Aeronautics Design Project

Stability and Control

T. Andrianne

APRI0004 2024-2025

Content of the course

- Introduction
- Control surfaces
- Geometry of airplane
- Static stability
- Longitudinal stability
- Lateral stability
- Dynamic stability

Stability and Control

Stability = ability to **keep** the aircraft in the air in the chosen flight attitude and to counteract disturbances

Control = ability to **change** the flight direction and attitude of the aircraft

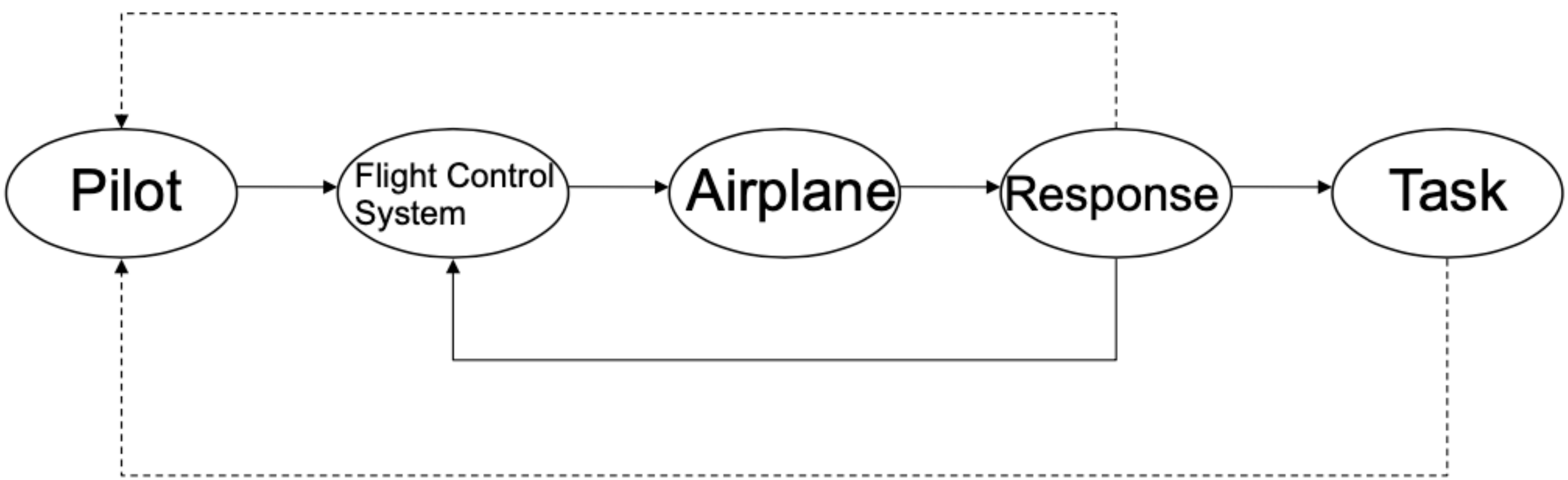
Both issues are :
- Not design criteria (aircraft are designed for performance)
- Investigated in the preliminary design process

Stability and control refer to “Flight Mechanics”

Flight mechanics ensure:
- to design an airplane able to accomplish efficiently a mission
- to make the task of the pilot easier (good handling in flight)
- to avoid unwanted/unexpected phenomena

Aircraft controls

Pilot :
- controls only the flight control system
- tailors the inputs on the FCS by observing aircraft's response
- always keeping an eye on the task at hand



Aircraft degrees of freedom

6 DOFs around the center of gravity (c.g.)

3 displacements:

x = horizontal motion

y = side motion

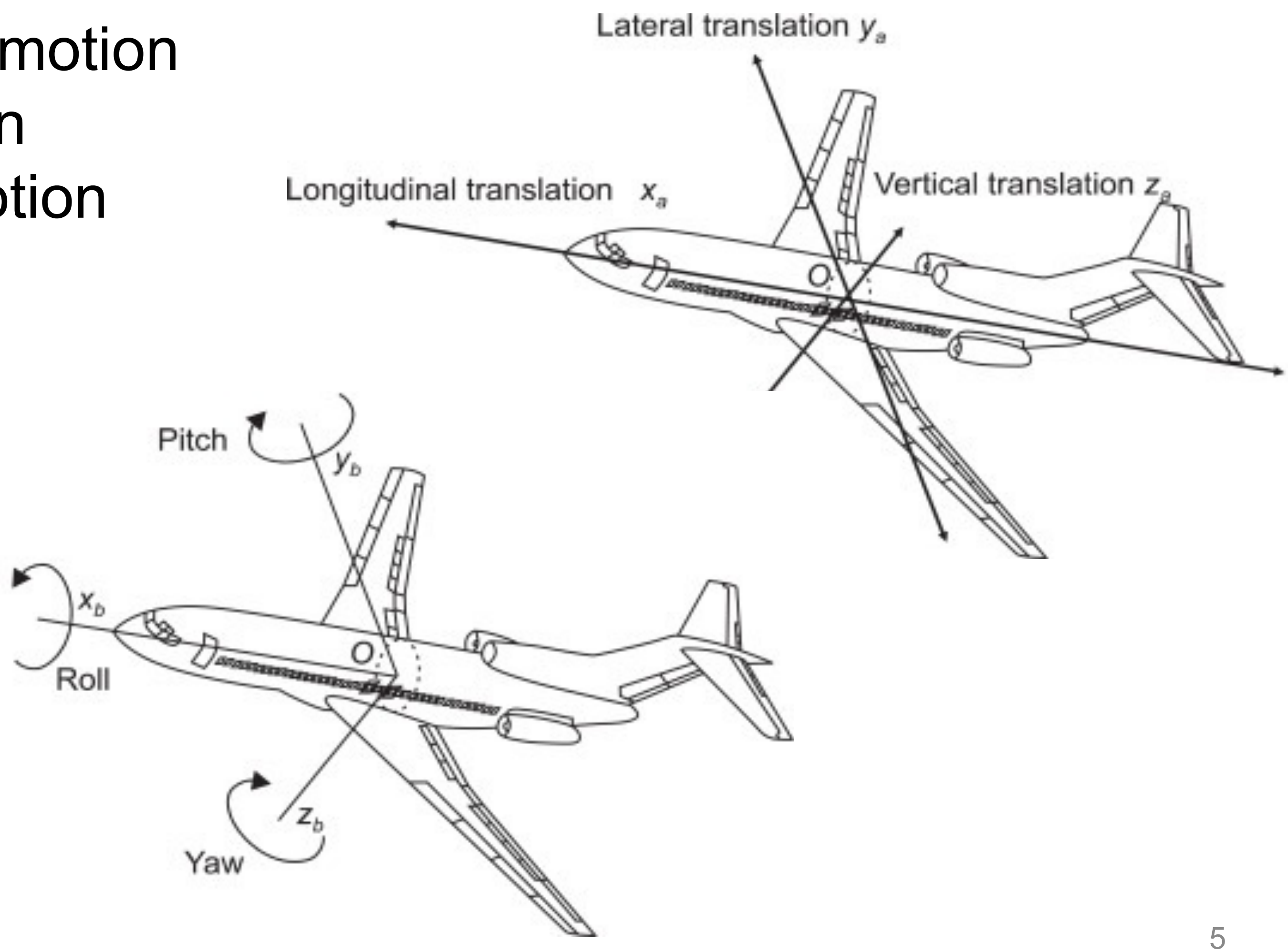
z = vertical motion

3 rotations:

x = roll

y = pitch

z = yaw



Control surfaces

Aircraft is controlled via control surfaces and power:
- Ailerons (used in pairs to control roll)
- Elevators (on the tail to control pitch)
- Rudder (on the fin to control yaw)
- Throttle (adjust the thrust of engine(s))

