Aircraft Design

Lecture 2: Aerodynamics

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Introduction

• Aerodynamics is the study of the loads exerted by the flow of air over an aircraft (there are other applications but they are boring)

• There are six loads:
  – Three forces:
    • Lift
    • Drag
    • Sideforce
  – Three moments:
    • Pitch
    • Roll
    • Yaw
Why study aerodynamics?

• Anything can fly, as long as you put a big enough rocket engine under it. But:
  – That’s the most expensive and dangerous solution
  – There are still stability, control and other problems that can only be resolved through a good aerodynamic study
  – There are several much better solutions. A few of them are listed in the next couple of slides. They all require a good understanding of aerodynamics.
Air vehicles

- **Airship**
  - A hydrostatic force provides lift
  - Motor(s) provide forward acceleration

- **Airplane**
  - A lifting surface (wing) provides lift
  - Motor(s) provide forward acceleration

- **Helicopter**
  - A rotor provides lift
  - The same rotor rotor provides forward acceleration

- **Autogyro**
  - A rotor provides lift
  - Another rotor provides forward acceleration
More air vehicles

- **Glider**
  - A lifting surface (wing) provides lift
  - There is no forward acceleration

- **Missile**
  - Several small lifting surfaces provide lift
  - A motor provides forward acceleration

- **Hot air balloon**
  - A hydrostatic force provides lift
  - There is no forward acceleration

- **Lifting body**
  - A lifting body provides lift
  - A motor provides forward acceleration (optional)
Airplane

• In this course we will mostly talk about airplanes
• The most popular airplane configuration is wing+fuselage+tail
• The configuration mirrors birds. The Wright brothers and others before them were inspired by bird flight
• Each component has a distinct role:
  – The wing provides lift
  – The fuselage holds cargo, passengers etc
  – The tail provides stability and control
Wings

- The role of the wing is to generate lift
- Lift creation can be described in two ways:
  - Pressure differential: The air pressure on the bottom surface of the wing is higher than the air pressure of the top surface. This pressure difference creates a net force upwards.
  - Newton’s third law: The wing pushes air downwards. As a consequence, the air itself pushes the wing upwards.
  - Either way, the laws of conservation apply: mass, momentum and energy.
Lift generation

- Lift generation of wings depends on their cross-sectional shape
- The Wright brothers were the first to study the effects of different cross-sectional shape
- They determined that the airfoil is the optimum cross-sectional shape for a wing
Airfoils

• Flow visualization

Pulsed jets show that the flow moves faster over the top surface.

By Bernoulli’s principle, faster flow speeds mean lower pressure.

Hence the pressure differential causing lift.
Airfoils continued

- But why is the flow accelerated on the top surface?
- Because it must separate at the trailing edge.
- Therefore, aircraft can fly because of viscosity.

\[ U = \text{airspeed} \]
\[ \alpha = \text{angle of attack} \]
\[ c = \text{chord} \]
\[ \Gamma = \text{circulation} \]
Shape effect (camber)

Camber increases the amount of lift produced by the airfoil.

Symmetric airfoil - no lift at 0° aoa

Cambered airfoil - produces lift at 0° aoa
The static pressure around a lifting airfoil looks something like:

L.E. Stagnation point

T.E. Stagnation point

Minimum pressure point

Pressure coefficient

\[ c_p = \frac{p - p_{ref}}{1/2 \rho U^2} \]

The lift is the integral of the pressure difference over the chord:

\[ l = \int_0^c (p_l(x) - p_u(x)) \, dx \]
Lift

- The amount of lift produced by an airfoil is also proportional to the total circulation required for the flow to separate at the trailing edge.

\[ l = \rho U \Gamma \]

- Where \( \rho \) is the air density, \( U \) the free stream airspeed and \( \Gamma \) the circulation.

- For flat plates and small angles of attack this result simplifies to

\[ l = \pi \rho U^2 \alpha \]
Lift coefficient

- A lift coefficient is a non-dimensional quantity defined as

\[ c_l = \frac{l}{\frac{1}{2} \rho U^2 c} \]

- For a flat plate, the lift coefficient can be obtained from

\[ c_l = 2\pi \alpha \]
True lift

As shown in the video earlier the flow cannot remain attached to the wing’s surface at high angles of attack.

