Aircraft Design
Conceptual Design

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Goals of the classes

• Design stages
  – **Conceptual design**
    • Purposes
      – Define the general configuration (tail or canard, high or low wing, …)
      – Analyze the existing technologies
      – Estimate performances for the different flight stages
      – Accurate estimation of the total weight, fuel weight, engine thrust, lifting surfaces, …
    • How
      – Limited number of variables (tens): span, airfoil profile, …
      – Accurate simple formula & abacuses
  – Preliminary study
    • Higher number of variables (hundreds)
    • Starting point: conceptual design
    • Numerical simulations
  – Detailed study
    • Each component is studied in details
**Fuselage**

- **Cross-section**
  - Seat width
    - Economy: ~20 inches*
      
      *1 inch = 2.54 cm
    - Business: ~24 inches
    - First: ~26.5 inches
  - Aisle width
    - Economy: ~19 inches
    - Business: ~19 inches
    - First: ~21 inches
  - Fuselage thickness
    - ~4% of $H_{int}$
Fuselage

- Cross-section (2)
  - Other arrangements
    - Business jets
      - More freedom
    - Elliptic section
      - A380
    - Non-pressurized cabin
      - Rectangular cross-section
Fuselage

- **Length**
  - Seat pitch
    - Economy: ~34 inches
    - First: ~40 inches
  - Toilets
    - Length: ~38 inches
    - >1 per 40 passengers
  - Pressurized cabin can extend back in the tail
    - Different seat layouts
    - Shortens the plane length (reduced weight)

187 passengers (12 first class, 35 business class, 140 economy class)
Fuselage

- **Length (2)**
  - **Doors**
    - Type I: ~36 inches
    - Type II: ~20 inches
    - Type III & IV: ~18 inches
**Fuselage**

- **Length (3)**
  - Ratio nose length/diameter $N_F$
    - $>1.5$ due to pressurization
    - Large enough to avoid divergence
  - Ratio tail length/diameter $A_F$
    - $\sim 1.8-2$
    - Closure angle $\sim 28-30^\circ$
    - Upsweep $\sim 14^\circ$: rotation during take off

  ![Boeing 777](image)

  - Part of the tail can be pressurized and used for the payload

  ![187 passengers](image)
Fuselage

- **Method**
  - **Inputs**
    - $N_{\text{seats}}$, layout, $N_F$, $A_F$,
  - **Outputs**
    - **Shape**

\[
\begin{align*}
\text{height}_{\text{fus}} &= \text{width}_{\text{fus}} = H_{\text{ext}} \\
\text{length}_{\text{fus}} &= \text{length}_{\text{body}} + \text{height}_{\text{fus}} \times (A_F + N_F) \\
S_{\text{fus,wetted}} &\approx \text{length}_{\text{body}} \text{height}_{\text{fus}} \pi + \pi \frac{\text{height}_{\text{fus}}^2}{4} \sqrt{1 + 4A_F^2} + \frac{\pi \text{height}_{\text{fus}}^2}{4} \left( 1 + 2N_F \frac{\arcsin \sqrt{1 - \frac{1}{4N_F^2}}}{\sqrt{1 - \frac{1}{4N_F^2}}} \right) \end{align*}
\]

1/2 prolate
Wing

- **Airfoils**
  - Which one?
    - Minimum drag during cruise
    - Depends on Reynolds number $R = \frac{Uc}{\nu}$
  - Properties
    - Airfoil lift coefficient $c_l = c_{l\alpha} [\alpha - \alpha_{l0}]$
    - Pitching moment
      - Aerodynamic centre
        \[ \frac{\partial c_m}{\partial c_l} \bigg|_{\alpha_{ac}} = 0 \]
      - Moment around $ac \sim$ constant at low attack angle $\alpha$

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2013-2014 Aircraft Design – Conceptual Design
Wing

- **Airfoils (2)**
  - Empirical formula
    - Lift coefficient: \( c_{l\alpha} \approx 6.1 \) (if \( t/c \sim 10-20\% \))
    - Zero-lift angle of attack (in °):
      \[ \alpha_{l0} = \{-\% cambrure, -4c_{Li}, -6c_{Li} \} \]
      for \{NACA-4, 5, 6\} airfoils
    - Design coefficient: \( c_{Li} \approx 0.4 \)
  - Moment (low \( \alpha \)):
    \[ c_m \approx -\pi \frac{c_{\text{max}}}{c} \]
• **Airfoils (3)**

  – **Numerical methods**

    • Do not predict stall velocities
    
    • Panda (be careful: if \(|c_p| > |c_p^*|\) then the solution is not accurate)
      
      

    • xfoil
      

  – **Experimental methods**

    • Curves on next slides

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2013-2014  Aircraft Design – Conceptual Design  11
Wing

- NACA 0009
Wing

• **NACA 0012**
Wing

- **NACA 1410**
Wing

- NACA 2415
- **NACA 64208**

![Graphs showing lift and moment coefficients against section angle of attack and lift coefficient.](image)
- NACA 64209
Wing

- NACA 641-012
Wing

- NACA 64₁-112
• NACA 64\textsubscript{1}-212
• NACA 64₁-412
• NASA SC(2)-0012 (0.8 Mach - supercritical)
  – No experiment close to stall
  – http://ntrs.nasa.gov/search.jsp?N=0
Wing

- NASA SC(2)-0714 (0.75 Mach - supercritical)
Wing

- **Geometry**
  - **Main parameters**
    - Span \( b = 2s \)
    - Aspect ratio \( AR = \frac{b^2}{S} \sim 7-9 \)
    - Total (gross) area \( S \)
    - Taper ratio \( \lambda = \frac{c_{tip}}{c_{root}} \)
    - Quarter chord sweep \( A_{1/4} \)
    \[
    \tan \Lambda_{e2} = \tan \Lambda_{e1} + \frac{4}{AR} \frac{1 - \lambda}{1 + \lambda} (\varepsilon_1 - \varepsilon_2)
    \]
    - Geometrical twist \( \varepsilon_{g\,\text{tip}} \)
Wing

- **Geometry (2)**
  - Aerodynamic center

\[
\text{MAC} = \bar{c} = \frac{2}{S} \int_{0}^{\frac{b}{2}} c^2 \, dy
\]

\[
y_{ac} = \frac{2}{S} \int_{0}^{\frac{b}{2}} c \, y \, dy
\]
Wing

• Geometry (3)
  – Aerodynamic center
    • Position $x_{ac}$ depends on compressibility effects
      
      $$\beta = \sqrt{1 - M^2}$$

      ![Diagram showing wing geometry and aerodynamic center position](image)

      - $\beta AR = 10$
      - $\beta AR = 8$
      - $\beta AR = 6$
      - $\beta AR = 4$
      - $\beta AR = 2$
• **Geometry (4)**
  
  – Allow to compute
    
    • Maximum thickness at \( s/2 \)

    \[
    \frac{\bar{t}}{c} = \frac{3}{10M} \sqrt[3]{\frac{1}{M \cos \Lambda_{1/4}}} - M \cos \Lambda_{1/4} \left[ 1 - \left( \frac{5 + M^2 \cos^2 \Lambda_{1/4}}{5 + (M^*)^2} \right)^{3.5} \right]^{\frac{2}{3}}
    \]

