Université de Liège
Département d’Aérospatiale et de Mécanique

Aircraft Design
Conceptual Design

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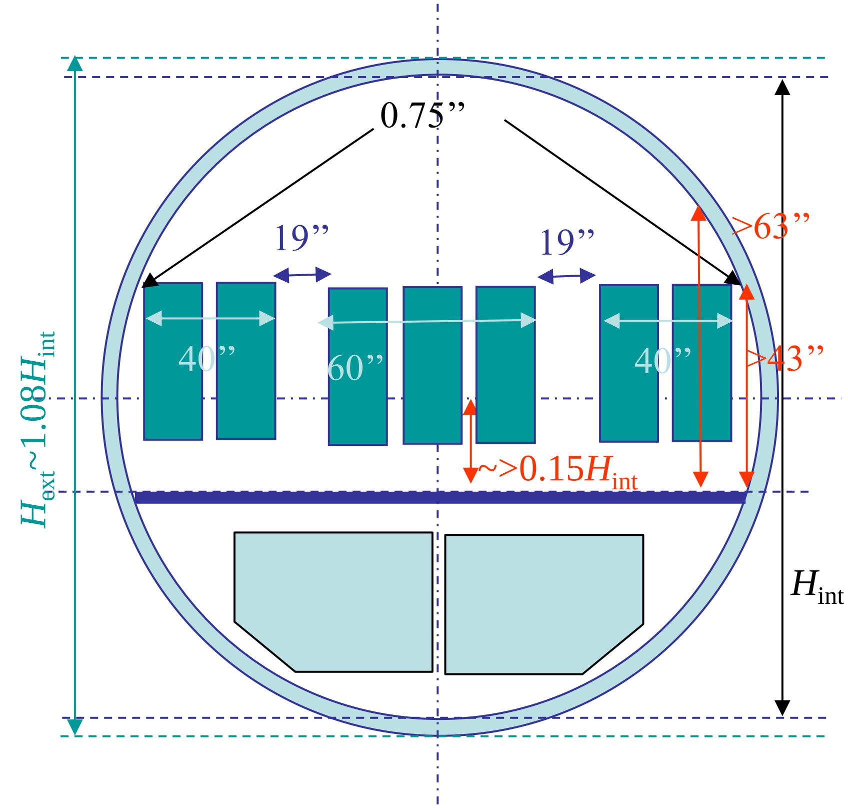
Goals of the classes

- Design stages
  - **Conceptual design**
    - Purposes
      - Define the general configuration (tail or canard, high or low wing, …)
      - Analyze the existing technologies
      - Estimate performances for the different flight stages
      - Accurate estimation of the total weight, fuel weight, engine thrust, lifting surfaces, …
    - How
      - Limited number of variables (tens): span, airfoil profile, …
      - Accurate simple formula & abacuses
  - Preliminary study
    - Higher number of variables (hundreds)
    - Starting point: conceptual design
    - Numerical simulations
  - Detailed study
    - Each component is studied in details

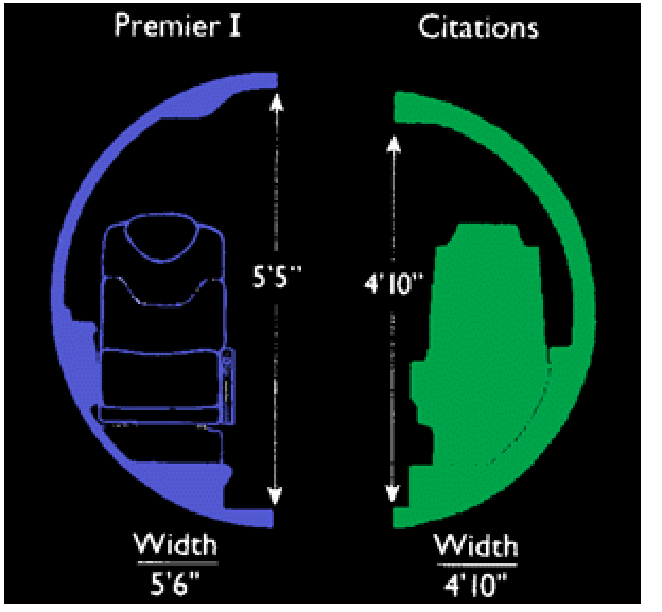
Fuselage

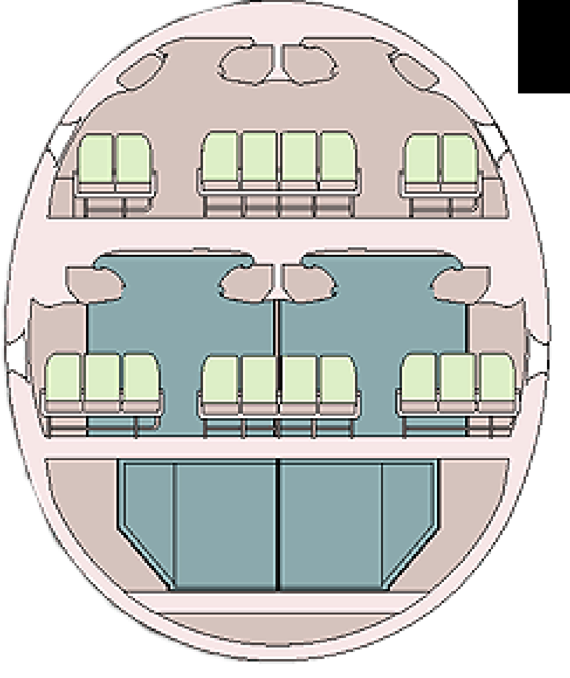
- Cross-section
  - Seat width
    - Economy: ~20 inches*

      *1 inch = 2.54 cm
    - Business: ~24 inches
    - First: ~26.5 inches
  - Aisle width
    - Economy: ~19 inches
    - Business: ~19 inches
    - First: ~21 inches
  - Fuselage thickness
    - ~ 4% of H_{int}



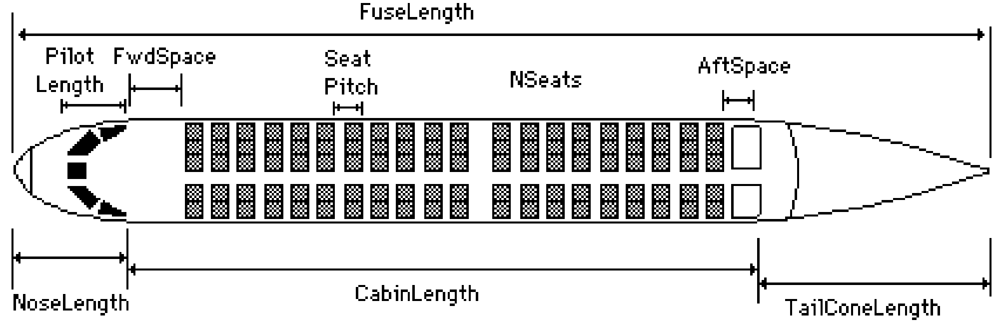
- Cross-section (2)
  - Other arrangements
    - Business jets
      - More freedom
    - Elliptic section
      - A380
    - Non-pressurized cabin
      - Rectangular cross-section

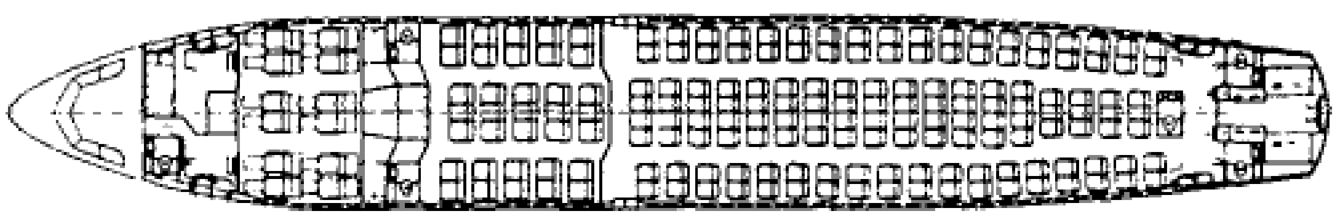




Fuselage

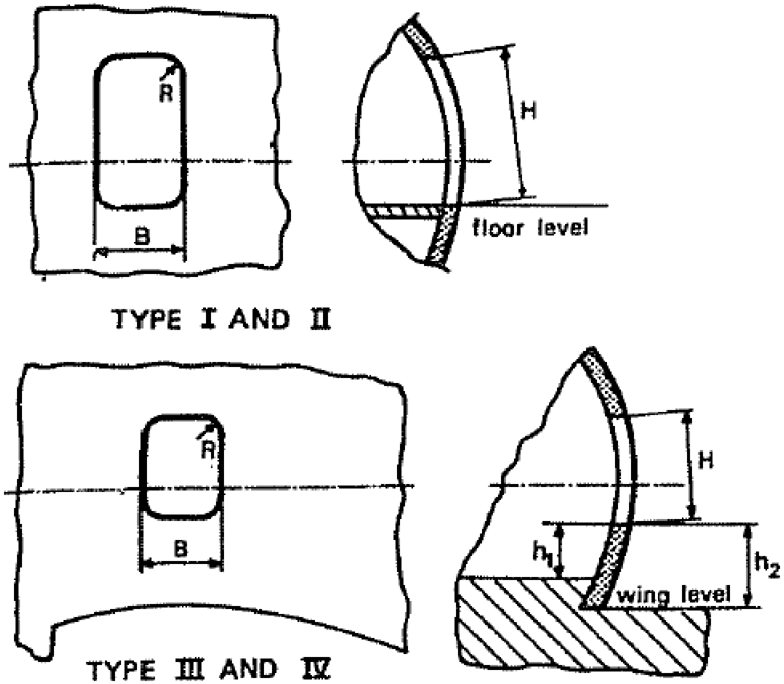
- Length
  - Seat pitch
    - Economy: ~34 inches
    - First: ~40 inches
  - Toilets
    - Length: ~38 inches
    - >1 per 40 passengers
  - Pressurized cabin can extend back in the tail
    - Different seat layouts
    - Shortens the plane length (reduced weight)





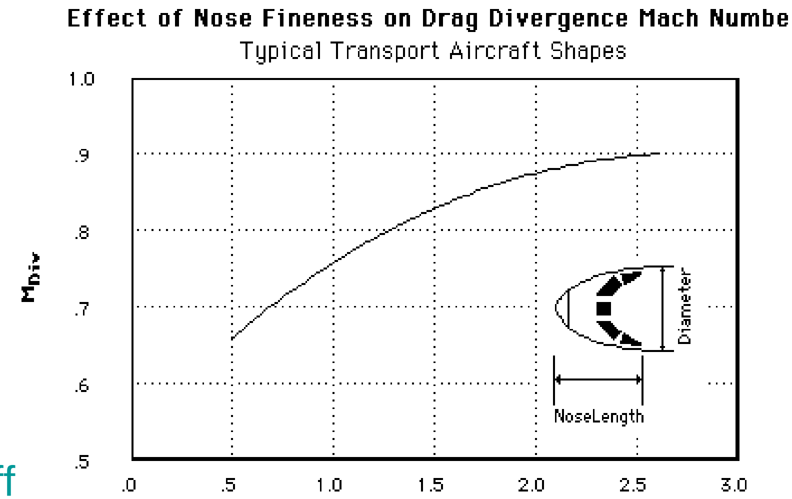
187 passengers (12 first class, 35 business class, 140 economy class)

- Length (2)
  - Doors
    - Type I: ~36 inches
    - Type II: ~20 inches
    - Type III & IV: ~18 inches

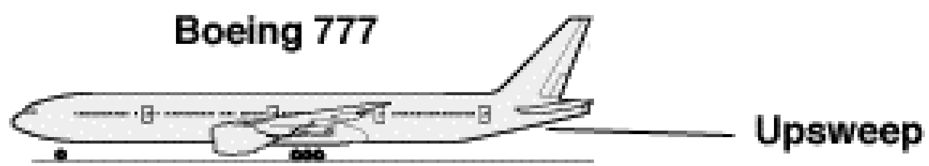


SEATING CAPACITY (EXCL. CABIN STAFF)	NUMBER OF EXITS REQUIRED ON EACH SIDE OF THE FUSELAGE			
	TYPE I	TYPE II	TYPE III	TYPE IV
1 through 10	-	-	-	1
11 through 19	-	-	1	-
20 through 39	-	1	-	1
40 through 59	1	-	-	1
60 through 79	1	-	1	-
80 through 109	1	-	1	1
110 through 139	2	-	1	-
140 through 179	2	-	2	-

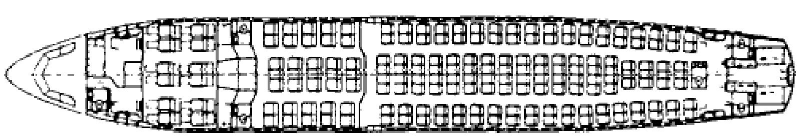
- Length (3)
  - Ratio nose length/diameter N_F
    - >1.5 due to pressurization
    - Large enough to avoid divergence
  - Ratio tail length/diameter A_F
    - ~1.8-2
    - Closure angle ~28-30°
    - Upsweep ~ 14°: rotation during take off



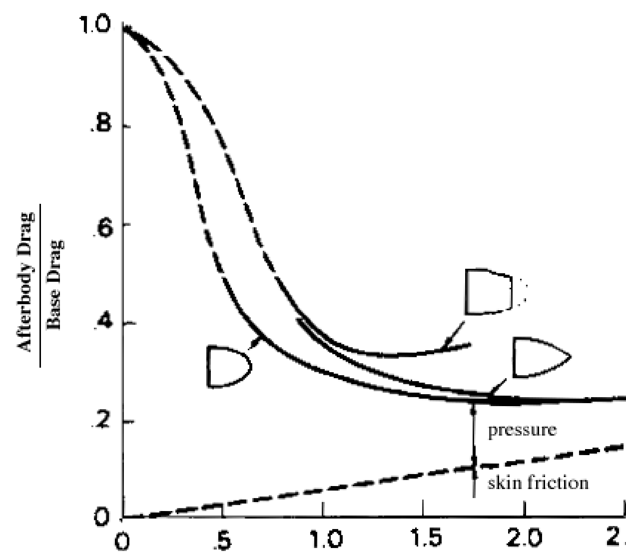
N_F = Nose Length/D



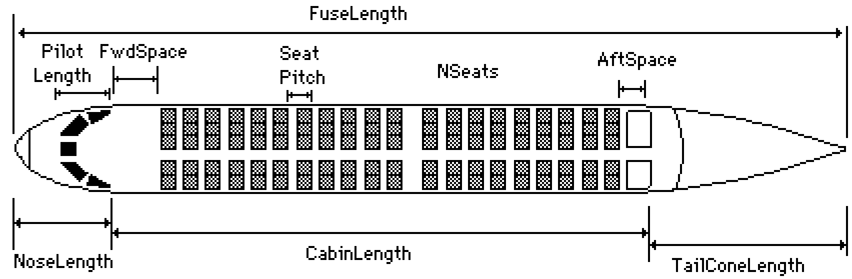
    - Part of the tail can be pressurized and used for the payload



187 passengers (12 first class, 35 business class, 140 economy class)



A_F = Aft Length/D



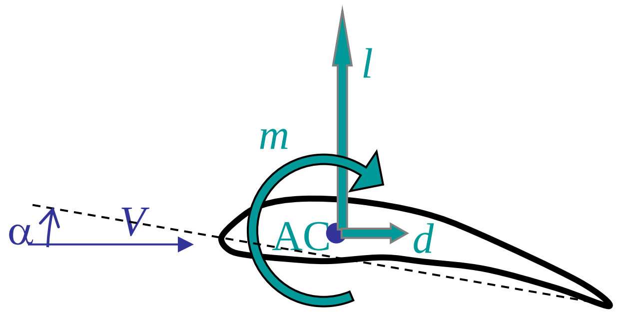
- Method
  - Inputs
    - N_{seats}, layout, N_F, A_F,
  - Outputs
    - Shape

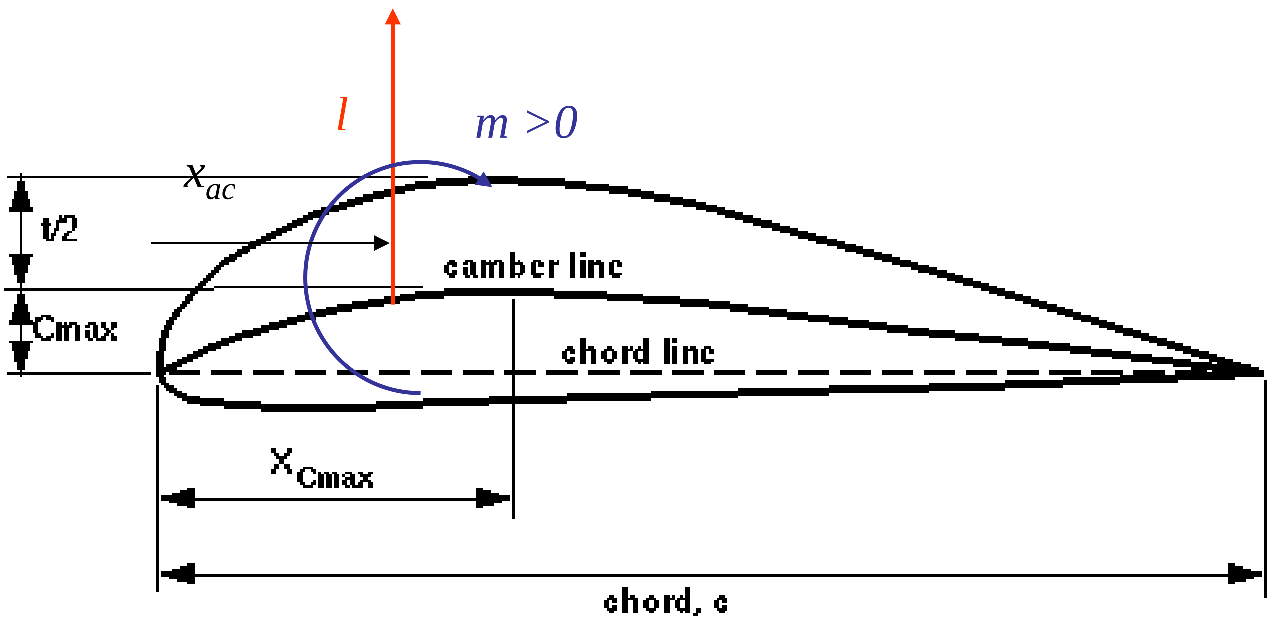
$$\text{height}_{\text{fus}}=\text{width}_{\text{fus}}= \text{H}_{\text{ext}}$$

$$\text{length}_{\text{fus}} = \text{length}_{\text{body}} + \text{height}_{\text{fus}} * (A_F + N_F)$$

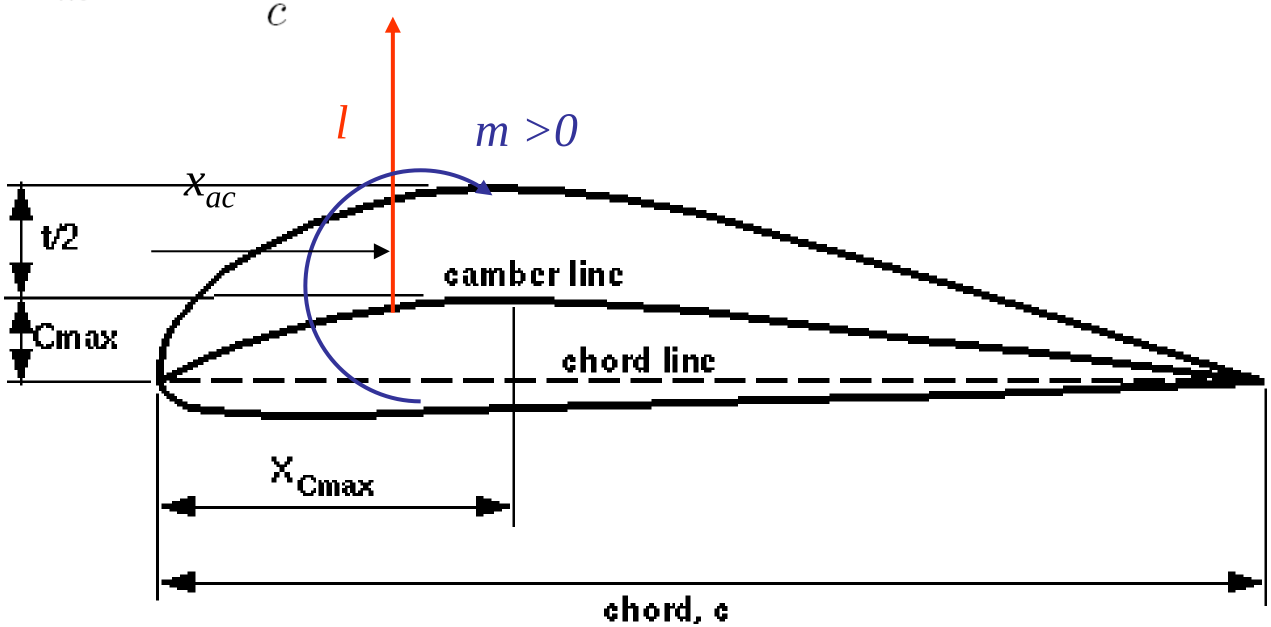
$$S_{\text{fus,wetted}} \simeq \underbrace{\text{length}_{\text{body}}\,\text{height}_{\text{fus}}\,\pi}_{\text{cylindre}} + \underbrace{\pi\,\frac{\text{height}_{\text{fus}}^2}{4}\sqrt{1+4A_F^2}}_{\text{cône}} + \underbrace{\frac{\pi\,\text{height}_{\text{fus}}^2}{4}\left(1+2N_F\frac{\arcsin\sqrt{1-\frac{1}{4N_F^2}}}{\sqrt{1-\frac{1}{4N_F^2}}}\right)}_{1/2\ \text{prolate}}$$

- Airfoils
  - Which one?
    - Minimum drag during cruise
    - Depends on Reynolds number $R = Uc / \nu$
  - Properties
    - Airfoil lift coefficient $c_l = c_{l_\alpha}[\alpha - \alpha_{l_0}]$
    - Pitching moment
      - Aerodynamic centre $\left.\frac{\partial c_m}{\partial c_l}\right|_{x_{ac}} = 0$
      - Moment around ac ~ constant at low attack angle α



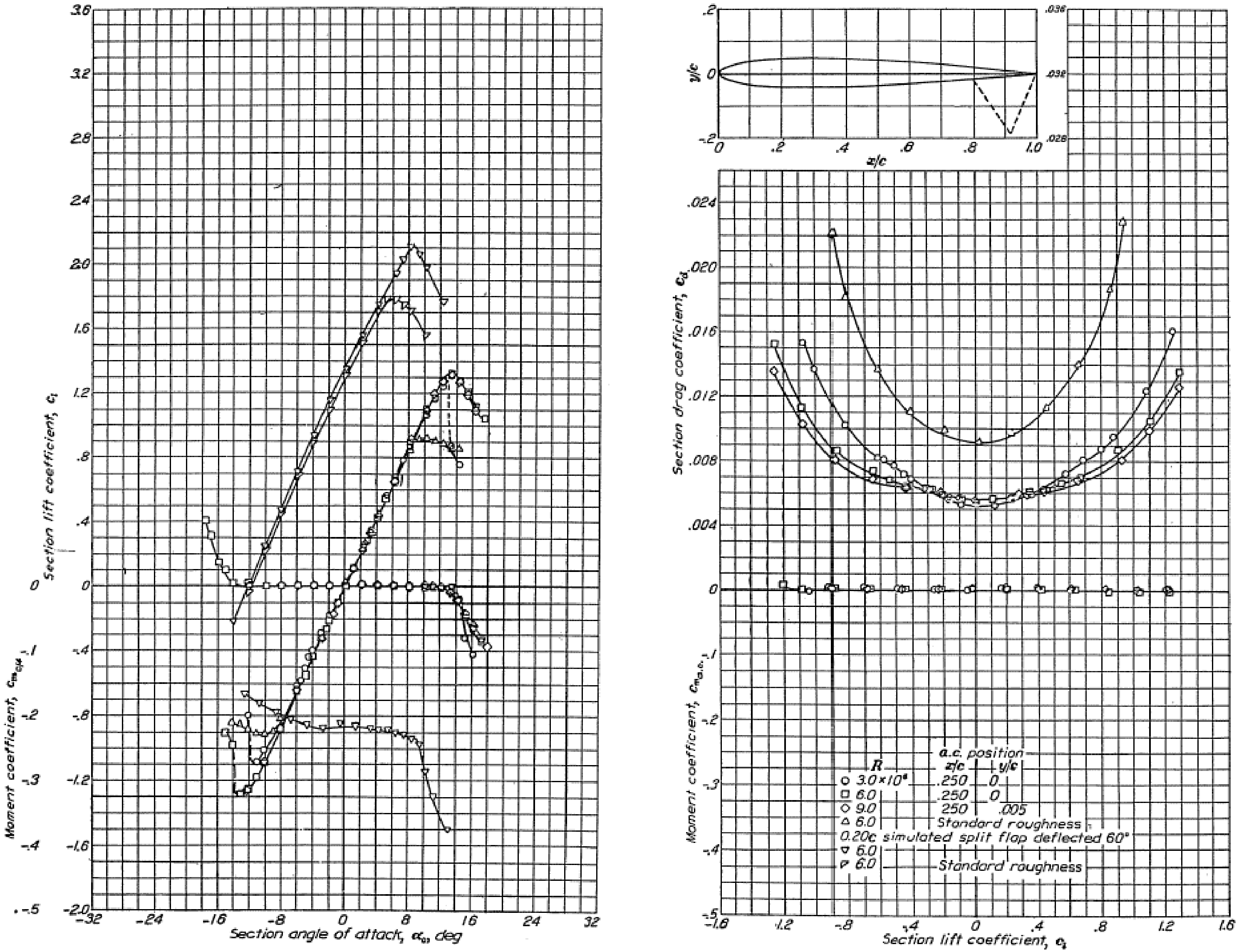


- Airfoils (2)
  - Empirical formula
    - Lift coefficient $c_{l\alpha} \simeq 6.1$ (if t/c ~10-20 %)
    - Zero-lift angle of attack (in °)
      - $\alpha_{l_0} = \{-\%\text{cambrure},\ -4{c_l}_i,\ -6{c_l}_i\}$ for {NACA-4, 5, 6} airfoils
      - Design coefficient ${c_l}_i \simeq 0.4$
    - Moment (low α): $c_m \simeq -\pi \dfrac{c_{\max}}{c}$

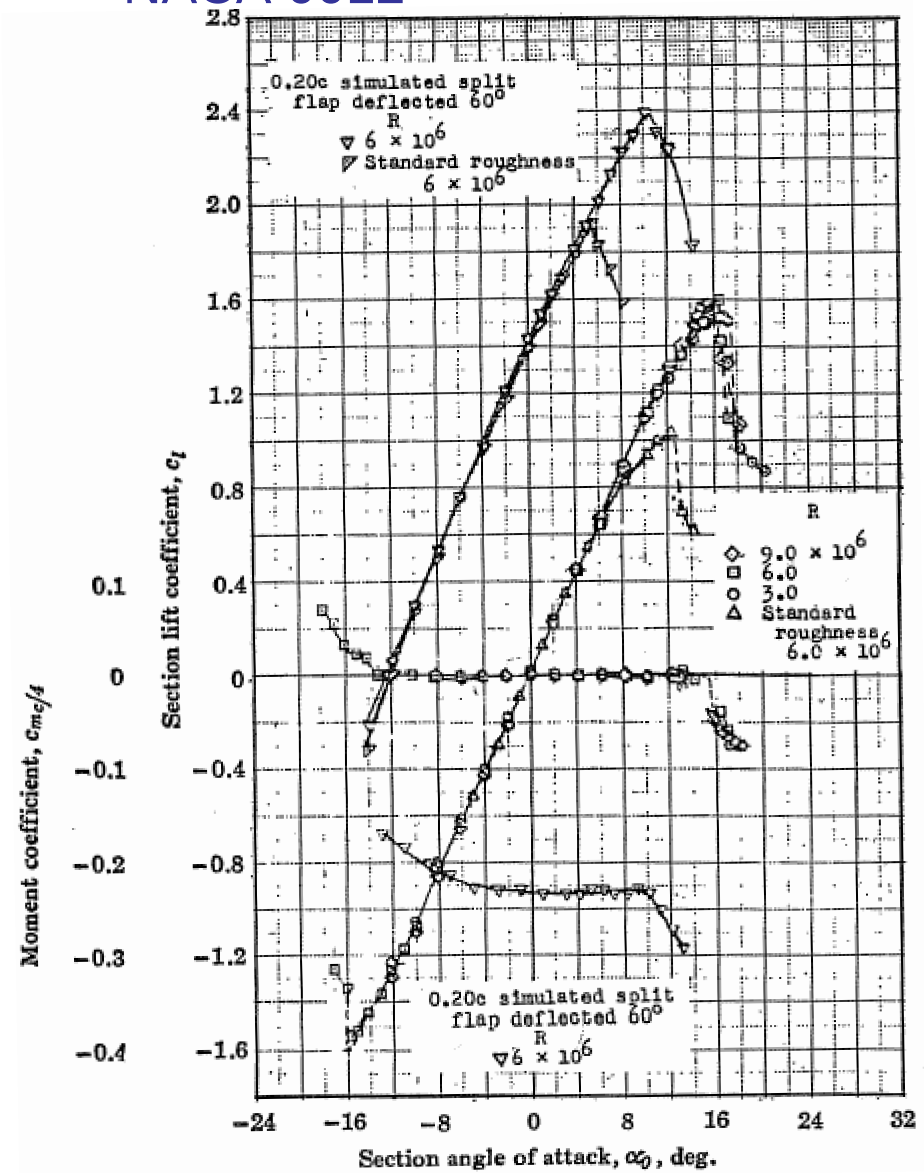


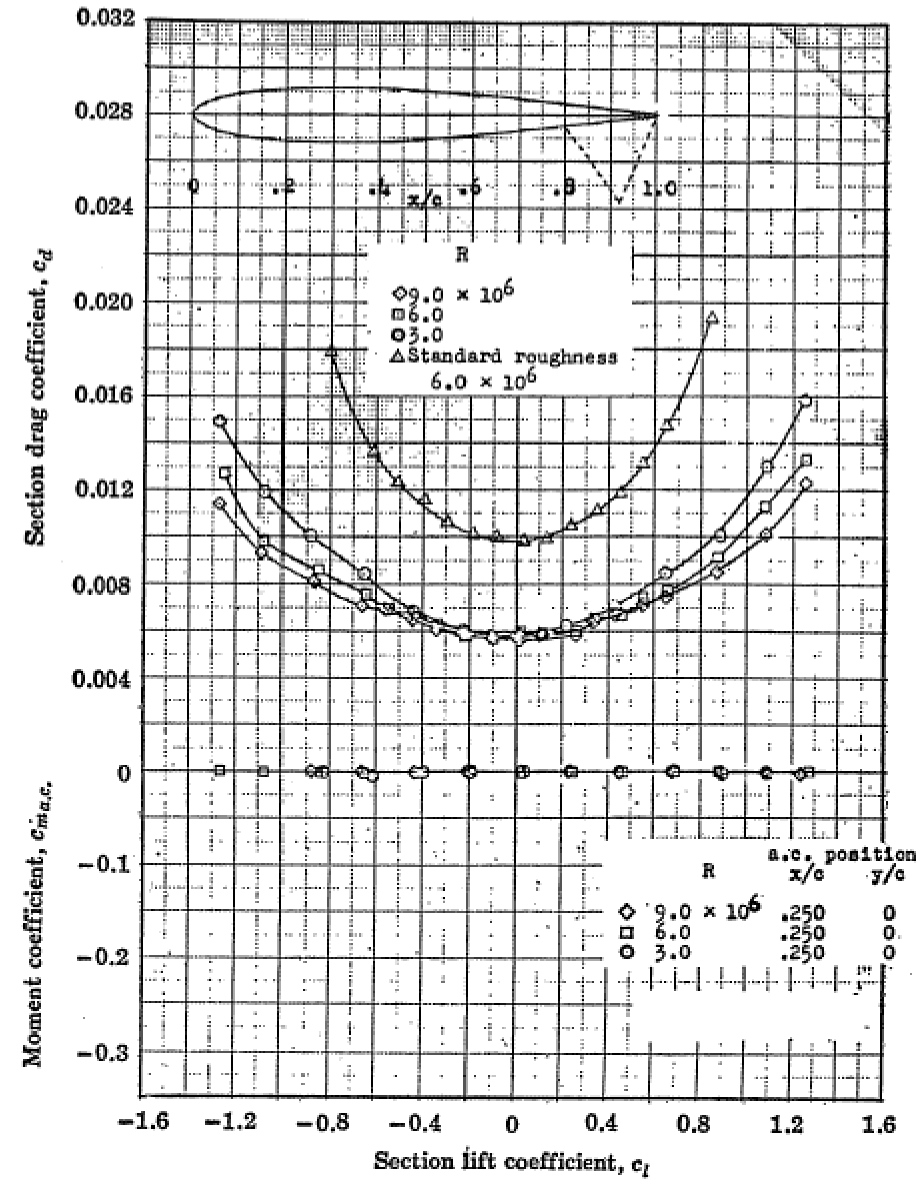
- Airfoils (3)
  - Numerical methods
    - Do not predict stall velocities
    - Panda (be careful: if $|c_p| > |c_p^*|$ then the solution is not accurate)
      - http://adg.stanford.edu/aa241/airfoils/panda.html
      - http://www.desktopaero.com/manuals/PandaManual/PandaManual.html
    - xfoil
      - http://web.mit.edu/drela/Public/web/xfoil/
  - Experimental methods
    - Curves on next slides