Flow separation results in loss of lift. Important characteristics:

- Maximum lift coefficient
- Maximum lift angle of attack
The lift is a single force but it is caused by a continuous pressure distribution. Therefore, it must have a point of application. This point is called the Centre of Pressure (cp). The moment caused by the lift acting at the cp around the leading edge is called the pitching moment. Pitching moment coefficient:

\[
C_m = \frac{m}{\frac{1}{2} \rho U^2 c^2}
\]
Unfortunately, all bodies in a real airflow are subjected to a drag force.

There are no easy expressions for the calculation of drag. There are many sources of drag and few of them are easily modeled.

Drag: \( d \)

Drag coefficient:

\[
C_d = \frac{d}{\frac{1}{2} \rho U^2 c}
\]
The boundary layer

- Drag in two dimensions cannot be modeled at all using inviscid assumptions.
- Prandtl was the first to realize that viscous effects are very important in a thin layer of flow near the wing’s surface.
- Skin friction and momentum deficit within the boundary layer are some of the major sources of drag.
Momentum deficit

- Momentum deficit can be visualized as:

- The flow in front of the airfoil and just after it looks like:

- In other words, momentum has been lost. Loss of momentum = drag force.
Graphical representation

Flow accelerates

Flow decelerates

High pressure

Low pressure

Very high pressure

Very low pressure

Separation

Boundary layer
Stall

**Type I: Trailing Edge Stall**

- Graphs showing pressure distributions and growth of boundary layer.
- Illustrations of flow separation regions.

**Type II: Leading Edge Stall**

- Similar graphs to Type I, focusing on leading edge behavior.

**Type III: Thin Airfoil Stall**

- Graphs showing pressure distributions and lift and pitching moment curves.

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**a.** Upper surface pressure distributions, growth of the boundary layer and separation regions and lift and pitching moment curves.

**b.** Stalling characteristics correlated with Reynolds number and airfoil geometry (Ref. 7-81)
Coordinate system

- The values of lift and drag also depend on the coordinate system we choose.
- The most consistent definition of lift and drag is:
  - Lift is a force perpendicular to the free stream.
  - Drag is a force parallel to the free stream.
- Therefore, the integral of the pressure distribution around the airfoil is not the lift but the force acting normal to the surface:
  \[ n = \int_0^c (p_l(x) - p_a(x))dx \]
- So that the actual lift is \( l = n \cos \alpha \). In general, if the normal, \( n \), and tangential, \( n \), forces are known:
  \[ l = n \cos \alpha - t \sin \alpha \]
  \[ d = t \cos \alpha + n \sin \alpha \]
# Airfoil characteristics

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Performance Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length, (c)</td>
<td>Lift curve slope (c_{l\alpha})</td>
</tr>
<tr>
<td>Thickness, (t)</td>
<td>Zero-lift angle (c_{l0})</td>
</tr>
<tr>
<td>Camber, (dz/dx)</td>
<td>Maximum lift angle (\alpha_{c_{l_{\text{max}}}})</td>
</tr>
<tr>
<td>Shape (e.g. NACA 0012)</td>
<td>Maximum lift coefficient (c_{l_{\text{max}}})</td>
</tr>
<tr>
<td></td>
<td>Minimum drag coefficient (c_{d_{\text{min}}}))</td>
</tr>
</tbody>
</table>
NACA 4-digit Airfoils

- These airfoils were developed in the 20s and 30s based on earlier Göttingen and Clark Y sections
- They use a very specific terminology
NACA 4-digit airfoils

- Defined by

\[
\pm y_t = \frac{t}{c} \left(0.2969 \sqrt{x} - 0.1260 x - 0.3516 x^2 + 0.2843 x^3 - 0.1015 x^4\right)
\]

\[
y_c = \frac{c_{\text{max}}}{c} \left(2 x_{c_{\text{max}}} x - x^2\right) \quad \text{from } x = 0 \text{ to } x = x_{c_{\text{max}}}
\]

\[
y_c = \frac{c_{\text{max}}}{(1 - p)^2} \left((1 - 2 x_{c_{\text{max}}}) + 2 x_{c_{\text{max}}} x - x^2\right) \quad \text{from } x = x_{c_{\text{max}}} \text{ to } x = c
\]

- Where \( y_{\text{upper}} = y_t + y_c \), \( y_{\text{lower}} = -y_t + y_c \), \( y_t \) is the thickness shape and \( y_c \) is the camber shape, \( x_{c_{\text{max}}} \) is the chord-wise position of maximum camber, \( t \) is the maximum thickness and \( c_{\text{max}} \) is the maximum camber.
Wings

- A wing can be seen as an extrusion of an airfoil in the $y$-direction.