  – Divergence is avoided at \( M \) cruise

  – With

  \[
  M^* = \{1, 1.05, 1.15\} - \frac{C'_L}{4 \cos^2 \Lambda_{1/4}}
  \]

  for \{normal, peaky, supercritical\} airfoils
Wing

• Geometry (5)
  – Allow to compute (2)
    • Fuel volume in the wing
      \[ V_{\text{fuel}} = 0.54 \frac{S^2}{b} \left( \frac{t}{c} \right)_{\text{root}} \frac{1 + \lambda \sqrt{\tau} + \lambda^2 \tau}{(1 + \lambda)^2} \quad \text{with} \quad \tau = \frac{(t/c)_{\text{tip}}}{(t/c)_{\text{root}}} \]
    – If too large, use \( c_{\text{root}}, c_{\text{tip}}, b \) & \( S \) corresponding to a reduced part of the wing

• Wetted surface
  – Surface in contact with the fluid
    \[ S_{\text{wetted}} = 2S_{\text{exp}} \left( 1 + \frac{1}{4} \left( \frac{t/c}{c_{\text{root}}} + \frac{t/c}{c_{\text{tip}} \lambda} \right) \right) \]
Wing

- **Lift**
  - Cruise (reduced angle of attack)
    - Wing lift coefficient
      \[ C_{Lw} = C_{Lw\alpha} \left[ \alpha_{\text{root}} - \alpha_{L0_{\text{root}}} \right] = a \left[ \alpha_{\text{root}} - \alpha_{L0_{\text{root}}} \right] \]
      - \( \alpha_{\text{root}} \): Angle of attack at root of the wing (rad)
      - \( \alpha_{L0_{\text{root}}} \): Angle of attack at root leading to a zero lift of the wing
    » See next slide
  - Slope of wing lift coefficient \((\text{rad}^{-1})\)

\[
\beta = \sqrt{1 - M^2} \\
k = \frac{\beta c_{l\alpha}}{2\pi} \\
\tan \Lambda_{\beta} = \frac{\tan \Lambda_{1/4}}{\sqrt{1 - M^2}}
\]

\[
\beta a = \frac{2}{\beta AR} + \sqrt{\left( \frac{1}{k \cos \Lambda_{\beta}} \right)^2 + \left( \frac{2}{\beta AR} \right)^2}
\]
Wing

• Lift (2)
  – Cruise (reduced angle of attack) (2)
    • Zero-lift angle of attack at root
      \[ \alpha_{L_{0\text{root}}} = \alpha_{l_{0\text{root}}} + \alpha_{01} \varepsilon_{atip} \]
    – Geometrical twist
      » Example: lofted
      \[ \varepsilon_{g} = \varepsilon_{g_{\text{tip}}} \frac{\lambda \frac{y}{s}}{1 - (1 - \lambda) \frac{y}{s}} \]
    – Local aerodynamic twist \( \alpha_{01} \)
      » \(-\alpha_{01}\) see picture
**Wing**

- **Lift (3)**
  - Cruise (reduced angle of attack) (3)
  - Zero-lift angle of attack at root
    \[ \alpha_{L0_{\text{root}}} = \alpha_{l0_{\text{root}}} + \alpha_{01}\varepsilon_{a_{\text{tip}}} \]
    - Aerodynamic twist \[ \varepsilon_{a_{\text{tip}}} = \varepsilon_{g_{\text{tip}}} + \alpha_{l0_{\text{root}}} - \alpha_{l0_{\text{tip}}} \]
      - <0 pour un washout
      - Zero-lift angle of attack of the airfoil \( \alpha_{l0} \) can change between root and tip if the airfoil has an evolving shape
    - Purpose: Stall initiated at \( \sim 0.4 \) s
• **Maximum lift**
  - Maximum lift coefficient in approach or at takeoff ($M << 1$)
  - Curves without high-lift devices
    \[
    C_{L_{\text{max}}} = \cos \Theta_{1/4} \left\{ 0.88, 0.95 \right\} \frac{(C_{l_{\text{max}}})_{\text{root}} + (C_{l_{\text{max}}})_{\text{tip}}}{2} \quad \{ \lambda = 1, \lambda \neq 1 \}
    \]
    - Airfoil NACA-4 5 6 digits, see pictures
    - Supercritical airfoil with rear loading: 10% larger than NACA-5
### Wing

- **Maximum lift (2)**
  - Maximum lift coefficient in approach or at takeoff \( M \ll 1 \) (2)
  - With high lift devices
    - Device & angle depend on
      - Approach
      - Landing
      - Takeoff (drag has to be reduced)

#### Table: High-Lift Device Parameters

<table>
<thead>
<tr>
<th>HIGH-LIFT DEVICE</th>
<th>TYPICAL FLAP ANGLE</th>
<th>( \frac{C_{l_{max}}}{\cos \alpha} \times 0.25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAILING EDGE</td>
<td>LEADING EDGE</td>
<td>TAKEOFF</td>
</tr>
<tr>
<td>PLAIN</td>
<td>-</td>
<td>20°</td>
</tr>
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<td>-</td>
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</tr>
<tr>
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<td>20°</td>
</tr>
<tr>
<td></td>
<td>SLAT</td>
<td>20°</td>
</tr>
</tbody>
</table>
Wing

- Maximum lift (3)
  - Maximum lift coefficient in approach or at takeoff ($M << 1$) (3)
    - With high lift devices (2)
      - Stall (equivalent) velocities
        $$V_s(0) = \sqrt{\frac{W(0)}{S} \frac{2}{\rho} \frac{1}{1.133C_{L_{\max}}(0)}}$$
      - $V_s$: flaps down (out)
      - $V_{s0}$: flaps in approach configuration (weight $W_0$ at landing)

### Table: High-Lift Devices and Typical Flap Angles

<table>
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<th>$C_{L_{\max}} / \cos \alpha_{25}$</th>
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</table>

Lost of velocity resulting from a maneuver
Stability

- **Longitudinal balance**
  
  - Lift
    
    \[ C_L = C_{Lw} + C_{LT} \frac{S_T}{S} \quad \Rightarrow \quad C_L = (C_{L\alpha})_{plane} \left( \alpha_f - (\alpha_{L0})_f \right) \]
    
  - Angle of attack of the fuselage \( \alpha_f \)
  - Zero-lift angle of attack of the fuselage \( (\alpha_{L0})_f \)
Stability

- **Longitudinal balance (2)**
  - Moment
    - Moment around gravity center
      \[
      C_m = C_{m0} + C_{L_w} \frac{x_{cg} - x_{acw}}{\bar{c}} + C_{mT} - C_{LT} \frac{S_T l_T}{\bar{c}S}
      \]
    - Pitching moment of the wing
      \[
      C_{m0} = \frac{2}{S \bar{c}} \int_0^{\frac{\alpha}{2}} c_m c^2 dy
      \]
      Zero for symmetrical airfoils
Stability