- NACA 0009

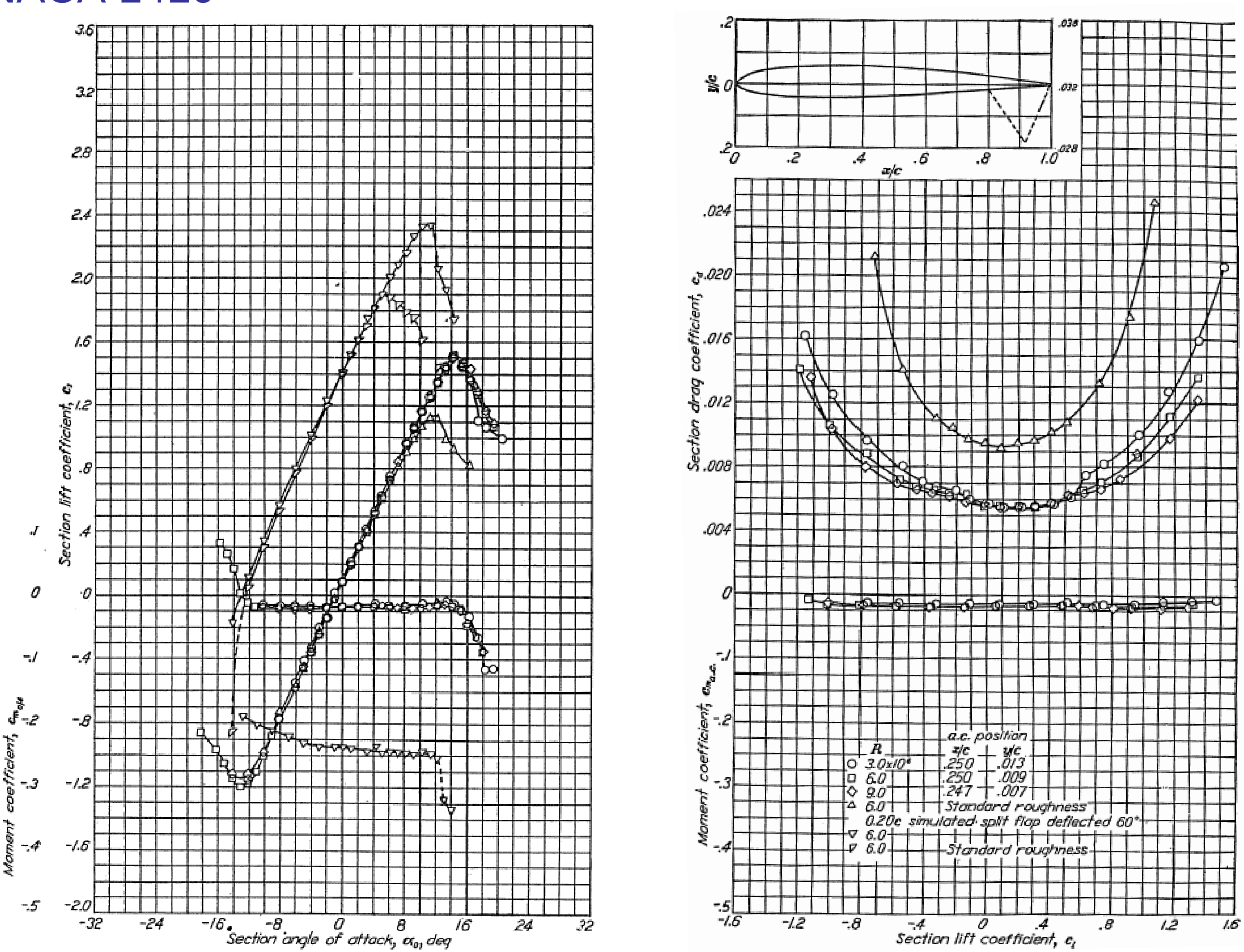

Section angle of attack, α_0, deg
Section lift coefficient, c_l
Moment coefficient, $c_{m_{c/4}}$
Section drag coefficient, c_d
Section lift coefficient, c_l
Moment coefficient, $c_{m_{a.c.}}$
y/c
x/c
a.c. position
R
x/c
y/c
3.0×10⁶ .250 0
6.0 .250 0
9.0 .250 .005
6.0 Standard roughness
0.20c simulated split flap deflected 60°
6.0
6.0 Standard roughness

- NACA 0012

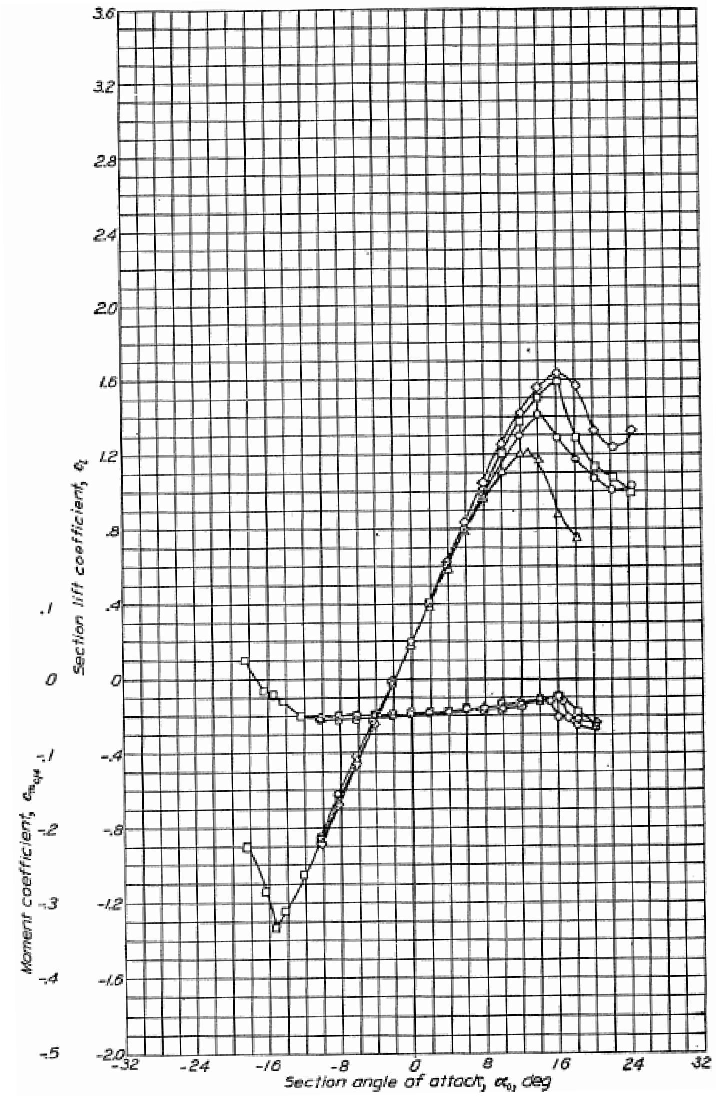

0.20c simulated split flap deflected 60°
R
6 × 10^6
Standard roughness 6 × 10^6
R
9.0 × 10^6
6.0
3.0
Standard roughness 6.0 × 10^6
0.20c simulated split flap deflected 60°
R
6 × 10^6
Moment coefficient, c_{m c/4}
Section lift coefficient, c_l
Section angle of attack, α_0, deg.
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
−0.4
−0.8
−1.2
−1.6
0.1
0
−0.1
−0.2
−0.3
−0.4
−24
−16
−8
0
8
16
24
32

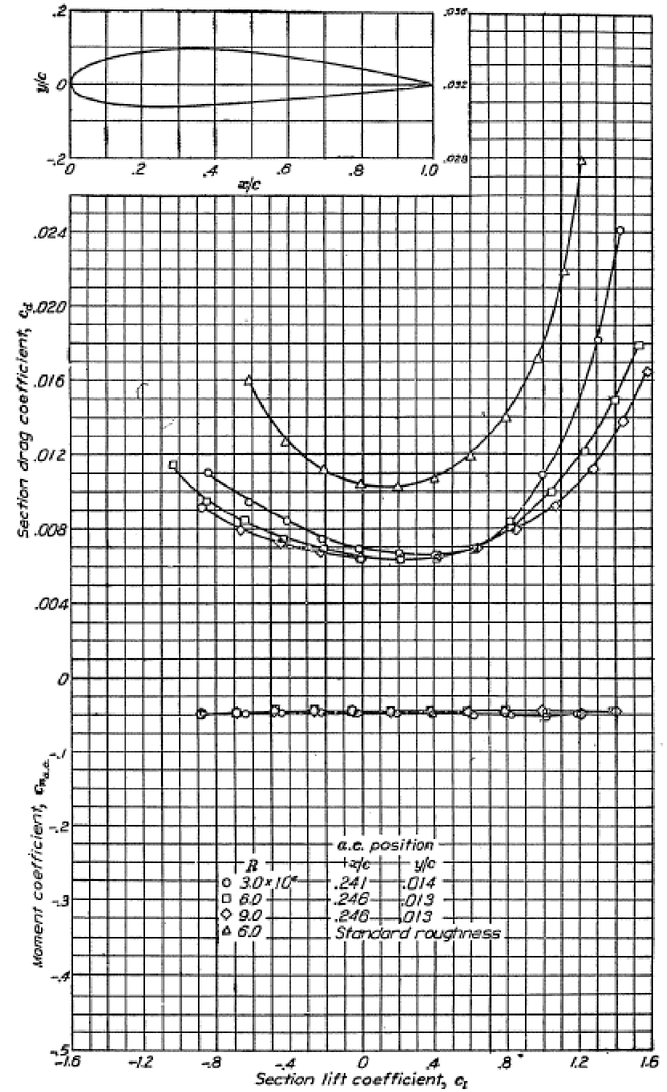

x/c
0
.2
.4
.6
.8
1.0
R
9.0 × 10^6
6.0
3.0
Standard roughness 6.0 × 10^6
Section drag coefficient, c_d
Moment coefficient, c_{m a.c.}
a.c. position
R
x/c
y/c
9.0 × 10^6
.250
0
6.0
.250
0
3.0
.250
0
Section lift coefficient, c_l
0.032
0.028
0.024
0.020
0.016
0.012
0.008
0.004
0
−0.1
−0.2
−0.3
−1.6
−1.2
−0.8
−0.4
0
0.4
0.8
1.2
1.6

- NACA 1410


Section lift coefficient, c_l
Moment coefficient, $c_{m_{c/4}}$
Section angle of attack, α_0, deg
y/c
x/c
Section drag coefficient, c_d
Moment coefficient, $c_{m_{a.c.}}$
Section lift coefficient, c_l
a.c. position
R
x/c
y/c
○ 3.0x10⁶ .250 .013
□ 6.0 .250 .009
◇ 9.0 .247 .007
△ 6.0 Standard roughness
0.20c simulated split flap deflected 60°
▽ 6.0
▽ 6.0 Standard roughness

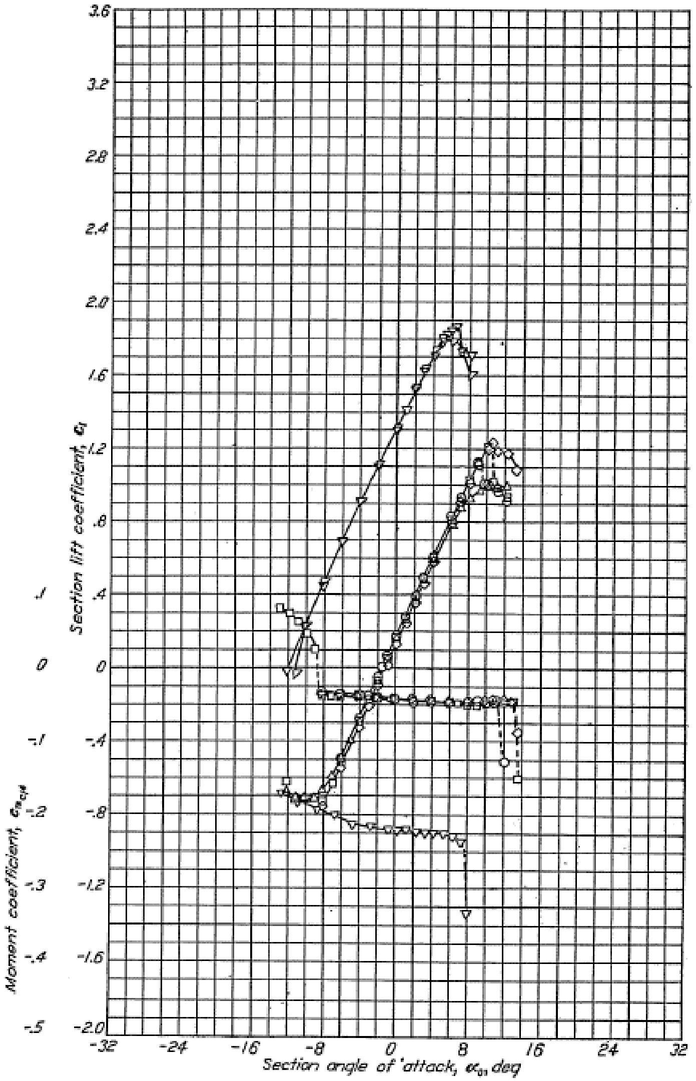
- NACA 2415

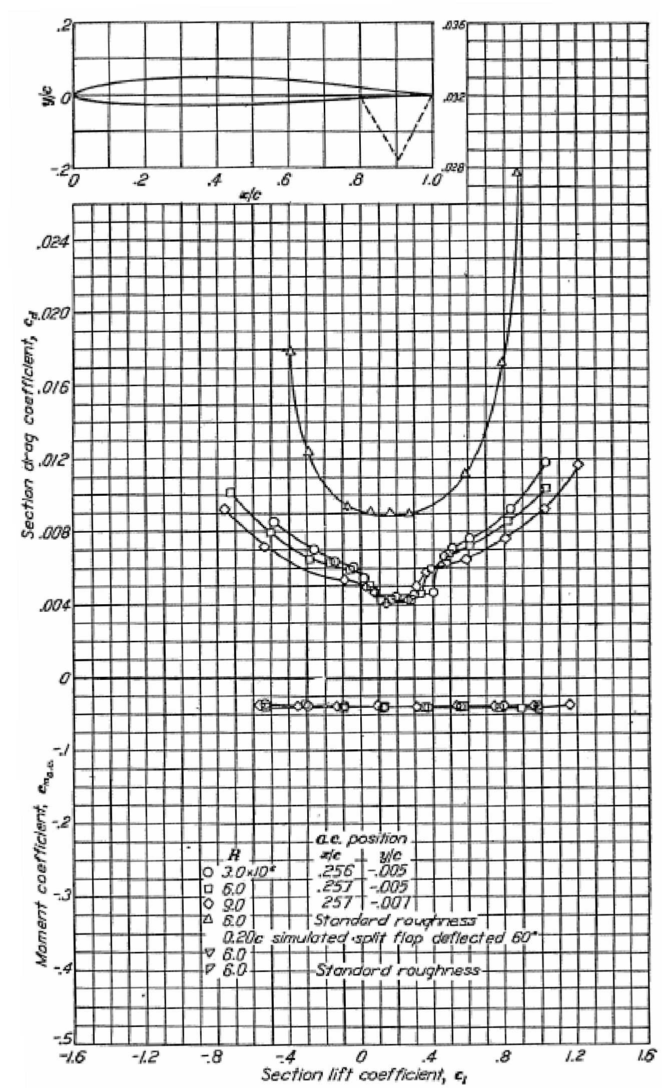

Section lift coefficient, c_l
Moment coefficient, $c_{m_{c/4}}$
Section angle of attack, α_0, deg


y/c
x/c
Section drag coefficient, c_d
Moment coefficient, $c_{m_{a.c.}}$
Section lift coefficient, c_l
a.c. position
R
x/c
y/c
o 3.0×10⁶ .241 .014
□ 6.0 .246 .013
◇ 9.0 .246 .013
△ 6.0 Standard roughness

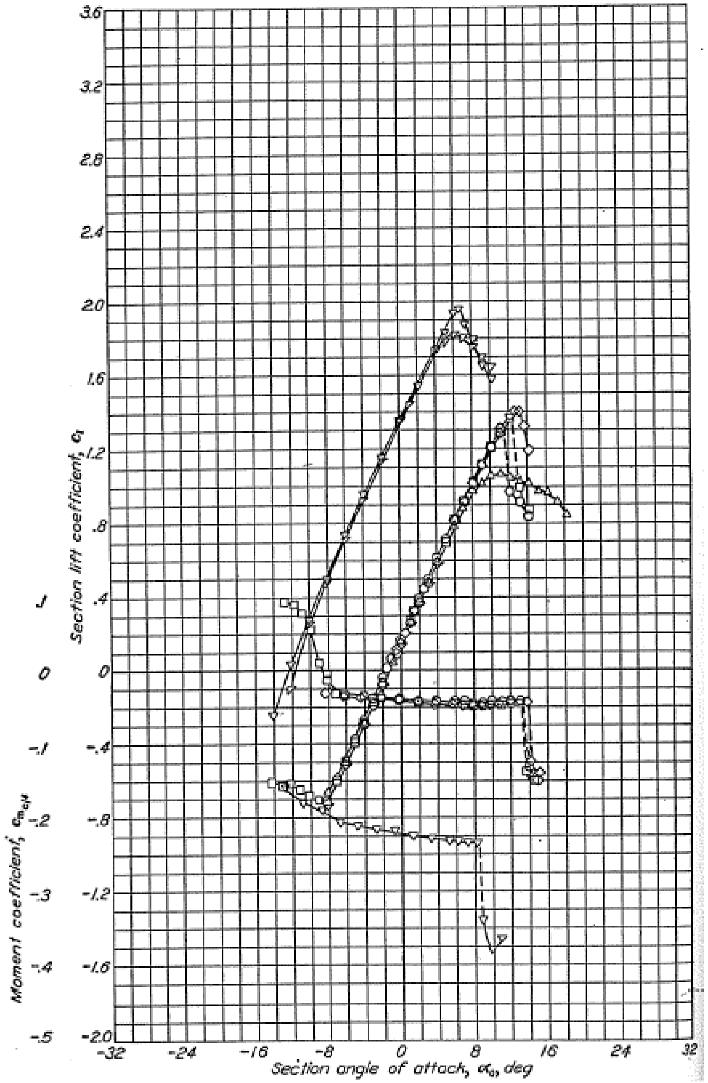
Wing

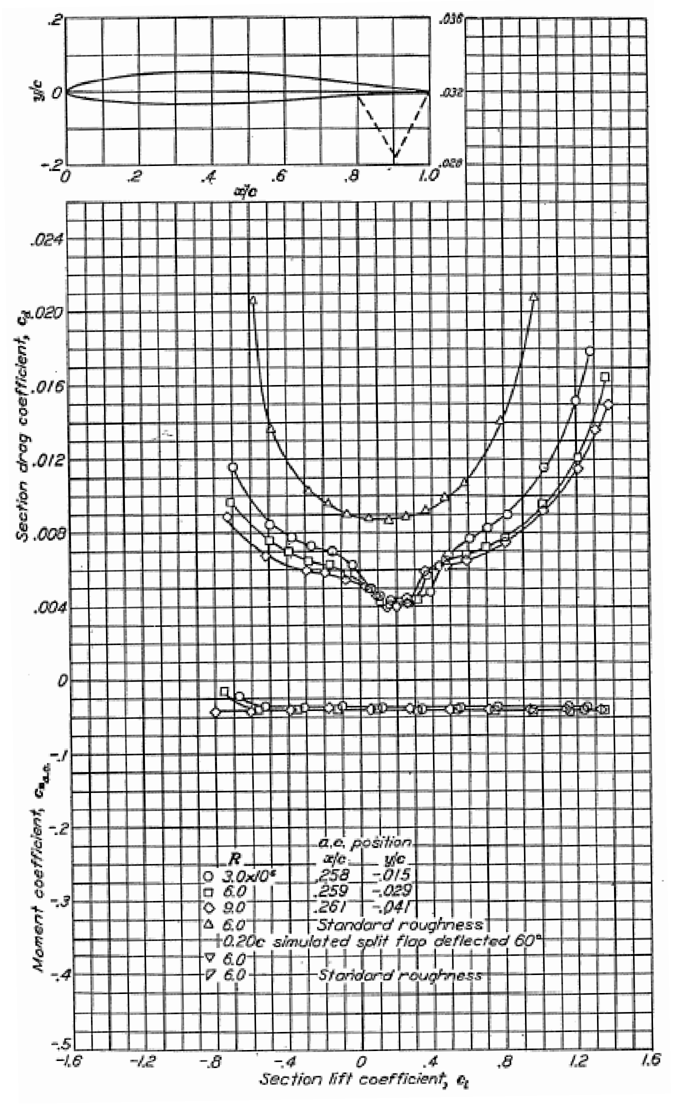
- NACA 64208





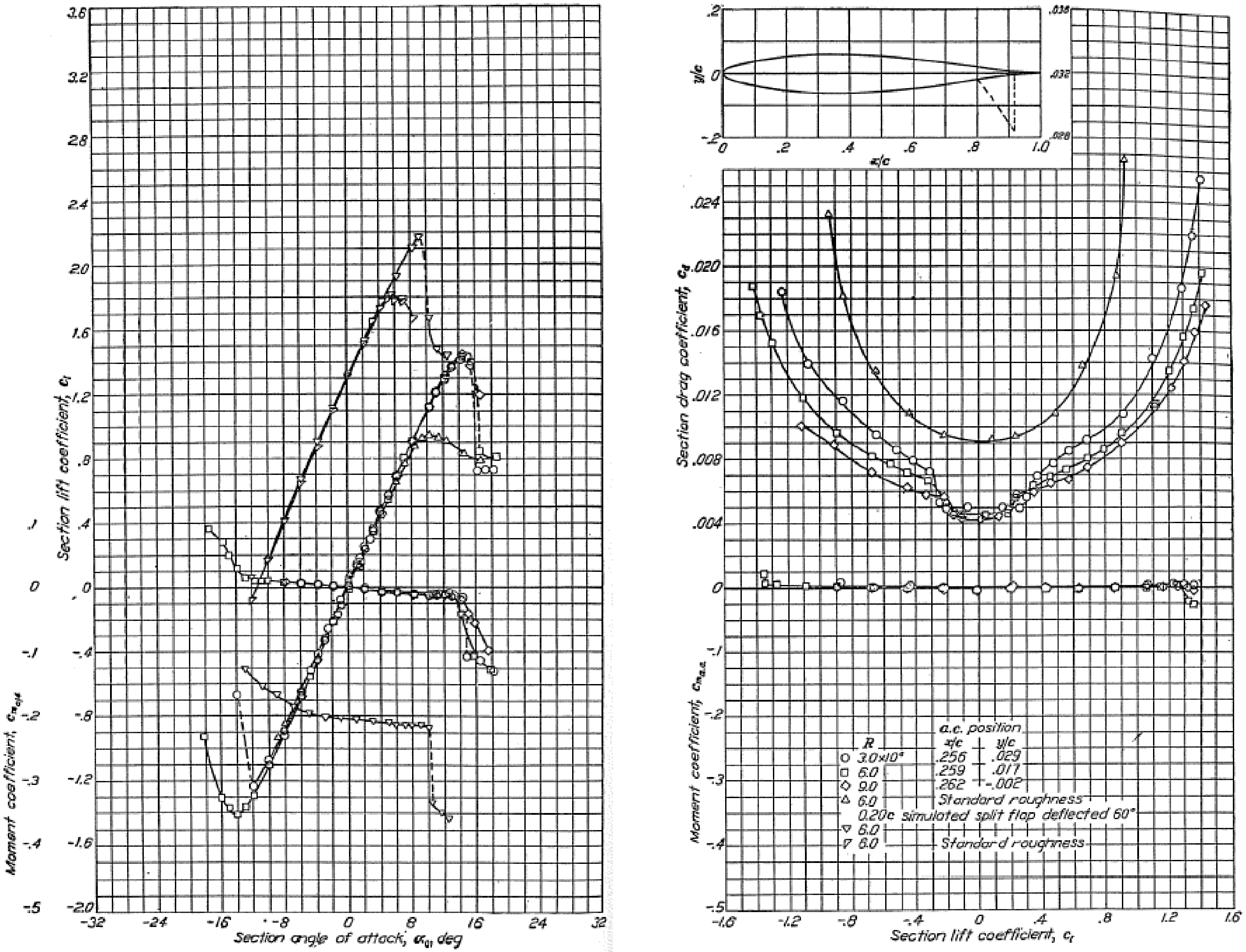
- NACA 64209



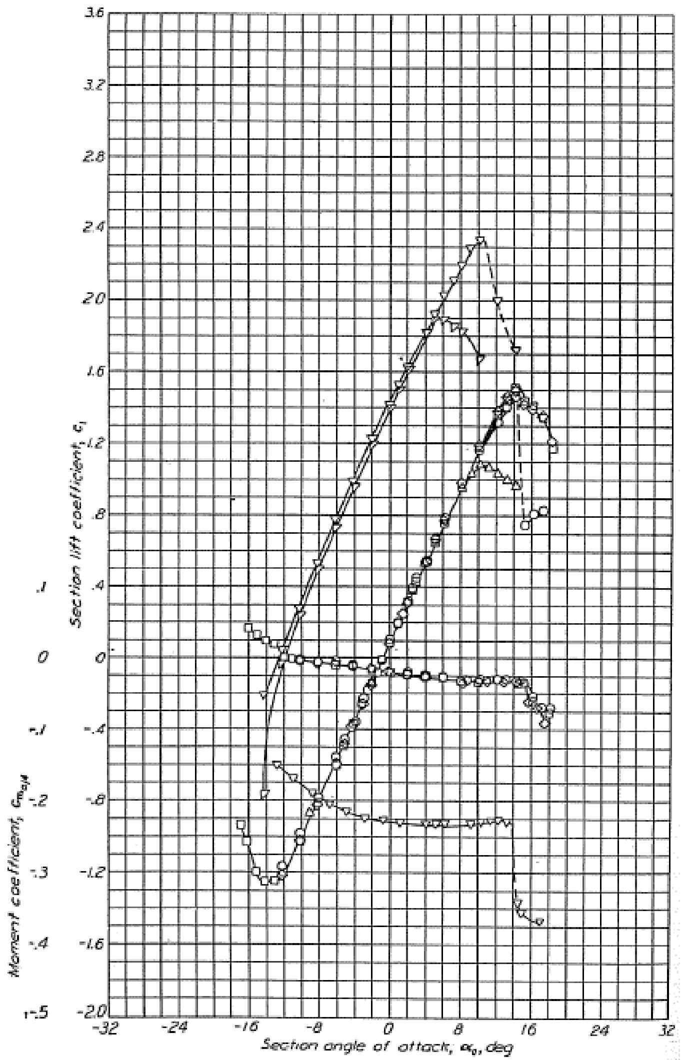

a.c. position
R
x/c
y/c
3.0×10⁶
.258
-.015
6.0
.259
-.029
9.0
.261
-.041
6.0
Standard roughness
0.20c simulated split flap deflected 60°
6.0
6.0
Standard roughness
Section angle of attack, α₀, deg
Section lift coefficient, cₗ
Section drag coefficient, c_d
Moment coefficient, c_m c/4
Moment coefficient, c_m a.c.
x/c
y/c

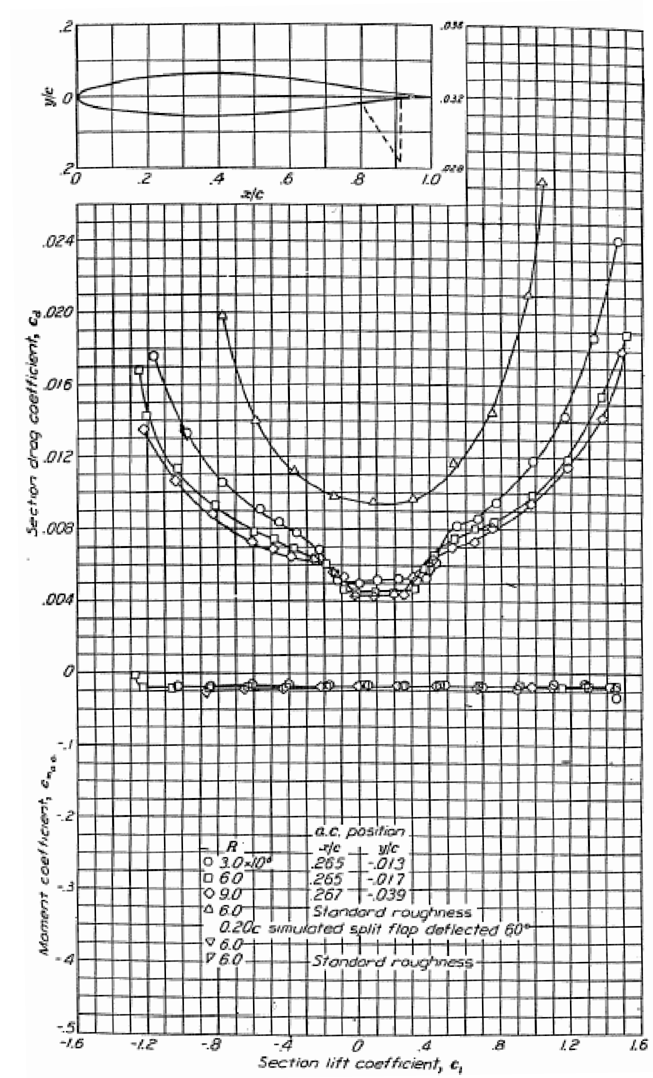
Wing

- NACA 64_1-012

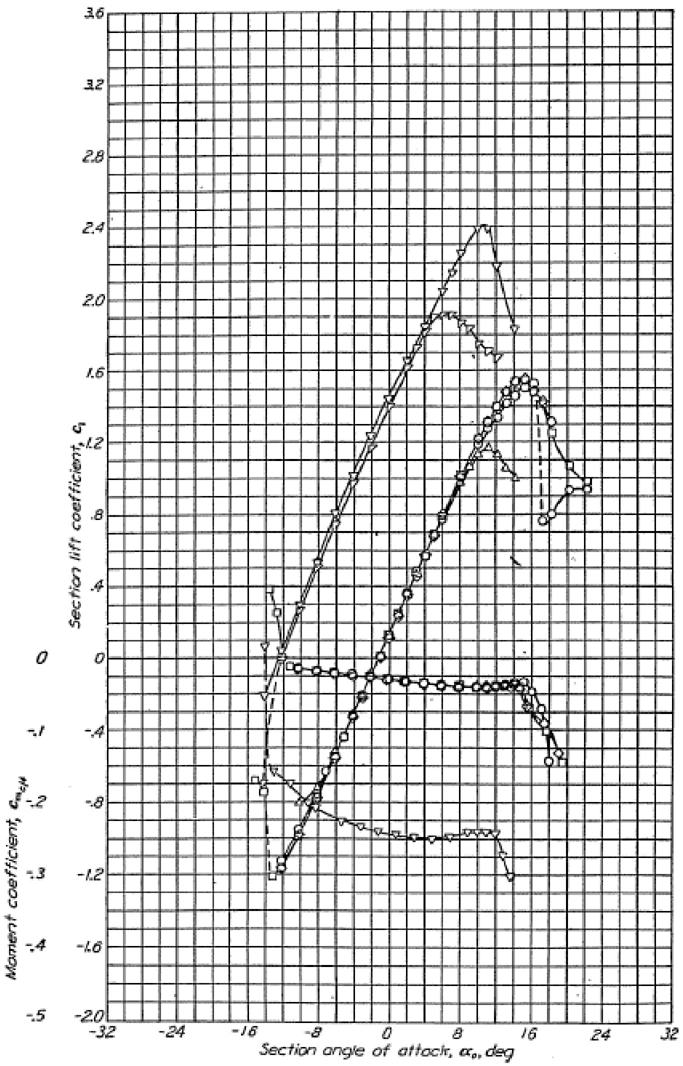


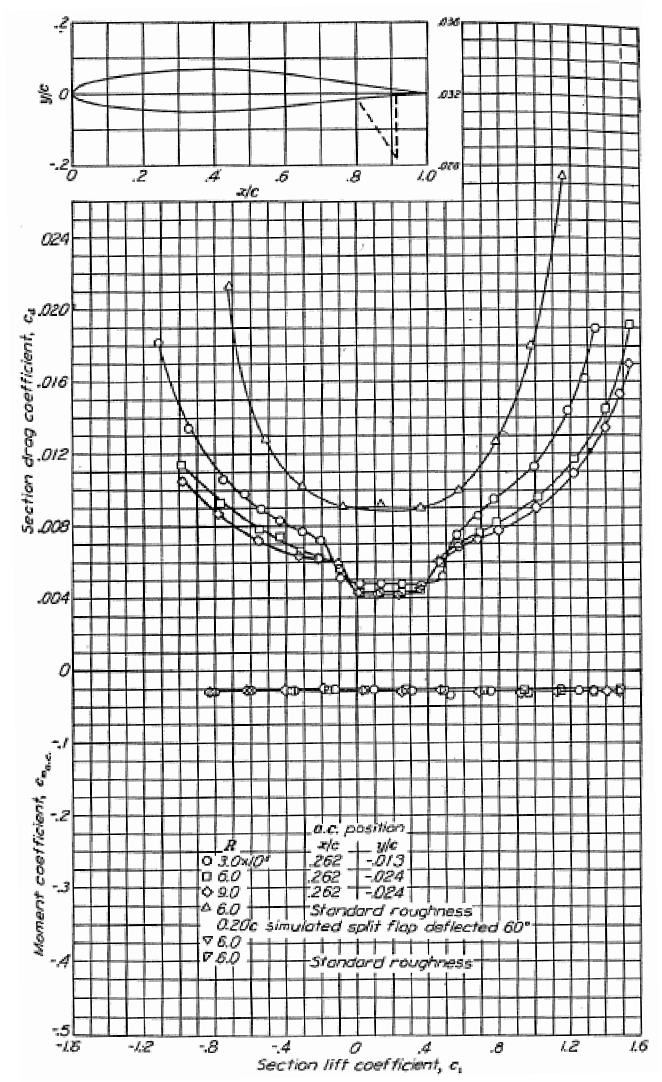
- NACA 64_1-112


Section lift coefficient, c_l
Moment coefficient, $c_{m_{c/4}}$
Section angle of attack, α_0, deg