Wing span: $b$

Wing half-span: $s = b / 2$

Wing area over full span: $S$

Aspect ratio: $AR = b^2 / S$
Forces on wings

• In principle, it is simple to calculate the aerodynamic lift acting on a wing: it is simply the integral of the sectional lift over the span:

\[ L = \int_{-s}^{s} l(y)dy, \text{ or, } C_L = \int_{-s}^{s} c_l(y)dy, \text{ where } C_L = \frac{L}{1 / 2 \rho U^2 S} \]

• Unfortunately, the sectional lift variation is not easy to calculate
2D vs 3D force coefficients

- **2D lift, drag, moment etc. coefficients:**
  - Use lowercase letters: $c_l$, $c_d$, $c_m$, etc.
  - Use chord or chord$^2$ to normalize, e.g.

\[
\frac{d}{\frac{1}{2} \rho U^2 c}, \quad \frac{m}{\frac{1}{2} \rho U^2 c^2}
\]

- **3D lift, drag, moment etc. coefficients:**
  - Use uppercase letters: $C_L$, $C_D$, $C_M$, etc.
  - Use surface area or surface*chord to normalize, e.g.

\[
\frac{D}{\frac{1}{2} \rho U^2 S}, \quad \frac{M}{\frac{1}{2} \rho U^2 Sc}
\]
**Lifting line theory**

The wing is modeled mathematically by a single horseshoe-shaped vortex (black line).

Flow visualization showing that the horse-shoe vortex is a good approximation of real flow.

The stronger the vortex, the higher the lift.
Wingtip vortex visualization

- Tip vortices of large aircraft can cause serious problems in airports.
Induced drag

- Three-dimensional wings feature one very clear and simple to quantify source of drag: induced drag.
- In essence, the fact that a wing produces lift means that it also produces drag.
- The drag is proportional to the square of the lift.
- The source of induced drag is a downwash flow velocity created on the wing by the trailing vortices.
The effective angle of attack is the difference between the geometric angle of attack and the angle induced by the downwash.

\[ \alpha_e = \alpha_\infty - \varepsilon = \alpha_\infty - \frac{w}{U} \]
Induced drag (2)

The lift component perpendicular to the free stream is approximately equal to $l$ for small downwash angles.

The lift component parallel to the free stream is the induced drag, given by

$$d = l\varepsilon$$

The lift acting on a 3D wing is not perpendicular to the free stream, it is perpendicular to the total airspeed due to the free stream and downwash.
Elliptical lift distribution

- It can be shown that a wing with an elliptical planform has an elliptical wing distribution.
- Furthermore, this wing distribution causes the minimum lift-induced drag.

For such a wing the total lift and induced drag can be easily calculated:

$$C_L = \frac{2\pi AR}{AR + 2} \alpha$$
$$C_D = \frac{C_L^2}{\pi AR}$$

Aspect ratio decreases $C_D$. 

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Introduction to Aircraft Design
More on lift distributions

- Few well-known aircraft ever featured an elliptical wing.
- Minimizing lift-induced drag is only one consideration in the design of wings.
- Most wings have non-elliptical lift distributions.
- The lift and induced drag can be calculated for such wings using lifting line theory.
Sources of drag

• Parasite drag:
  – Skin friction drag (friction)
  – Form drag (also known as pressure or profile drag)
  – Interference drag (vortices created at the intersection of surfaces)

• Induced drag (due to downwash of wings)

• Wave drag (supersonic drag)
Wing characteristics

- Aspect Ratio
- Twist: angle of attack at root is not the same as angle of attack at tip
- Taper: chord at tip is smaller than chord at root
- Thickness ratio: Wing thickness varies over the span
- Airfoil: may change over wingspan
- Sweep: wingtip lies behind or in front of wing root
Aspect Ratio

Low aspect ratio wing
F-15

High aspect ratio wing
B-52
Aspect Ratio

• The Aspect Ratio is one of the basic design parameters for transport aircraft.
• It has a significant effect on aircraft performance. This will be discussed at a later lecture.
• Large values of AR tend to make the flow around the wind more 2D.
• Remember that 2D flows do not cause induced drag.
• Therefore, a very high aspect ratio increases the lift coefficient and decreases the drag coefficient.
Twisted wings
Wing twist

- **Wash-in:** $\alpha_{\text{tip}} > \alpha_{\text{root}}$
- **Wash-out:** $\alpha_{\text{tip}} < \alpha_{\text{root}}$

The effect of twist is usually modeled by defining the basic lift distribution (no twist) and the additional lift distribution (lift due only to twist).