• Trimmed configuration
  – Equations

  \[ C_L = C_{Lw} + C_{LT} \frac{S_T}{S} \]

  \[ C_m = C_{m0} + C_{Lw} \frac{x_{cg} - x_{acw}}{\bar{c}} + C_{MT} - C_{LT} \frac{S_T l_T}{\bar{c} S} \]

  – At equilibrium (steady flight)

  \[ C_m = 0 \quad \Rightarrow \quad C_L = C_{Lw} + C_{LT} \frac{S_T}{S} = C_{Lw} \left( 1 + (h - h_0) \frac{\bar{c}}{l_T} \right) + \frac{\bar{c}}{l_T} C_{m0} \]
Stability

- **Trimmed configuration (2)**
  - Angle of incidence of the wing $i_w$
    - Angle between the fuselage and the root chord
    - In cruise
      - $\alpha_f \approx 0$ so the fuselage is horizontal
      - Lift is known from the weight $C_L (\alpha_f = 0) = C_{L0}$
Stability

- **Trimmed configuration (3)**
  - Angle of incidence of the wing $i_w$ (2)

- **Equations**

\[
\begin{align*}
C_L (\alpha_f = 0) &= C_{L0} \\
C_L &= C_{Lw} + C_{LT} \frac{S_T}{S} = C_{Lw} \left( 1 + (h - h_0) \frac{\bar{c}}{l_T} \right) + \frac{\bar{c}}{l_T} C_{m0} \\
C_{Lw} &= a \left[ \alpha_{\text{root}} - \alpha_{L0,\text{root}} \right] \\
\alpha_{\text{root}} &= \alpha_f + i_w \\
\alpha_{L0,\text{root}} &= \alpha_{l0,\text{root}} + \alpha_{01} \varepsilon_{a \text{tip}} \\
i_w &= \frac{C_{Lw}^*}{a} + \alpha_{01} \varepsilon_{a \text{tip}} + (\alpha_{l0})_{\text{root}} \\
C_{Lw}^* &= \frac{C_{L0} - \frac{\bar{c}}{l_T} C_{m0}}{1 + (h - h_0) \frac{\bar{c}}{l_T}}
\end{align*}
\]
Stability

- Trimmed configuration (4)
  - Value $\alpha_f = 0$ is obtained for one single value of the lift, so for a given weight
  - But weight changes during flight, as well as the cg location
  - To define $i_w$, values of $C_{L0}$ & $x_{cg}$ are taken for
    - 50% of maximum payload
    - 50% of fuel capacity
  - Lift curve of a trimmed aircraft

\[
\begin{align*}
C_L &= C_{Lw} + C_{LT} \frac{S_T}{S} = C_{Lw} \left( 1 + (h - h_0) \frac{\bar{c}}{l_T} \right) + \frac{\bar{c}}{l_T} C_{m0} \\
C_L &= (C_{L\alpha})_{plane} \left( \alpha_f - (\alpha_{L0})_f \right) \\
\frac{(\alpha_{L0})_f}{C_{L\alpha_{plane}}} &= - \frac{C_{L0}}{C_{L\alpha_{plane}}} \\
C_{L\alpha_{plane}} &= \left( 1 + (h - h_0) \frac{\bar{c}}{l_T} \right) a
\end{align*}
\]
Stability

• **Stick-fixed neutral point** \( x_n = h_n \bar{c} \)
  - CG position for which \( \frac{\partial C_m}{\partial C_{Lw}} = 0 \) with elevators blocked

• When elevators are blocked, stability requires \( \frac{\partial C_m}{\partial \alpha} < 0 \)

• As \( C_L \sim \text{proportional to } \alpha \), the stability limit is approximated by \( \frac{\partial C_m}{\partial C_{Lw}} = 0 \)

• But as \( C_m = C_{m0} + C_{Lw} \frac{x_{cg} - x_{acw}}{\bar{c}} + C_{mT} - C_{LT} \frac{S_T l_T}{\bar{c} S} \)

  the stability depends on the cg position

• Neutral point is the position of the cg leading to \( \frac{\partial C_m}{\partial C_{Lw}} = 0 \)
Stability

- **Stick-fixed neutral point** \( x_n = h_n \bar{c} \) (2)

  - **Definition**

    - As
      \[
      C_m = C_{m0} + C_{Lw} \frac{x_{cg} - x_{acw}}{\bar{c}} + C_{mT} - C_{LT} \frac{S_T l_T}{\bar{c}S}
      \]

    \[
    h_n = h_0 + \frac{dC_{LT}}{dC_{Lw}} \frac{l_T S_T}{\bar{c}S}
    \]

  - But this not correct as fuselage is destabilizing (low momentum but high derivative)
Stability

- **Stick-fixed neutral point** \( x_n = h_n \bar{c} \) (3)
  - Definition (2)
  - Fuselage effect

\[
C_m = C_{m0} + C_{Lw} \frac{x_{cg} - x_{acw}}{\bar{c}} + C_{mT} - C_{LT} \frac{S_T l_T}{\bar{c} S} + C_{m_{fus}}
\]

\[
h_n = h_0 + \frac{dC_{LT}}{dC_{Lw}} \frac{l_T S_T}{\bar{c} S} - \frac{dC_{m_{fus}}}{dC_{Lw}}
\]
Stability

- **Stick-fixed neutral point**  \( x_n = h_n \bar{c} \)  
  
  - Position  \( h_n = h_0 + \frac{dC_{LT}}{dC_{Lw}} l_T S_T - \frac{dC_{m_{fus}}}{dC_{Lw}} \)

- **Stick-fixed tail lift slope** \((\eta, \beta_\eta \text{ constant})\)

- **Tail lift**  \( C_{LT} = a_1 (\alpha_T - \alpha_{T0}) + a_2 \eta + a_3 \beta_\eta \)

- **Attack angle of horizontal tail in terms of downwash** \(\varepsilon\):  \( \alpha_T = \alpha_{\text{root}} - \varepsilon + \eta_T \)

  with  \( \varepsilon \approx \frac{d\varepsilon}{d\alpha} (\alpha_{\text{root}} - \alpha_{L0\text{root}}) \)

- **As**  \( C_{Lw} = a (\alpha_{\text{root}} - \alpha_{L0\text{root}}) \Rightarrow \alpha_T \approx \frac{C_{Lw}}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) + \alpha_{L0\text{root}} + \eta_T \)

- **Eventually**  \( \left(\frac{dC_{LT}}{dC_{Lw}}\right)^{\text{stick-fixed}} = \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \)
**Stability**

- **Stick-fixed neutral point**  \( x_n = h_n \bar{c} \)  \( (5) \)

  - **Downwash**

    - **Gradient of downwash resulting from the wing vortex**

      \[
      \frac{\partial \varepsilon}{\partial \alpha} = 1.75 \frac{a}{\pi AR \left( \frac{2\lambda l_t}{b} \right)^{\frac{1}{4}} (1 + |m|)}
      \]

      \( l_t = rb/2 \)

      \( l_t = \) distance between ac of wing and ac of horizontal tail
Stability

- Stick-fixed neutral point \( x_n = h_n \bar{c} \) (6)
- Fuselage effect
  - Empirical method NACA TR711

\[
\frac{dC_{m_fus}}{dC_{Lw}} = \frac{k_{fus width_fus}^2 length_fus}{S \bar{c} a}
\]