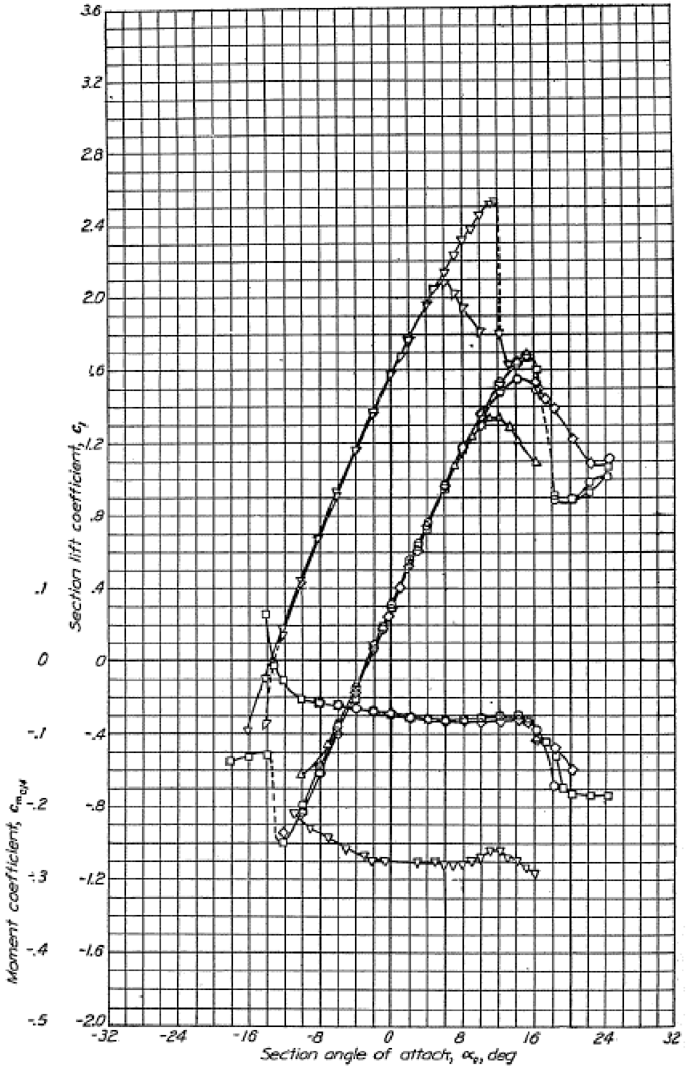

y/c
x/c
Section drag coefficient, c_d
Moment coefficient, $c_{m_{a.c.}}$
Section lift coefficient, c_l
a.c. position
R
x/c
y/c
3.0×10⁶ .265 -.013
6.0 .265 -.017
9.0 .267 -.039
6.0 Standard roughness
0.20c simulated split flap deflected 60°
6.0
6.0 Standard roughness

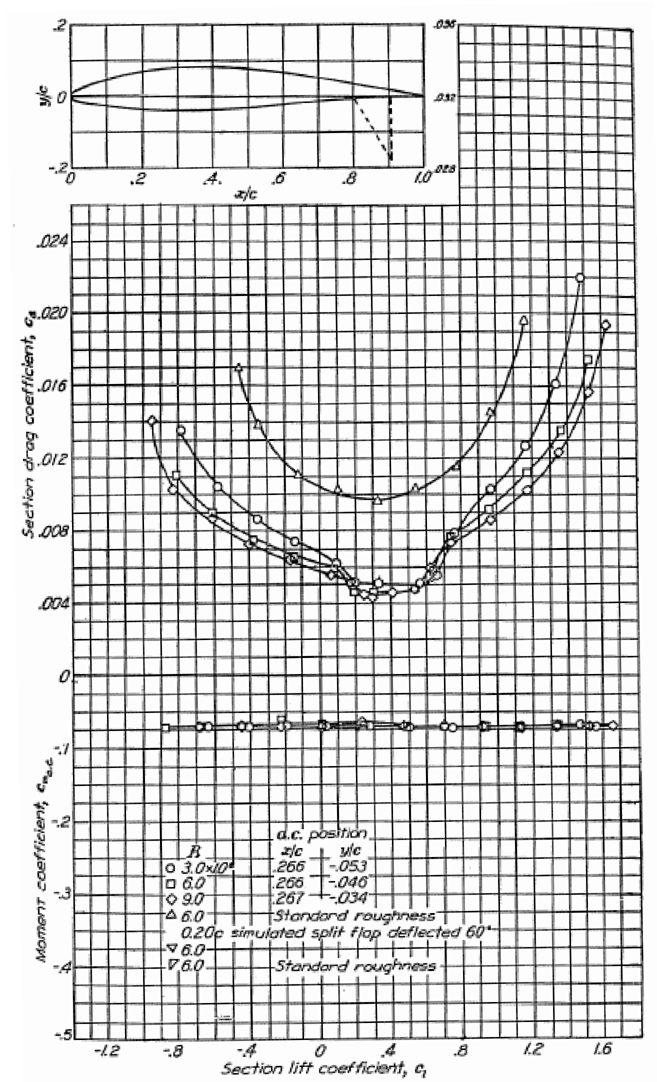
- NACA 64_1-212


Section lift coefficient, c_l
Moment coefficient, $c_{m_{c/4}}$
Section angle of attack, α_0, deg

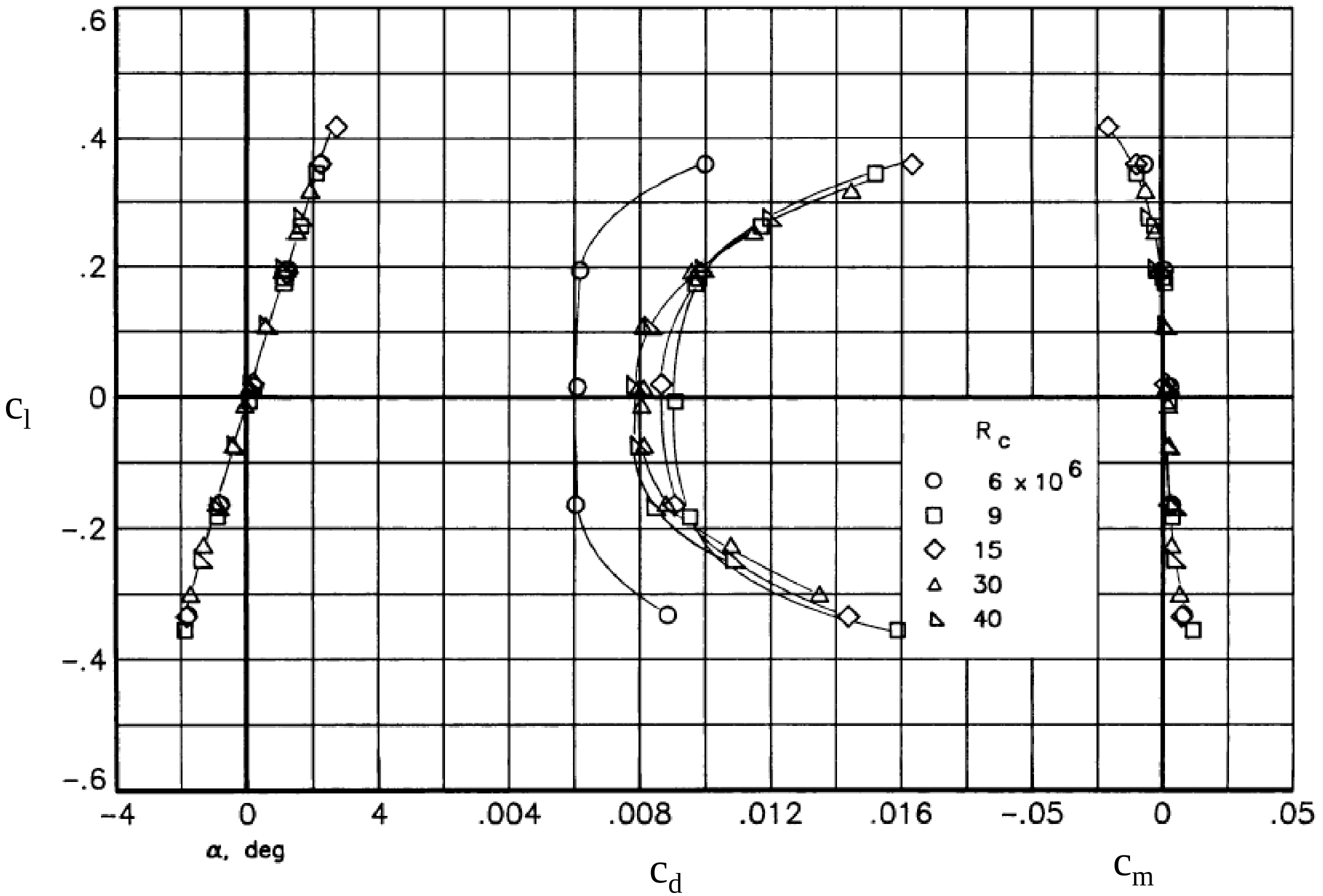

y/c
x/c
Section drag coefficient, c_d
Moment coefficient, $c_{m_{a.c.}}$
Section lift coefficient, c_l
a.c. position
R
x/c
y/c
3.0×10⁶
.262
-.013
6.0
.262
-.024
9.0
.262
-.024
6.0
Standard roughness
0.20c simulated split flap deflected 60°
6.0
6.0
Standard roughness

- NACA 64_1-412

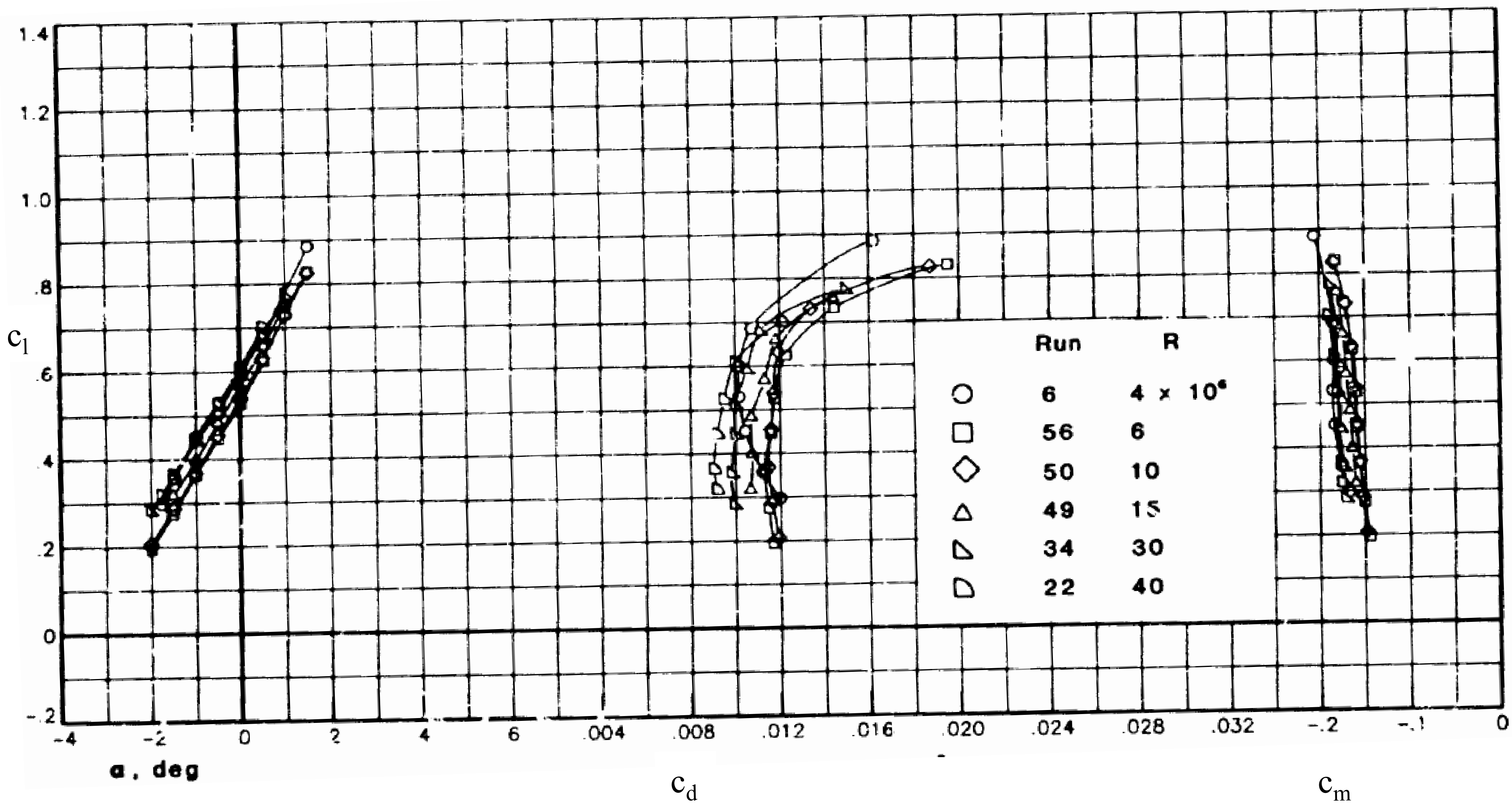

Section lift coefficient, c_l
Moment coefficient, c_m c/4
Section angle of attack, α_0, deg
-32 -24 -16 -8 0 8 16 24 32
3.6 3.2 2.8 2.4 2.0 1.6 1.2 .8 .4 0 -.4 -.8 -1.2 -1.6 -2.0
.1 0 -.1 -.2 -.3 -.4 -.5


y/c
x/c
0 .2 .4 .6 .8 1.0
Section drag coefficient, c_d
Moment coefficient, c_m a.c.
Section lift coefficient, c_l
-1.2 -.8 -.4 0 .4 .8 1.2 1.6
a.c. position
R
x/c
y/c
3.0×10⁶ .266 -.053
6.0 .266 -.046
9.0 .267 -.034
6.0 Standard roughness
0.20c simulated split flap deflected 60°
6.0
6.0 Standard roughness

- NASA SC(2)-0012 (0.8 Mach - supercritical)
  - No experiment close to stall
  - http://ntrs.nasa.gov/search.jsp?N=0



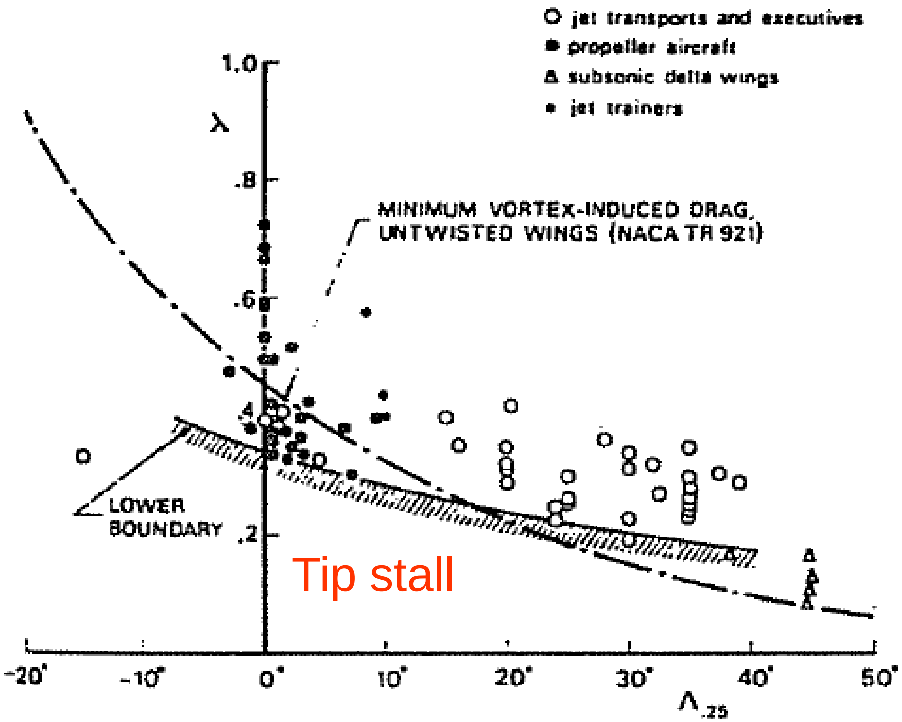
- NASA SC(2)-0714 (0.75 Mach - supercritical)

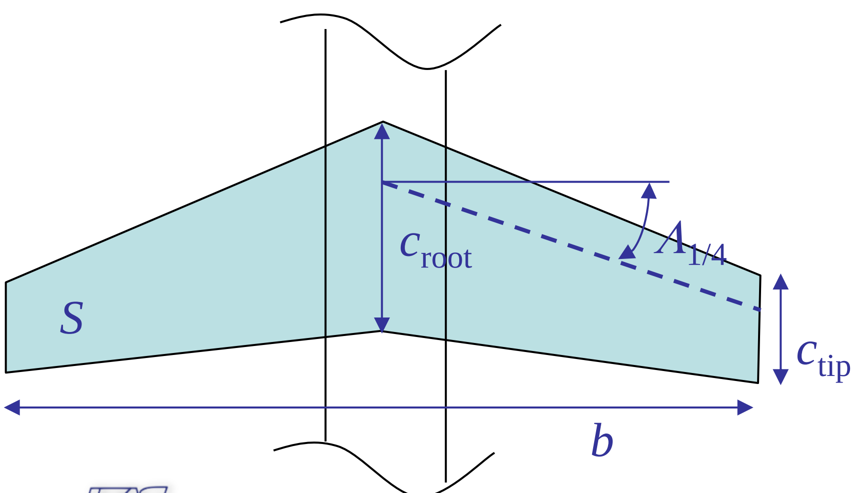


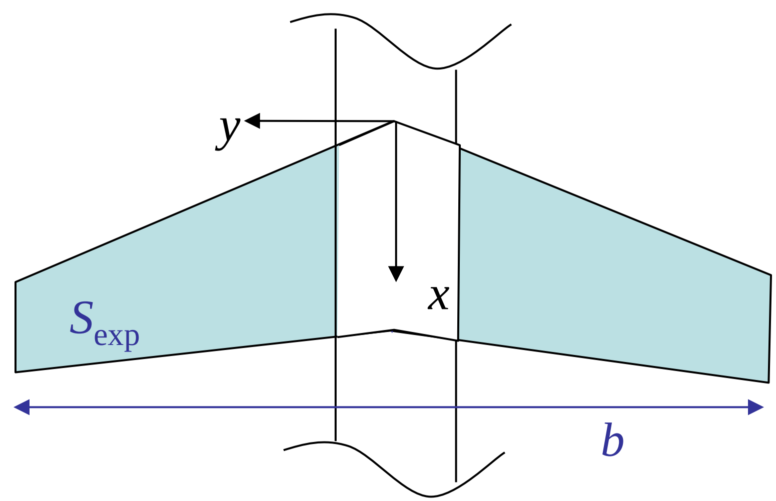
- Geometry
  - Main parameters
    - Span $b=2s$
    - Aspect ratio $AR = b^2/S$ ~ 7-9
    - Total (gross) area S
    - Taper ratio $\lambda = c_{\text{tip}}/c_{\text{root}}$
    - Quarter chord sweep $\Lambda_{1/4}$

$$\tan\Lambda_{e_2} = \tan\Lambda_{e_1} + \frac{4}{AR}\frac{1-\lambda}{1+\lambda}(e_1 - e_2)$$

- Geometrical twist $\varepsilon_{g\ \text{tip}}$

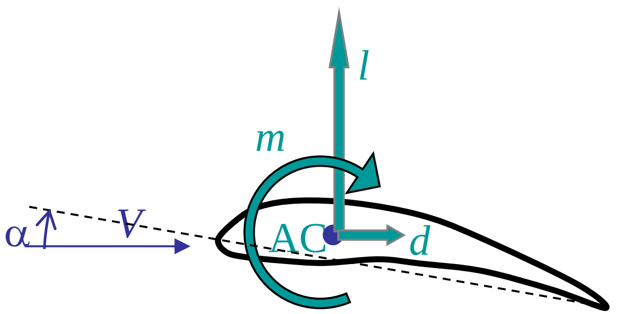


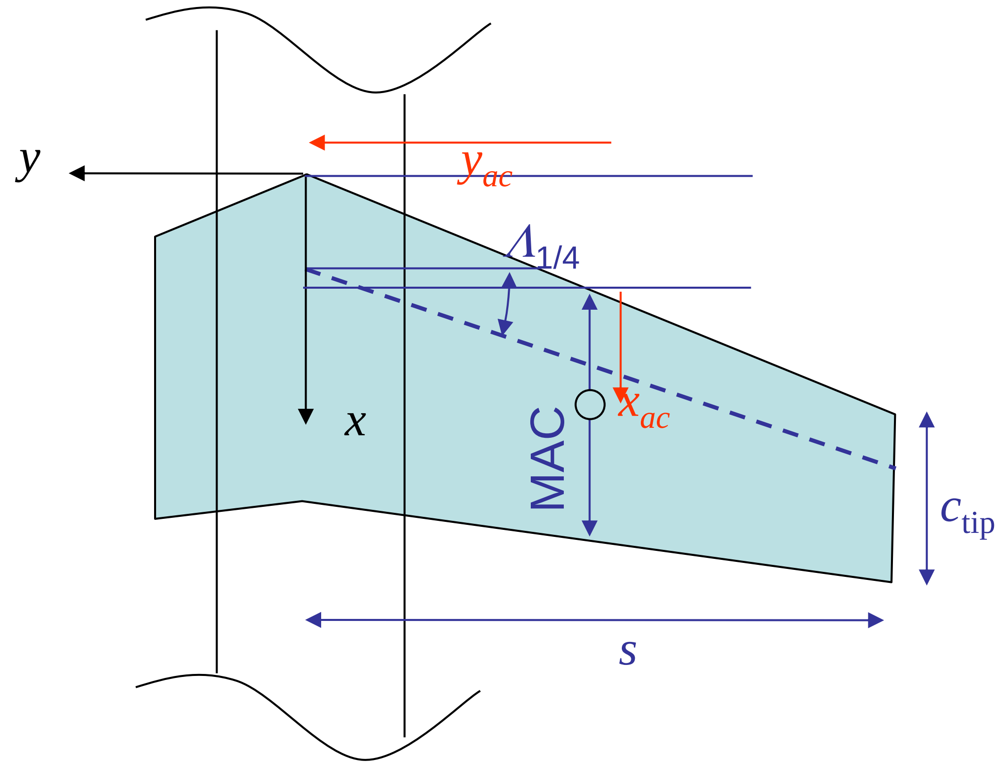




- Geometry (2)
  - Aerodynamic center
    - $\mathrm{MAC} = \bar{\bar{c}} = \frac{2}{S}\int_0^{\frac{b}{2}} c^2 dy$

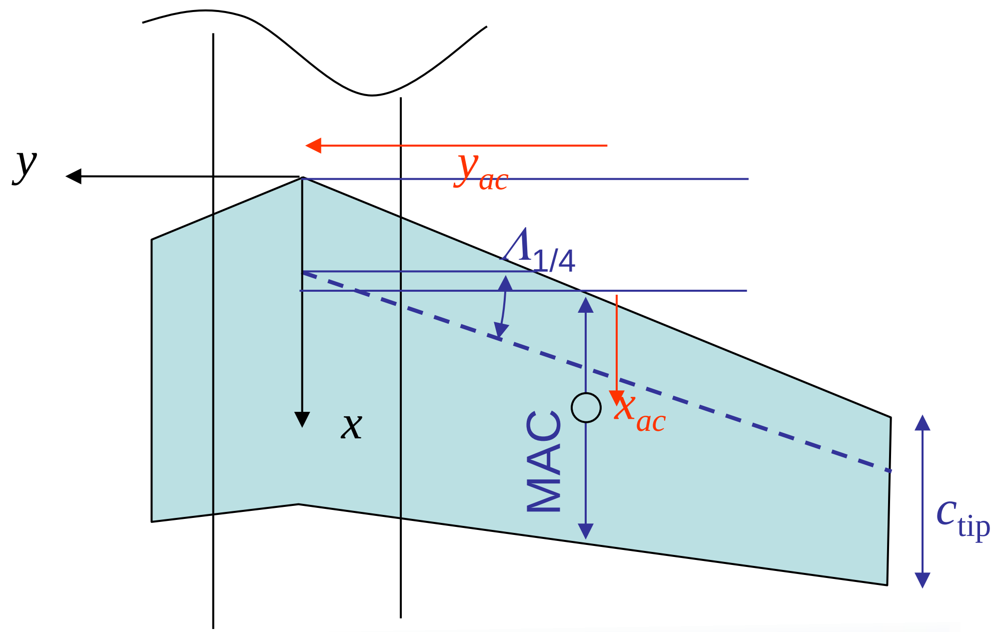
$$y_{ac} = \frac{2}{S}\int_0^{\frac{b}{2}} c\, y\, dy$$

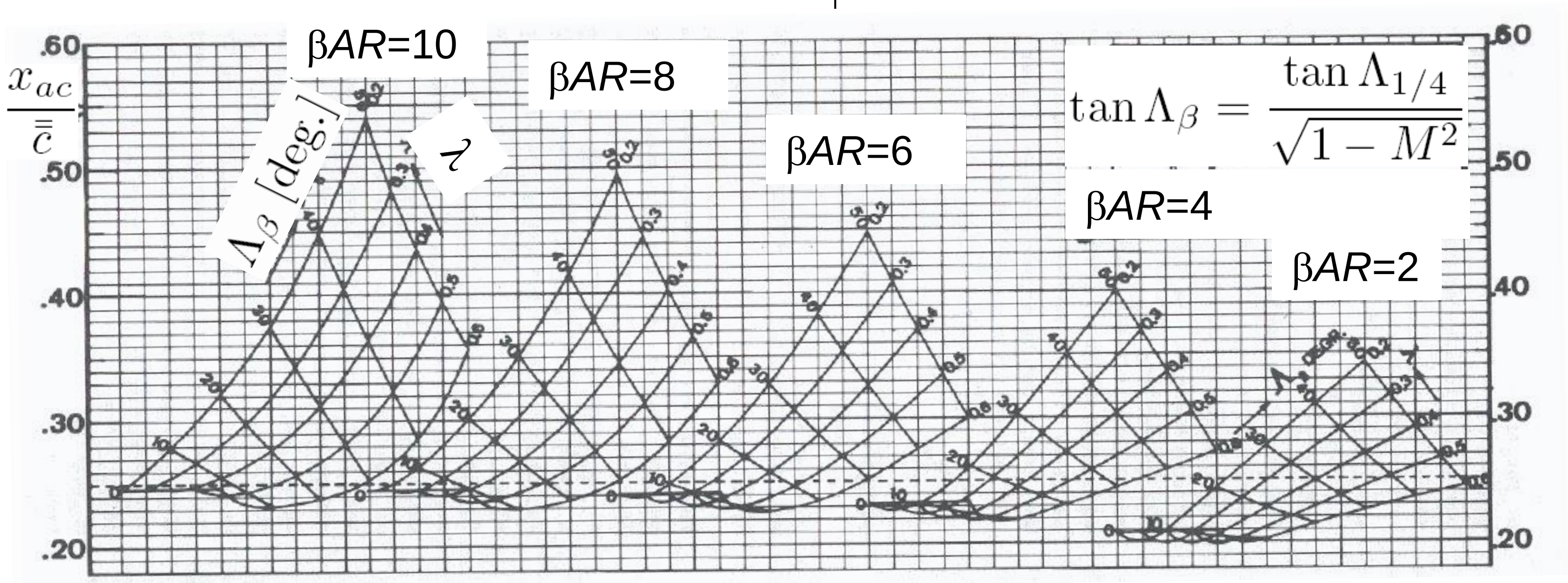




- Geometry (3)
  - Aerodynamic center
    - Position x_{ac} depends on compressibility effects

$$\beta = \sqrt{1 - M^2}$$





- Geometry (4)
  - Allow to compute
    - Maximum thickness at $s/2$

$$\frac{\bar{t}}{c} = \frac{3}{10M}\sqrt[3]{\frac{1}{M\cos\Lambda_{1/4}} - M\cos\Lambda_{1/4}}\left[1 - \left(\frac{5 + M^2\cos^2\Lambda_{1/4}}{5 + (M*)^2}\right)^{3.5}\right]^{\frac{2}{3}}$$

  - Divergence is avoided at M cruise
  - With

$$M* = \{1,\ 1.05,\ 1.15\} - \frac{C_L}{4\cos^2\Lambda_{1/4}}$$

for {normal , peaky, supercritical} airfoils

- Geometry (5)
  - Allow to compute (2)
    - Fuel volume in the wing

$$V_{\text{fuel}} = 0.54\frac{S^2}{b}\left(\frac{t}{c}\right)_{\text{root}} \frac{1+\lambda\sqrt{\tau}+\lambda^2\tau}{(1+\lambda)^2} \quad \text{with} \quad \tau = \frac{\left(\frac{t}{c}\right)_{\text{tip}}}{\left(\frac{t}{c}\right)_{\text{root}}}$$

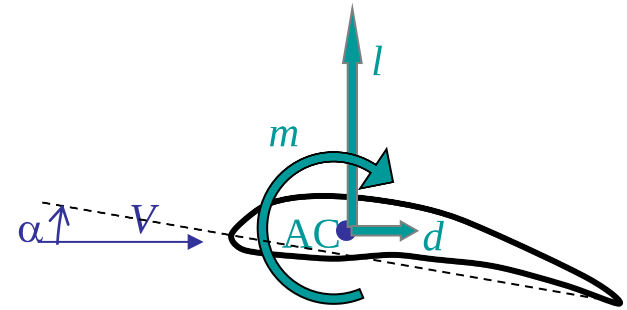
      - If too large, use c_{root}, c_{tip}, b & S corresponding to a reduced part of the wing

    - Wetted surface
      - Surface in contact with the fluid

$$S_{\text{wetted}\,w} = 2S_{\text{exp}}\left(1+\frac{1}{4}\frac{(t/c)_{\text{root}}+(t/c)_{\text{tip}}\,\lambda}{1+\lambda}\right)$$

- **Lift**
  - Cruise (reduced angle of attack)
    - Wing lift coefficient

$$C_{Lw} = C_{Lw\alpha}\left[\alpha_{\text{root}} - \alpha_{L_{0_{\text{root}}}}\right] = a\left[\alpha_{\text{root}} - \alpha_{L_{0_{\text{root}}}}\right]$$



      - α_{root}: Angle of attack at root of the wing (rad)
      - $\alpha_{L_{0_{\text{root}}}}$: Angle of attack at root leading to a zero lift of the wing
        - » See next slide
    - Slope of wing lift coefficient (rad^{-1})

$$\left.\begin{aligned} \beta &= \sqrt{1-M^2} \\ k &= \frac{\beta c_{l\alpha}}{2\pi} \\ \tan\Lambda_\beta &= \frac{\tan\Lambda_{1/4}}{\sqrt{1-M^2}} \end{aligned}\right\} \Longrightarrow \beta a = \frac{2\pi}{\frac{2}{\beta AR} + \sqrt{\left(\frac{1}{k\cos\Lambda_\beta}\right)^2 + \left(\frac{2}{\beta AR}\right)^2}}$$

- Lift (2)
  - Cruise (reduced angle of attack) (2)
    - Zero-lift angle of attack at root

$$\alpha_{L_{0_{\text{root}}}} = \alpha_{l_{0_{\text{root}}}} + \alpha_{01}\varepsilon_{a\,\text{tip}}$$