Wings often have wash-out to reduce structural weight and improve stall characteristics.
More on wing twist

- The point of initial stalling should be sufficiently inboard, around \(0.4s\) from the wing root.
- This can be achieved with suitable twist. If the stall point is too far outboard, a little washout will bring it inboard.
- However, a washout of more than \(5^\circ\) results in an unacceptable increase in induced drag.
Tapered Wings

• Most World War II fighter aircraft had tapered wings
The taper ratio is defined as \( \lambda = \frac{c_{\text{tip}}}{c_{\text{root}}} \).
Taper reduces the amount of lift produced near the wing-tip.
Consequently, the tip vortex is weaker and the induced drag is decreased.
Taper also reduces structural weight.
As the chord at the root is unchanged the maximum lift is not severely affected by taper.
If the taper is not too high, the stalling characteristics are acceptable, even without twist.
Effect of wing taper on lift distribution

Taper increases sectional lift coefficient!
Non-linear taper

- Taper doesn’t have to be linear

P-51 Mustang

Cessna 150
No taper

- Untapered wings are easy and cheap to manufacture but aerodynamically inferior to tapered wings
Thickness-to-chord ratio

- High Aspect Ratio is good for transport aircraft: it decreases the induced drag coefficient.
- For such wings to be structurally sound, they must be very thick near the root.
- This is usually achieved by increasing the thickness ratio near the wing.
- Thickness affects also the profile drag. Too much increase in thickness can cancel the decrease in induced drag due to high AR.
Thickness-to-chord ratio

- Thickness ratio also affects the maximum lift
- Optimal thickness ratios:
  - 15-20% near the wing root
  - 10-15% near the wing tip
- Higher than 20% is not good

Maximum $C_L$ for NACA airfoils
Airfoil selection

- The airfoil section does not affect the wing lift distribution at small angles of attack.

- It affects mainly:
  - The curve of local $c_{l_{\text{max}}}$
  - The profile drag $c_f$
Airfoil choices

- **NACA four-digit airfoils:**
  - Drag increase with lift is gradual
  - Cambered sections have good maximum lift and docile stall
  - Gradual changes in drag and pitching moment
  - They are used in light aircraft (mostly wingtips and tailplanes) and trainers (gradual changes are good for training aircraft).

- **NACA five-digit**
  - Better maximum lift than 4-digit but very abrupt stall
  - They are sometimes used inboard, combined with 4-digit airfoils near the wingtip.
More airfoils

• NACA 6-series
  – Also known as ‘laminar flow’ series
  – Designed to have low profile drag at low lift coefficients - the ‘low drag bucket’ region
  – They also have high critical Mach numbers
  – Lower maximum lift than 4- and 5-digit series
  – Extensively tested and very well documented
Airfoil selection considerations

• In short, airfoil selection should be made using the following considerations:
  – An airfoil with low profile drag at the design flight conditions must be chosen
  – The airfoil must be capable of giving the desired maximum lift to the wing (with flaps if needed)
  – Compressibility issues must be addressed
Compressibility issues

- High subsonic cruise Mach numbers can be attained by:
  - Using sweepback
  - Reducing the thickness-to-chord ratio
  - Using improved airfoil sections (‘supercritical airfoils’)
  - Optimizing spanwise camber and twist variation
Zero sweep angle

- BAe ATP: Maximum airspeed of 137 m/s at 25000 ft, i.e. maximum $M=0.44$
- Straight tapered wings
High sweep angle

- Airbus A380: Maximum airspeed of 265m/s at 35000ft, i.e. maximum M=0.89
- Highly swept tapered wings
Types of sweep

- Straight quarter-line chord
- Reduced sweep at inboard section
- Increased sweep at inboard section

$c/4$
Effect of sweep

A component of the free stream airspeed, $V_T$ is tangent to the wing. Therefore, the airspeed seen by the airfoil is only $V_N$

$$V_N = V_\infty \cos \Lambda$$

The effective Mach number seen by the wing’s airfoil is

$$M_{\text{eff}} = M_\infty \cos \Lambda$$

A higher $V_\infty$ is required to achieve the sonic conditions
Using sweepback

• Up to $M=0.65$ or $M=0.7$, straight wings with appropriate thickness ratio are sufficient.
• At higher Mach numbers, sweepback is required
• A sweep angle of 35 degrees is rarely exceeded
It is clear that sweep angle increases with Mach number.