Rudder

Elevator

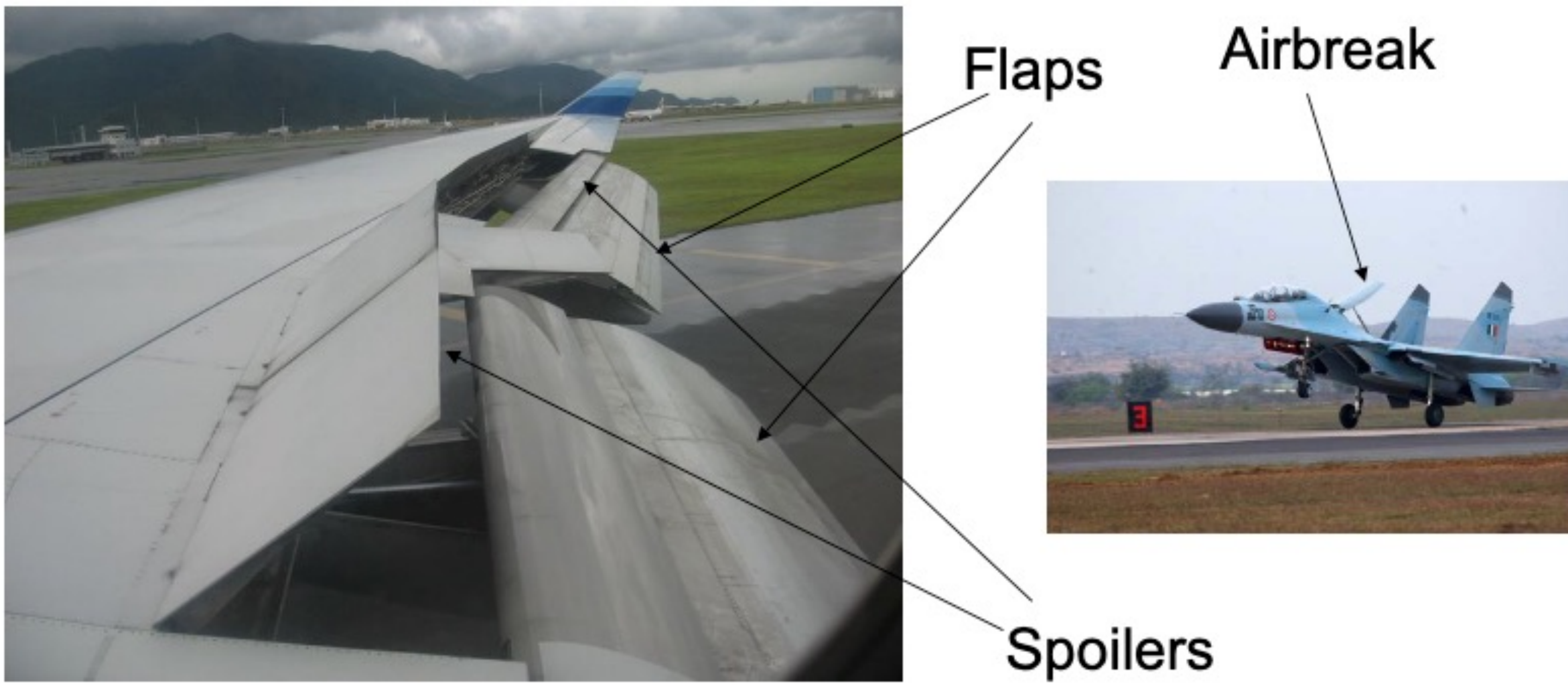
Aileron

Throttle

Elevon
(elevator+aileron)

Rudderon
(rudder+aileron)

Other control surfaces



Vectored thrust

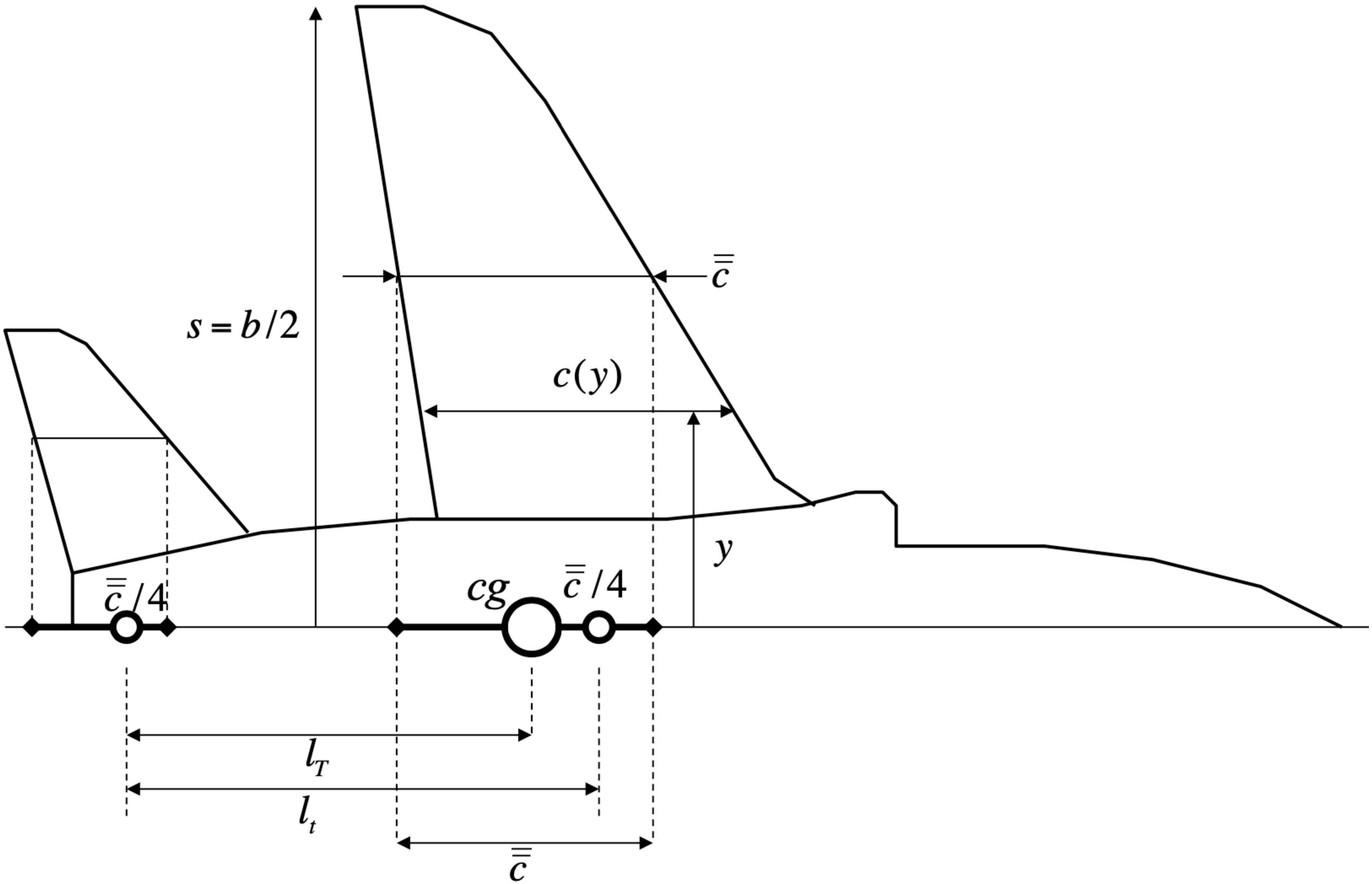
Combination of control surfaces

Decelerons

Spoileron

Flaperon

Airplane geometry



Wing geometry

- Standard mean chord (smc)

$$\bar{c} = \int_{-s}^{s} c(y)dy \Big/ \int_{-s}^{s} dy$$

- Mean aerodynamic chord (mac)