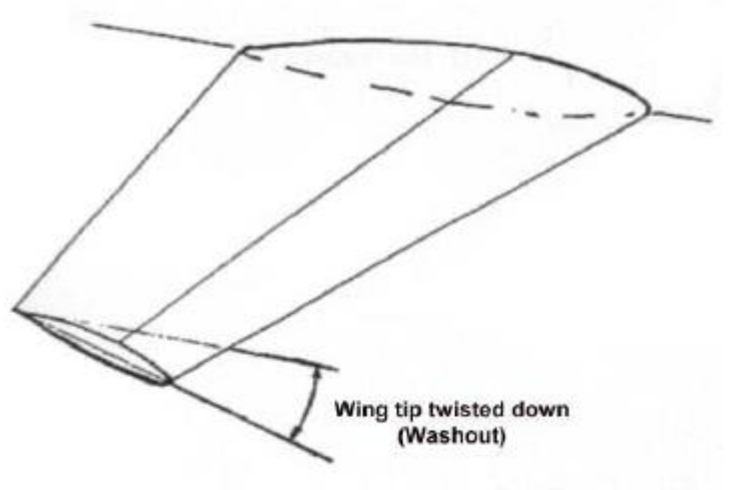
– Geometrical twist

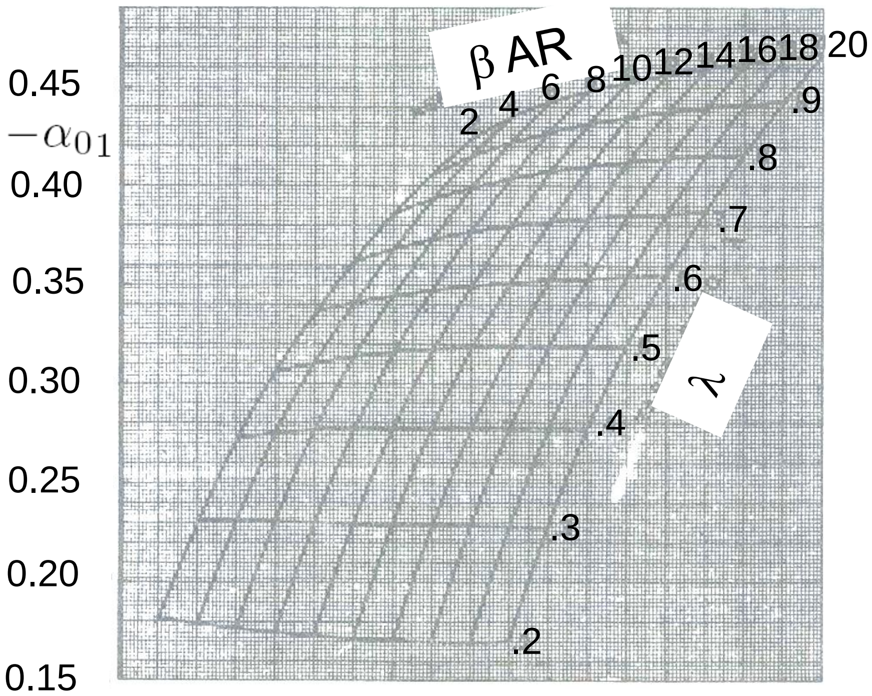
» Example: lofted

$$\varepsilon_g = \varepsilon_{g\,\text{tip}} \frac{\lambda \frac{y}{s}}{1-(1-\lambda)\frac{y}{s}}$$

– Local aerodynamic twist α_{01}

» $-\alpha_{01}$ see picture





- Lift (3)
  - Cruise (reduced angle of attack) (3)
    - Zero-lift angle of attack at root

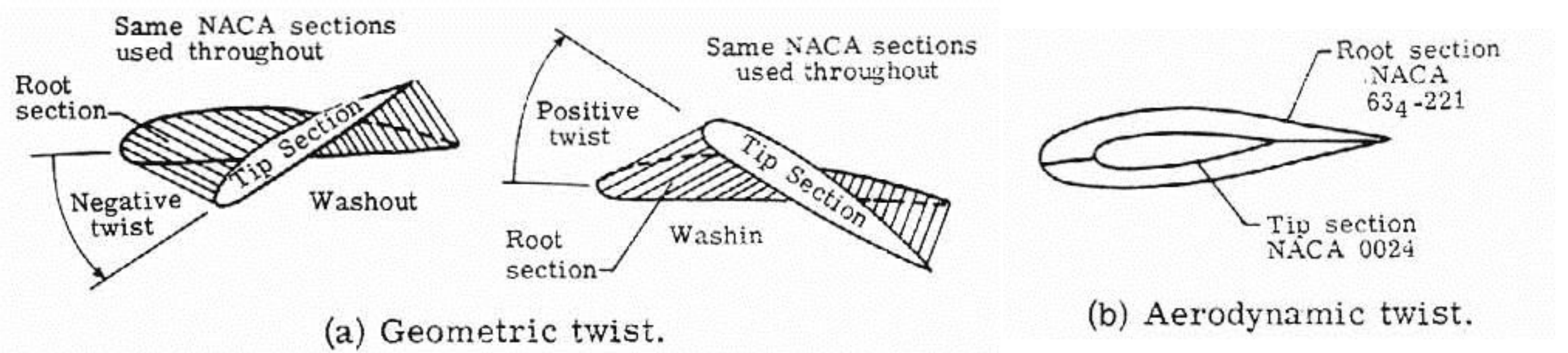
$$\alpha_{L0_{\text{root}}} = \alpha_{l0_{\text{root}}} + \alpha_{01}\varepsilon_{a\,\text{tip}}$$

– Aerodynamic twist $\varepsilon_{a\,\text{tip}} = \varepsilon_{g\,\text{tip}} + \alpha_{l0_{\text{root}}} - \alpha_{l0_{\text{tip}}}$

» <0 pour un washout

» Zero-lift angle of attack of the airfoil α_{l0} can change between root and tip if the airfoil has an evolving shape

– Purpose: Stall initiated at ~ 0.4 *s*



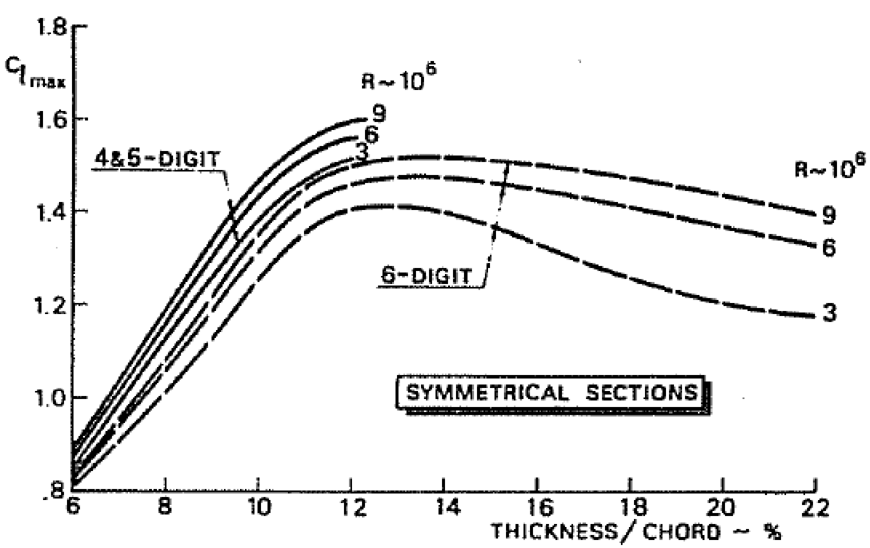
(a) Geometric twist.

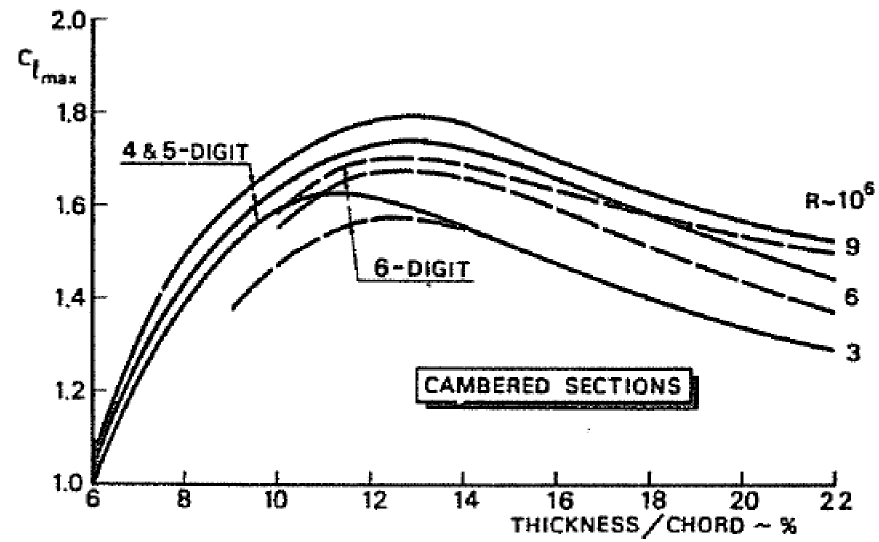
(b) Aerodynamic twist.

- Maximum lift
  - Maximum lift coefficient in approach or at takeoff ($M << 1$)
    - Curves without high-lift devices

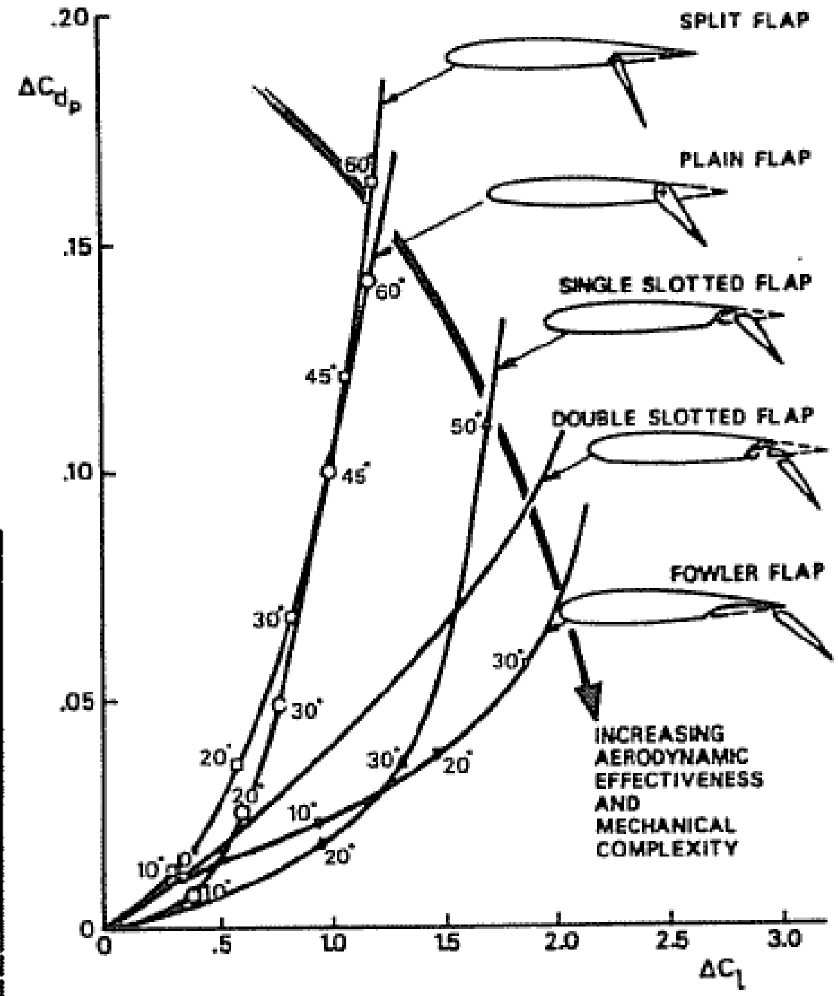
$$C_{L\max} = \cos\Lambda_{1/4}\,\{0.88,\ 0.95\}\,\frac{(c_{l\max})_{\text{root}} + (c_{l\max})_{\text{tip}}}{2}$$ { λ =1, $\lambda \neq 1$ }

      - Airfoil NACA-4 5 6 digits, see pictures
      - Supercritical airfoil with rear loading: 10% larger than NACA-5





- Maximum lift (2)
  - Maximum lift coefficient in approach or at takeoff (M << 1) (2)
    - With high lift devices
      - Device & angle depend on
        - » Approach
        - » Landing
        - » Takeoff (drag has to be reduced)



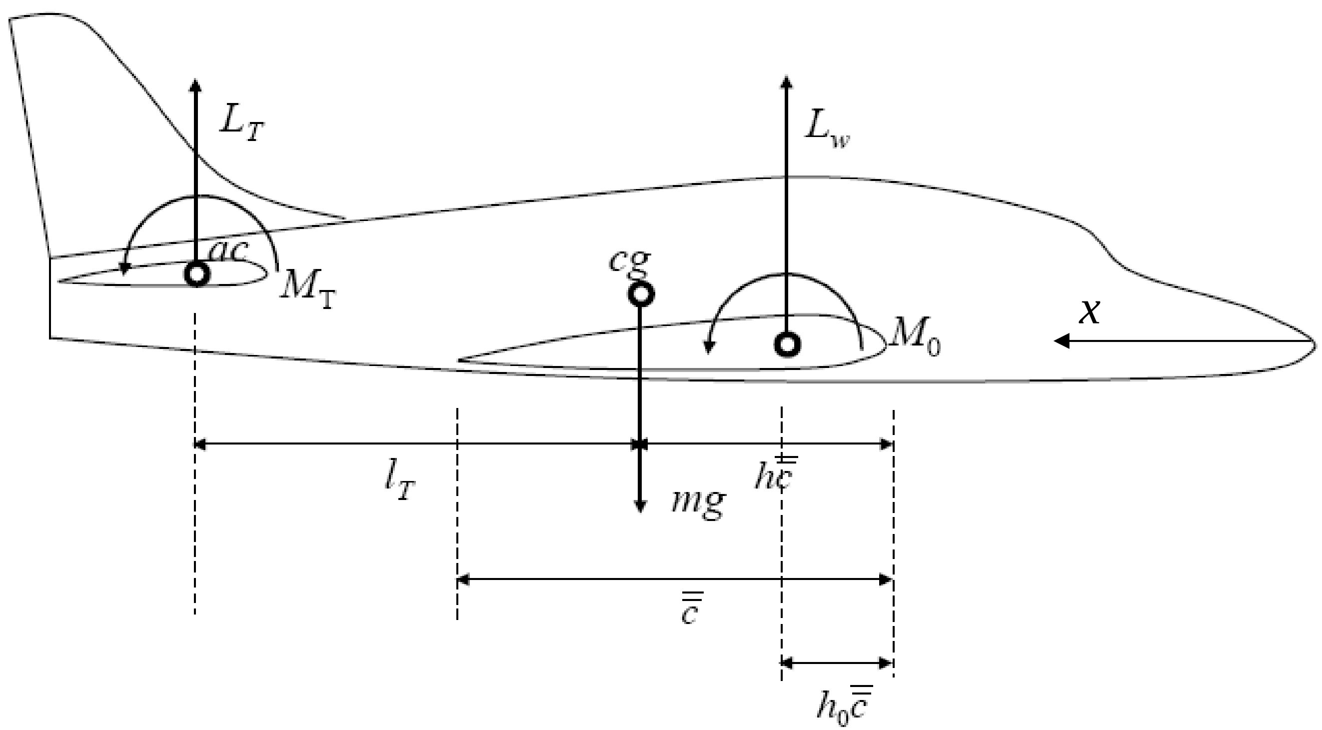
HIGH-LIFT DEVICE		TYPICAL FLAP ANGLE		$C_{l_{max}}/\cos\Lambda_{.25}$	
TRAILING EDGE	LEADING EDGE	TAKEOFF	LANDING	TAKEOFF	LANDING
PLAIN	-	20°	60°	1.40-1.60	1.70-2.00
SINGLE SLOTTED	-	20°	40°	1.50-1.70	1.80-2.20
FOWLER*	-	15°	40°	2.00-2.20	2.50-2.90
DOUBLE SLOTTED**	-	20°	50°	1.70-1.95	2.30-2.70
	SLAT			2.30-?.60	2.80-3.20
TRIPLE SLOTTED**	SLAT	20°	40°	2.40-2.70	3.20-3.50

- Maximum lift (3)
  - Maximum lift coefficient in approach or at takeoff (M << 1) (3)
    - With high lift devices (2)
      - Stall (equivalent) velocities $$V_{s(0)} = \sqrt{\frac{W_{(0)}}{S}\frac{2}{\rho}\frac{1}{1.133 C_{L\max(0)}}}$$
      - V_s: flaps down (out)
      - V_{s0}: flaps in approach configuration (weight W_0 at landing)

Lost of velocity resulting from a maneuver

HIGH-LIFT DEVICE		TYPICAL FLAP ANGLE		$C_{L_{max}}/\cos\Lambda_{.25}$	
TRAILING EDGE	LEADING EDGE	TAKEOFF	LANDING	TAKEOFF	LANDING
PLAIN	-	20°	60°	1.40-1.60	1.70-2.00
SINGLE SLOTTED	-	20°	40°	1.50-1.70	1.80-2.20
FOWLER*	-	15°	40°	2.00-2.20	2.50-2.90
DOUBLE SLOTTED**	-	20°	50°	1.70-1.95	2.30-2.70
	SLAT			2.30-2.60	2.80-3.20
TRIPLE SLOTTED**	SLAT	20°	40°	2.40-2.70	3.20-3.50

- Longitudinal balance
  - Lift
    - $C_L = C_{Lw} + C_{LT}\frac{S_T}{S} \Longrightarrow C_L = (C_{L\alpha})_{\text{plane}}\left(\alpha_f - (\alpha_{L0})_f\right)$
    - Angle of attack of the fuselage α_f
    - Zero-lift angle of attack of the fuselage $(\alpha_{L0})_f$

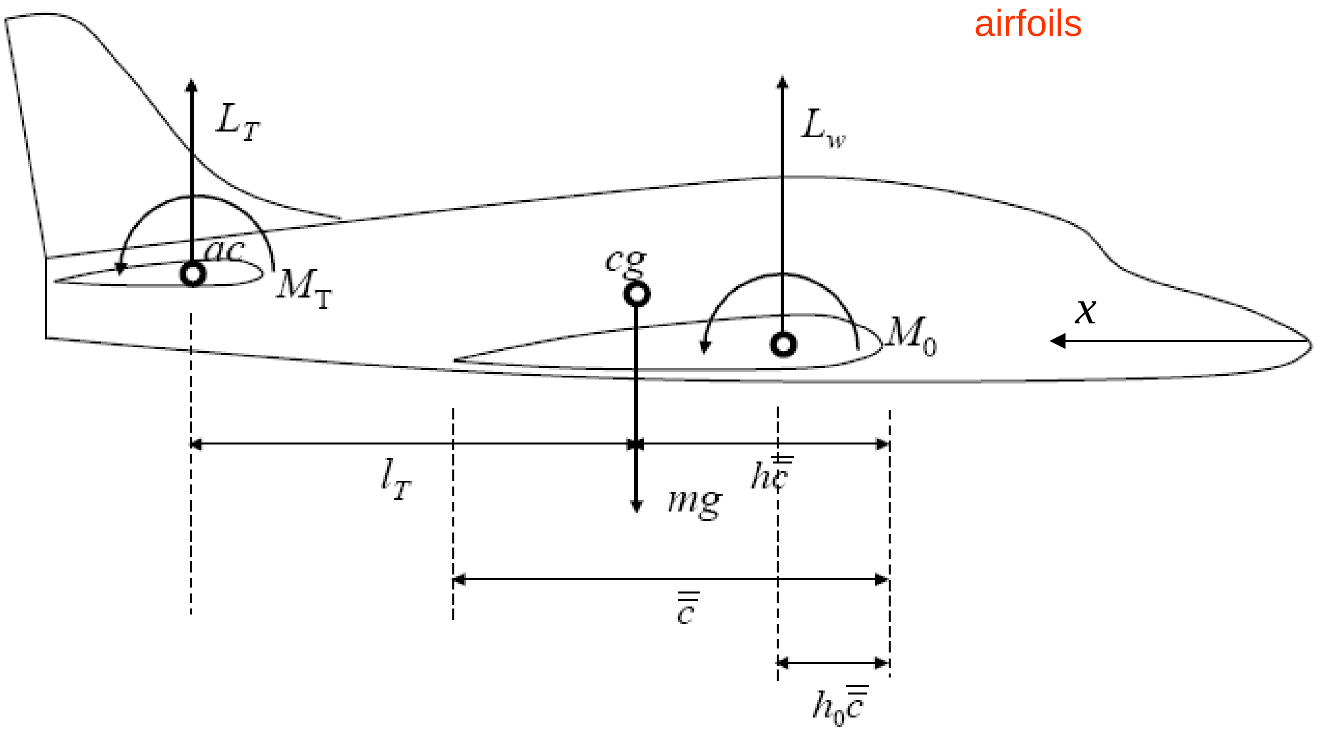


- Longitudinal balance (2)
  - Moment
    - Moment around gravity center

$$C_m = C_{m0} + C_{Lw}\frac{x_{cg} - x_{acw}}{\bar{\bar{c}}} + C_{mT} - C_{LT}\frac{S_T l_T}{\bar{\bar{c}} S}$$

    - Pitching moment of the wing $C_{m0} = \frac{2}{S\,\bar{\bar{c}}}\int_0^{\frac{b}{2}} c_m c^2 dy$

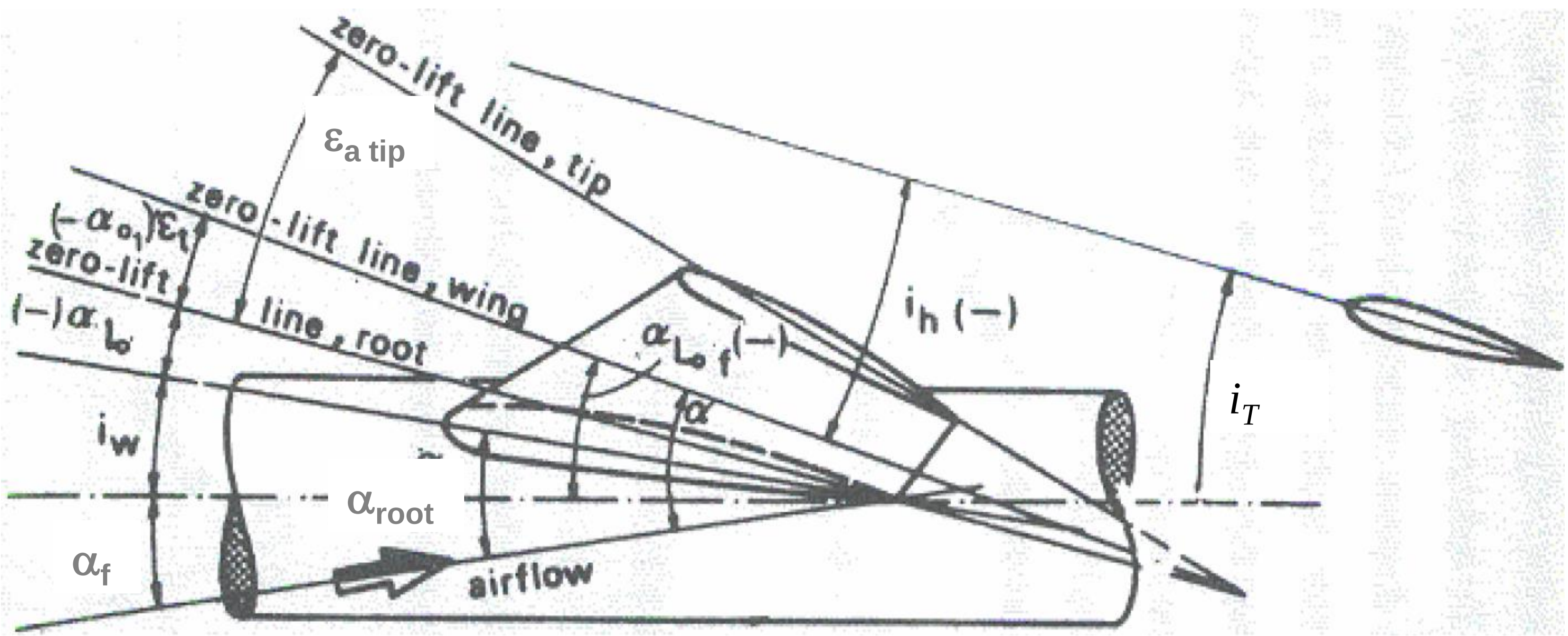
Zero for symmetrical airfoils



- Trimmed configuration
  - Equations
    - $C_L = C_{Lw} + C_{LT}\dfrac{S_T}{S}$
    - $C_m = C_{m0} + C_{Lw}\dfrac{x_{cg} - x_{acw}}{\bar{\bar{c}}} + C_{mT} - C_{LT}\dfrac{S_T l_T}{\bar{\bar{c}} S}$
  - At equilibrium (steady flight)
    - $C_m = 0 \implies C_L = C_{Lw} + C_{LT}\dfrac{S_T}{S} = C_{Lw}\left(1 + (h - h_0)\dfrac{\bar{\bar{c}}}{l_T}\right) + \dfrac{\bar{\bar{c}}}{l_T} C_{m0}$

Stability

- Trimmed configuration (2)
  - Angle of incidence of the wing i_w
    - Angle between the fuselage and the root chord
    - In cruise
      - α_f ~0 so the fuselage is horizontal
      - Lift is known from the weight $C_L\left(\alpha_f = 0\right) = C_{L0}$



Stability

- Trimmed configuration (3)
  - Angle of incidence of the wing i_w (2)
    - Equations

$$
\begin{cases}
C_L\left(\alpha_f = 0\right) = C_{L0} \\
C_L = C_{Lw} + C_{LT}\dfrac{S_T}{S} = C_{Lw}\left(1 + \left(h - h_0\right)\dfrac{\bar{\bar{c}}}{l_T}\right) + \dfrac{\bar{\bar{c}}}{l_T} C_{m0} \\
C_{Lw} = a\left[\alpha_{\text{root}} - \alpha_{L0_{\text{root}}}\right] \\
\alpha_{\text{root}} = \alpha_f + i_w \\
\alpha_{L0_{\text{root}}} = \alpha_{l0_{\text{root}}} + \alpha_{01}\varepsilon_{a\,\text{tip}}
\end{cases}
$$

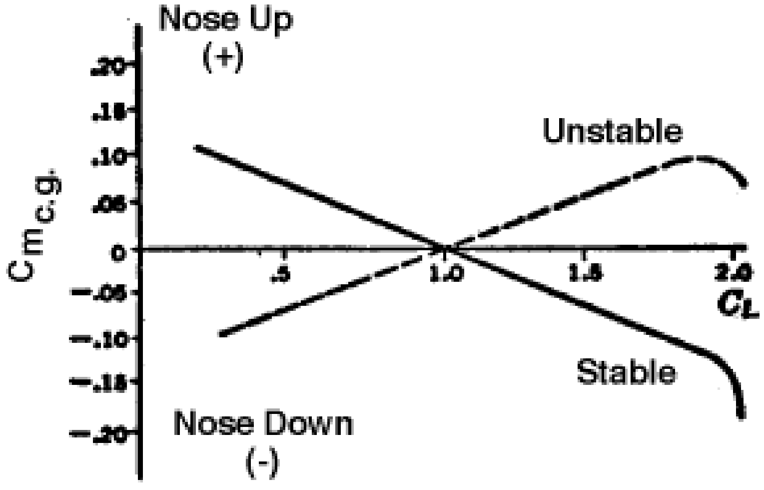
$$
\begin{cases}
i_w = \dfrac{C_{Lw}^{*}}{a} + \alpha_{01}\varepsilon_{a\,\text{tip}} + \left(\alpha_{l0}\right)_{\text{root}} \\
C_{Lw}^{*} = \dfrac{C_{L0} - \frac{\bar{\bar{c}}}{l_T} C_{m0}}{1 + \left(h - h_0\right)\frac{\bar{\bar{c}}}{l_T}}
\end{cases}
$$

- Trimmed configuration (4)
  - Value $\alpha_f = 0$ is obtained for one single value of the lift, so for a given weight
  - But weight changes during flight, as well as the cg location
  - To define i_w, values of C_{L0} & x_{cg} are taken for
    - 50% of maximum payload
    - 50% of fuel capacity
  - Lift curve of a trimmed aircraft

$$\begin{cases} C_L = C_{Lw} + C_{LT}\dfrac{S_T}{S} = C_{Lw}\left(1 + (h - h_0)\dfrac{\bar{\bar{c}}}{l_T}\right) + \dfrac{\bar{\bar{c}}}{l_T} C_{m0} \\ C_L = (C_{L\alpha})_{\text{plane}}\left(\alpha_f - (\alpha_{L0})_f\right) \end{cases}$$

$$\Rightarrow \begin{cases} (\alpha_{L0})_f = -\dfrac{C_{L0}}{C_{L\alpha\text{plane}}} \\ C_{L\alpha\text{plane}} = \left(1 + (h - h_0)\dfrac{\bar{\bar{c}}}{l_T}\right) a \end{cases}$$

- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$
  - CG position for which $\frac{\partial C_m}{\partial C_{Lw}} = 0$ with elevators blocked
    - When elevators are blocked, stability requires $\frac{\partial C_m}{\partial \alpha} < 0$



- As C_L ~ proportional to α, the stability limit is approximated by $\frac{\partial C_m}{\partial C_{Lw}} = 0$
- But as $C_m = C_{m0} + C_{Lw}\frac{x_{cg} - x_{acw}}{\bar{\bar{c}}} + C_{mT} - C_{LT}\frac{S_T l_T}{\bar{\bar{c}} S}$

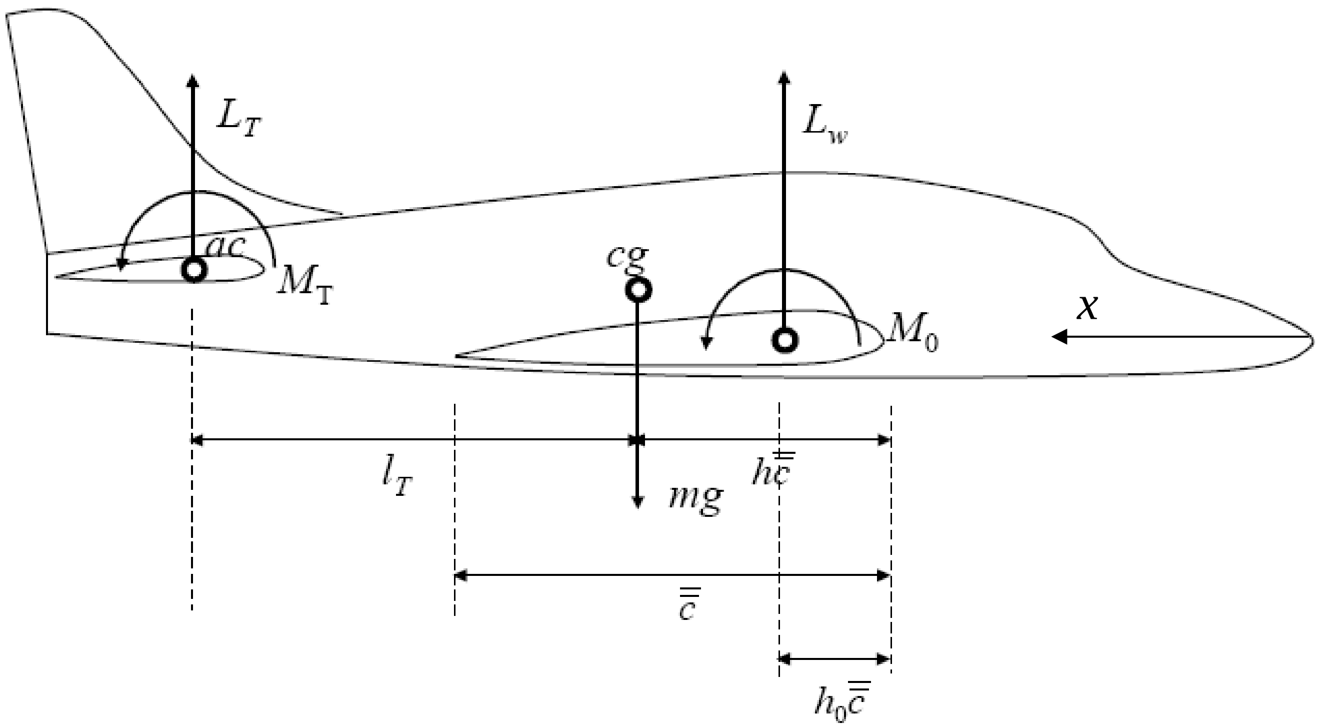
  the stability depends on the cg position
- Neutral point is the position of the cg leading to $\frac{\partial C_m}{\partial C_{Lw}} = 0$

Stability

- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$ (2)
  - Definition
    - As $$C_m = C_{m0} + C_{Lw} \frac{x_{cg} - x_{acw}}{\bar{\bar{c}}} + C_{mT} - C_{LT} \frac{S_T l_T}{\bar{\bar{c}} S}$$

$$\Longrightarrow \quad h_n = h_0 + \frac{dC_{LT}}{dC_{Lw}} \frac{l_T S_T}{\bar{\bar{c}} S}$$