Taper ratio also, generally, increases with Mach number.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Aspect Ratio</th>
<th>Taper Ratio</th>
<th>Sweep Angle</th>
<th>Maximum Mach</th>
</tr>
</thead>
<tbody>
<tr>
<td>VFW-Fokker 614</td>
<td>7.22</td>
<td>0.402</td>
<td>15°</td>
<td>0.65</td>
</tr>
<tr>
<td>Yakovlev Yak 40</td>
<td>9.00</td>
<td>0.396</td>
<td>0°</td>
<td>0.70</td>
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<tr>
<td>Fokker-VFW F 28</td>
<td>7.27</td>
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<td>16°</td>
<td>0.75</td>
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<tr>
<td>BAC 1-11 200/400</td>
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<td>0.321</td>
<td>20°</td>
<td>0.78</td>
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<tr>
<td>Aerospatialle Caravelle</td>
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<td>20°</td>
<td>0.81</td>
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<tr>
<td>Boeing 737 100/200</td>
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<tr>
<td>MDD DC-8 10/50/60</td>
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<td>0.244</td>
<td>30°</td>
<td>0.88</td>
</tr>
<tr>
<td>Boeing 707</td>
<td>7.11</td>
<td>0.293</td>
<td>35°</td>
<td>0.90</td>
</tr>
<tr>
<td>Boeing 747</td>
<td>6.96</td>
<td>0.309</td>
<td>37°30’</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Shock waves

• Shock waves can be formed on the upper (and even lower) surfaces of wings travelling in transonic (i.e. M<1) flow. For example:
Airfoils for Transonic Conditions

- a. Conventional section with roof-top pressure distribution
- b. Peaky upper surface pressure distribution
- c. Supercritical upper surface pressure distribution
- d. Rear loading airfoil compared with conventional airfoil (lower surface)
Transonic airfoils

- Rooftop: They have a relatively flat pressure distribution on the forward upper surface. The flow is not accelerated in this region and the advent of M=1 is delayed. NACA 6-series

- Peaky upper surface: Supersonic velocities and suction near the leading edge, followed by a weak shock wave. The drag rise is postponed to higher airspeeds. BAC 1-11, VC-10 and DC-9 have used this.
Transonic airfoils

• Supercritical airfoils: They have a flat upper surface, creating shock-free supersonic flow region. This region is much greater than that of the peaky distribution.

• Rear loading: The rear lower surface is highly cambered so that a lot of lift is generated near the rear of the airfoil. When combined with a flat upper surface (supercritical airfoil) a large decrease in drag for the same Mach number and lift coefficient can be obtained.

• This combination has been used by Airbus A300 and all subsequent civil transports.
A value for the most appropriate thickness ratio for a particular Mach number can be obtained from:

\[
t / c = 0.3 \left\{ 1 - \left( \frac{5 + M^2}{5 + M'^*} \right)^{3.5} \left( \frac{\sqrt{1 - M^2}}{M^2} \right) \right\}^{2/3}
\]

Where \( M'^* = 1 \) for conventional airfoils, \( M'^* = 1.05 \) for peaky airfoils and \( M'^* = 1.15 \) for supercritical airfoils.

\( M \cos \Lambda \) can be used for sweptback wings.
Wing lift coefficient
(unswept wings)

- The lift wing coefficient can be estimated by

\[ C_L = C_{L\alpha} \alpha \]

\[ C_{L\alpha} = 0.995 \frac{c_{l\alpha}}{E + c_{l\alpha} / \pi AR} \]

\[ E = 1 + \frac{2 \lambda}{AR(1 + \lambda)} \]

- \( C_{L\alpha} \)=wing lift curve slope
- \( c_{l\alpha} \)=sectional lift curve slope
- \( E \)=Jone’s correction
- \( \lambda \)=taper ratio
- \( AR \)=Aspect ratio
Wing lift coefficient (unswept wings)

• For a sectional lift curve slope of $2\pi$

$$C_{L\alpha} = \frac{2\pi}{1 + \frac{2}{AR} \left(\frac{1+2\lambda}{1+\lambda}\right)}$$