$$\bar{\bar{c}} = \int_{-s}^{s} c^2(y)dy \Big/ \int_{-s}^{s} c(y)dy$$

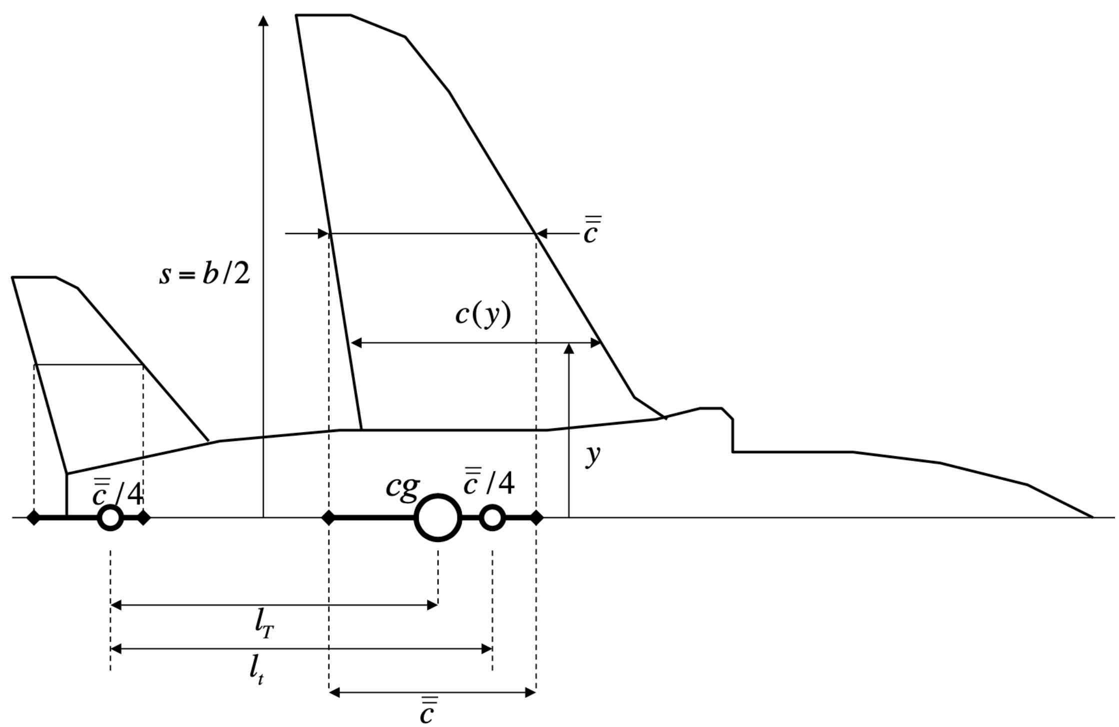
- Wing area : $\mathrm{S} = b\bar{c}$

- Aspect Ratio : $\mathrm{AR} = b^2/S$

Tail geometry

- Tailplane area : S_T
- Tailplane moment arm : l_T
- Tailplane volume ratio ($\bar{V}_T$) is a measure of the aerodynamic effectiveness of the tail

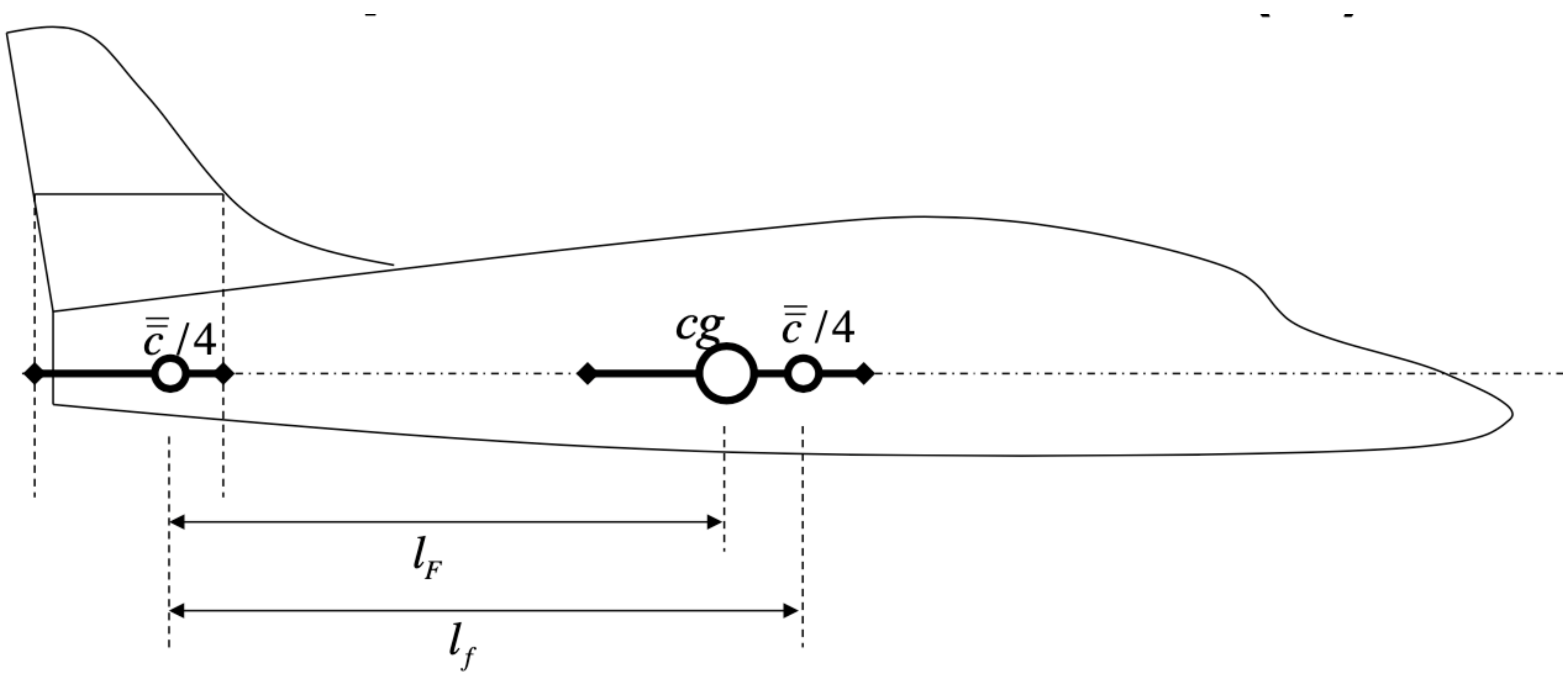
$$\bar{V}_T = \frac{S_T l_T}{S\bar{\bar{c}}}$$



Fin geometry

- Fin moment arm : l_F

- Fin volume ratio / effectiveness : $\bar{V}_F = \frac{S_F l_F}{S\bar{\bar{c}}}$



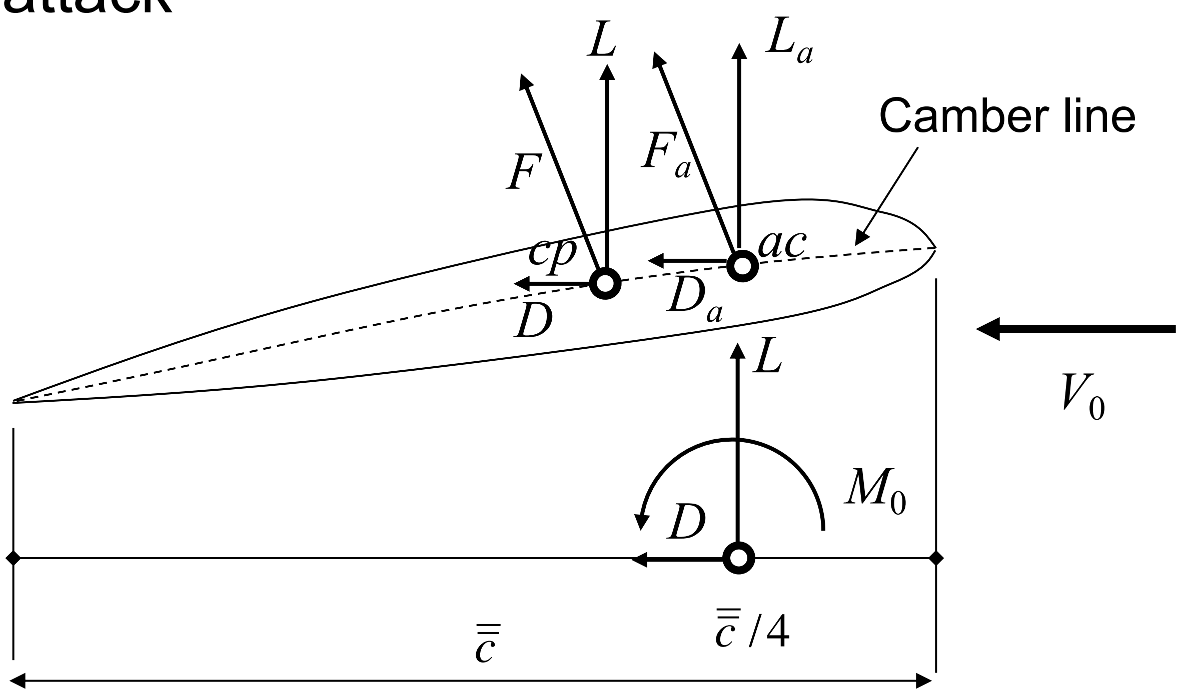
Aerodynamic centers

Centre of pressure (*cp*)

Position of the aerodynamic resultant force, where no aerodynamic moment applies

Quarter-chord = aerodynamic centre (*ac*)

Point at which aerodynamic force due to angle of attack, F_a, acts. Aerodynamic moment, M_o, is independent of angle of attack



Static stability

Stability conditions:

- **At fixed flight conditions :**

→ All forces and moments around c.g. are balanced

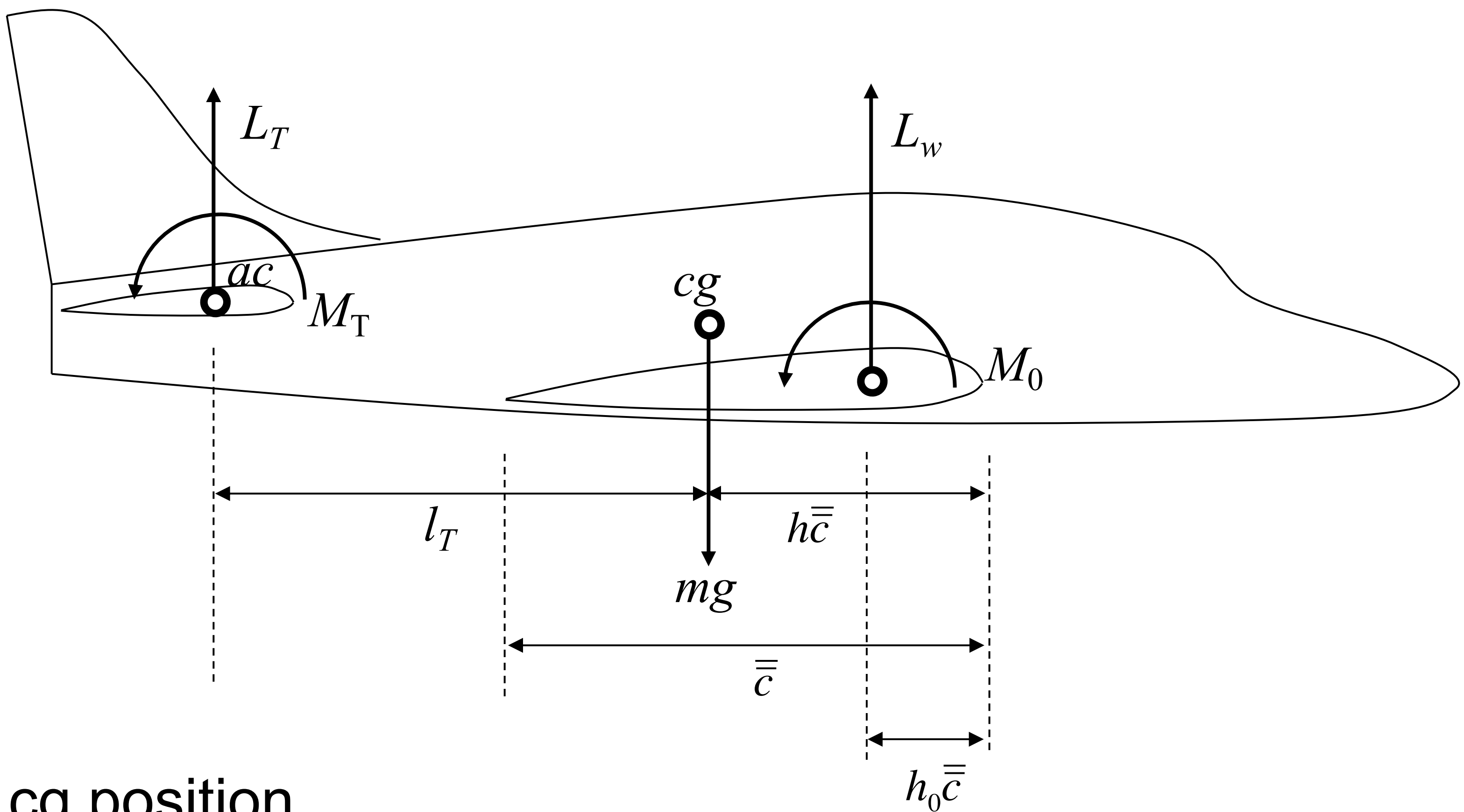
= TRIM position (adjusted using trim tabs)

- **After any small perturbation in flight attitude**

→ Aircraft returns to its equilibrium position

Aircraft static stability must be ensured along:
- Longitudinal axis
- Lateral axis

Longitudinal stability



h = cg position

h_0 = position of the aerodynamic center of the wing

l_T = (dimensional) position of the aerodynamic center of the tail

Pitching moment equation

Assuming:
- Steady level flight
- Thrust balances drag and both pass by the c.g.

Vertical balance:

$$L_w + L_T - mg = 0$$

Pitching moment (around c.g.):

$$\mathrm{M} = M_0 + L_w(h - h_0)\bar{\bar{c}} - L_T l_T + M_T = 0$$

(positive nose-up)

Stability

Equilibrium point can be **stable**, **unstable** or **neutrally stable**

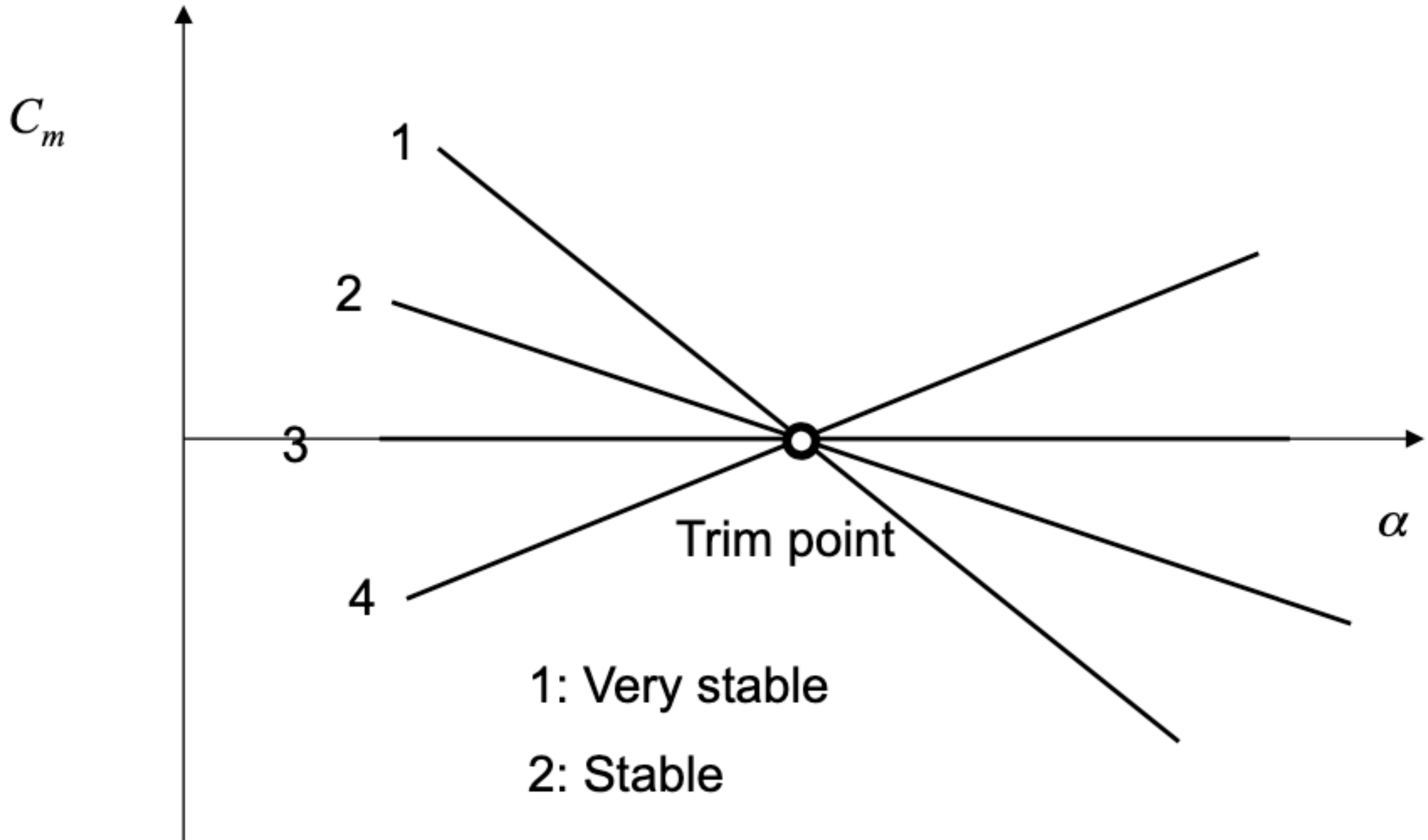
Stable equilibrium point is characterized by

$$M = 0 \quad \text{and} \quad \frac{dM}{d\alpha} < 0$$

A more general condition, taking into account compressibility effects, is

$$M = 0 \quad \text{and} \quad \frac{dM}{dL} < 0 \qquad \text{or} \qquad C_m = 0 \quad \text{and} \quad \frac{dC_m}{dC_L} < 0$$