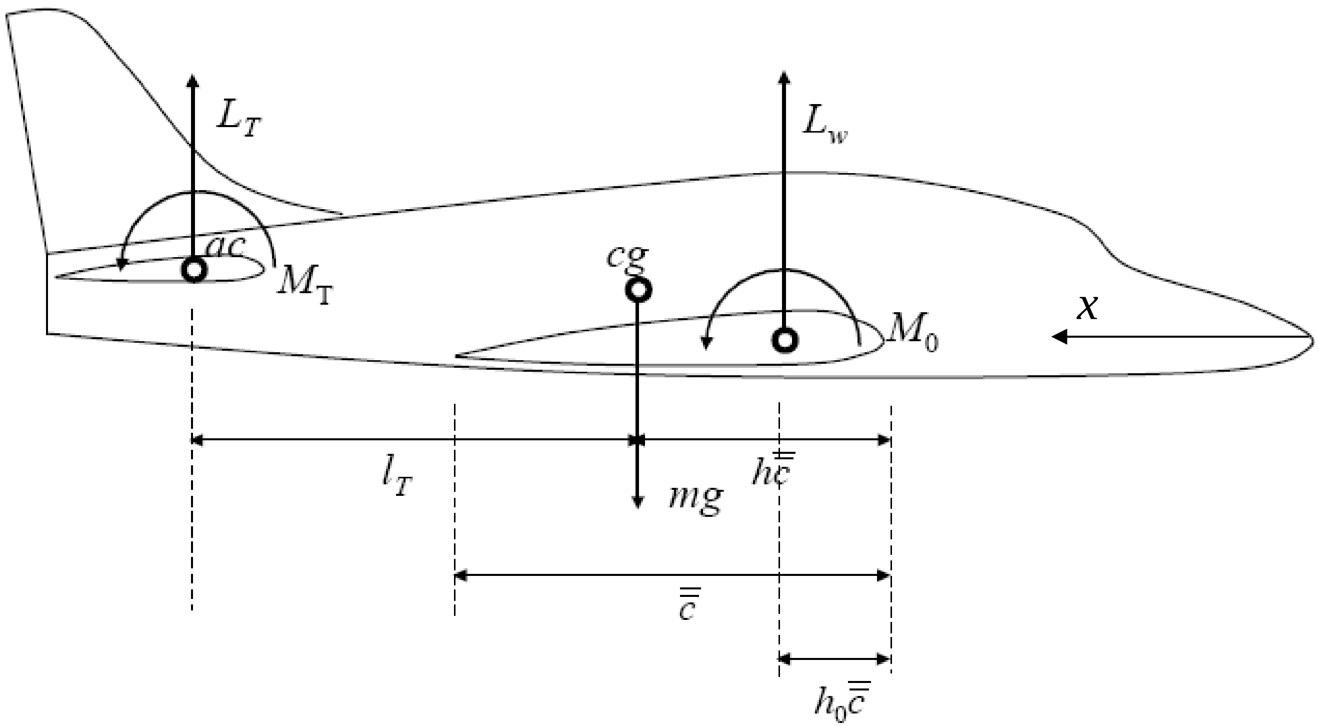
    - But this not correct as fuselage is destabilizing (low momentum but high derivative)



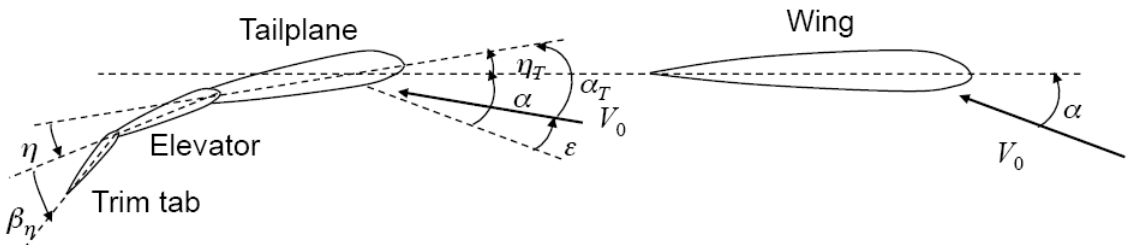
- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$ (3)
  - Definition (2)
    - Fuselage effect

$$C_m = C_{m0} + C_{Lw}\frac{x_{cg} - x_{acw}}{\bar{\bar{c}}} + C_{mT} - C_{LT}\frac{S_T l_T}{\bar{\bar{c}} S} + C_{m\,\text{fus}}$$

$$\Longrightarrow \quad h_n = h_0 + \frac{dC_{LT}}{dC_{Lw}}\frac{l_T S_T}{\bar{\bar{c}} S} - \frac{dC_{m\,\text{fus}}}{dC_{Lw}}$$



- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$ (4)
  - Position $h_n = h_0 + \frac{dC_{L\,T}}{dC_{L\,w}} \frac{l_T S_T}{\bar{\bar{c}} S} - \frac{dC_{m\,\mathrm{fus}}}{dC_{L\,w}}$
    - Stick-fixed tail lift slope (η, β_η constant)



- Tail lift $C_{L\,T} = a_1 (\alpha_T - \alpha_{T\,0}) + a_2 \eta + a_3 \beta_\eta$
- Attack angle of horizontal tail in terms of downwash ε: $\alpha_T = \alpha_{\mathrm{root}} - \varepsilon + \eta_T$

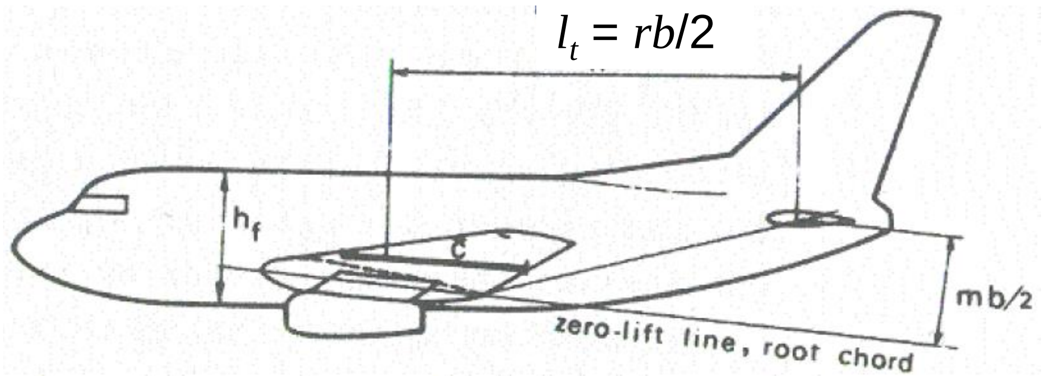
  with $\varepsilon \simeq \frac{d\varepsilon}{d\alpha} (\alpha_{\mathrm{root}} - \alpha_{L\,0\mathrm{root}})$
- As $C_{L\,w} = a (\alpha_{\mathrm{root}} - \alpha_{L\,0\mathrm{root}}) \Longrightarrow \alpha_T \simeq \frac{C_{L\,w}}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) + \alpha_{L\,0\mathrm{root}} + \eta_T$

- Eventually $\left(\frac{dC_{L\,T}}{dC_{L\,w}}\right)^{\text{stick-fixed}} = \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$

- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$ (5)
  - Downwash
    - Gradient of downwash resulting from the wing vortex

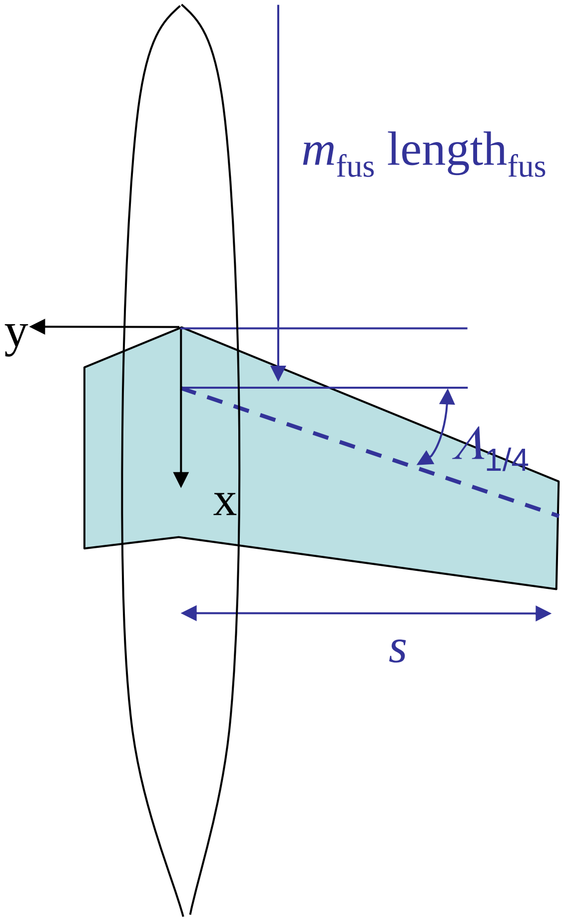
$$\frac{\partial \varepsilon}{\partial \alpha} = 1.75 \frac{a}{\pi AR \left(\frac{2\lambda l_t}{b}\right)^{\frac{1}{4}} (1 + |m|)}$$

l_t = distance between ac of wing and ac of horizontal tail



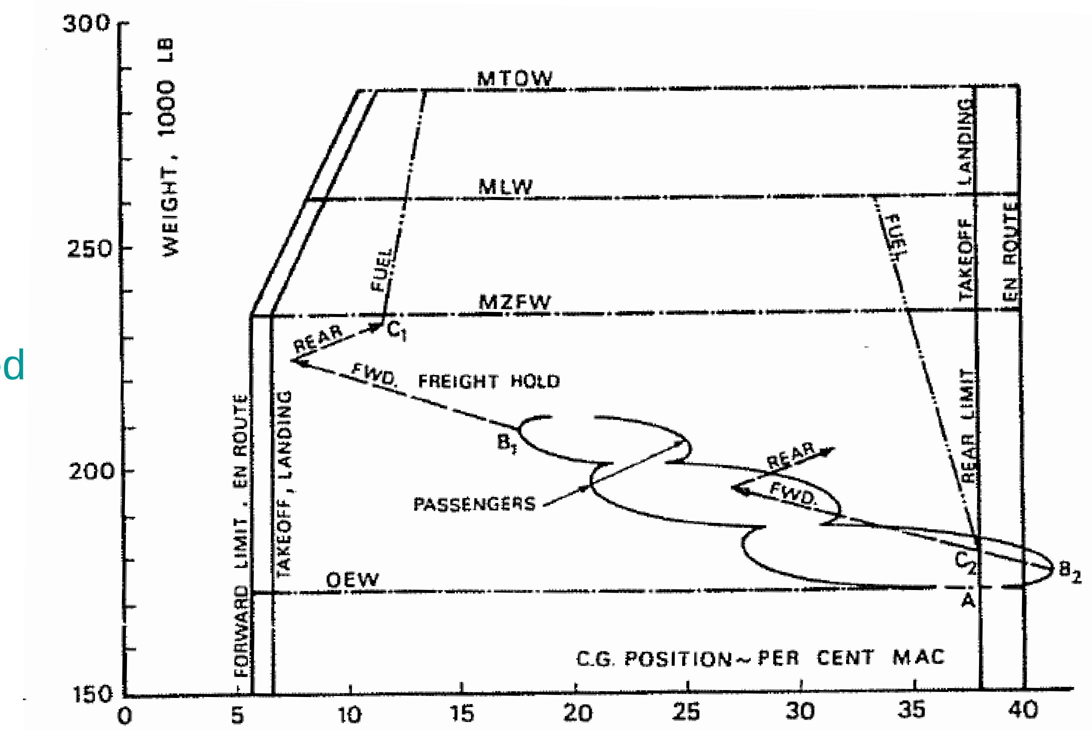
- Stick-fixed neutral point $x_n = h_n \bar{\bar{c}}$ (6)
  - Fuselage effect
    - Empirical method NACA TR711 $$\frac{dC_{m\,\mathrm{fus}}}{dC_{L\,w}} = \frac{k_{\mathrm{fus}} \mathrm{width}_{\mathrm{fus}}^2 \mathrm{length}_{\mathrm{fus}}}{S \bar{\bar{c}} a}$$

m_{fus}	k_{fus}
0.1	0.115
0.2	0.172
0.3	0.344
0.4	0.487
0.5	0.688
0.6	0.888
0.7	1.146

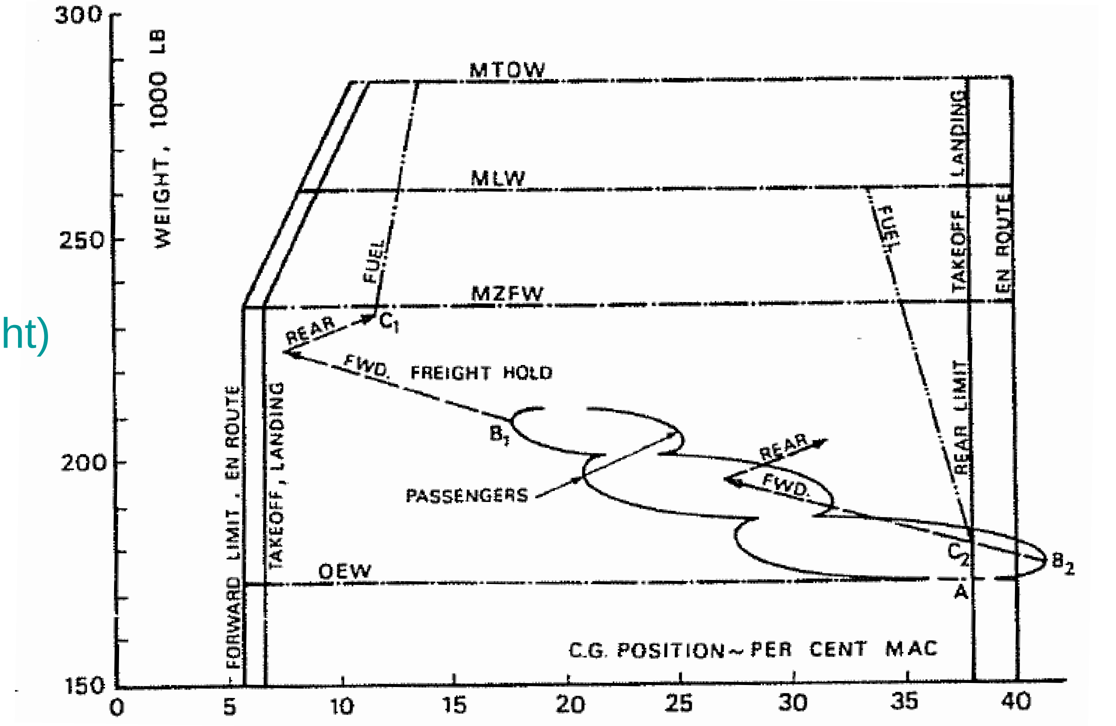


- Stability margin
  - Stability requires $\dfrac{\partial C_m}{\partial \alpha} < 0$
  - The stability is measured by the stability margin
    - $K_n = \dfrac{x_n - x_{cg}}{\bar{\bar{c}}} = h_n - h > 0$
  - FAA requirement
    - Stable enough $\Longrightarrow$ $K_n > 5\%$
  - Enough maneuverability
    - K_n <~ 10%
    - If T tail, in order of avoiding deep stall: 10% <~ K_n < 20%

- Stability margin (2)
  - $K_n = \dfrac{x_n - x_{cg}}{\bar{\bar{c}}} = h_n - h > 0$
  - Flight conditions
    - h_0 depends on velocity
    - CG location
      - Depends on payload
      - Changes during the flight as fuel is burned
  - Whatever the flight condition is K_n should remains > 5%

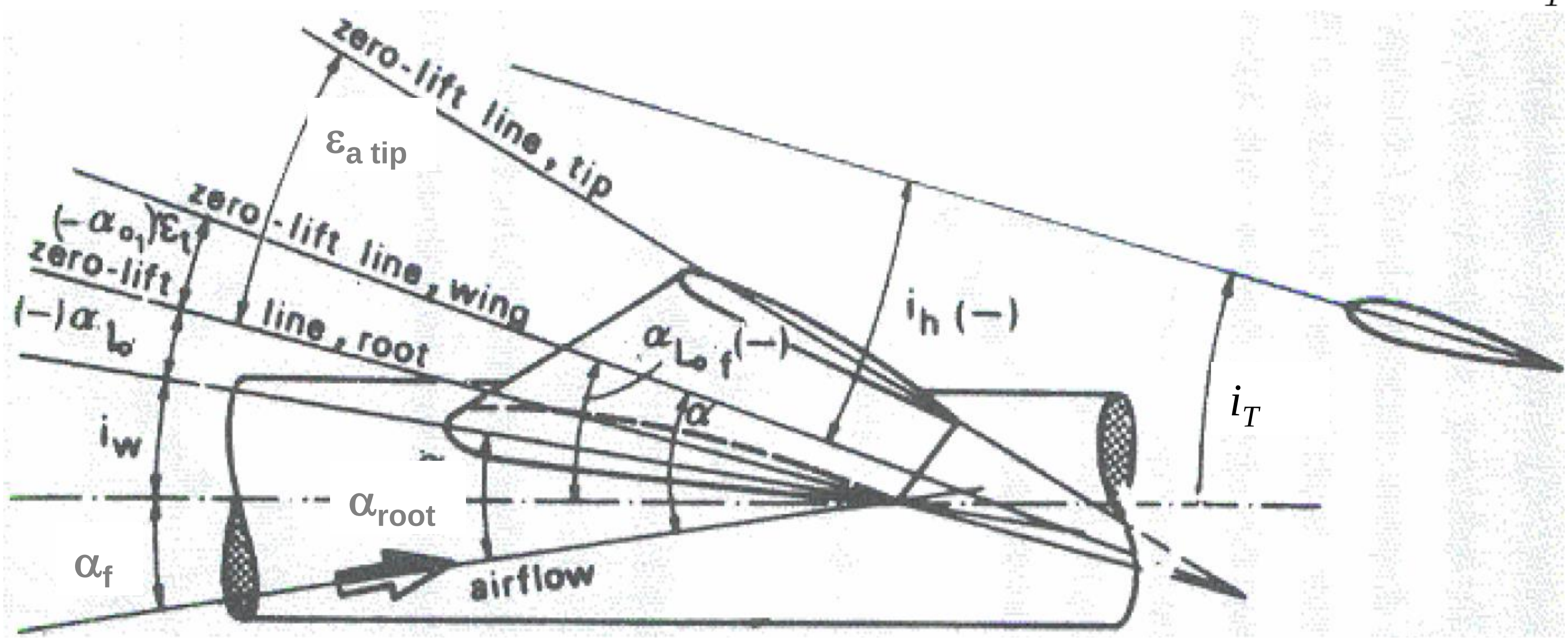


- Stability margin (3)
  - $K_n = \frac{x_n - x_{cg}}{\bar{\bar{c}}} = h_n - h > 0$
  - In general during cruise
    - CG close to 0.25 $\bar{\bar{c}}$
      - Allows reducing the drag due to the tail
    - Tail can act in negative lift (can reach 5% of the weight)



- Angle of incidence of horizontal tail i_T
  - Tail lift should be equal to $C_{LT} = a_1 \left[\frac{C_{Lw}}{a} \left(1 - \frac{d\epsilon}{d\alpha} \right) + \eta_T + \alpha_{L0\text{root}} \right]$ for trimmed cruise ($\alpha_f = 0$) & $\alpha_{T0} = 0$, with

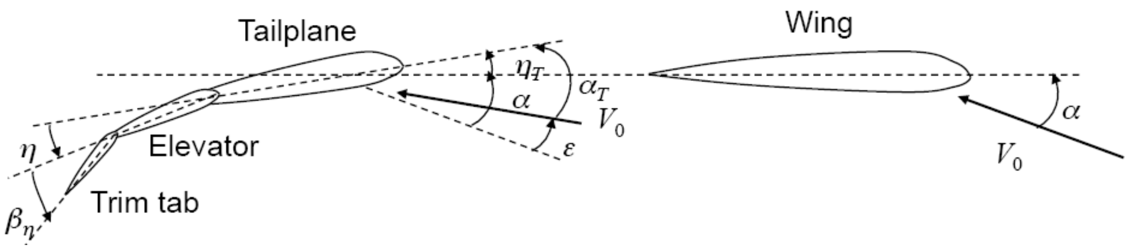
$$\begin{cases} C_L = C_{L0} = -C_{L\alpha\text{plane}} \left(\alpha_{L0}\right)_f \\ C_{Lw} = C^*_{Lw} = \dfrac{C_{L0} - \frac{\bar{\bar{c}}}{l_T} C_{m0}}{1 + (h - h_0)\frac{\bar{\bar{c}}}{l_T}} \\ C_{LT} = \left[C_{m0} + C^*_{Lw}(h - h_0)\right] \dfrac{S\bar{\bar{c}}}{S_T l_T} \end{cases}$$



- Angle of incidence of horizontal tail i_T (2)
  - Equations

$$\begin{cases} C_{LT} = a_1 \left[\frac{C_{L\,w}}{a} \left(1 - \frac{d\epsilon}{d\alpha} \right) + \eta_T + \alpha_{L0\mathrm{root}} \right] \\ i_w = \frac{C^*_{L\,w}}{a} + \alpha_{01}\varepsilon_{a\mathrm{tip}} + (\alpha_{l0})_{\mathrm{root}} \\ C_{L\,w} = C^*_{L\,w} = \frac{C_{L0} - \frac{\bar{\bar{c}}}{l_T} C_{m0}}{1 + (h - h_0) \frac{\bar{\bar{c}}}{l_T}} \\ C_{LT} = \left[C_{m0} + C^*_{L\,w} (h - h_0) \right] \frac{S\bar{\bar{c}}}{S_T l_T} \end{cases}$$

  - Tail incidence angle
    - From η_T

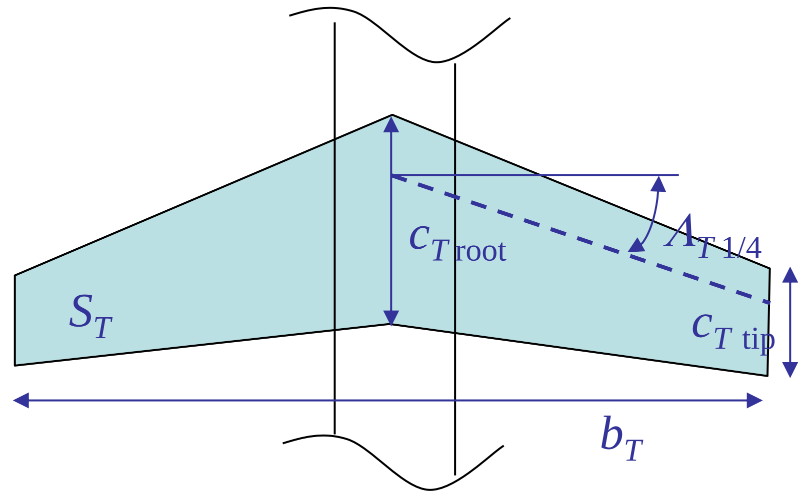


$$\eta_T = i_T - i_w = i_T - \frac{C^*_{L\,w}}{a} - \alpha_{L0\mathrm{root}}$$

$$\Longrightarrow \quad i_T = \frac{C_{m0} + C^*_{L\,w} (h - h_0)}{a_1 \frac{S_T l_T}{S\bar{\bar{c}}}} + \frac{\frac{d\varepsilon}{d\alpha}}{a} C^*_{L\,w}$$

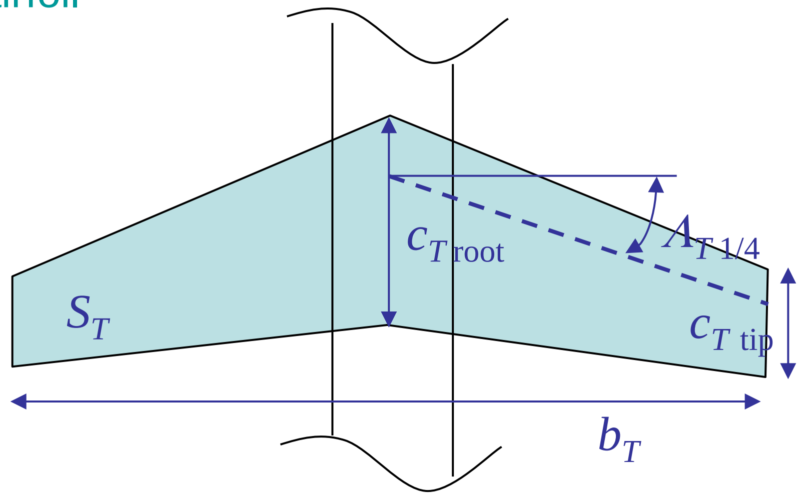
    - Generally i_T such that $\alpha_T < \alpha_{\mathrm{root}}$

- Geometry
  - Parameters
    - Span $b_T=2s_T$
    - Aspect ratio $AR_T = b_T^2/S_T$ ~ 3-6
    - Taper ratio $\lambda_T = c_{T\,\mathrm{tip}}/c_{T\,\mathrm{root}}$ ~0.3-0.5
      - Reduced weight
    - Sweep angle $\Lambda_{T1/4}$
      - 5° more than wings in order to avoid shock waves
    - Airfoil: symmetrical, reduced thickness (e.g. NACA0012)

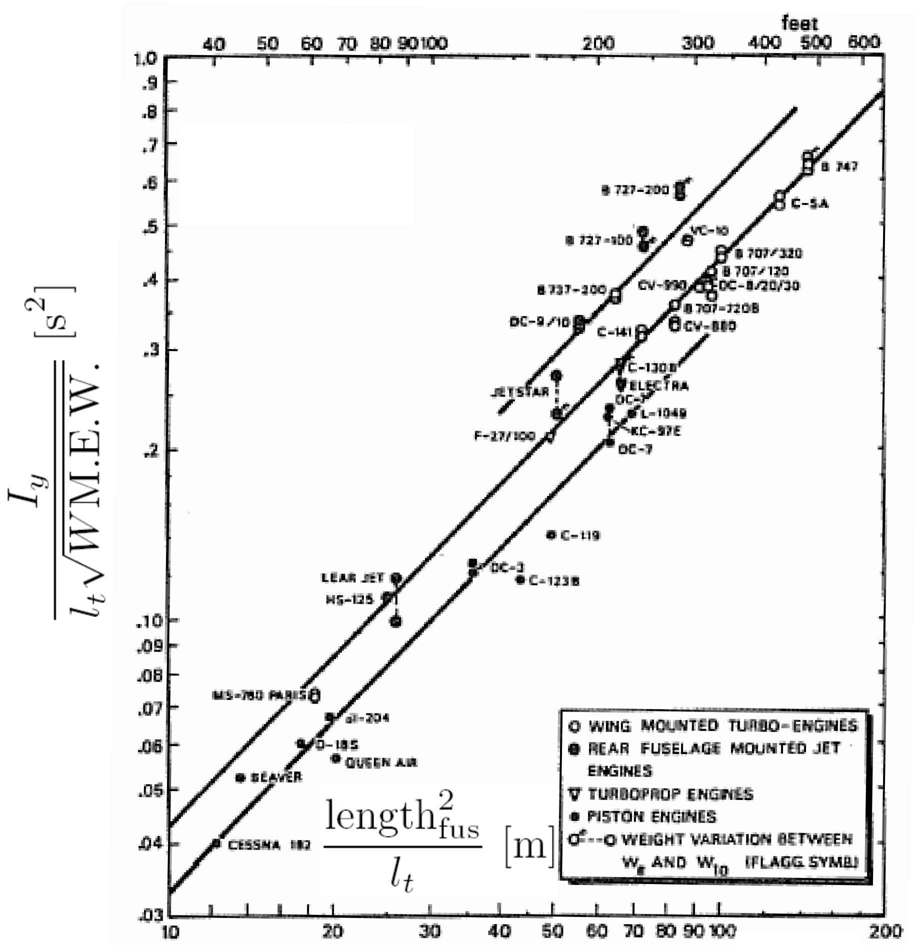


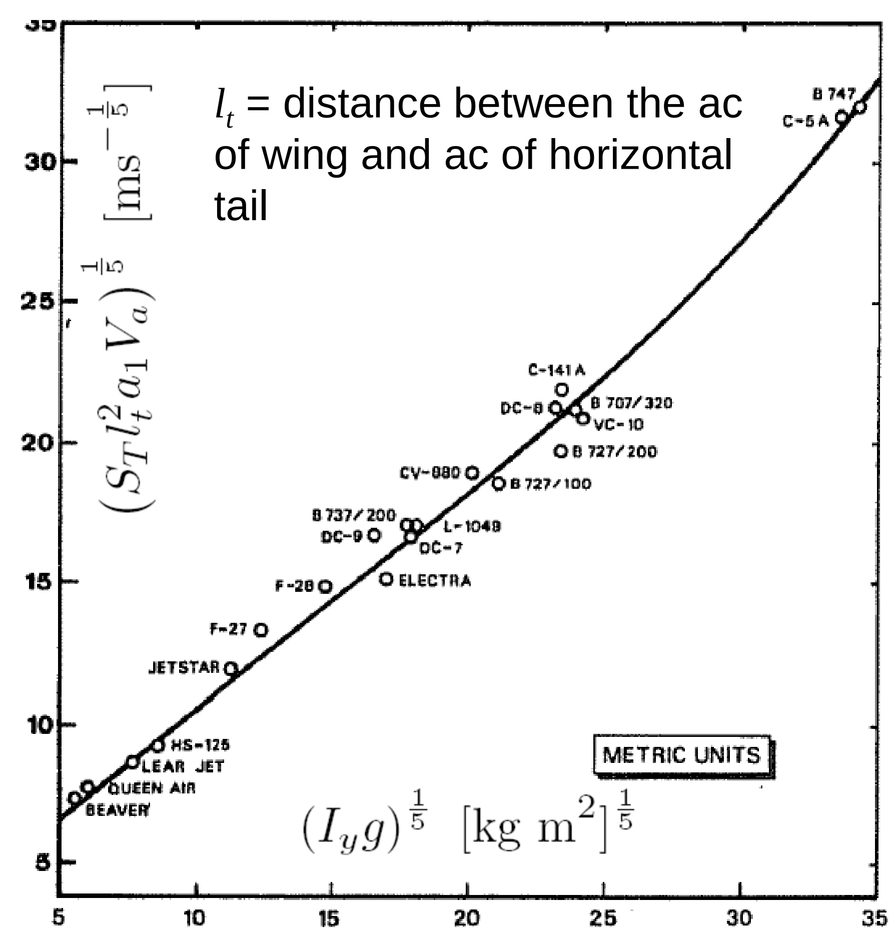
  - Design criteria
    - Longitudinal static equilibrium
    - Longitudinal stability
      - Damping for short period & Phugoïd modes
    - Powerful enough to allow maneuvers
      - Rotation at take off
    - Should stall after the wing

- Outputs
  - Proceed as for wings
    - Thickness to remain below critical Mach number
    - Lift coefficient slope as for wing
    - Lift coefficient
      - Should account for wing downwash effect
      - $C_{L\,T} = a_1 \left[\frac{C_{L\,w}}{a} \left(1 - \frac{d\epsilon}{d\alpha} \right) + \eta_T + \alpha_{L\,0_{\text{root}}} \right]$ if symmetrical airfoil
    - Aerodynamic center computed as for wing
    - No pitching moment if symmetrical airfoil
    - No aerodynamic twist (neglected)