<table>
<thead>
<tr>
<th>( m_{fus} )</th>
<th>( k_{fus} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.115</td>
</tr>
<tr>
<td>0.2</td>
<td>0.172</td>
</tr>
<tr>
<td>0.3</td>
<td>0.344</td>
</tr>
<tr>
<td>0.4</td>
<td>0.487</td>
</tr>
<tr>
<td>0.5</td>
<td>0.688</td>
</tr>
<tr>
<td>0.6</td>
<td>0.888</td>
</tr>
<tr>
<td>0.7</td>
<td>1.146</td>
</tr>
</tbody>
</table>
Stability

- Stability margin
  - Stability requires \( \frac{\partial C_m}{\partial \alpha} < 0 \)
  - The stability is measured by the stability margin
    \[ K_n = \frac{x_n - x_{cg}}{c} = h_n - h > 0 \]
  - FAA requirement
    - Stable enough \( \iff \) \( K_n > 5\% \)
  - Enough maneuverability
    - \( K_n <~ 10\% \)
    - If T tail, in order of avoiding deep stall: \( 10\% <~ K_n < 20\% \)
Stability

- Stability margin (2)
  - \[ K_n = \frac{x_n - x_{cg}}{c} = h_n - h > 0 \]
  - Flight conditions
    - \( h_0 \) depends on velocity
    - CG location
      - Depends on payload
      - Changes during the flight as fuel is burned
    - Whatever the flight condition is \( K_n \) should remains > 5%
**Stability**

- **Stability margin (3)**
  
  \[ K_n = \frac{x_n - x_{cg}}{c} = h_n - h > 0 \]

- In general during cruise
  - CG close to 0.25 $\bar{c}$
    - Allows reducing the drag due to the tail
  - Tail can act in negative lift (can reach 5% of the weight)
Stability

• Angle of incidence of horizontal tail $i_T$
  
  - Tail lift should be equal to $C_{LT} = a_1 \left[ \frac{C_L}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T + \alpha_{L0root}\right]$ for trimmed cruise ($\alpha_f = 0$) & $\alpha_{T0} = 0$, with

\[
\begin{align*}
C_L &= C_{L0} = -C_{L\alpha\text{plane}}(\alpha_{L0})_f \\
C_{Lw} &= C_{Lw}^* = \frac{C_{L0} - \frac{\bar{c}}{l_T} C_{m0}}{1 + (h - h_0) \frac{\bar{c}}{l_T}} \\
C_{LT} &= [C_{m0} + C_{Lw}^* (h - h_0)] \frac{S\bar{c}}{S_T l_T}
\end{align*}
\]
Stability

• Angle of incidence of horizontal tail \( i_T \) (2)

\[
C_{LT} = a_1 \left[ \frac{C_{Lw}}{a} \left( 1 - \frac{d \varepsilon}{d \alpha} \right) + \eta_T + \alpha_{L0\text{root}} \right]
\]

\[
i_w = \frac{C_{Lw}^*}{a} + \alpha_{01} \varepsilon_{atip} + (\alpha_{t0})_{\text{root}}
\]

\[
C_{Lw} = C_{Lw}^* = \frac{C_{L0} - \frac{c}{l_T} C_{m0}}{1 + (h - h_0) \frac{c}{l_T}}
\]

\[
C_{LT} = \left[ C_{m0} + C_{Lw}^* (h - h_0) \right] \frac{S \bar{c}}{S_T l_T}
\]

- Equations

- Tail incidence angle

  • From \( \eta_T \)

\[
\eta_T = i_T - i_w = i_T - \frac{C_{Lw}^*}{a} - \alpha_{L0\text{root}}
\]

\[
i_T = \frac{C_{m0} + C_{Lw}^* (h - h_0)}{a_1 \frac{S_T l_T}{S \bar{c}}} + \frac{d \varepsilon}{d \alpha} C_{Lw}^*
\]

- Generally \( i_T \) such that \( \alpha_T < \alpha_{\text{root}} \)
Horizontal tail

- **Geometry**
  - Parameters
    - Span \( b_T = 2s_T \)
    - Aspect ratio \( AR_T = b_T^2/S_T \approx 3-6 \)
    - Taper ratio \( \lambda_T = c_{T \text{tip}}/c_{T \text{root}} \approx 0.3-0.5 \)
      - Reduced weight
    - Sweep angle \( \Lambda_{T1/4} \)
      - 5° more than wings in order to avoid shock waves
    - Airfoil: symmetrical, reduced thickness (e.g. NACA0012)
  - Design criteria
    - Longitudinal static equilibrium
    - Longitudinal stability
      - Damping for short period & Phugoïd modes
    - Powerful enough to allow maneuvers
      - Rotation at take off
    - Should stall after the wing
Horizontal tail

- **Outputs**
  - Proceed as for wings
    - Thickness to remain below critical Mach number
    - Lift coefficient slope as for wing
    - Lift coefficient
      - Should account for wing downwash effect

\[
C_{LT} = a_1 \left[ \frac{C_{Lw}}{a} \left( 1 - \frac{d\epsilon}{d\alpha} \right) + \eta T + \alpha L_{0\text{root}} \right]
\]

  - if symmetrical airfoil

- Aerodynamic center computed as for wing
- No pitching moment if symmetrical airfoil
- No aerodynamic twist (neglected)
• **Quick design**
  
  − Stability depends mainly on $S_T/S \sim 0.2-0.4$
  − Maneuverability depends mainly on $\frac{l_T S_T}{c S} \sim 0.5-1.2$
  
• Approach velocity $V_a = 1.3 V_{so}$
• Geometry
  – Parameters
    • Span $b_F$
    • Aspect ratio $AR_F = b_F^2/S_F$
      – ~ 0.7
      – For T tail ~ 2
    • Taper ratio $\lambda_F = c_{F_{\text{tip}}} / c_{F_{\text{root}}}$
    • Sweep angle $\Lambda_{F_{1/4}} : 30$ to $40^\circ$
  – Airfoil
    – Symmetrical
    – Low thickness (e.g. NACA0012)
    – No twist
  • Distance between cg and fin ac $l_F$
  – Design criteria
    • No stall at maximum rudder deflection
    • Maneuverability ensured after engine failure
    • Landing with side wind of 55 km/h
    • Lateral static & dynamic stabilities (Dutch roll)
### Fin

- **Loadings**
  - Lift coefficient: \( C_{LF} = \frac{L_F}{\frac{1}{2} \rho V^2 S_F} \)
  - Yaw coefficient: \( C_N = C_{LF} \frac{S_F l_F}{S_b} \)
  - Slope with respect to yaw angle \( \beta \): \( C_{N\beta} = \partial_{\beta} C_N \)
Fin

- **Quick design**
  - Lateral stability (most severe criterion for engines attached on fuselage)
    \[
    C_{N\beta_f} = -K_{\beta} \frac{S_{fs} length_{fus}}{Sb} \left( \frac{h_{f1}}{h_{f2}} \right)^{\frac{1}{2}} \left( \frac{b_{f2}}{b_{f1}} \right)^{\frac{1}{3}}
    \]
    \[
    K_{\beta} = 0.3 \frac{l_{cg}}{length_{fus}} + 0.75 \frac{h_{f_{max}}}{length_{fus}} - 0.105
    \]
  - {High, mid, low}-mounted wing effect
    \[
    C_{N\beta_i} = \{-0.017, 0.012, 0.024\}
    \]

