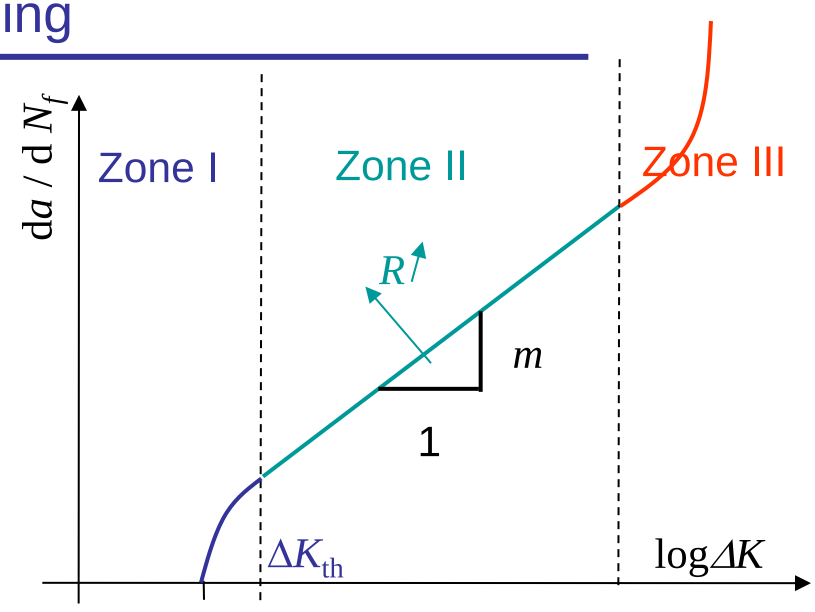
Interval of striations

- There is ΔK_{th} such that if $\Delta K \sim \Delta K_{th}$:
  - The crack has a growth rate lower than one atomic spacing per cycle (statistical value)
  - Dormant crack

- Crack growth rate
  - Zone I
    - Stage I fatigue crack growth
    - ΔK_{th} depends on R
  - Zone II
    - Stage II fatigue crack growth: striation
    - 1963, Paris-Erdogan

$$\frac{da}{dN_f} = C\Delta K^m$$

      - Depends on the geometry, the loading, the frequency
      - Steel: ΔK_{th} ~ 2-5 MPa $m^{1/2}$, C ~ 0.07-0.11 10^{-11} [m (MPa $m^{1/2})^{-m}$], m ~ 4
      - Steel in sea water: ΔK_{th} ~ 1-1.5 MPa $m^{1/2}$, C ~ 1.6 10^{-11} [idem], m ~ 3.3
    - Be careful: K depends on a $\Longrightarrow$ integration required to get $a(N_f)$
      - Mode I : $K_I = \sigma_\infty \sqrt{\pi a}$ $\Longrightarrow$ $\Delta K = (\sigma_{\infty,\max} - \sigma_{\infty,\min})\sqrt{\pi a}$
  - Zone III
    - Rapid crack growth until failure
    - Static behavior (cleavage) due to the effect of $K_{max}(a)$
    - There is failure once a_f is reached, with a_f such that $K_{max}(a_f) = K_c$



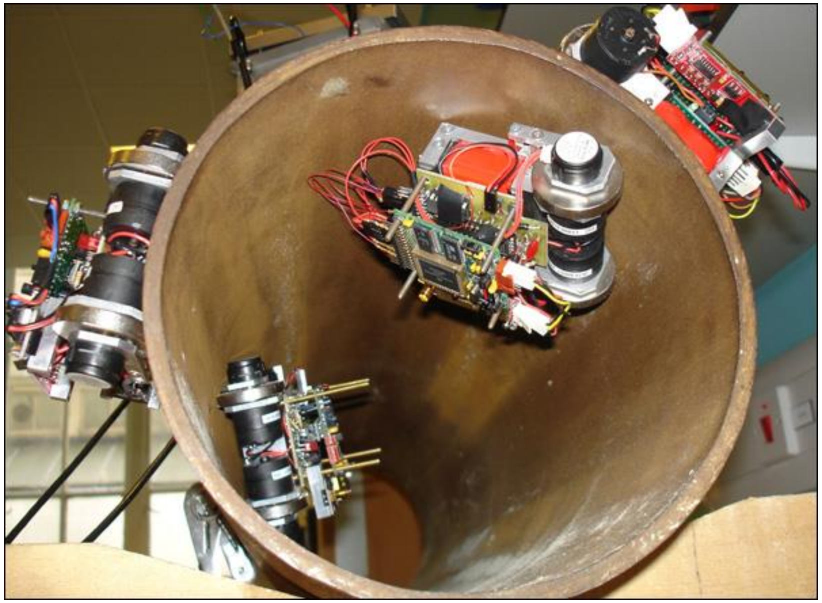
- « Infinite life design »
  - $\sigma_a < \sigma_e$: « infinite » life
  - Economically deficient
- « Safe life design »
  - No crack before a determined number of cycles
    - At the end of the expected life the component is changed even if no failure has occurred
    - Emphasis on prevention of crack initiation
    - Approach theoretical in nature
      - Assumes initial crack free structures
  - Use of $\sigma_a - N_f$ curves (stress life)
    - Add factor of safety
  - Components of rotating structures vibrating with the flow cycles (blades)
    - Once cracks form, the remaining life is very short due to the high frequency of loading

Fatigue design

- « Fail safe design »
  - Even if an individual member of a component fails, there should be sufficient structural integrity to operate safely
  - Load paths and crack arresters
  - Mandate periodic inspection
  - Accent on crack growth rather than crack initiation
  - Example: 1988, B737, Aloha Airlines 243
    - 2 fuselage plates not glued
    - Sea water $\Longrightarrow$ rust and volume increased
    - Fatigue of the rivets
    - The crack followed a predefined path allowing a safe operation

Fatigue design

- « Damage tolerant design »
  - Assume cracks are present from the beginning of service
  - Characterize the significance of fatigue cracks on structural performance
    - Control initial crack sizes through manufacturing processes and (non-destructive) inspections
    - Estimate crack growth rates during service (Paris-Erdogan) & plan conservative inspection intervals (e.g. every so many years, number of flights)
    - Verify crack growth during these inspections
    - Predict end of life (a_f)
    - Remove old structures from service before predicted end-of-life (fracture) or implement repair-rehabilitation strategy



  - Non-destructive inspections
    - Optical
    - X-rays
    - Ultrasonic (reflection on crack surface)

References

- Lecture notes
  - Lecture Notes on Fracture Mechanics, Alan T. Zehnder, Cornell University, Ithaca,
- Other references
  - « on-line »
    - Fracture Mechanics, Piet Schreurs, TUe, http://www.mate.tue.nl/~piet/edu/frm/sht/bmsht.html
  - Book
    - Fracture Mechanics: Fundamentals and applications, D. T. Anderson. CRC press, 1991.