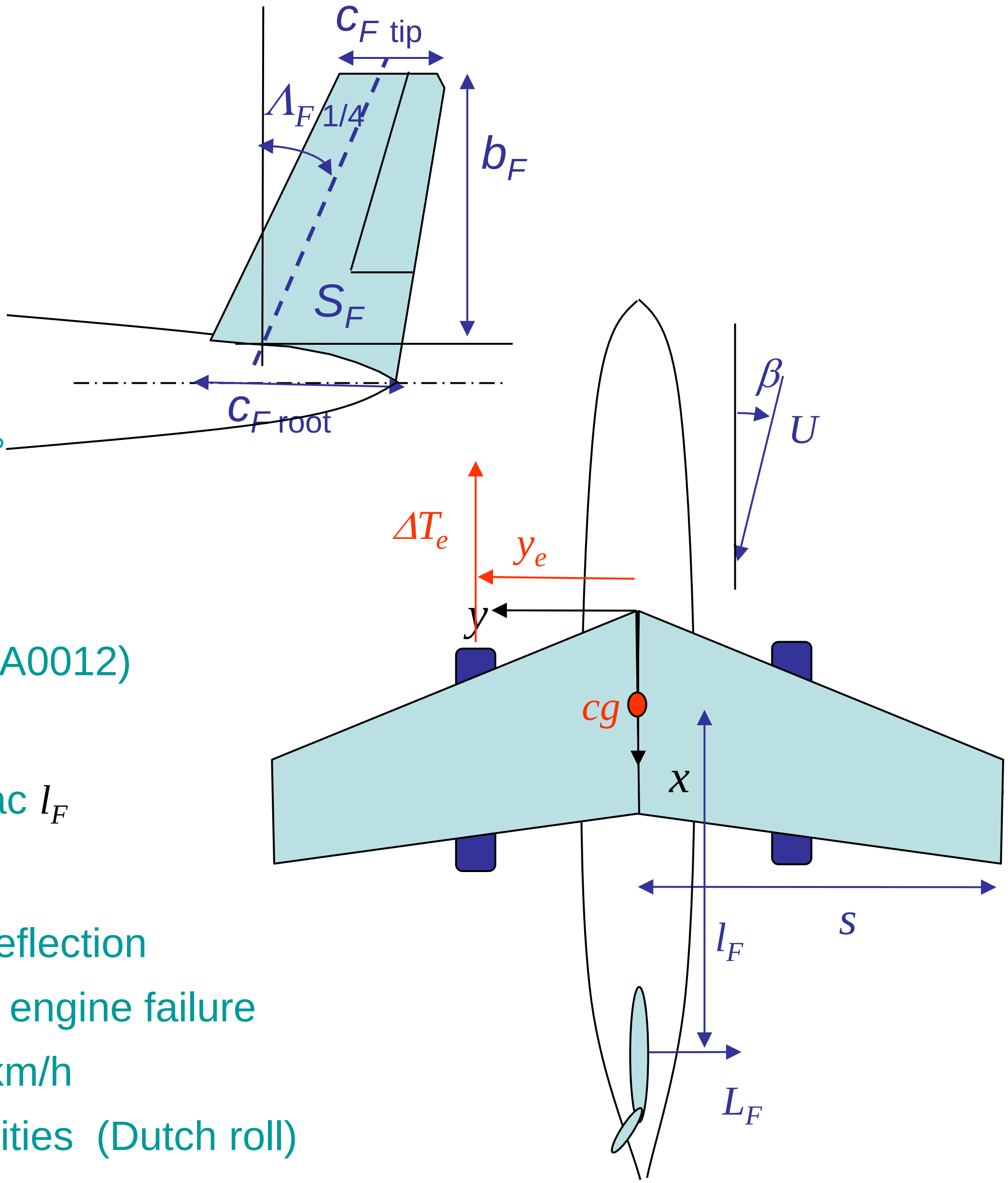
- Quick design
  - Stability depends mainly on S_T / S ~ 0.2-0.4
  - Maneuverability depends mainly on $\dfrac{l_T S_T}{\bar{\bar{c}} S}$ ~ 0.5-1.2
    - Approach velocity V_a =1.3 V_{so}



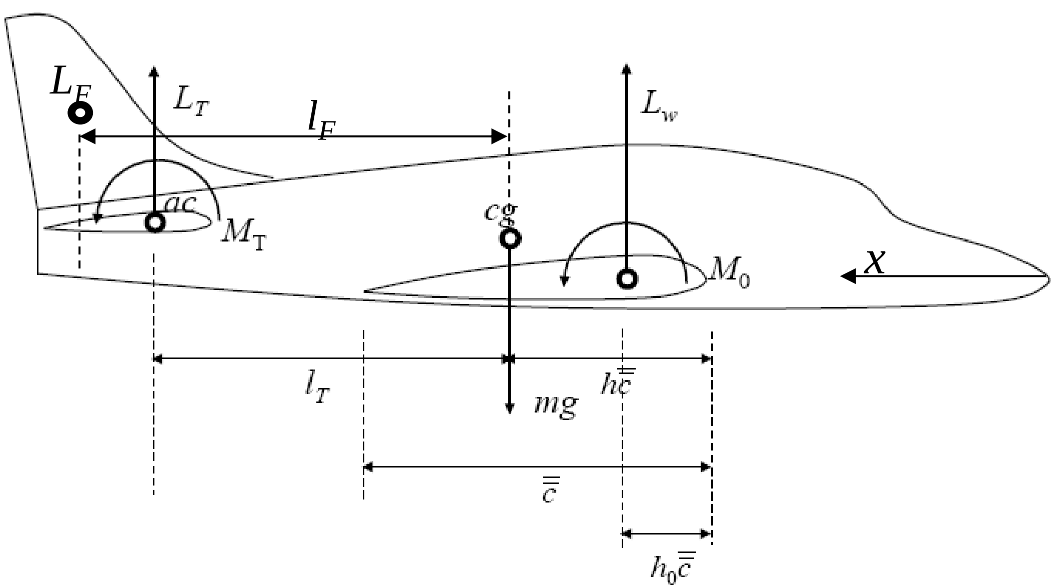


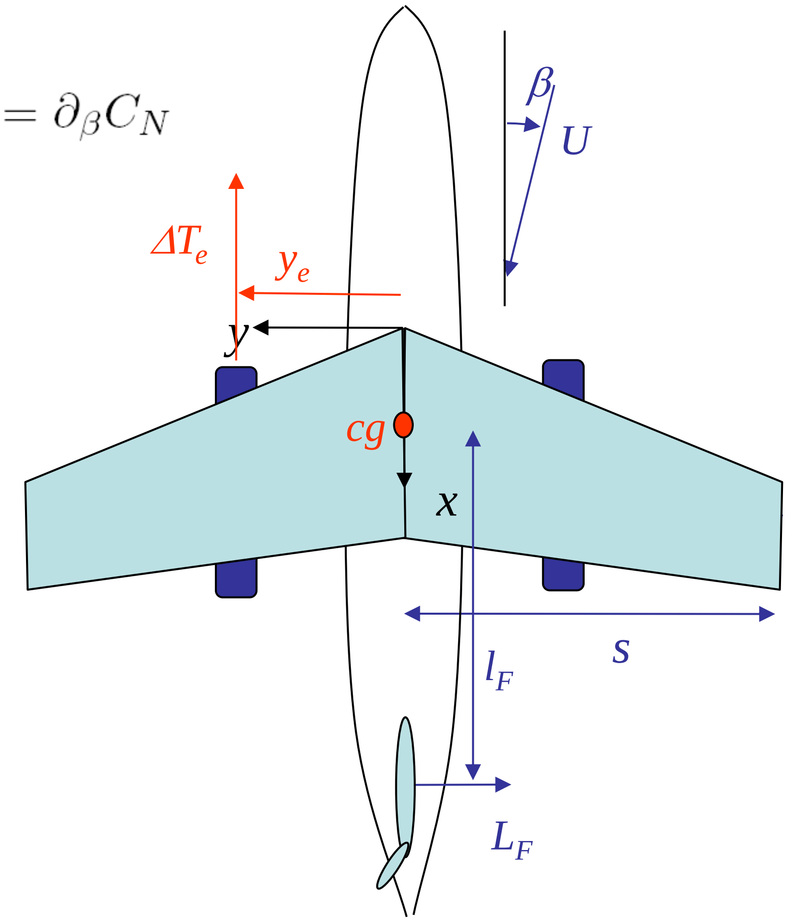
Fin

- Geometry
  - Parameters
    - Span b_F
    - Aspect ratio $AR_F = b_F^{\,2}/S_F$
      - ~ 0.7
      - For T tail ~ 2
    - Taper ratio $\lambda_F = c_{F\ \mathrm{tip}}/c_{F\ \mathrm{root}}$
    - Sweep angle $\Lambda_{F\,1/4}$: 30 to 40°
    - Airfoil
      - Symmetrical
      - Low thickness (e.g. NACA0012)
      - No twist
    - Distance between cg and fin ac l_F
  - Design criteria
    - No stall at maximum rudder deflection
    - Maneuverability ensured after engine failure
    - Landing with side wind of 55 km/h
    - Lateral static & dynamic stabilities (Dutch roll)



- Loadings
  - Lift coefficient $C_{L\,F} = \dfrac{L_F}{\frac{1}{2}\rho V^2 S_F}$
  - Yaw coefficient $C_N = C_{L\,F}\dfrac{S_F l_F}{S b}$
  - Slope with respect to yaw angle β $\quad C_{N\,\beta} = \partial_\beta C_N$

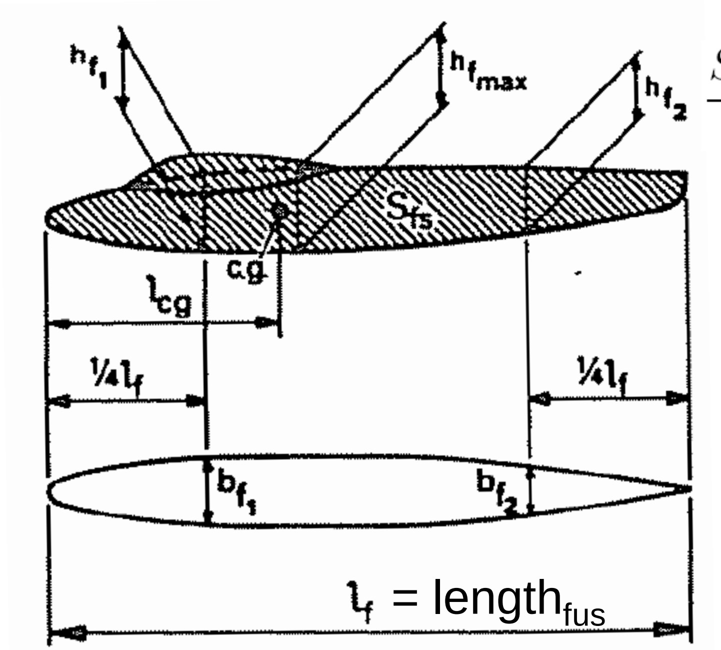


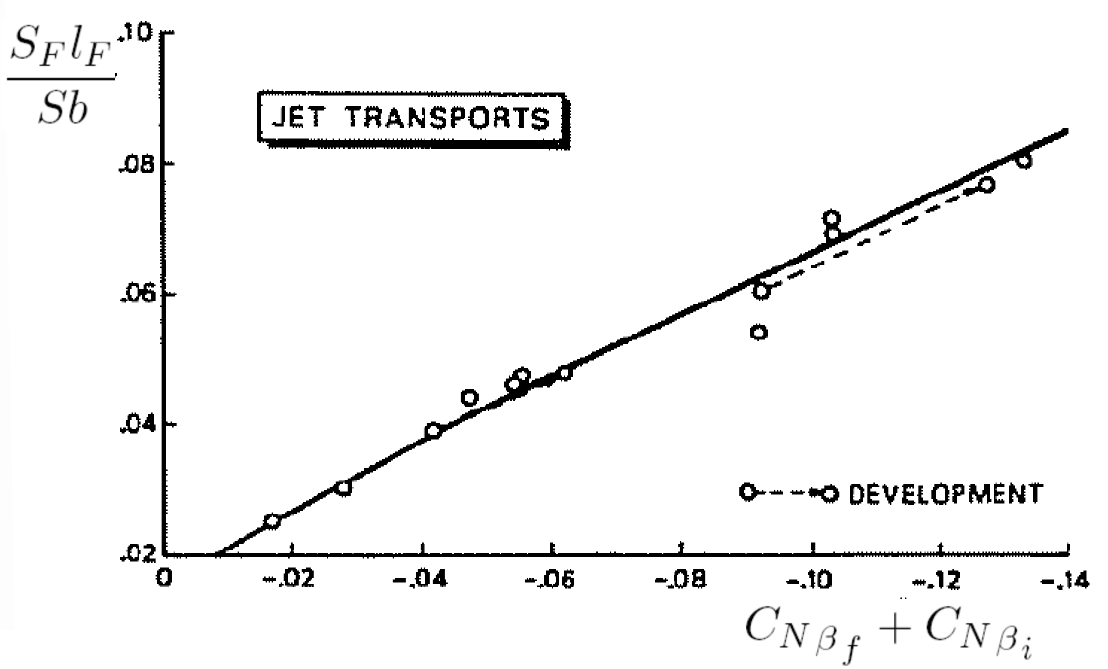


- Quick design
  - Lateral stability (most severe criterion for engines attached on fuselage)
    - Fuselage effect

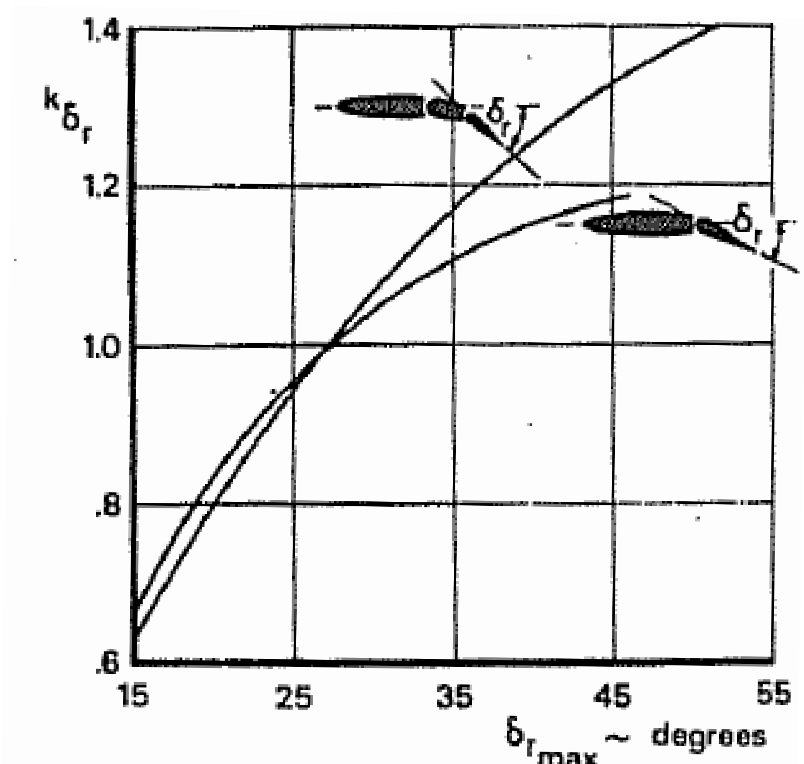
$$\begin{cases} C_{N\beta_f} = -K_\beta \dfrac{S_{fs}\text{length}_{\text{fus}}}{Sb}\left(\dfrac{h_{f1}}{h_{f2}}\right)^{\frac{1}{2}}\left(\dfrac{b_{f2}}{b_{f1}}\right)^{\frac{1}{3}} \\ K_\beta = 0.3\dfrac{l_{cg}}{\text{length}_{\text{fus}}} + 0.75\dfrac{h_{f\,\text{max}}}{\text{length}_{\text{fus}}} - 0.105 \end{cases}$$

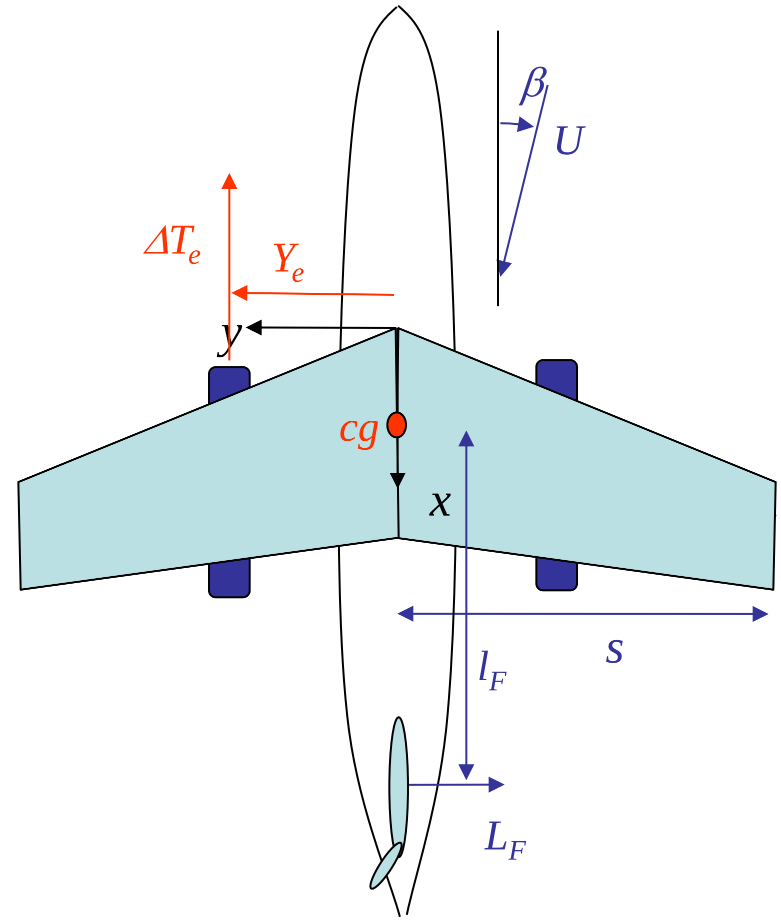
    - {High, mid, low}-mounted wing effect $C_{N\beta_i} = \{-0.017,\ 0.012,\ 0.024\}$



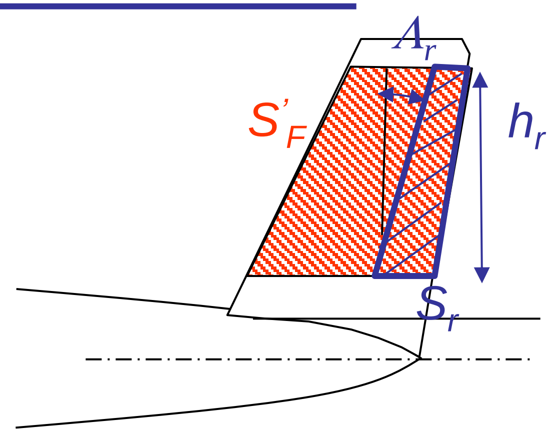


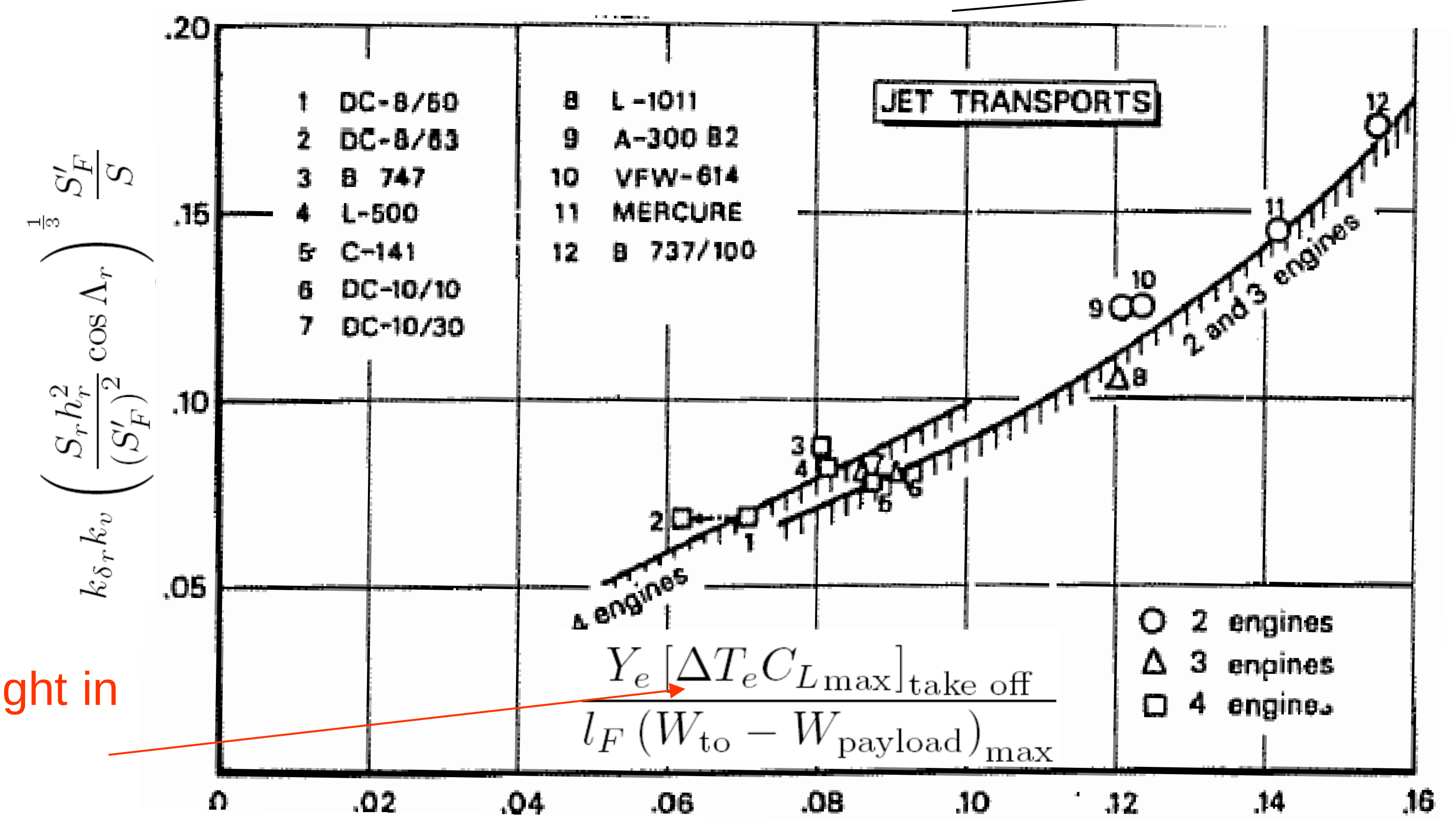
- Quick design (2)
  - Engine failure (most severe criterion for wing-mounted engines)
    - Takeoff configuration (critical as larger thrust)
    - Engine thrust ΔT_e at Y_e from fuselage axis
    - Maximal rudder deflection $\delta_{r\,\mathrm{max}}$ ~30°
    - Effect of rudder measured by $k_{\delta\,r}$





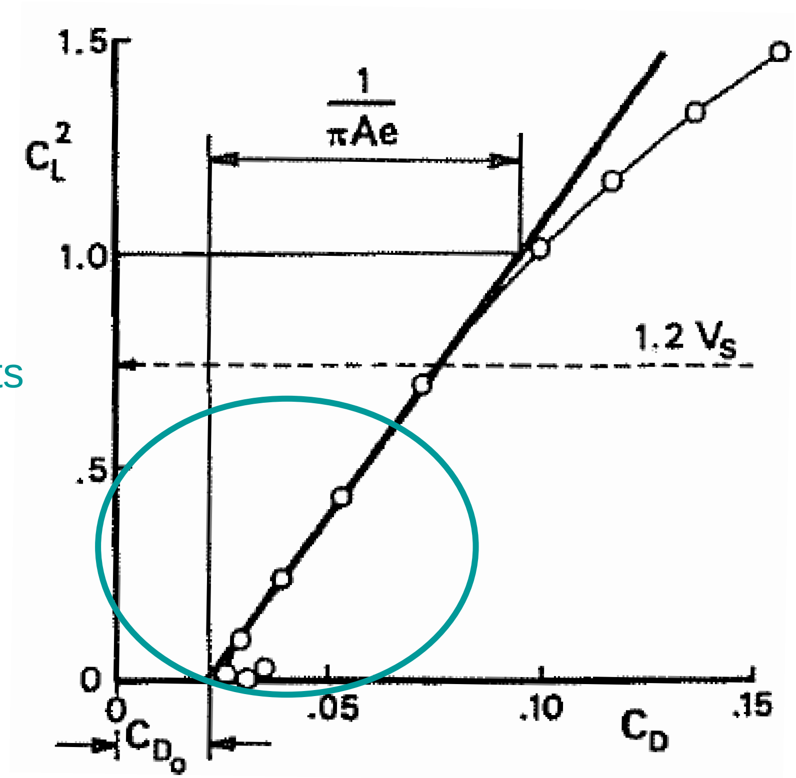
- Quick design (3)
  - Engine failure (wing-mounted engines) (2)
    - Effect of fin: k_v = 1.1 for T-tail, 1 for other tails





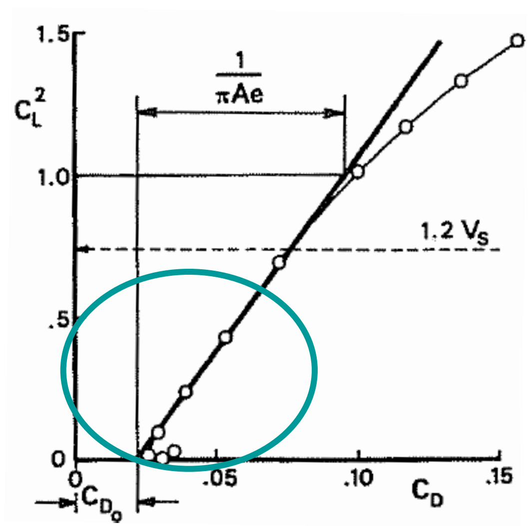
Thrust & weight in kg or N

- In cruise
  - Cruise drag is critical to compute
    - Required thrust
    - Fuel consumption
  - Detailed method
    - Compute contribution of each aircraft component on
      - Induced drag (due to vortex)
      - Profile drag (friction & pressure)
      - Interference drag
        - » Interaction between components
        - » Account for $C_{Lw} \neq C_L$ during normalization
  - Polar of the aircraft
    - Drag can be plotted in term of lift
    - $C_D = C_{D0} + \frac{{C_L}^2_{\text{plane}}}{e\pi AR}$



Drag

- In cruise (2)
  - Quick method
    - $$C_D = C_{D0} + \frac{{C_L}^2_{\text{plane}}}{e\pi AR}$$
    - With e and C_{D0} from statistics



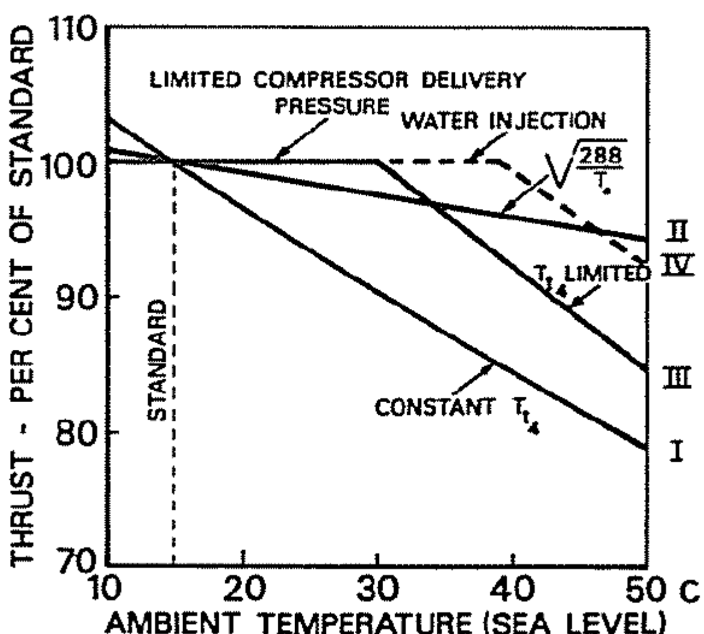
	C_{D_0}	e
high-subsonic jet aircraft	.014 - .020	.75 - .85*
large turbopropeller aircraft	.018 - .024	.80 - .85
twin-engine piston aircraft	.022 - .028	.75 - .80
small single engine aircraft		
retractable gear	.020 - .030	.75 - .80
fixed gear	.025 - .040	.65 - .75
agricultural aircraft:		
- spray system removed	.060	.65 - .75
- spray system installed	.070 - .080	.65 - .75

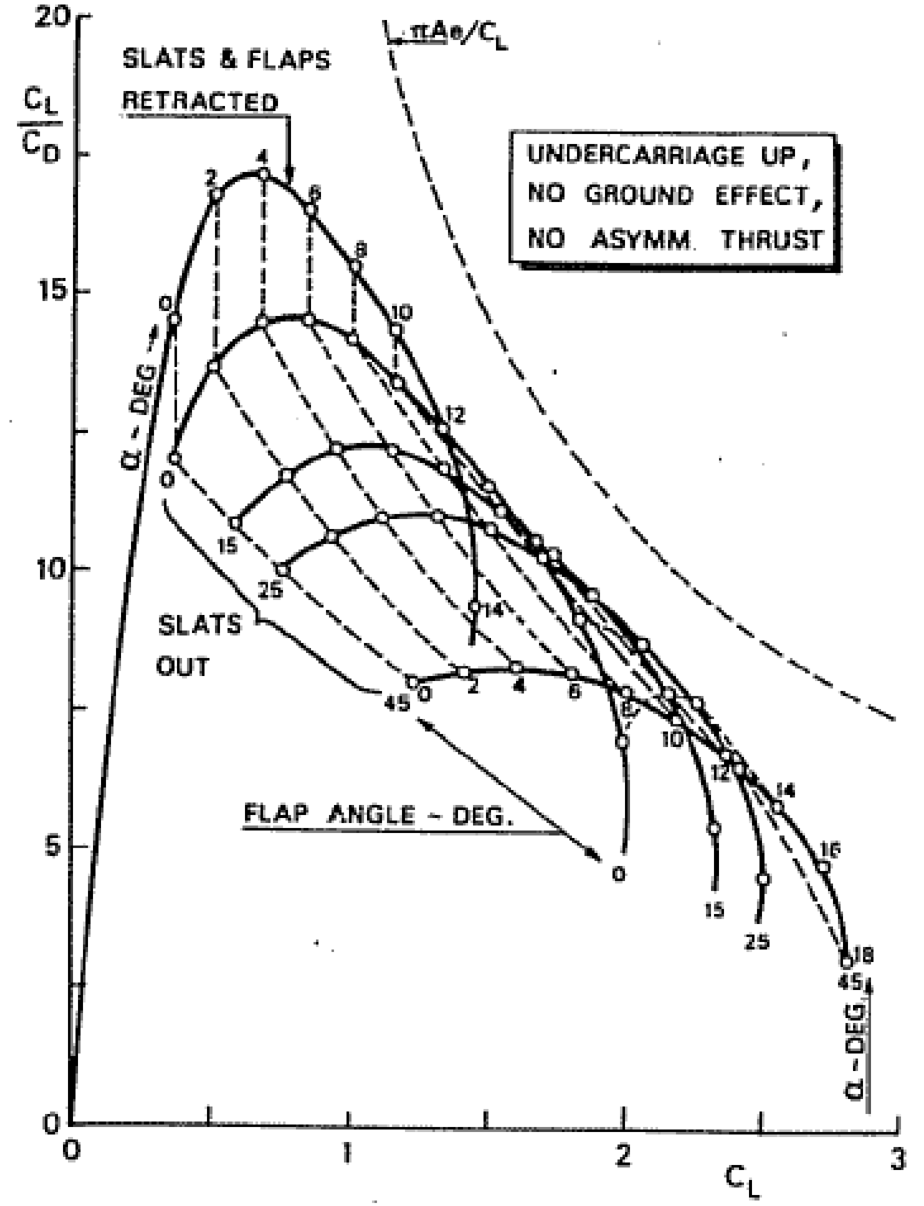
* The higher the sweep angle, the lower the e-factor

- Meaningful only if the design is correct
  - A wrong design would lead to higher drag
  - This would not appear with this method

- In cruise (3)
  - Compressibility effect
    - Low if correct wing design
      - Divergence Mach larger than cruise Mach (t/c small enough)
    - In this case, add, to the drag coefficient, the compressibility effect obtained by
      - $$\Delta_{\text{comp}} C_D = \begin{cases} 0.0005 & \text{long range conditions} \\ 0.002 & \text{high speed conditions} \end{cases}$$

- Landing & takeoff
  - Low velocity drag (flaps down) is critical to compute
    - Thrust required at takeoff
    - Maximum payload
      - Can depend on the airport
        - » Temperature
        - » Runaway

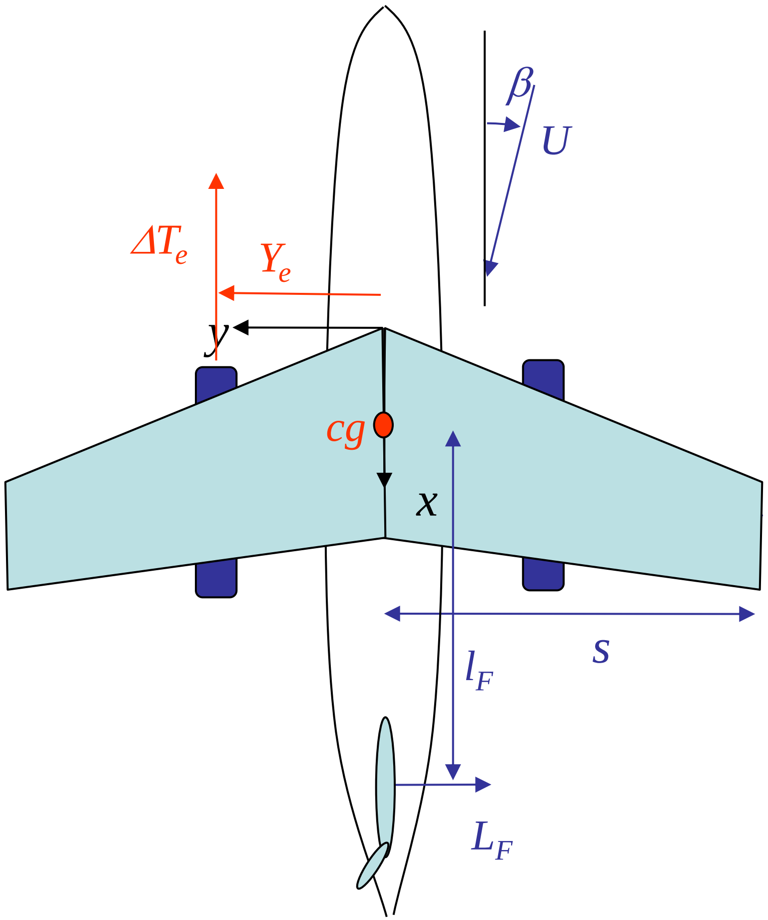




- Landing & takeoff (2)
  - Plane velocity
    - Takeoff & landing safety speed
      - At 35 ft altitude
      - $V_2 = 1.2\ V_{s(0)}$
  - Polar
    - $$C_{D\,V_2} = C_0 + \frac{C_{L\,V_2}^{\,2}}{E\pi AR}$$
    - Slats out
      - $C_0 = 0.018$
      - $E = 0.7$
    - Slats in
      - $C_0 = 0.005$
      - $E = 0.61$
    - C_L with high lift devices