[Diagram showing aircraft design with annotations for fuselage effect and lateral stability criteria.]
• **Quick design (2)**
  - Engine failure (most severe criterion for wing-mounted engines)
    - Takeoff configuration (critical as larger thrust)
    - Engine thrust $\Delta T_e$ at $Y_e$ from fuselage axis
    - Maximal rudder deflection $\delta_{r,\text{max}} \sim 30^\circ$
    - Effect of rudder measured by $k_{\delta r}$
Fin

- Quick design (3)
  - Engine failure (wing-mounted engines) (2)
    - Effect of fin: $k_v = 1.1$ for T-tail, 1 for other tails

Thrust & weight in kg or N

\[
Y_e \left[ \Delta T e C_{L\text{max}} \right]_{\text{take off}} = \frac{1}{l_F} \left( W_{\text{to}} - W_{\text{payload}} \right)_{\text{max}}
\]
Drag

• In cruise
  – Cruise drag is critical to compute
    • Required thrust
    • Fuel consumption
  – Detailed method
    • Compute contribution of each aircraft component on
      – Induced drag (due to vortex)
      – Profile drag (friction & pressure)
      – Interference drag
        » Interaction between components
        » Account for $C_{Lw} \neq C_L$ during normalization
  – Polar of the aircraft
    • Drag can be plotted in term of lift
      • $C_D = C_{D0} + \frac{C_{L\text{plane}}^2}{e\pi AR}$
Drag

- In cruise (2)
  - Quick method
    \[ C_D = C_{D0} + \frac{C_{L}^2 \text{plane}}{e\pi AR} \]
  - With \( e \) and \( C_{D0} \) from statistics

- Meaningful only if the design is correct
  - A wrong design would lead to higher drag
  - This would not appear with this method

<table>
<thead>
<tr>
<th></th>
<th>( C_{D0} )</th>
<th>( e )</th>
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<tbody>
<tr>
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<td>large turbopropeller</td>
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<td>agricultural aircraft:</td>
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<tr>
<td>- spray system removed</td>
<td>.060</td>
<td>.65 - .75</td>
</tr>
<tr>
<td>- spray system installed</td>
<td>.070 - .080</td>
<td>.65 - .75</td>
</tr>
</tbody>
</table>

* The higher the sweep angle, the lower the e-factor
Drag

- In cruise (3)
  - Compressibility effect
    - Low if correct wing design
    - Divergence Mach larger than cruise Mach ($t/c$ small enough)
  - In this case, add, to the drag coefficient, the compressibility effect obtained by
    \[
    \Delta_{\text{comp}} C_D = \begin{cases} 
    0.0005 & \text{long range conditions} \\
    0.002 & \text{high speed conditions}
    \end{cases}
    \]
Drag

- Landing & takeoff
  - Low velocity drag (flaps down) is critical to compute
    - Thrust required at takeoff
    - Maximum payload
      - Can depend on the airport
        - Temperature
        - Runaway
Drag

- **Landing & takeoff (2)**
  - Plane velocity
    - Takeoff & landing safety speed
      - At 35 ft altitude
      - $V_2 = 1.2 \ V_s(0)$
  - Polar

\[ C_{DV_2} = C_0 + \frac{C_{L}^2 V_2^2}{E \pi AR} \]

- Slats out
  - $C_0 = 0.018$
  - $E = 0.7$

- Slats in
  - $C_0 = 0.005$
  - $E = 0.61$

- $C_L$ with high lift devices

<table>
<thead>
<tr>
<th>HIGH-LIFT DEVICE</th>
<th>TYPICAL FLAP ANGLE</th>
<th>$C_{L_{\max}} / \cos \alpha_{\text{25}}$</th>
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<tbody>
<tr>
<td>TRAILING EDGE</td>
<td>LEADING EDGE</td>
<td>TAKEOFF</td>
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</tr>
<tr>
<td>Single Slotted</td>
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<tr>
<td>Fowler</td>
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<td>15°</td>
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<tr>
<td>Double Slotted</td>
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<td>20°</td>
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<td>Triple Slotted</td>
<td>SLAT</td>
<td>20°</td>
</tr>
<tr>
<td></td>
<td>SLAT</td>
<td></td>
</tr>
</tbody>
</table>
Drag

- **Takeoff with one engine**
  - Corrected polar
    
    \[ C_{D_{V_2}} = C_0 + \frac{C_L^2}{E \pi AR} \]

- If low thrust (landing)
  - Reduce \( E \) by
    - 4% for wing-mounted engines
    - 2% for engines on the fuselage

- If high thrust (takeoff)
  - Compute explicitly effects of
    - Wind-milling
    - Drag due to the rudder
Drag

• **Takeoff with one engine (2)**
  - Method to compute the drag leads to coefficients of the form $C_D S$
    - Has to be divided by the gross wing area $S$ to get back to $C_D$
    - The terms have to be added to the $C_D$ obtained
      
      \[ C_{DV_2} = C_0 + \frac{C_{L_2V_2}^2}{E \pi AR} \]

      - 2 parts: wind-millings and rudder
      \[ (C_D S)_{ef} = (C_D S)_{wm} + (C_D S)_{rud} \]

    - Wind-milling
      - \[ (C_D S)_{wm} \approx 0.0785 D_{inlet}^2 \]
Drag

- **Takeoff with one engine (3)**
  - **Rudder**
    - Moment due to
      - Thrust unbalance $\Delta T_e$
      - Acting at $Y_e$ from fuselage axis
    - Balanced by rudder load
      \[ C_{Y_F} = \frac{\Delta T_e \cdot Y_e}{qS_F \cdot l_F} \]
    - Leads to a drag
      - Induced part (vortex)
        \[ (C_D S)_{v_r u d} = \frac{C_{Y_F}^2 \cdot S_F}{\pi A R_F} \]
      - Profile part (friction & pressure)
        \[ (C_D S)_{p_r u d} = \frac{2.3}{\pi A R_F^{\frac{4}{3}}} \sqrt{S_F S_r} \left( \cos \Lambda_F \frac{1}{4} \right)^{\frac{1}{3}} C_{Y_F}^2 \]
Engine performance

- **Data**

<table>
<thead>
<tr>
<th>Engine</th>
<th>SLS thrust (KN)</th>
<th>Cruise thrust (KN)</th>
<th>SLS specific fuel consumption (sfc) (kg/daN.h)</th>
<th>Cruise specific fuel consumption (sfc) (kg/daN.h)</th>
<th>By pass ratio</th>
<th>Diameter (mm)</th>
<th>Length (mm)</th>
<th>Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF6-80C2</td>
<td>262.4</td>
<td>46.7</td>
<td>0.356</td>
<td>0.585</td>
<td>5.09</td>
<td>2362</td>
<td>4036</td>
<td>4058</td>
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<tr>
<td>CF34-3A</td>
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<td>6.8</td>
<td>0.357</td>
<td>0.718</td>
<td>6.2</td>
<td>1118</td>
<td>2616</td>
<td>737</td>
</tr>
</tbody>
</table>