Degree of stability



1: Very stable

2: Stable

3: Neutrally stable

4: Unstable

Pitching stability

Pitching moment equation

$$\mathrm{M} = M_0 + L_w(h - h_0)\bar{\bar{c}} - L_T l_T + M_T = 0$$

Assuming a symmetric tailplane so that $M_T = 0$

The equation can be re-written

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - C_{L_T}\bar{V}_T = 0$$

where

$$C_m = \frac{M}{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}} \qquad C_{L_w} = \frac{L_w}{\frac{1}{2}\rho V_0^2 S} \qquad C_{L_T} = \frac{L_T}{\frac{1}{2}\rho V_0^2 S_T}$$

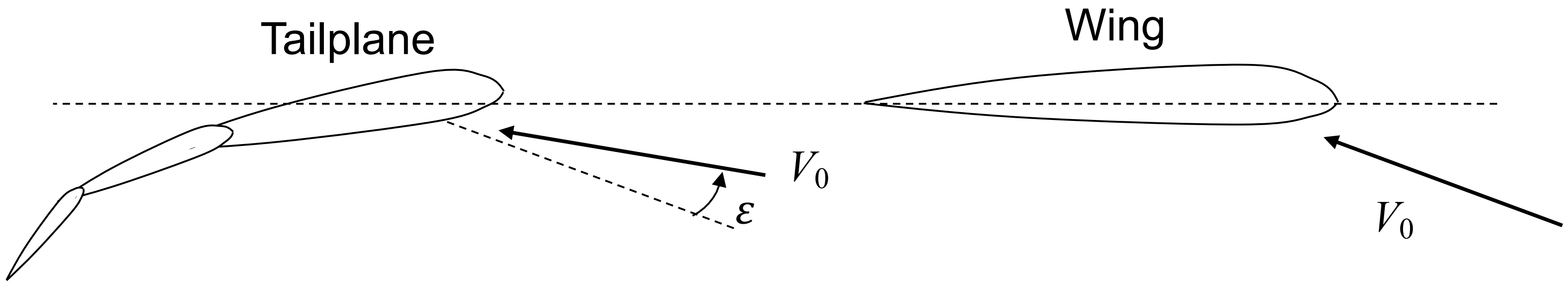
Pitching stability

Static stability if $\frac{dC_m}{dC_L} < 0$ or approximately $\frac{dC_m}{dC_{L_w}} < 0$

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - C_{L_T}\bar{V}_T$$

$$\rightarrow \frac{dC_m}{dC_{L_w}} = \underbrace{\frac{dC_{m_0}}{dC_{L_w}}}_{=0} + (h - h_0) - \underbrace{\frac{dC_{L_T}}{dC_{L_w}}}_{\text{must be calculated}}\bar{V}_T$$

Tailplane is impacted by the downwash effect from the wing
→ downwash angle **ε**



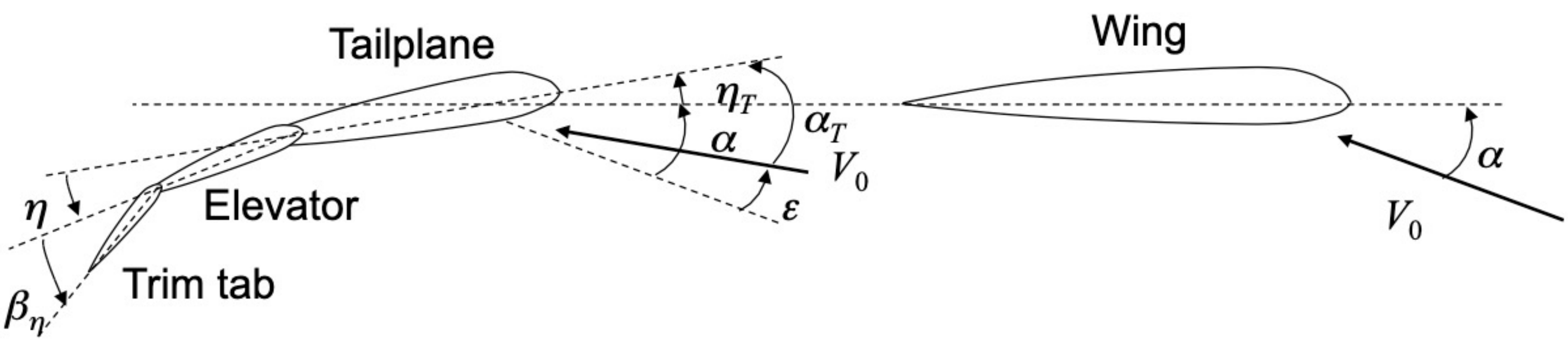
Wing-tail flow

Total angle of attack of the tailplane

$$\alpha_T = \alpha - \varepsilon + \eta_T$$

Lift coefficient of the tailplane

$$C_{L_T} = \alpha_0 + a_1\alpha_T + a_2\eta + a_3\beta_\eta$$



Wing-tail flow

Downwash on the tailplane, for small disturbances

→ ε is a linear function of wing incidence α

→ $\varepsilon = \frac{d\varepsilon}{d\alpha}\alpha$

The lift coefficient of the wing is also a linear function of α

$C_{L_w} = \mathrm{a}\,\alpha$ → $\alpha = C_{L_w}/a$

So that, $\alpha_T = \alpha - \varepsilon + \eta_T = \frac{C_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T$

Wing-tail flow

Lift coefficient of the tailplane becomes

$$C_{L_T} = \alpha_0 + a_1 \frac{C_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta$$

The derivation over C_{L_w} yields

$$\frac{dC_{L_T}}{dC_{L_w}} = \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_2\frac{d\eta}{dC_{L_w}} + a_3\frac{d\beta_\eta}{dC_{L_w}}$$ (since $\boldsymbol{\eta}_T$ is constant)

The derivative of the pitching moment coefficient becomes:

$$\frac{dC_m}{dC_{L_w}} = (h - h_0) - \left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_2\frac{d\eta}{dC_{L_w}} + a_3\frac{d\beta_\eta}{dC_{L_w}}\right]\bar{V}_T$$

Controls

Two manners to control the aircraft:
- fixed/locked controls → $\boldsymbol{\eta}$ and $\boldsymbol{\beta_\eta}$ are constant
- free controls → $\boldsymbol{\eta}$ and $\boldsymbol{\beta_\eta}$ are variable

→ the pitching stability $\frac{dC_m}{dC_{L_w}} < 0$ will take different forms

Fixed controls

Assuming a trimmed aircraft

What is the effect of a small perturbation (e.g. gust) ?

The pitching moment equation becomes

$$\frac{dC_m}{dC_{L_W}} = (h - h_0) - \left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_2 \cancel{\frac{d\eta}{dC_{L_W}}} + a_3 \cancel{\frac{d\beta_\eta}{dC_{L_W}}}\right]\bar{V}_T$$

$$\rightarrow \frac{dC_m}{dC_{L_W}} = (h - h_0) - \bar{V}_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Controls fixed stability margin, $K_n = -\frac{dC_m}{dC_{L_W}} = h_n - h$

where h_n is the controls fixed neutral point, $h_n = h_0 + \bar{V}_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$