HIGH-LIFT DEVICE		TYPICAL FLAP ANGLE		$C_{L_{max}}/\cos\Lambda_{.25}$	
TRAILING EDGE	LEADING EDGE	TAKEOFF	LANDING	TAKEOFF	LANDING
PLAIN	-	20°	60°	1.40-1.60	1.70-2.00
SINGLE SLOTTED	-	20°	40°	1.50-1.70	1.80-2.20
FOWLER*	-	15°	40°	2.00-2.20	2.50-2.90
DOUBLE SLOTTED**	-	20°	50°	1.70-1.95	2.30-2.70
	SLAT			2.30-2.60	2.80-3.20
TRIPLE SLOTTED**	SLAT	20°	40°	2.40-2.70	3.20-3.50

- Takeoff with one engine
  - Corrected polar
    - $$C_{D\,V_2} = C_0 + \frac{C_{L\,V_2}^{\;2}}{E \pi AR}$$
    - If low thrust (landing)
      - Reduce E by
        - » 4 % for wing-mounted engines
        - » 2 % for engines on the fuselage
    - If high thrust (takeoff)
      - Compute explicitly effects of
        - » Wind-milling
        - » Drag due to the rudder



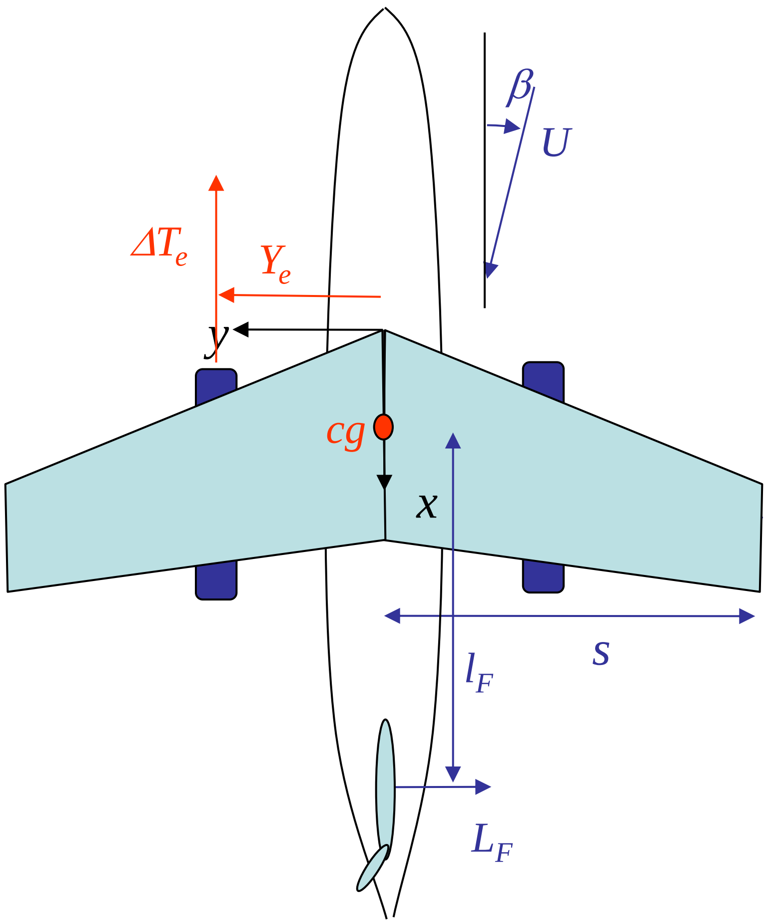
- Takeoff with one engine (2)
  - Method to compute the drag leads to coefficients of the form $C_D S$
    - Has to be divided by the gross wing area S to get back to C_D
    - The terms have to be added to the C_D obtained with high lift devices out, $C_{D V_2} = C_0 + \frac{C_{L V_2}^2}{E \pi AR}$
    - 2 parts: wind-millings and rudder

$$(C_D S)_{ef} = (C_D S)_{wm} + (C_D S)_{rud}$$

  - Wind-milling
    - $(C_D S)_{wm} \simeq 0.0785 D_{\text{inlet}}^2$

- Takeoff with one engine (3)
  - Rudder
    - Moment due to
      - Thrust unbalance ΔT_e
      - Acting at Y_e from fuselage axis
    - Balanced by rudder load

$$C_{Y\,F} = \frac{\Delta T_e}{q S_F} \frac{Y_e}{l_F}$$



- Leads to a drag
  - Induced part (vortex)

$$\left(C_D S\right)_{v\,rud} = \frac{C_{Y\,F}^{\,2} S_F}{\pi A R_F}$$

  - Profile part (friction & pressure)

$$\left(C_D S\right)_{p\,rud} = \frac{2.3}{\pi A R_F^{\frac{4}{3}}} \sqrt{S_F S_r} \left(\cos \Lambda_{F\,\frac{1}{4}}\right)^{\frac{1}{3}} C_{Y\,F}^{\,2}$$

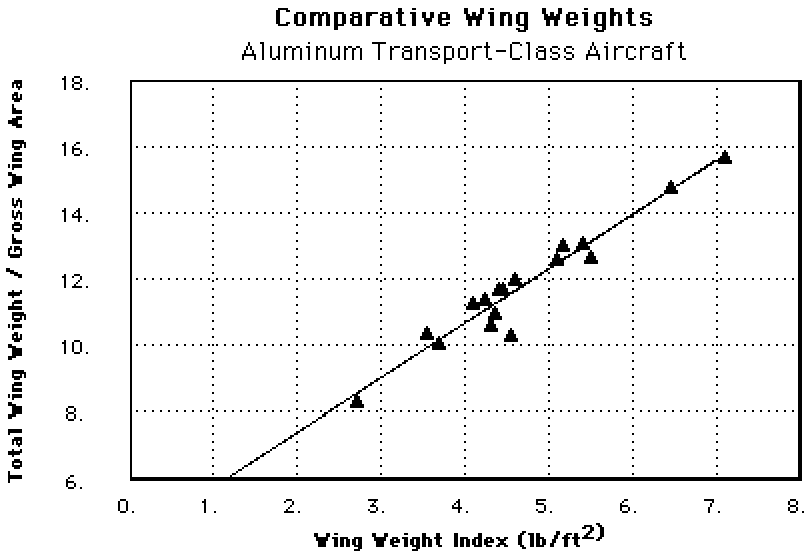
- Data

Engine	SLS thrust (KN)	Cruise thrust (KN)	SLS specific fuel consumption (sfc) (kg/daN.h)	Cruise specific fuel consumption (sfc) (kg/daN.h)	By pass ratio	Diameter (mm)	Length (mm)	Weight (kg)
CF6-80C2	262.4	46.7	0.356	0.585	5.09	2362	4036	4058
CF34-3A	41	6.8	0.357	0.718	6.2	1118	2616	737

- Sea Level Static
  - $M = 0$
  - Standard atmospheric conditions at sea level
  - SLS thrust: T_{to} (to is for takeoff)
  - Correction for $M > 0$ $$\frac{T}{T_{\text{to}}} \simeq 1 - \frac{0.45M\,(1+\text{BPR})}{\sqrt{1+0.75\text{BPR}}} + (0.6+0.11\text{BPR})\,M^2$$
- Cruise
  - Standard atmosphere at a given altitude
- Specific Fuel Consumption
  - Fuel consumption
    - Per unit of thrust and
    - Per unit of time

Structural weight

- Component weight can be estimated
  - For conceptual design
  - Based on statistical results of traditional aluminum structures
  - Example: wing



- Structural weight [lbs]
  - Wing with ailerons

$$W_w = 4.22\,S + 1.642\,10^{-6}\,\frac{n_{\mathrm{ultim}} b^3 \sqrt{W_{\mathrm{to}} \mathrm{ZFW}}\,(1+2\lambda)}{\frac{t}{c}\Big|_{\mathrm{avg}} \cos^2\Lambda\, S\,(1+\lambda)}$$

S: gross area of the wing [ft^2]

W_{to}: take off weight [lb]

ZFW: zero fuel weight [lb]

b: span [ft]

Λ: sweep angle of the structural axis

λ: taper (c_{tip}/c_{root}),

t: airfoil thickness [ft]

c: chord [ft]

  - Horizontal empennage & elevators

$$W_T = 5.25\,S_{T\,\mathrm{exp}} + 0.8\,10^{-6}\,\frac{n_{\mathrm{ultim}} b_T^3 W_{\mathrm{to}} \bar{\bar{c}} \sqrt{S_{T\,\mathrm{exp}}}}{\frac{t_T}{c_T}\Big|_{\mathrm{avg}} \cos^2\Lambda_T\, l_T\, S_T^{\frac{3}{2}}}$$

$S_{T\,exp}$: exposed empennage area [ft^2]

l_T: distance plane CG to empennage CP [ft]

$\bar{\bar{c}}$: average aerodynamic chord of the wing [ft]

S_T: gross empennage area [ft^2]

b_T: empennage span [ft]

t_T: empennage airfoil thickness [ft]

c_T : empennage chord [ft]

Λ_T: sweep angle of empennage structural axis

- Structural weight [lbs] (2)
  - Fin without rudder

$$W_{F'} = 2.62\, S_F + 1.5\, 10^{-5}\, \frac{n_{\text{ultim}} b_F^3 \left(8 + 0.44 \frac{W_{\text{to}}}{S}\right)}{\left.\frac{t_F}{c_F}\right|_{\text{avg}} \cos^2 \Lambda_F}$$

S_F: fin area [ft^2]
t_F: fin airfoil thickness [ft]
Λ_F: sweep angle of fin structural axis
b_F: fin height [ft]
c_F: fin chord [ft]
S: gross surface of wing [ft^2]

  - Rudder: $W_r / S_r \sim 1.6\, W_{F'} / S_F$
  - Fuselage
    - Pressure index $I_{\text{p}} = 1.5\, 10^{-3}\, \Delta p_{\text{max}} \text{width}_{\text{fus}}$
    - Δp [lb/ft^2] (cabin pressure ~2600m)
    - Bending index

$$I_{\text{b}} = 1.91\, 10^{-4} n_{\text{limit at ZFW}} \left(\text{ZFW} - W_w - W_{\text{wing-mounted engines}}\right) \frac{\text{length}_{\text{fus}}}{\text{height}_{\text{fus}}^2}$$

    - Weight depends on wetted area S_{wetted} [ft^2] (area in direct contact with air)

$$W_{\text{fus}} = (1.051 + 0.102\, I_{\text{fus}})\ S_{\text{fus,wetted}}$$

$$I_{\text{fus}} = \begin{cases} I_{\text{p}} & \text{if } I_{\text{p}} > I_{\text{b}} \\ \frac{\left(I_{\text{p}}^2 + I_{\text{b}}^2\right)}{2 I_{\text{b}}} & \text{if } I_{\text{p}} < I_{\text{b}} \end{cases}$$

- Structural weight [lbs] (3)
  - Systems
    - Landing gear $W_{gear} = 0.04\ W_{to}$
    - Hydromechanical system of control surfaces $W_{SC} = I_{SC}\ (S_{T\exp} + S_F)$

      I_{sc} [lb/ft^2] : 3.5, 2.5 or 1.7 (fully, partially or not powered)
    - Propulsion $W_{prop} = 1.6 W_{eng} \sim 0.6486\ T_{to}^{0.9255}$

      T_{to} : Static thrust (M 0) at sea level [lbf], *1lbf ~ 4.4 N
    - Equipment
      - APU $W_{APU} = 7\ N_{seats}$
      - Instruments (business, domestic, transatlantic) $W_{inst} = 100, 800, 1200$
      - Hydraulics $W_{hydr} = 0.65\ S$
      - Electrical $W_{elec} \sim 13\ N_{seats}$
      - Electronics (business, domestic, transatlantic) $W_{etronic} = 300, 900, 1500$
      - Furnishing if < 300 seats $W_{furn} \sim (43.7 - 0.037\ N_{seats})\ N_{seats} + 46\ N_{seats}$

        if > 300 seats $W_{furn} \sim (43.7 - 0.037{*}300)\ N_{seats} + 46\ N_{seats}$
      - AC & deicing $W_{AC} = 15\ N_{seats}$
  - Payload ($W_{payload}$)
    - Operating items (class dependant) $W_{oper} = [17 - 40]\ N_{pass}$
    - Flight crew $W_{crew} = (190 + 50)\ N_{crew}$
    - Flight attendant $W_{attend} = (170 + 40)\ N_{atten}$
    - Passengers (people and luggage) $W_{pax} = 225\ N_{pass}$
  - Definitions
    - ZFW: Sum of these components $ZFW = \Sigma\ W_i$

- Structural weight [lbs] (4)
  - Examples

Aircraft System	CITATION-500	MDAT-30	MDAT-50	F-28	MDAT-70	DC-9-10	BAC-111	DC-9-30	737-200	727-100
Wing System	1,020	3,143	4,360	7,526	5,910	9,366	9,817	11,391	11,164	17,682
Tail System	288	1,010	1,193	1,477	1,505	2,619	2,470	2,790	2,777	4,148
Body System	930	4,276	5,692	6,909	7,118	9,452	11,274	11,118	11,920	17,589
Alighting Gear System	425	1,379	1,874	2,564	2,440	3,640	3,465	4,182	4,038	7,244
Nacelle System	241	948	1,294	866	1,684	1,462	1,191	1,462	1,515	2,226
Propulsion System (less Dry Engine)	340	1,140	1,338	988	1,702	1,478	1,788	2,190	1,721	3,052
Flight Controls System (less Auto Pilot)	196	600	699	1,404	805	1,102	1,655	1,434	2,325	2,836
Auxiliary Power System	0	343	400	320	460	805	719	817	855	0
Instrument System	76	300	300	267	300	490	504	575	518	723
Hydraulic and Pneumatic System	94	257	300	406	345	681	1,391	753	835	1,054
Electrical System	361	617	825	953	1,040	1,631	1,610	1,715	2,156	2,988
Avionics System (incl. Auto Pilot)	321	586	586	923	586	1,039	1,368	1,108	1,100	1,844
Furnishings and Equipment System	794	2,657	3,548	3,535	4,772	6,690	7,771	8,594	9,119	11,962
Air Conditioning System	188	325	435	520	550	1,016	1,062	1,110	1,084	1,526
Anti-icing System	101	384	448	520	511	472	234	474	113	639
Load and Handling System	2	20	20	--	20	19	9	57	--	15
Empty Weight (less Dry Engine)	5,377	17,985	23,312	29,178	29,748	41,962	46,328	49,770	51,240	75,528
Dry Engine Weight	1,002	2,480	3,373	4,327	4,392	6,113	5,434	6,160	6,212	9,322
Empty Weight (M.E.W.)	6,379	20,465	26,685	33,505	34,140	48,075	51,762	55,930	57,452	84,850
Takeoff Gross Weight	11,650	34,480	46,850	62,000	61,000	86,300	99,650	108,000	104,000	161,000

Manufacturer empty weight

- Structural weight [lbs] (5)
  - Examples

Aircraft System	727-200	707-320	DC-8-55	DC-8-62	DC-10-10	L-1011	DC-10-40	747	SCAT-15*
Wing System	18,529	28,647	34,909	36,247	48,990	47,401	57,748	88,741	83,940
Tail System	4,142	6,004	4,952	4,930	13,657	8,570	14,454	11,958	8,590
Body System	22,415	22,299	22,246	23,704	44,790	49,432	46,522	68,452	54,322
Alighting Gear System	7,948	11,216	11,682	11,449	18,581	19,923	25,085	32,220	28,720
Nacelle System	2,225	3,176	4,644	6,648	8,493	8,916	9,328	10,830	15,650
Propulsion System (less Dry Engine)	3,022	5,306	9,410	7,840	7,673	8,279	13,503	9,605	6,310
Flight Controls System (less Auto Pilot)	2,984	2,139	2,035	2,098	5,120	5,068	5,188	6,886	10,777
Auxiliary Power Plant System	849	0	0	0	1,589	1,202	1,592	1,797	--
Instrument System	827	550	1,002	916	1,349	1,016	1,645	1,486	3,400
Hydraulic and Pneumatic Group	1,147	1,557	2,250	1,744	4,150	4,401	4,346	5,067	10,670
Electrical System	2,844	3,944	2,414	2,752	5,366	5,490	5,293	5,305	6,002
Avionics System (incl. Auto Pilot)	1,896	1,815	1,870	2,058	2,827	2,801	3,186	4,134	4,178
Furnishings and Equipment System	14,702	16,875	15,884	15,340	38,072	32,829	33,114	48,007	20,615
Air Conditioning System	1,802	1,602	2,388	2,296	2,386	3,344	2,527	3,634	2,820
Anti-icing System	666	626	794	673	416	296	555	413	210
Load and Handling System	19	--	55	54	62	--	62	228 -896**	--
Empty Weight (less Dry Engine)	86,017	105,756	116,535	118,749	203,521	198,968	224,148	297,867	256,204
Dry Engine Weight	9,678	19,420	16,936	17,316	23,229	30,046	25,587	35,700	45,020
Empty Weight (M.E.W.)	95,695	125,176	133,471	136,065	226,750	229,014	249,735	333,567	301,224
Takeoff Gross Weight	175,000	312,000	325,000	335,000	430,000	430,000	565,000	775,000	631,000

Manufacturer empty weight

Structural weight

- CG locations
  - Wing: 30% chord at wing MAC
  - Horizontal tail: 30% chord at 35% semi-span
  - Fin: 30% chord at 35% of vertical height
  - Surface controls: 40% chord on wing MAC
  - Fuselage: 45% of fuselage length
  - Main gear: located sufficiently aft of aft c.g. to permit 5% - 8% of load on nose gear
  - Hydraulics: 75% at wing c.g., 25% at tail c.g.
  - AC / deicing: End of fuse nose section
  - Propulsion: 50% of nacelle length for each engine
  - Electrical: 75% at fuselage center, 25% at propulsion c.g.
  - Electronics and Instruments: 40% of nose section
  - APU: Varies
  - Furnishings, passengers, baggage, cargo, operating items, flight attendants: From layout. Near 51% of fuselage length
  - Crew: 45% of nose length
  - Fuel: Compute from tank layout

Fuel weight

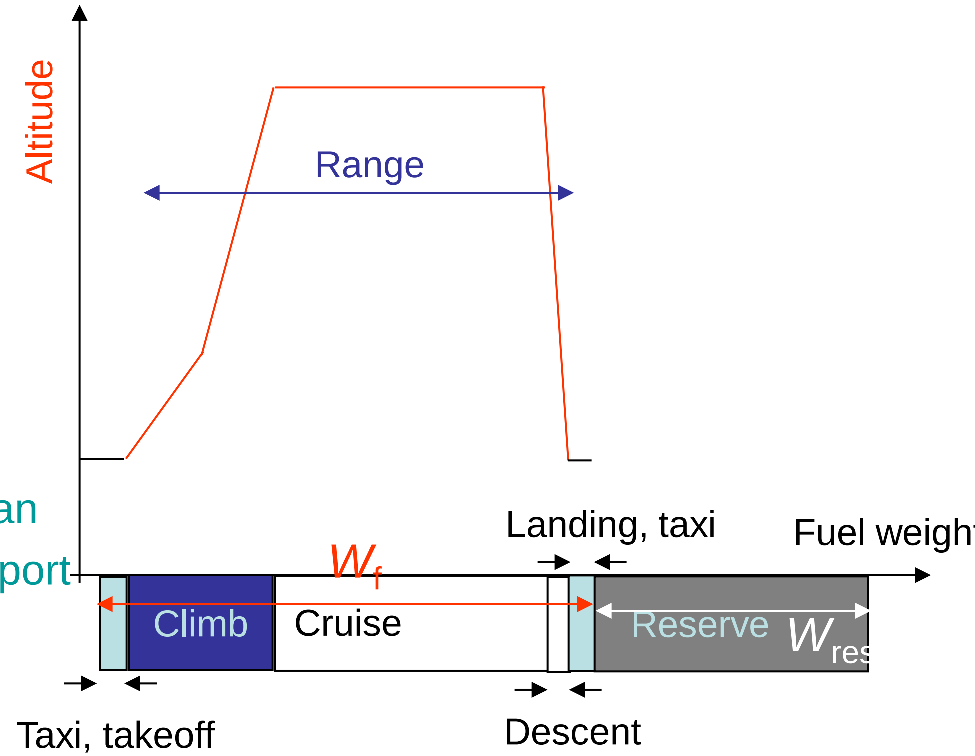
- For a given mission
  - Taxi & takeoff

    $W_{taxi} = 0.0035\, W_{to}$
  - Landing & taxi

    $W_{land} = 0.0035\, W_{to}$
  - Reserve
    - Should allow
      - Deviations from the flight plan
      - Diversion to an alternate airport
    - Airliners
      - $W_{res} \sim 0.08$ ZFW
    - Business jet
      - W_{res} fuel consumption for ¾-h cruise
  - Climbing (angle of ~ 10°)

$$\frac{W_{\mathrm{climb}}}{W_{\mathrm{TO}}} \simeq \frac{1}{100}\left[\frac{\text{cruise altitude [ft]}}{31600\text{ [ft]}} + \frac{1}{2}M^2_{\mathrm{cruise}}\right]$$

  - Descend: ~ same fuel consumption than cruise
  - Take Off Weight (TOW): W_{to} =ZFW + W_{res} +W_f
  - Landing weight: ZFW + W_{res} + 0.0035 W_{to}



- For a given mission (2)
  - Cruise
    - Bréguet equation

$$R_{\text{cruise}} = \int_{W_i}^{W_i - W_{\text{cruise}}} V\,dt = -\int_{W_i}^{W_i - W_{\text{cruise}}} \frac{V}{C_T T} dW = -\int_{W_i}^{W_i - W_{\text{cruise}}} \frac{VL}{C_T DW} dW$$

      - Specific Fuel Consumption C_T
        - » Consumption (of all the engines) per unit of thrust (of all the engines) per unit of time
      - Initial weight $W_i = W_{\text{to}} - W_{\text{taxi}} - W_{\text{climb}}$
      - Final weight $W_i - W_{\text{cruise}} = \text{ZFW} + W_{\text{land}} + W_{\text{res}}$
    - Flight with ratio C_D / C_L ~ constant

$$\frac{R_{\text{cruise}}}{a_0} = \frac{M_{\text{cruise}} \frac{C_L}{C_D}}{\frac{C_T}{\sqrt{\theta}}} \ln \frac{W_i}{W_i - W_{\text{cruise}}}$$

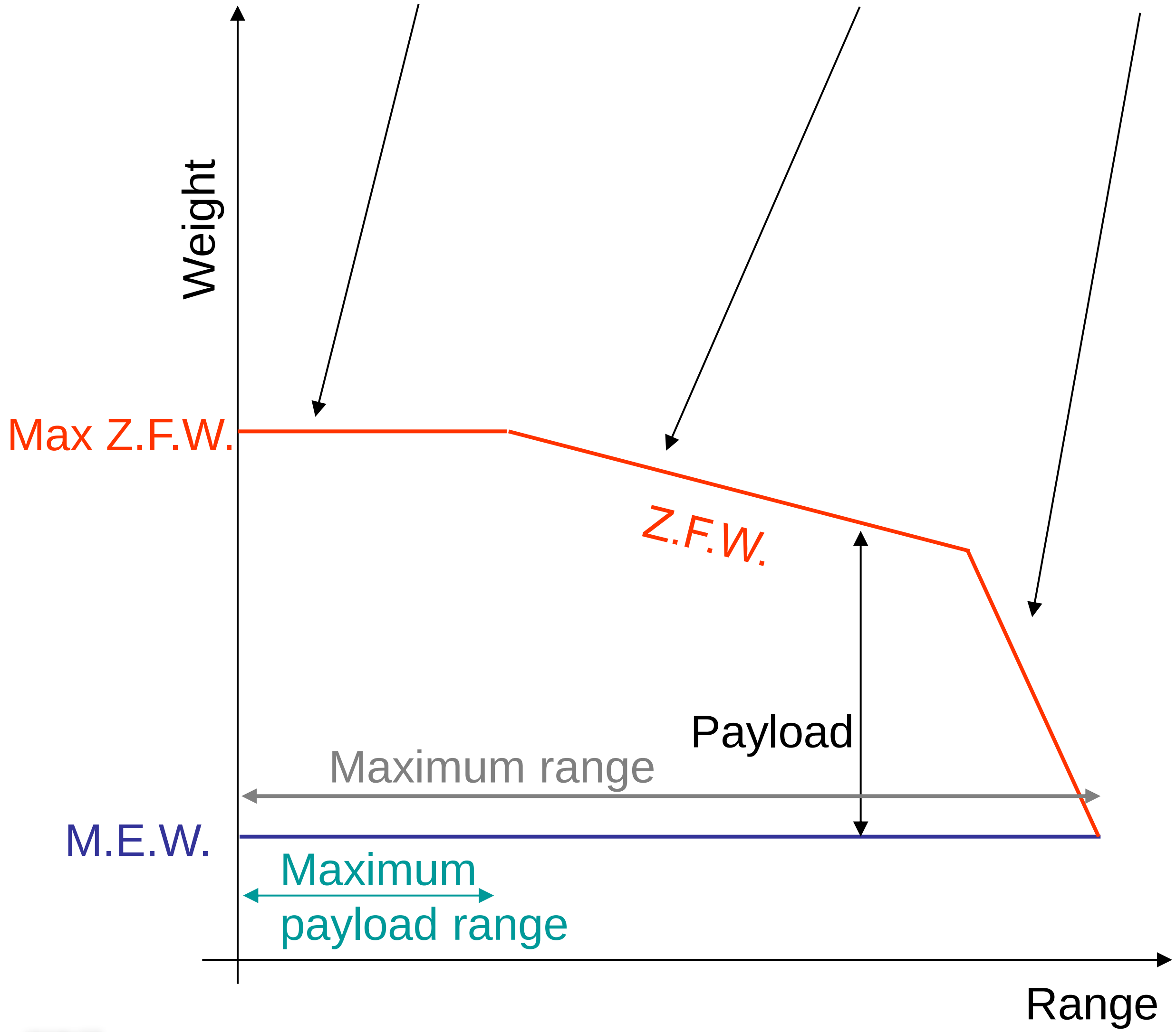
Sound speed at SL

Temperature/Temperature SL

  - Fuel weight (without reserve) $W_{\text{f}} = W_{\text{taxi}} + W_{\text{climb}} + W_{\text{cruise}} + W_{\text{land}}$

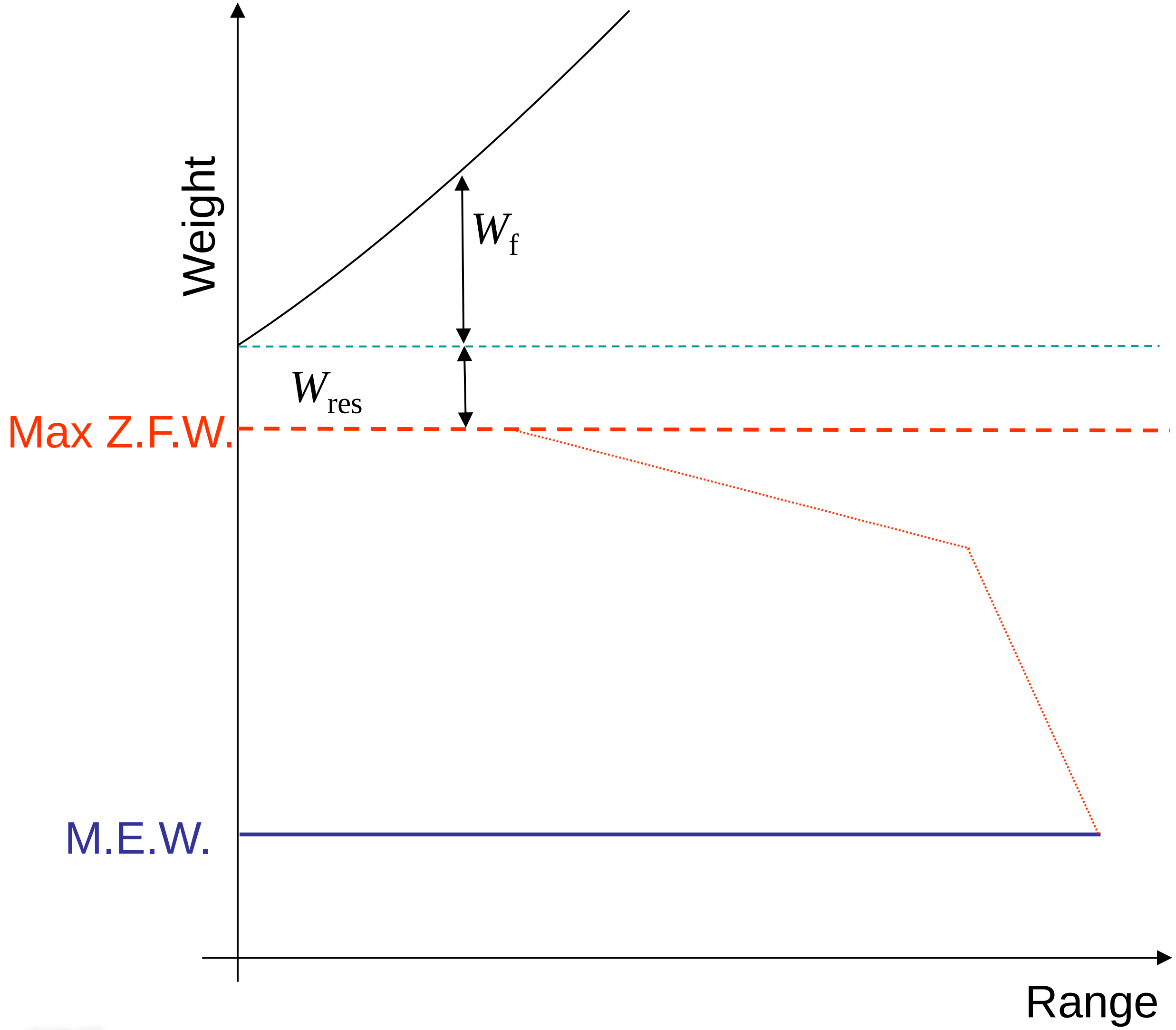
Payload-range diagram

- Maximum range depends on the payload
  - 3 zones: Max Payload, M.T.O.W. (structural), fuel capacity