- **Sea Level Static**
  - $M = 0$
  - Standard atmospheric conditions at sea level
  - SLS thrust: $T_{to}$ (to is for takeoff)
  
  \[ \frac{T}{T_{to}} \simeq 1 - \frac{0.45M (1 + BPR)}{\sqrt{1 + 0.75BPR}} + (0.6 + 0.11BPR) M^2 \]

- **Cruise**
  - Standard atmosphere at a given altitude

- **Specific Fuel Consumption**
  - Fuel consumption
    - Per unit of thrust and
    - Per unit of time
• **Component weight can be estimated**
  – For conceptual design
  – Based on statistical results of traditional aluminum structures
  – Example: wing

![Comparative Wing Weights](image-url)
Structural weight

- **Structural weight [lbs]**
  - **Wing with ailerons**
    
    \[ W_w = 4.22 S + 1.642 \times 10^{-6} \frac{n_{ultim} b^3 \sqrt{W_{to} ZFW}}{t/c_{avg}} \sqrt{1 + 2\lambda} \left(1 + \frac{\Lambda}{S}\right) \]
    
    \( S \): gross area of the wing [ft\(^2\)]
    \( W_{to} \): take off weight [lb]
    \( ZFW \): zero fuel weight [lb]
    \( b \): span [ft]
    \( \Lambda \): sweep angle of the structural axis
    \( \lambda \): taper (\( c_{tip}/c_{root} \)),
    \( t \): airfoil thickness [ft]
    
    - **Horizontal empennage & elevators**
      
      \[ W_T = 5.25 S_{T_{exp}} + 0.8 \times 10^{-6} \frac{n_{ultim} b_T^3 W_{to} \bar{c}}{t_T/c_T_{avg}} \sqrt{S_{T_{exp}}^{3/2}} \sqrt{1 + \frac{\Lambda_T}{l_T} \frac{S_T^2}{S_{T_{exp}}^{3/2}}} \]
      
      \( S_{T_{exp}} \): exposed empennage area [ft\(^2\)]
      \( l_T \): distance plane CG to empennage CP [ft]
      \( \bar{c} \): average aerodynamic chord of the wing [ft]
      \( S_T \): gross empennage area [ft\(^2\)]
      \( b_T \): empennage span [ft]
      \( t_T \): empennage airfoil thickness [ft]
      \( c_T \): empennage chord [ft]
      \( \Lambda_T \): sweep angle of empennage structural axis
Structural weight

- **Structural weight [lbs]** (2)
  - **Fin without rudder**
    \[ W_{F'} = 2.62 \, S_F + 1.5 \times 10^{-5} \, n_{\text{ultim}} \, b_F^3 \left( 8 + 0.44 \frac{W_\infty}{S} \right) \frac{t_F}{c_F} \, \text{avg} \, \cos^2 \Lambda_F \]

  \( S_F \): fin area [ft\(^2\)]

  \( t_F \): fin airfoil thickness [ft]

  \( \Lambda_F \): sweep angle of fin structural axis

  \( b_F \): fin height [ft]

  \( c_F \): fin chord [ft]

  \( S \): gross surface of wing [ft\(^2\)]

  \( W \): weight

  \( S \): gross surface

  \( L \): sweep angle of fin structural axis

  \( W \): weight

  \( S \): gross surface

- **Rudder:** \( \frac{W_r}{S_r} \sim 1.6 \, W_{F'}/S_F \)

- **Fuselage**
  - **Pressure index**
    \[ I_p = 1.5 \times 10^{-3} \, \Delta p_{\text{max}} \, \text{width}_{\text{fus}} \]

  \( \Delta p \) [lb/ft\(^2\)] (cabin pressure \( \sim 2600 \text{m} \))

  \( \text{width}_{\text{fus}} \): fuselage width

  \( \text{height}_{\text{fus}} \): fuselage height

  \( n_{\text{limit}} \) at ZFW

  \( ZFW = W_w - W_{\text{wing-mounted engines}} \)

  \( W_{\text{fus}} \): fuselage weight

  \( S_{\text{fus, wetted}} \): fuselage wetted surface

  \( I_{\text{fus}} \): fuselage index

- **Bending index**

  \[ I_b = 1.91 \times 10^{-4} \, n_{\text{limit}} \, \text{at ZFW} \, (ZFW - W_w - W_{\text{wing-mounted engines}}) \frac{\text{length}_{\text{fus}}}{\text{height}_{\text{fus}}^2} \]

- **Weight depends on wetted area** \( S_{\text{wetted}} \) [ft\(^2\)] (area in direct contact with air)

  \[ W_{\text{fus}} = (1.051 + 0.102 \, I_{\text{fus}}) \, S_{\text{fus, wetted}} \]

  \( I_{\text{fus}} \): fuselage index

  \[ I_{\text{fus}} = \begin{cases} 
  I_p & \text{if } I_p > I_b \\
  \frac{I_p^2 + I_b^2}{2I_b} & \text{if } I_p < I_b 
  \end{cases} \]

  \( I_p \): pressure index

  \( I_b \): bending index
Structural weight

- **Structural weight [lbs] (3)**
  - **Systems**
    - Landing gear
    - Hydromechanical system of control surfaces
      \[ W_{SC} = I_{SC} (S_{Texp} + S_F) \]
      \[ I_{sc} \text{ [lb/ft}^2\text{]} : 3.5, 2.5 \text{ or } 1.7 \text{ (fully, partially or not powered)} \]
    - Propulsion
      \[ W_{prop} = 1.6W_{eng} \sim 0.6486 \ T_{to}^{0.9255} \]
      \[ T_{to} : \text{Static thrust (M 0) at sea level [lbf], } *1lbf \sim 4.4 \text{ N} \]
  - **Propulsion**
    - APU
    - Instruments (business, domestic, transatlantic)
    - Hydraulics
    - Electrical
    - Electronics (business, domestic, transatlantic)
    - Furnishing  if < 300 seats
      \[ W_{furn} \sim (43.7-0.037 \ N_{seats}) \ N_{seats} + 46 \ N_{seats} \]
      if > 300 seats
      \[ W_{furn} \sim (43.7-0.037*300) \ N_{seats} + 46 \ N_{seats} \]
    - AC & deicing
  - **Payload** (\(W_{payload}\))
    - Operating items (class dependant)
    - Flight crew
    - Flight attendant
    - Passengers (people and luggage)
  - **Definitions**
    - ZFW: Sum of these components
      \[ ZFW = \Sigma W_i \]
### Structural weight