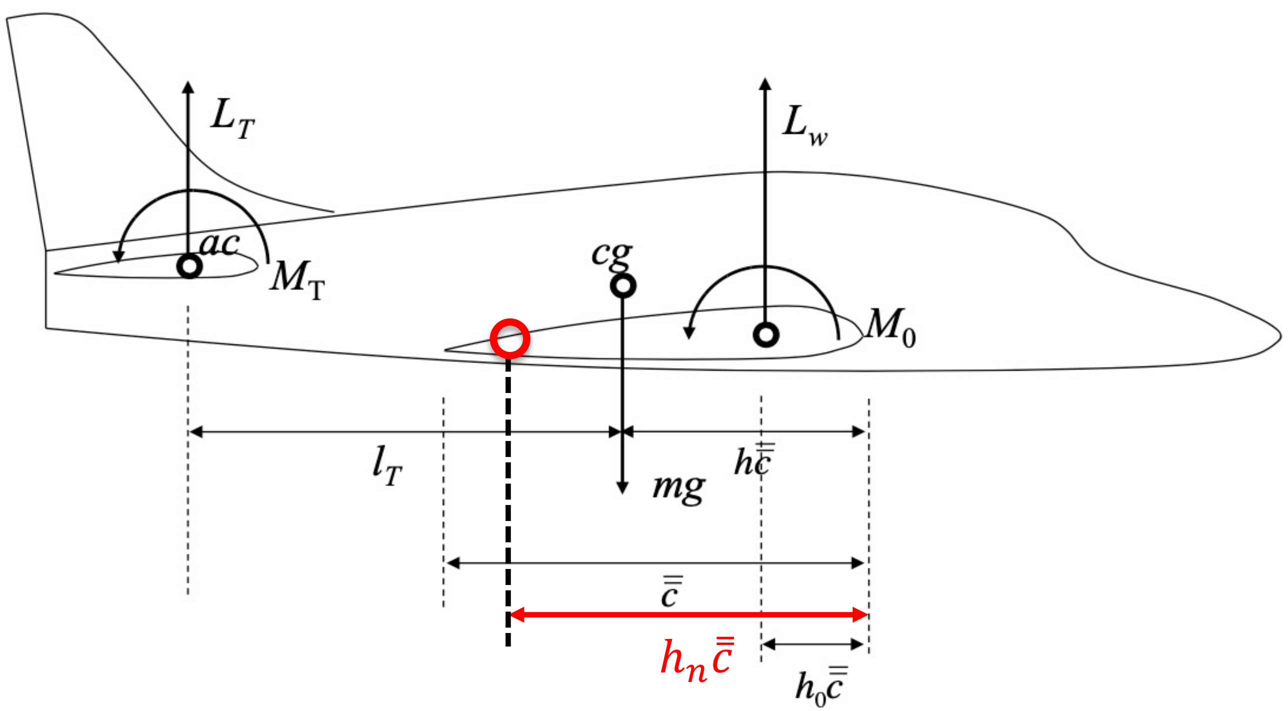
Fixed controls

A **stable** aircraft has a **positive stability margin** : $K_n > 0$

(the more positive, the more stable)

Stable aircraft if the cg position is ahead of the neutral point

→ $h_n - h > 0$



Fixed controls

Certification authorities specify that

$$K_n \geq 0.05 \text{ at all times}$$

Too much stability can be a bad thing !

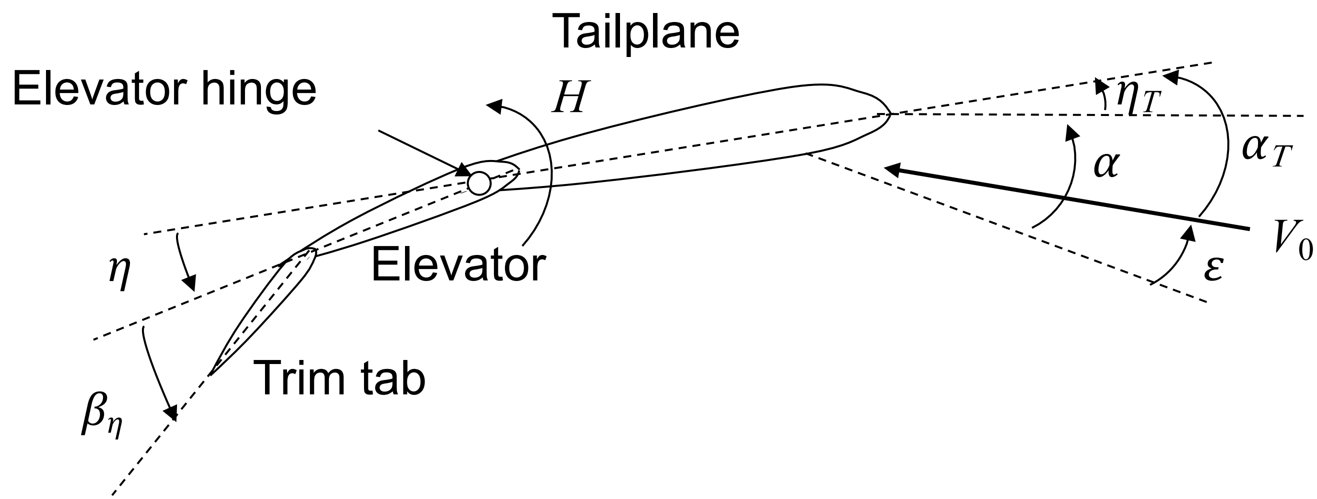
Stability margin may change if:
- fuel is burned
- payload is released
  (missiles, bombs, external fuel tanks, paratroopers, ..)

Free controls

Pilots don’t want to hold the controls throughout the flight

→ Trim tab ($\boldsymbol{\beta_\eta}$) can be adjusted such that if the elevator float freely, it will at an angle ($\boldsymbol{\eta}$) corresponding to the desired trim condition

= hands-off trim condition
(pilot does not need to adjust the elevator)



Free controls

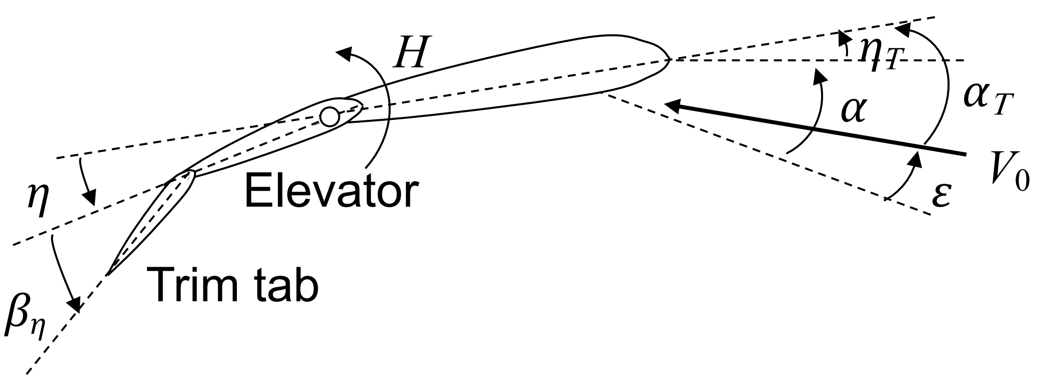
At hands-off trim condition

The derivative of the pitching moment with the wing lift is

$$\frac{dC_m}{dC_{L_w}} = (h - h_0) - \bar{V}_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