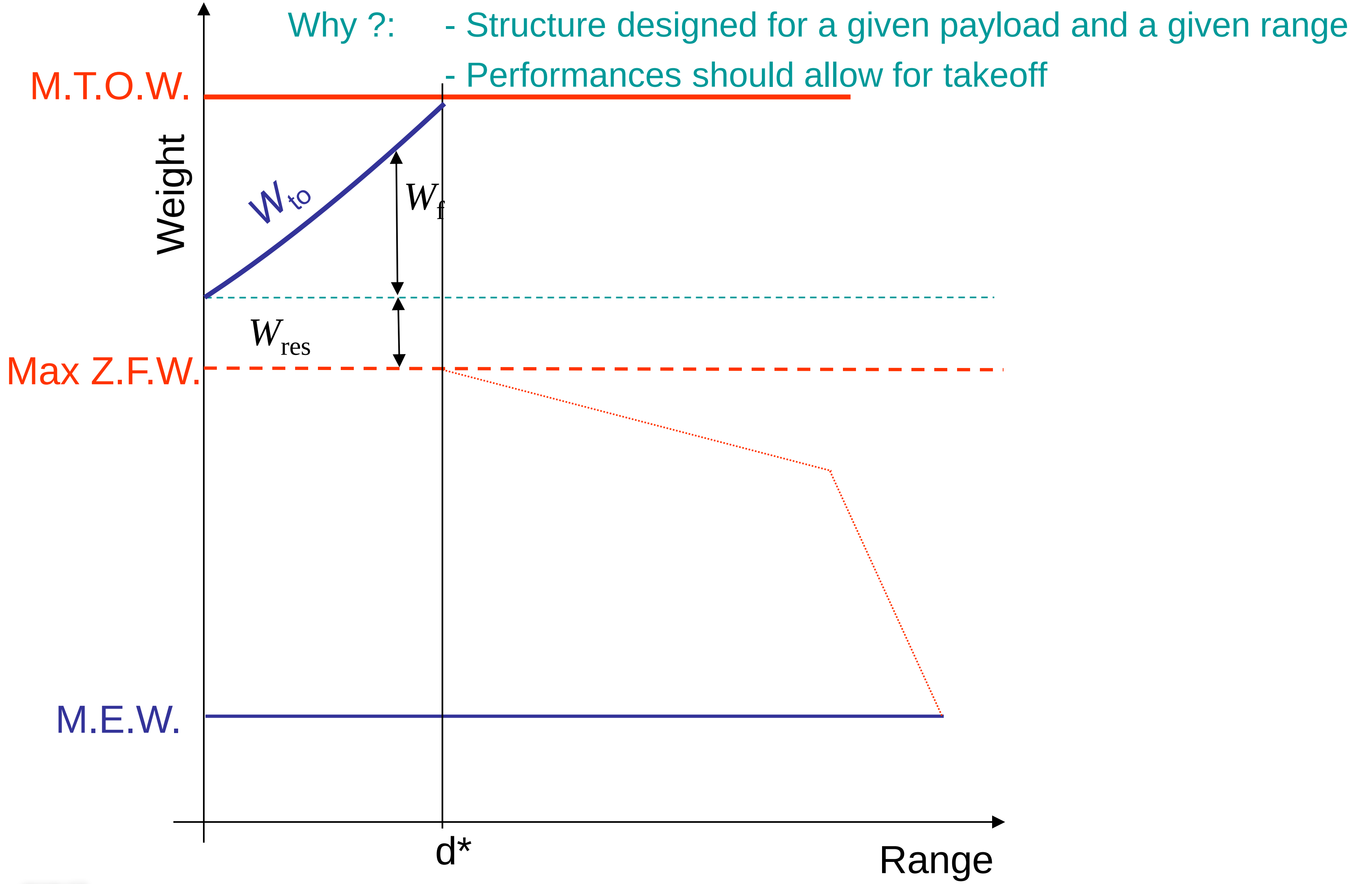


Payload-range diagram

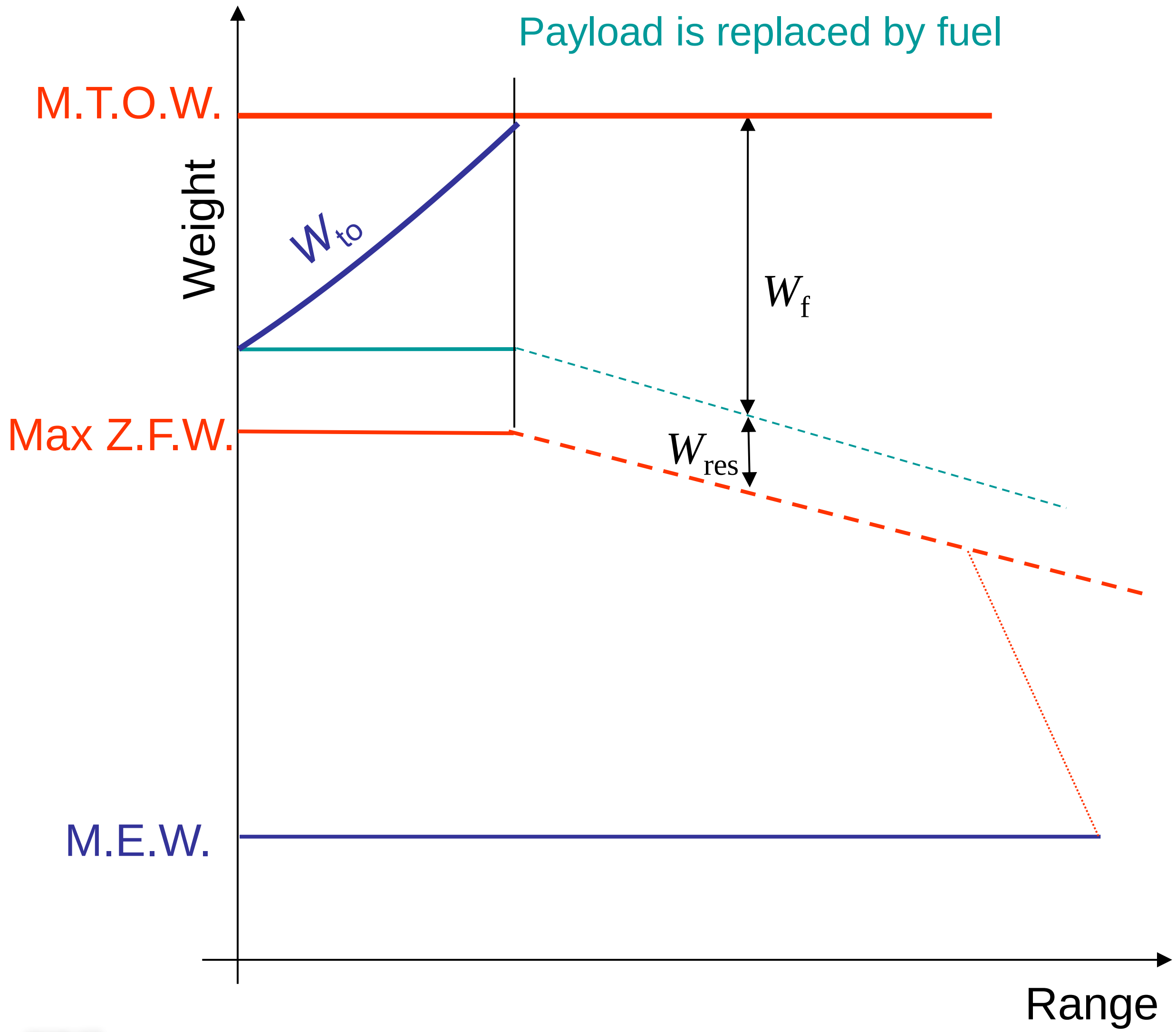
- Maximum range depends on the payload (2)
  - First step: add required fuel for the range at maximum payload



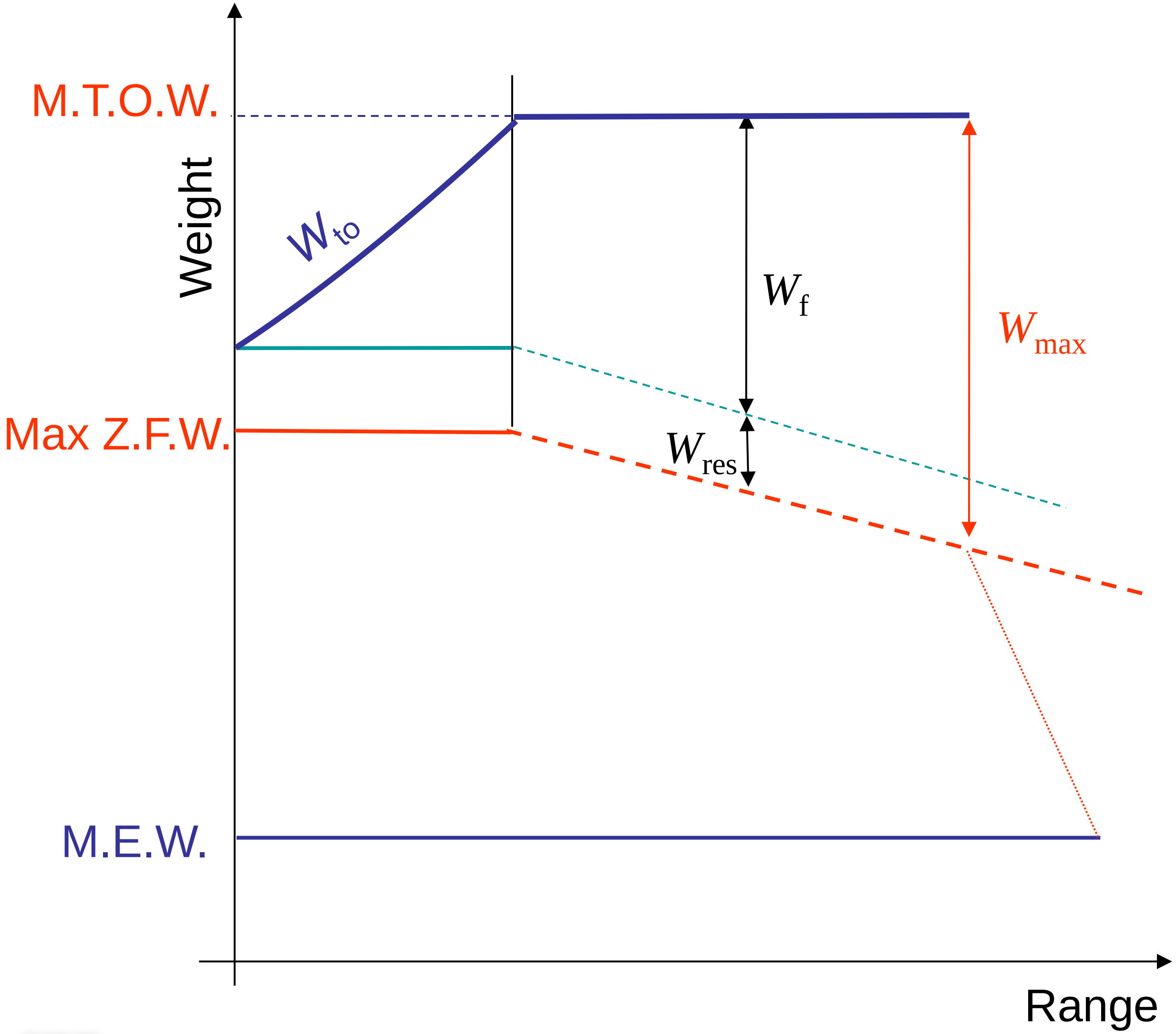
- Maximum range depends on the payload (3)
  - Second step: Threshold resulting from the maximum allowed TOW



- Maximum range depends on the payload (4)
  - Third step: Keep same M.T.O.W. and reduce payload when range increases

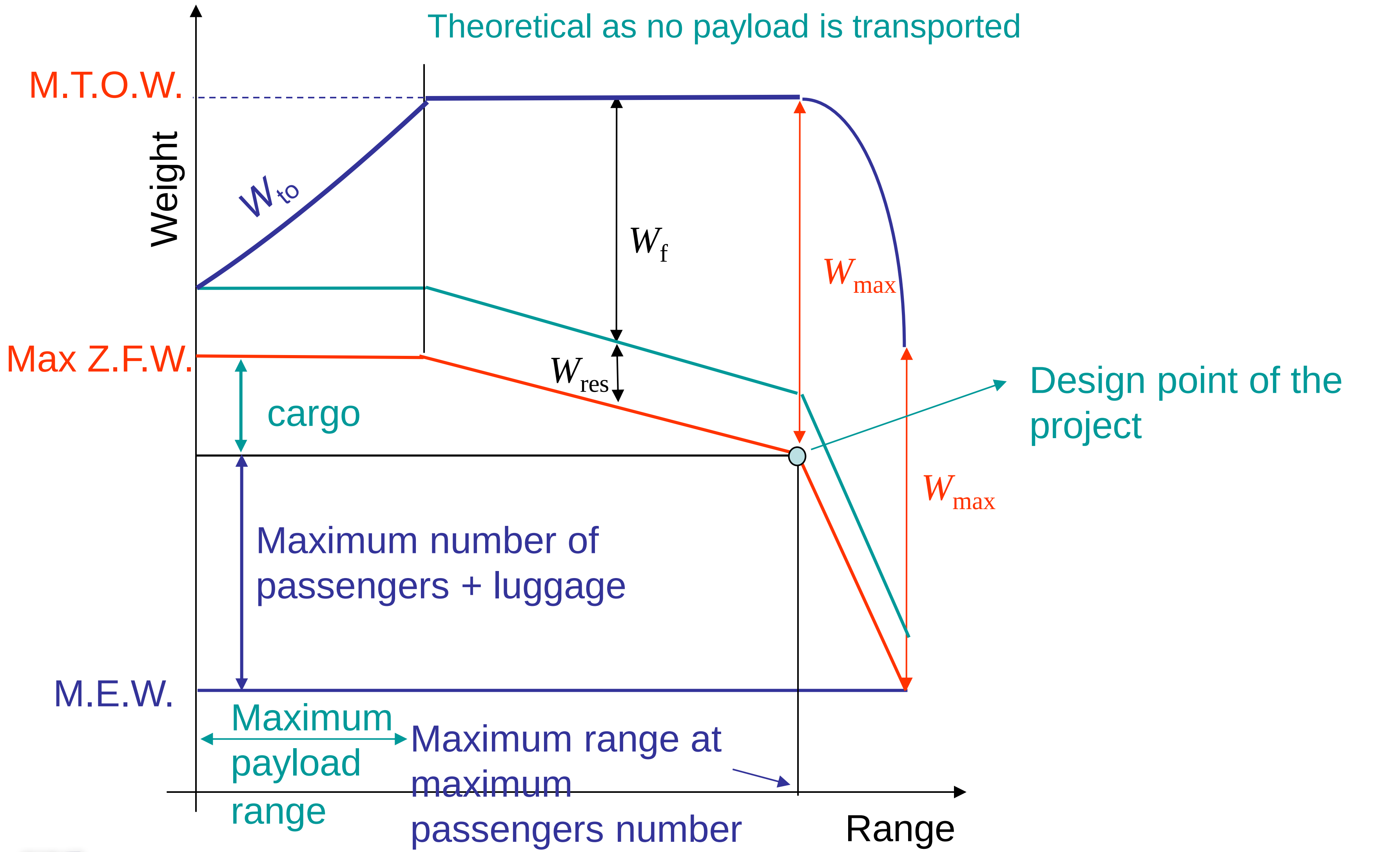


- Maximum range depends on the payload (5)
  - Fourth step: Maximum fuel tank capacity reached

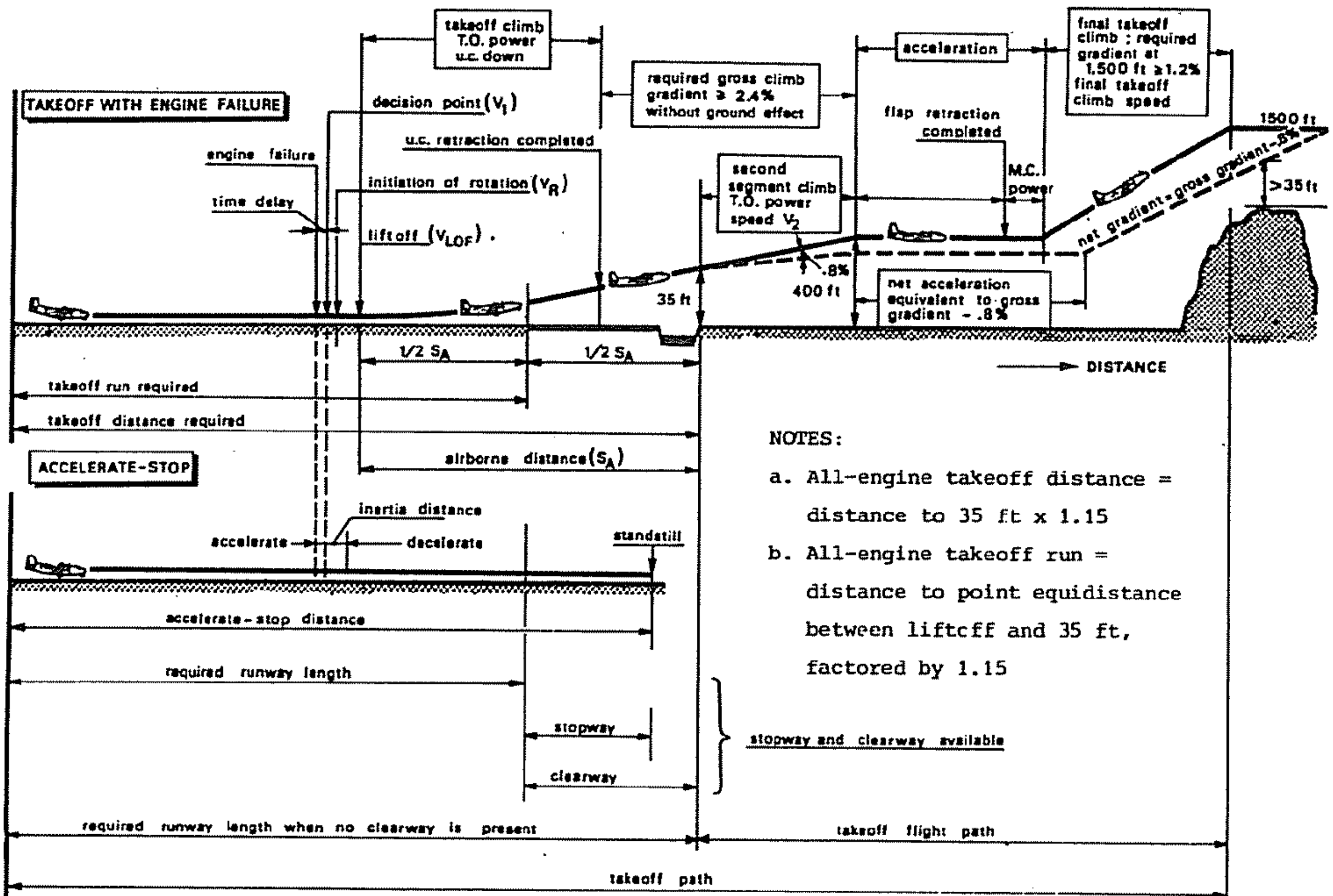


Payload-range diagram

- Maximum range depends on the payload (6)
  - Fifth step: Maximum range deduced at zero payload

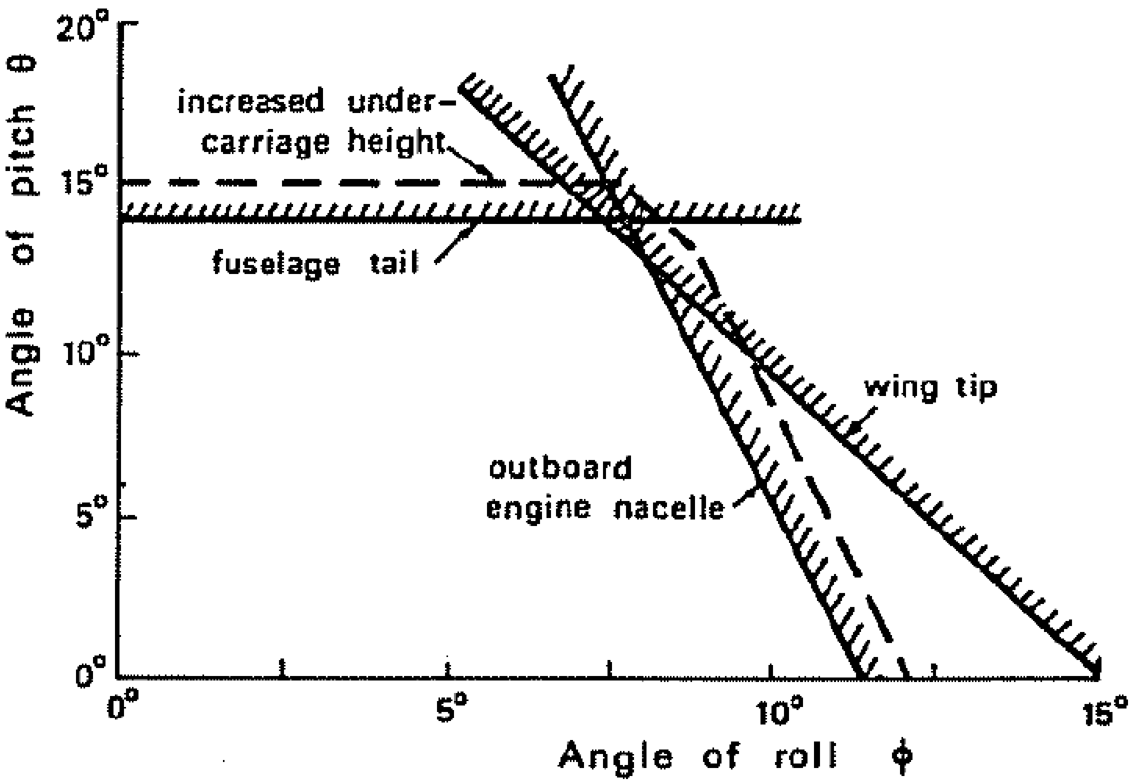


- Takeoff



TAKEOFF WITH ENGINE FAILURE
takeoff climb T.O. power u.c. down
decision point (V_1)
u.c. retraction completed
engine failure
time delay
initiation of rotation (V_R)
liftoff (V_{LOF})
required gross climb gradient ≥ 2.4% without ground effect
second segment climb T.O. power speed V_2
acceleration
flap retraction completed
M.C. power
final takeoff climb ; required gradient at 1.500 ft ≥1.2% final takeoff climb speed
1500 ft
net gradient = gross gradient - .8%
>35 ft
35 ft
400 ft
.8%
net acceleration equivalent to gross gradient - .8%
1/2 S_A
1/2 S_A
DISTANCE
takeoff run required
takeoff distance required
airborne distance (S_A)
ACCELERATE-STOP
inertia distance
accelerate
decelerate
standstill
accelerate-stop distance
required runway length
stopway
clearway
stopway and clearway available
required runway length when no clearway is present
takeoff flight path
takeoff path
NOTES:
a. All-engine takeoff distance = distance to 35 ft x 1.15
b. All-engine takeoff run = distance to point equidistance between liftoff and 35 ft, factored by 1.15

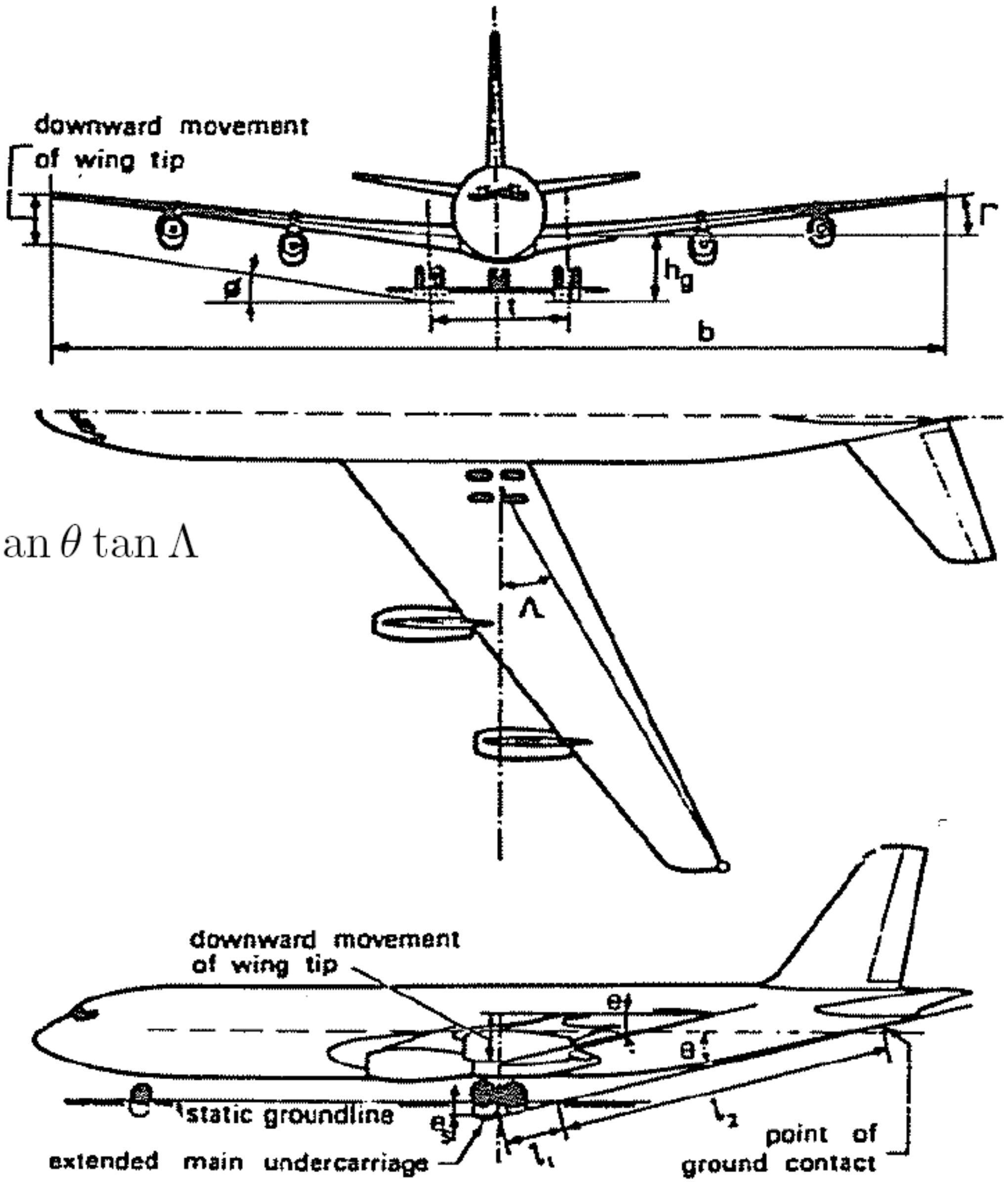
- Angles at takeoff
  - Only the wheels can be in contact with the ground
    - Plane geometry leads to maximum values of
      - Pitch angle θ
      - Roll angle ϕ



- Angles at takeoff (2)
  - Example:
    - Wing tip should not touch the ground during rotation θ even if the plane is experiencing a roll ϕ
    - Geometric considerations

$$\tan\phi = \tan\Gamma + \frac{2h_g}{b-t} - \tan\theta\tan\Lambda$$

    - Roll angle ϕ of 8° should be authorized
    - e_s: static deflection of shock absorber (e_s et l_1 ~ 0 as first approximation)



- Angles at takeoff (3)
  - Pitch angle at takeoff

$$\theta_{\mathrm{LOF}} = \alpha_{\mathrm{LOF}} + \frac{d\theta}{dt}\left(\frac{2l_1}{V_{\mathrm{LOF}}} + \sqrt{\frac{l_2 C_{L\,\mathrm{LOF}}}{ga}}\right)$$

Undercarriage fully extended

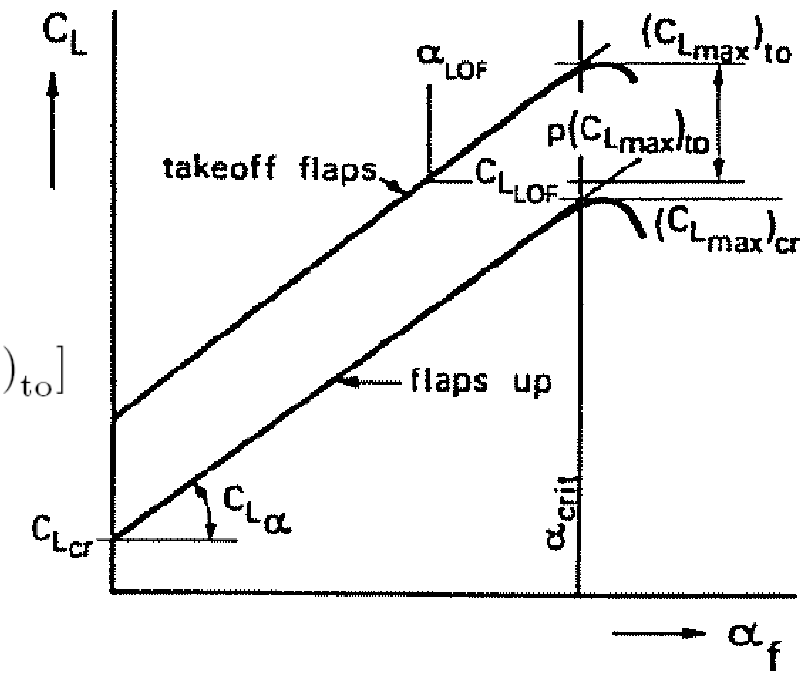
Climb of the undercarriage (from e_S)

Climb of the rear of the fuselage

- dθ/dt ~ 4 °/s
- α_{LOF}: maximum angle of attack of the fuselage expected during takeoff with flaps up

$$\alpha_{\mathrm{LOF}} = \frac{1}{C_{L\alpha}}\left[\left(C_{L\,\mathrm{max}}\right)_{\mathrm{cruise}} - C_{L\,\mathrm{cruise}} - p\left(C_{L\,\mathrm{max}}\right)_{\mathrm{to}}\right]$$

- $C_{L\,\mathrm{LOF}}$: maximum lift expected during takeoff with flaps down
- Margin p ~ 0.15
- Lift off velocity: V_{LOF} ~ 1.15 V_{s0}



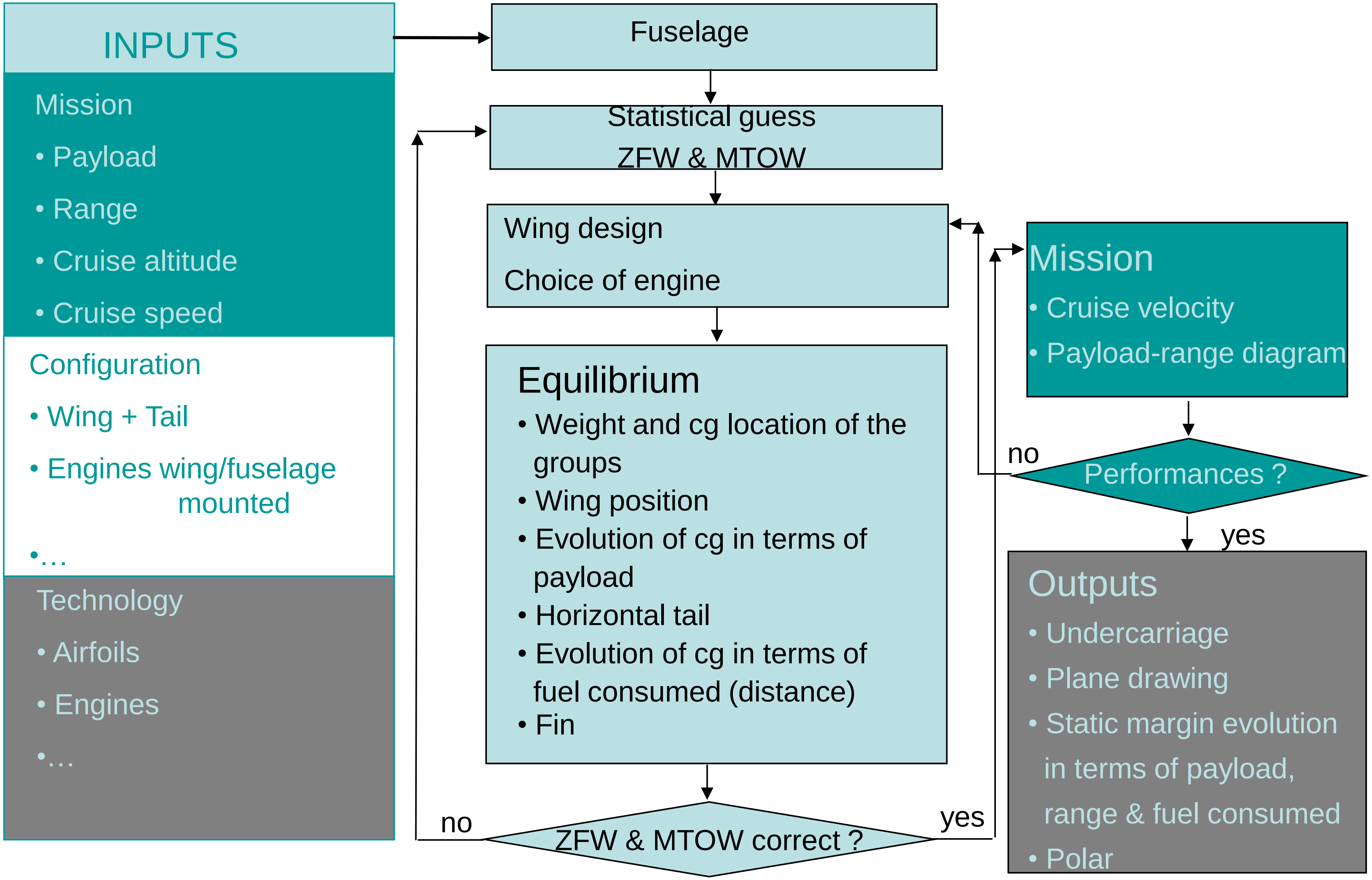
- Landing
  - Impact point of rear wheels behind projection of cg on the ground
    - If not, the plane would fall backward
    - Touchdown angle: $\theta_{TD} \sim \theta_{LOF}$
    - Distance l_m between cg and rear wheels

$$l_m \geq (|z_{cg}| + e_s) \tan \theta_{TD}$$

      - e_s: static deflection of shock absorber
      - z_{CG}: distance from cg to the ground

- Front wheels
  - About 8 to 15% of MTOW supported by front wheels
    - Lower than 8%: direction is not effective
    - More than 15%: difficulties at breaking
      - Now new devices are allowing to get more than 15%
  - CG location can change with the payload

Design steps



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  - Aircraft Design: Synthesis and Analysis, Ilan Kroo, Stanford University, http://adg.stanford.edu/aa241/AircraftDesign.html
- Other
  - Book
    - Synthesis of Subsonic Airplane Design, Egbert Torenbeek, Delft University Press, Kluwer Academic Publishers, The Netherlands, ISBN 90-246-2724-3, 1982.