- **Examples**

<table>
<thead>
<tr>
<th>Aircraft System</th>
<th>CITATION-500</th>
<th>MDAT-30</th>
<th>MDAT-50</th>
<th>F-28</th>
<th>MDAT-70</th>
<th>DC-9-10</th>
<th>BAC-111</th>
<th>DC-9-30</th>
<th>737-200</th>
<th>727-100</th>
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<tbody>
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<td>Wing System</td>
<td>1,020</td>
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<td>20</td>
<td>19</td>
<td>9</td>
<td>57</td>
<td>--</td>
<td>15</td>
</tr>
</tbody>
</table>

| Empty Weight (less Dry Engine)           | 5,377        | 17,985  | 23,312  | 29,178| 29,748  | 41,962  | 46,328  | 49,770  | 51,240  | 75,528  |
| Dry Engine Weight                        | 1,002        | 2,480   | 3,373   | 4,327| 4,392   | 6,113   | 5,434   | 6,160   | 6,212   | 9,322   |
| Empty Weight (M.E.W.)                    | 6,379        | 20,465  | 26,685  | 33,505| 34,140  | 48,075  | 51,762  | 55,930  | 57,452  | 84,850  |
| Takeoff Gross Weight                     | 11,650       | 34,480  | 46,850  | 62,000| 61,000  | 86,300  | 99,650  | 108,000 | 104,000 | 161,000 |
### Structural weight

**Examples**

<table>
<thead>
<tr>
<th>Aircraft System</th>
<th>727-200</th>
<th>707-320</th>
<th>DC-8-55</th>
<th>DC-8-62</th>
<th>DC-10-10</th>
<th>L-1011</th>
<th>DC-10-40</th>
<th>747</th>
<th>SCAT-15</th>
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<tbody>
<tr>
<td>Wing System</td>
<td>18,529</td>
<td>28,647</td>
<td>34,909</td>
<td>36,247</td>
<td>48,990</td>
<td>47,401</td>
<td>57,748</td>
<td>88,741</td>
<td>83,940</td>
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<td>Tail System</td>
<td>4,142</td>
<td>6,004</td>
<td>4,952</td>
<td>4,930</td>
<td>13,657</td>
<td>8,570</td>
<td>14,454</td>
<td>11,958</td>
<td>8,590</td>
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<td>Body System</td>
<td>22,415</td>
<td>22,299</td>
<td>22,246</td>
<td>23,704</td>
<td>44,790</td>
<td>49,432</td>
<td>46,522</td>
<td>68,452</td>
<td>54,322</td>
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<td>Lighting Gear System</td>
<td>7,948</td>
<td>11,216</td>
<td>11,682</td>
<td>11,449</td>
<td>18,581</td>
<td>19,923</td>
<td>25,085</td>
<td>32,220</td>
<td>28,720</td>
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<td>Nacelle System</td>
<td>2,225</td>
<td>3,176</td>
<td>4,644</td>
<td>6,648</td>
<td>8,493</td>
<td>8,916</td>
<td>9,328</td>
<td>10,830</td>
<td>15,650</td>
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<tr>
<td>Propulsion System (less Dry Engine)</td>
<td>3,022</td>
<td>5,306</td>
<td>9,410</td>
<td>7,840</td>
<td>7,673</td>
<td>8,279</td>
<td>13,503</td>
<td>9,605</td>
<td>6,310</td>
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<td>Flight Controls System (less Auto Pilot)</td>
<td>2,984</td>
<td>2,139</td>
<td>2,035</td>
<td>2,098</td>
<td>5,120</td>
<td>5,068</td>
<td>5,198</td>
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<td>849</td>
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<td>0</td>
<td>0</td>
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<td>1,202</td>
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<td>1,002</td>
<td>916</td>
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<td>Hydraulic and Pneumatic Group</td>
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<td>2,250</td>
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<td>Electrical System</td>
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<td>3,944</td>
<td>2,414</td>
<td>2,752</td>
<td>5,366</td>
<td>5,490</td>
<td>5,293</td>
<td>5,305</td>
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<tr>
<td>Avionics System (incl. Auto Pilot)</td>
<td>1,896</td>
<td>1,815</td>
<td>1,870</td>
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<td>2,827</td>
<td>2,801</td>
<td>3,186</td>
<td>4,134</td>
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<td>Furnishings and Equipment System</td>
<td>14,702</td>
<td>16,875</td>
<td>15,884</td>
<td>15,340</td>
<td>38,072</td>
<td>32,829</td>
<td>33,114</td>
<td>48,007</td>
<td>20,615</td>
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<td>Air Conditioning System</td>
<td>1,802</td>
<td>1,602</td>
<td>2,388</td>
<td>2,296</td>
<td>2,386</td>
<td>3,344</td>
<td>2,527</td>
<td>3,634</td>
<td>2,820</td>
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<tr>
<td>Anti-icing System</td>
<td>666</td>
<td>626</td>
<td>794</td>
<td>673</td>
<td>416</td>
<td>296</td>
<td>555</td>
<td>413</td>
<td>210</td>
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<tr>
<td>Load and Handling System</td>
<td>19</td>
<td>--</td>
<td>55</td>
<td>54</td>
<td>62</td>
<td>--</td>
<td>62</td>
<td>228**</td>
<td>--</td>
</tr>
</tbody>
</table>

| Empty Weight (less Dry Engine)  | 86,017  | 105,756 | 116,535 | 118,749 | 203,521 | 198,968 | 224,148 | 297,867 | 256,204 |
| Dry Engine Weight               | 9,678   | 19,420  | 16,936  | 17,316  | 23,229  | 30,046  | 25,587  | 35,700  | 45,020  |

| Empty Weight (M.E.W.)           | 95,695  | 125,176 | 133,471 | 136,065 | 226,750 | 229,014 | 249,735 | 333,567 | 301,224 |
| Takeoff Gross Weight            | 175,000 | 312,000 | 325,000 | 335,000 | 430,000 | 430,000 | 565,000 | 775,000 | 631,000 |
Structural weight

• CG locations
  – Wing: 30% chord at wing MAC
  – Horizontal tail: 30% chord at 35% semi-span
  – Fin: 30% chord at 35% of vertical height
  – Surface controls: 40% chord on wing MAC
  – Fuselage: 45% of fuselage length
  – Main gear: located sufficiently aft of aft c.g. to permit 5% - 8% of load on nose gear
  – Hydraulics: 75% at wing c.g., 25% at tail c.g.
  – AC / deicing: End of fuse nose section
  – Propulsion: 50% of nacelle length for each engine
  – Electrical: 75% at fuselage center, 25% at propulsion c.g.
  – Electronics and Instruments: 40% of nose section
  – APU: Varies
  – Furnishings, passengers, baggage, cargo, operating items, flight attendants: From layout. Near 51% of fuselage length
  – Crew: 45% of nose length
  – Fuel: Compute from tank layout
Fuel weight

For a given mission

- Taxi & takeoff
  \[ W_{\text{taxi}} = 0.0035 \, W_{\text{to}} \]
- Landing & taxi
  \[ W_{\text{land}} = 0.0035 \, W_{\text{to}} \]
- Reserve
  - Should allow
    - Deviations from the flight plan
    - Diversion to an alternate airport
  - Airliners
    - \[ W_{\text{res}} \sim 0.08 \, \text{ZFW} \]
  - Business jet
    - \[ W_{\text{res}} \] fuel consumption for \( \frac{3}{4} \)-h cruise
    - Climbing (angle of \( \sim 10^\circ \))
      \[ \frac{W_{\text{climb}}}{W_{\text{TO}}} \approx \frac{1}{100} \left[ \frac{\text{cruise altitude}}{31600 \, [\text{ft}]} \right] + \frac{1}{2} \, M_{\text{cruise}}^2 \]
    - Descend: \( \sim \) same fuel consumption than cruise
    - Take Off Weight (TOW):
      \[ W_{\text{to}} = \text{ZFW} + W_{\text{res}} + W_f \]
    - Landing weight:
      \[ \text{ZFW} + W_{\text{res}} + 0.0035 \, W_{\text{to}} \]
Fuel weight