• Compressibility effects can be taken into account by replacing $AR$ by $\beta AR$ and $C_{L\alpha}$ by $\beta C_{L\alpha}$, where

$$\beta = \sqrt{1 - M^2}$$
Wing lift coefficient (swept wings)

• For swept wings, an approximate expression for the wing lift curve slope is

\[
\beta C_{L\alpha} = \frac{2\pi}{2 + \sqrt{1 + \left(\frac{2}{\beta A}\right)^2}}
\]

where \( \tan \Lambda_\beta = \frac{\tan \Lambda_{1/2}}{\beta} \) and \( k = \frac{\beta c_{l\alpha}}{2\pi} \)

• \( \Lambda_{1/2} \) is the sweepback angle at the half-chord
Wing maximum lift coefficient

- The maximum lift coefficient of a wing can be approximated by

\[ C_{L_{\text{max}}} = k_s \frac{c_{l_{\text{max,root}}} + c_{l_{\text{max,tip}}}}{2} \]

- Where \( k_s = 0.88 \) for untapered wings and \( k_s = 0.95 \) for tapered wings.
The fuselage produces very little (if any) lift. However, some lift is carried over from the wing onto the fuselage.
Supersonic flight

• Supersonic flight requires different wing design because:
  – Lift generation mechanisms are different
  – Much more drag is produced
• Supersonic airfoils are usually very thin and sharp
• Supersonic wings are either trapezoidal or Delta-shaped
Trapezoidal wings

Bell X-1

F-104

F-22?
Delta Wings

Concorde

MiG-21

Mirage 2000
Supersonic flow over flat plate

Expansion waves

Shock wave (compression)

$p_u$ is lower than $p_l$ so that there is a net force upwards - lift
Lift on supersonic flat plate

• It is easy to calculate the lift on a flat plate airfoil in a supersonic flow

• The compression and expansion cause pressure coefficients of

\[ c_{p_l} = \frac{2\alpha}{\sqrt{M^2 - 1}} \quad c_{p_u} = -\frac{2\alpha}{\sqrt{M^2 - 1}} \]

• The force acting normal to the plate is

\[ c_n = \frac{1}{c} \int c_p \, dx = \frac{1}{c} \int (c_{p_l} - c_{p_u}) \, dx = \frac{4\alpha}{\sqrt{M^2 - 1}} \]

• Leading to a lift coefficient of:

\[ c_l = c_n \cos \alpha = \frac{4\alpha}{\sqrt{M^2 - 1}} \]
Drag on supersonic flat plate

- Unlike incompressible flow, supersonic flow causes drag, known as wave drag.
- The drag force is obtained from the normal force as

\[ c_d = c_n \sin \alpha = \frac{4 \alpha^2}{\sqrt{M^2 - 1}} \]
The difference in shock strengths causes the flow on the lower surface to be compressed more than the flow on the upper surface.
Lift is generated on a Delta wing by the creation of conical vortices. High speed flow under the vortices causes low pressure. Pressure difference with lower surface causes lift.
Lift on Delta Wing

- Lift contribution from potential and vortex

\[ C_L = C_{L,P} + C_{L,V} = \]
\[ K_p \sin \alpha \cos^2 \alpha + K_V \cos \alpha \sin^2 \alpha \]

Where \( \alpha \) is the angle of attack and \( K_P, K_V \) come from the drawing
Vortex burst

- At very high angles of attack the vortices can break down
- Loss of lift ensues
Delta Wings at low speeds

- Delta wings are quite inefficient at low speeds
- Concorde had to fly at an uncomfortably high angle of attack to take off and land
- Several solutions have been tried
  - Canards
  - Compound Delta
  - Swing-wing
  - Leading Edge Extensions
Canards

- This was the solution adopted by the Tu-144

The canards were there specifically to control the aircraft at low airspeeds
Compound Delta

Two Delta wings superimposed, with different sweeps. The highly swept section creates additional, stronger vortices that increase lift and serve to keep the flow attached at higher angles of attack.

Saab-35 Draken

F-16XL
Swing-Wing

- Quite a simple concept: at high speeds you get a Delta, at low speeds you get a high AR trapezoidal wing

F-14
Leading Edge Extensions

- Some aircraft combine the advantages of Delta wings with those of trapezoidal wings.
- Leading Edge Extensions (as on this F-18) create vortices just as Delta Wings.
- The vortices can keep the flow over the wing attached at very high angles of attack.