where b_1, b_2 and b_3 define the elevator hinge moment

$$C_H = b_1\alpha_T + b_2\eta + b_3\beta_\eta$$



Free controls

Controls free stability margin, $K'_n = -\frac{dC_m}{dC_{L_w}} = h'_n - h$

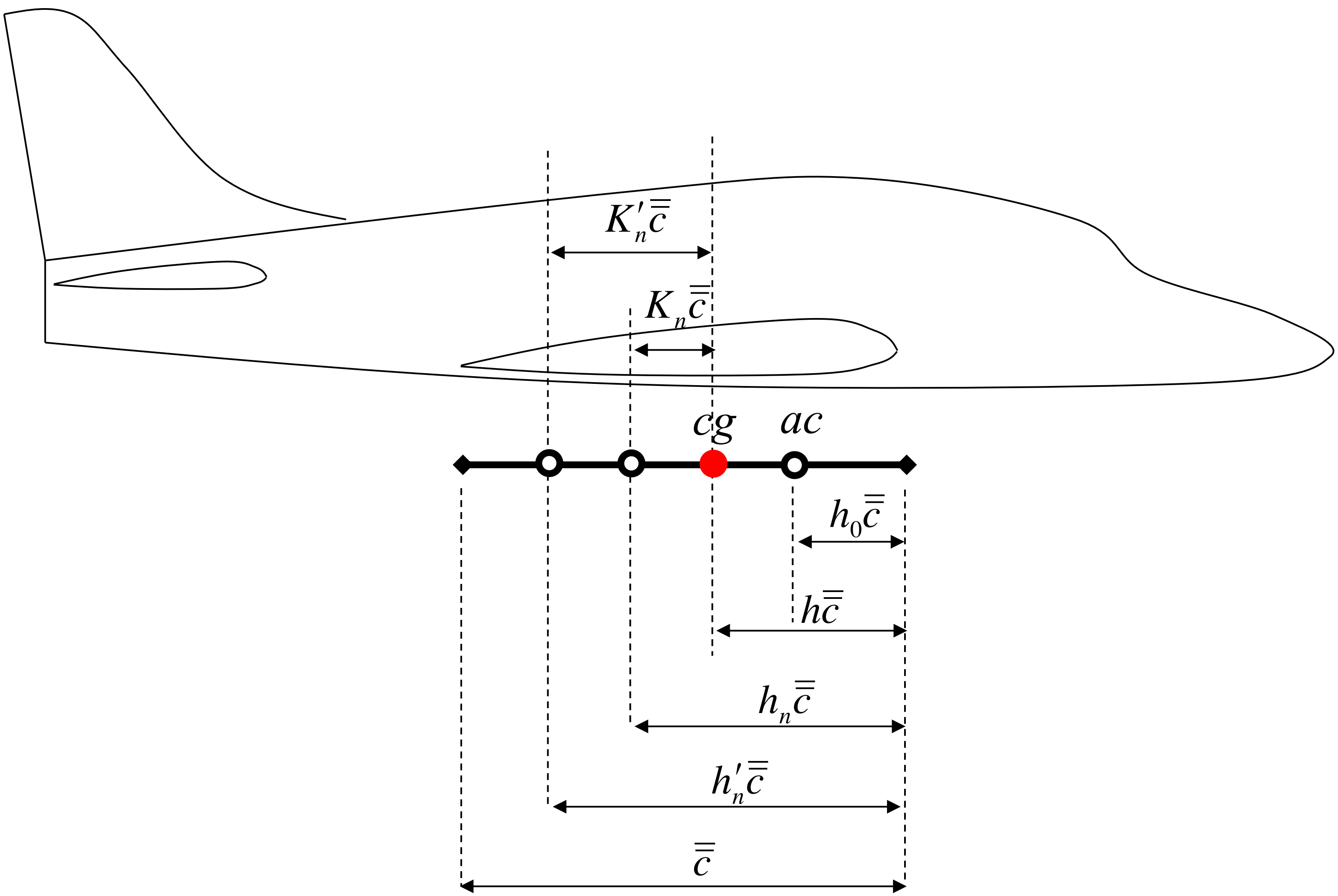
where h'_n is the controls free neutral point,

$$h'_n = h_0 + \bar{V}_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

or $$h'_n = h_n - \bar{V}_T \frac{a_2 b_1}{a b_2}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

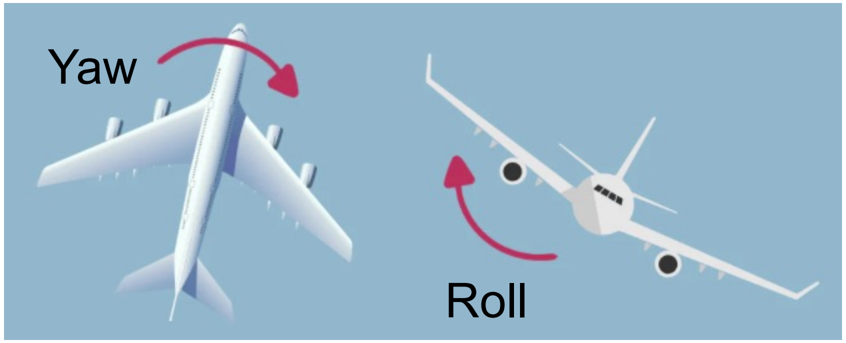
- As for fixed controls, $K'_n \geq 0$ → stable aircraft
- Centre of gravity position must be ahead of the controls free neutral point to have a stable aircraft
- Usually, $h'_n > h_n$
- An aircraft that is stable with controls fixed is usually also stable controls free

Summary of longitudinal stability



Lateral stability

Lateral flight → unsymmetrical flow around the aircraft
→ Sideslip



Roll and yaw are **always coupled**, because:

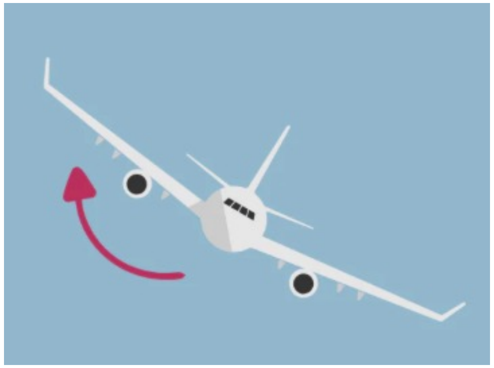
- Rolling produces sideslip
- Ailerons cause adverse yaw
- Dihedral
- Wingtip vortices
- Sweepback of wings
- Fin

The stability in yaw and roll must be ensured

Roll stability

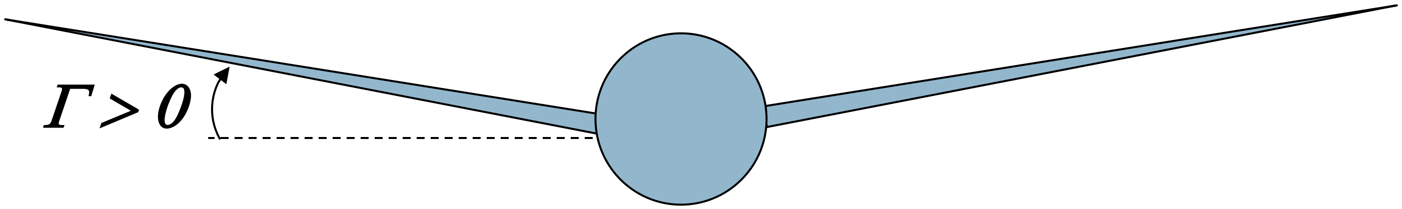
- **φ** rotation around longitudinal axis of the aircraft

- **No active** mechanism for roll control



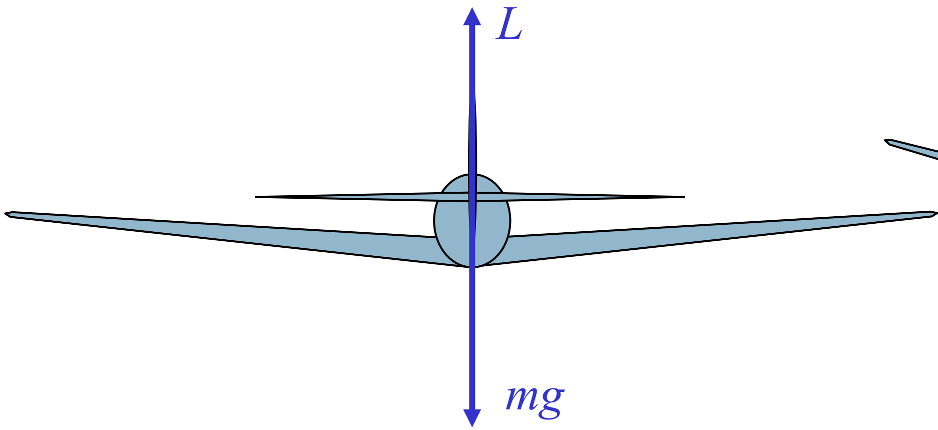
(such as tailplane/elevator for longitudinal stability or rudder for yaw stability)

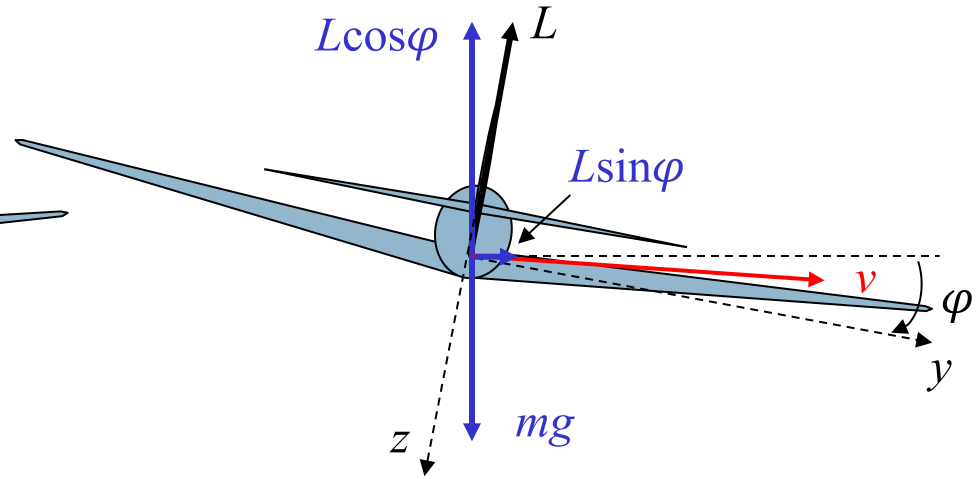
- Wing dihedral **Γ** is the only **passive** mechanism



The higher the dihedral, the more the stable the aircraft
As usual, too much stability can be a bad thing.