• For a given mission (2)
  – Cruise
    • Bréguet equation
      \[ R_{\text{cruise}} = \int_{W_i}^{W_i - W_{\text{cruise}}} V \, dt = - \int_{W_i}^{W_i - W_{\text{cruise}}} \frac{V}{C_T T} dW = - \int_{W_i}^{W_i - W_{\text{cruise}}} \frac{V L}{C_T D W} dW \]
    – Specific Fuel Consumption \( C_T \)
      » Consumption (of all the engines) per unit of thrust (of all the engines) per unit of time
      – Initial weight \( W_i = W_{\text{to}} - W_{\text{taxi}} - W_{\text{climb}} \)
      – Final weight \( W_i - W_{\text{cruise}} = ZFW + W_{\text{land}} + W_{\text{res}} \)
    • Flight with ratio \( C_D / C_L \sim \) constant
      \[ \frac{R_{\text{cruise}}}{a_0} = \frac{M_{\text{cruise}} C_L}{C_D} \frac{C_T}{\sqrt{\theta}} \ln \frac{W_i}{W_i - W_{\text{cruise}}} \]

Sound speed at SL

Temperature/Temperature SL

– Fuel weight (without reserve) \( W_f = W_{\text{taxi}} + W_{\text{climb}} + W_{\text{cruise}} + W_{\text{land}} \)
- Maximum range depends on the payload
  - 3 zones: Max Payload, M.T.O.W. (structural), fuel capacity

Payload-range diagram

- Max Z.F.W.
- Z.F.W.
- M.E.W.
- Maximum range
- Maximum payload range

Range

Weight

Maximum
payload
range

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Payload-range diagram

- Maximum range depends on the payload (2)
  - First step: add required fuel for the range at maximum payload

![Diagram showing weight vs. range with labels M.E.W., Max Z.F.W., W_f, and W_res.]
• Maximum range depends on the payload (3)
  – Second step: Threshold resulting from the maximum allowed TOW

Why ?:
- Structure designed for a given payload and a given range
- Performances should allow for takeoff
• Maximum range depends on the payload (4)
  – Third step: Keep same M.T.O.W. and reduce payload when range increases

Payload is replaced by fuel
Payload-range diagram

- Maximum range depends on the payload (5)
  - Fourth step: Maximum fuel tank capacity reached

![Graph showing the relationship between weight and range]

- M.T.O.W.
- Max Z.F.W.
- M.E.W.
- Weight
- Range
- $W_{\text{to}}$
- $W_{\text{f}}$
- $W_{\text{res}}$
- $W_{\text{max}}$
• Maximum range depends on the payload (6)
  – Fifth step: Maximum range deduced at zero payload

![Payload-range diagram](image)

- Maximum payload range
- Maximum range at maximum passengers number
- Maximum number of passengers + luggage
- Design point of the project
- Theoretical as no payload is transported

<table>
<thead>
<tr>
<th>Weight</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.T.O.W.</td>
<td>Design point of the project</td>
</tr>
<tr>
<td>Max Z.F.W.</td>
<td>Design point of the project</td>
</tr>
<tr>
<td>M.E.W.</td>
<td>Design point of the project</td>
</tr>
</tbody>
</table>

Maximum range at maximum passengers number

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Undercarriage

- Takeoff

NOTES:

a. All-engine takeoff distance = distance to 35 ft x 1.15
b. All-engine takeoff run = distance to point equidistance between lift off and 35 ft, factored by 1.15

Accelerate-stop distance

Required runway length

Stopway and clearway available

Required runway length when no clearway is present

Takeoff path
Undercarriage

• Angles at takeoff
  – Only the wheels can be in contact with the ground
  • Plane geometry leads to maximum values of
    – Pitch angle $\theta$
    – Roll angle $\phi$
• Angles at takeoff (2)
  – Example:
    • Wing tip should not touch the ground during rotation $\theta$ even if the plane is experiencing a roll $\phi$
    • Geometric considerations

$$\tan \phi = \tan \Gamma + \frac{2h_g}{b - t} - \tan \theta \tan \Lambda$$

• Roll angle $\phi$ of 8° should be authorized
• $e_s$: static deflection of shock absorber ($e_s$ et $l_1 \sim 0$ as first approximation)
Angles at takeoff (3)

- Pitch angle at takeoff

\[ \theta_{\text{LOF}} = \alpha_{\text{LOF}} + \frac{d\theta}{dt} \left( \frac{2l_1}{V_{\text{LOF}}} + \sqrt{\frac{l_2 C_{\text{LLOF}}}{ga}} \right) \]

- Pitch rate of climb

- Pitch angle at takeoff

\[ \alpha_{\text{LOF}} = \frac{1}{C_{L\alpha}} \left[ (C_{L_{\text{max}}}^{\text{cruise}} - C_{L_{\text{cruise}}} - p(C_{L_{\text{max}}}^{\text{to}}) \right] \]

- Lift coefficient for maximum lift expected during takeoff

- Margin

\[ p \sim 0.15 \]

Lift off velocity:

\[ V_{\text{LOF}} \sim 1.15 V_{s0} \]
Undercarriage

• **Landing**
  - Impact point of rear wheels behind projection of cg on the ground
   - If not, the plane would fall backward
   - Touchdown angle: $\theta_{TD} \sim \theta_{LOF}$
   - Distance $l_m$ between cg and rear wheels

\[
l_m \geq (|z_{cg}| + e_s) \tan \theta_{TD}
\]

  - $e_s$: static deflection of shock absorber
  - $z_{CG}$: distance from cg to the ground

• **Front wheels**
  - About 8 to 15% of MTOW supported by front wheels
    - Lower than 8%: direction is not effective
    - More than 15%: difficulties at breaking
      - Now new devices are allowing to get more than 15%
  - CG location can change with the payload
Design steps

**INPUTS**
- Mission
  - Payload
  - Range
  - Cruise altitude
  - Cruise speed
- Configuration
  - Wing + Tail
  - Engines wing/fuselage mounted
- Technology
  - Airfoils
  - Engines
  - …

**Fuselage**

**Statistical guess**
- ZFW & MTOW

**Wing design**
- Choice of engine

**Equilibrium**
- Weight and cg location of the groups
- Wing position
- Evolution of cg in terms of payload
- Horizontal tail
- Evolution of cg in terms of fuel consumed (distance)
- Fin

**Mission**
- Cruise velocity
- Payload-range diagram

**Outputs**
- Undercarriage
- Plane drawing
- Static margin evolution in terms of payload, range & fuel consumed
- Polar

**Performances ?**
- yes

**ZFW & MTOW correct ?**
- yes
- no
References

• Reference of the classes

• Other
  – Book