Roll motion





At roll angle **φ**

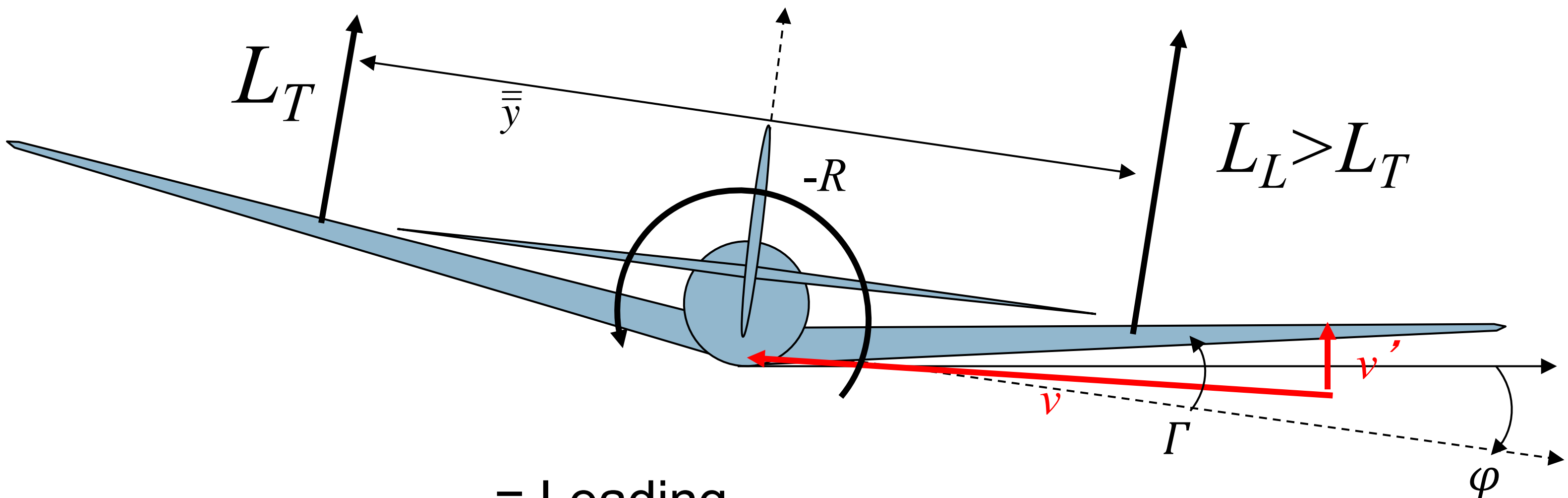
→ Lift is still perpendicular to wings and equal to *mg*

But along :
- Vertical axis : $L\cos\varphi < mg$ → Aircraft moves down
- Horizontal axis : $L\sin\varphi$ → Aircraft drifts (sideslip)

Dihedral

Restoring/stabilising moment *R*

$$R = (L_T - L_L)\bar{\bar{\bar{y}}}$$



$_L$ = Leading
$_T$ = Trailing

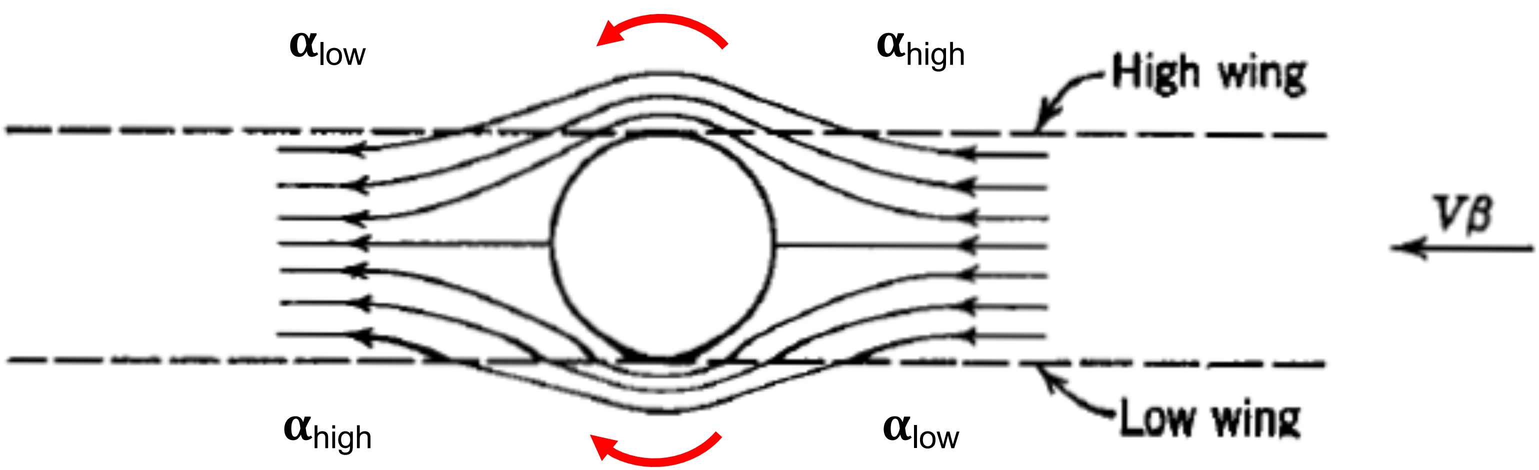
Effect of fuselage

Fuselage strongly impacts the flow around the wing

→ The position of the wing on the fuselage has a major effect:

- High wing → stabilising moment

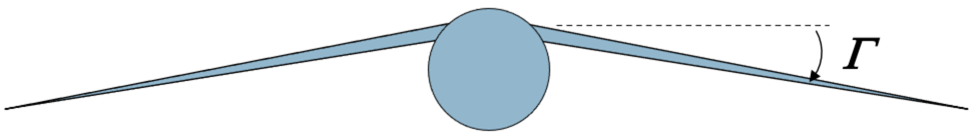
- Low wing → destabilising moment



Dihedral

High wing aircraft: very stable (too much)

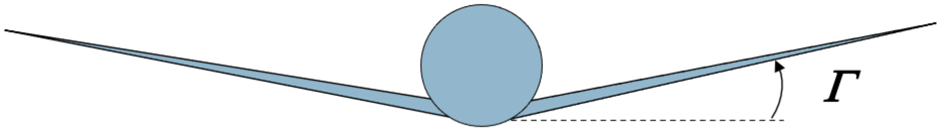
→ Negative dihedral (anhedral) can be used to **reduce stability** and **increase manoeuvrability**



Harrier

For **Low** wing aircraft: less stable

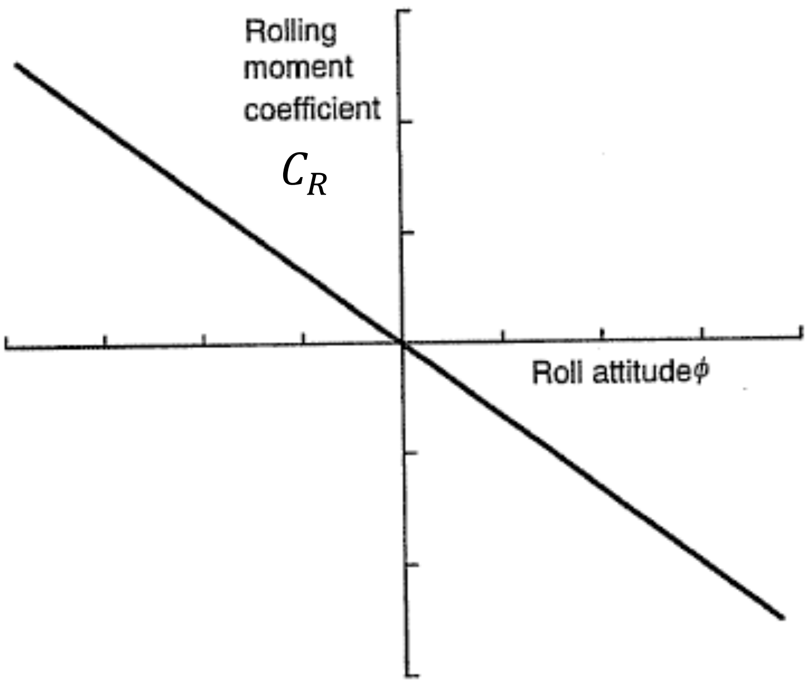
→ Positive dihedral ca be used to **increase** stability

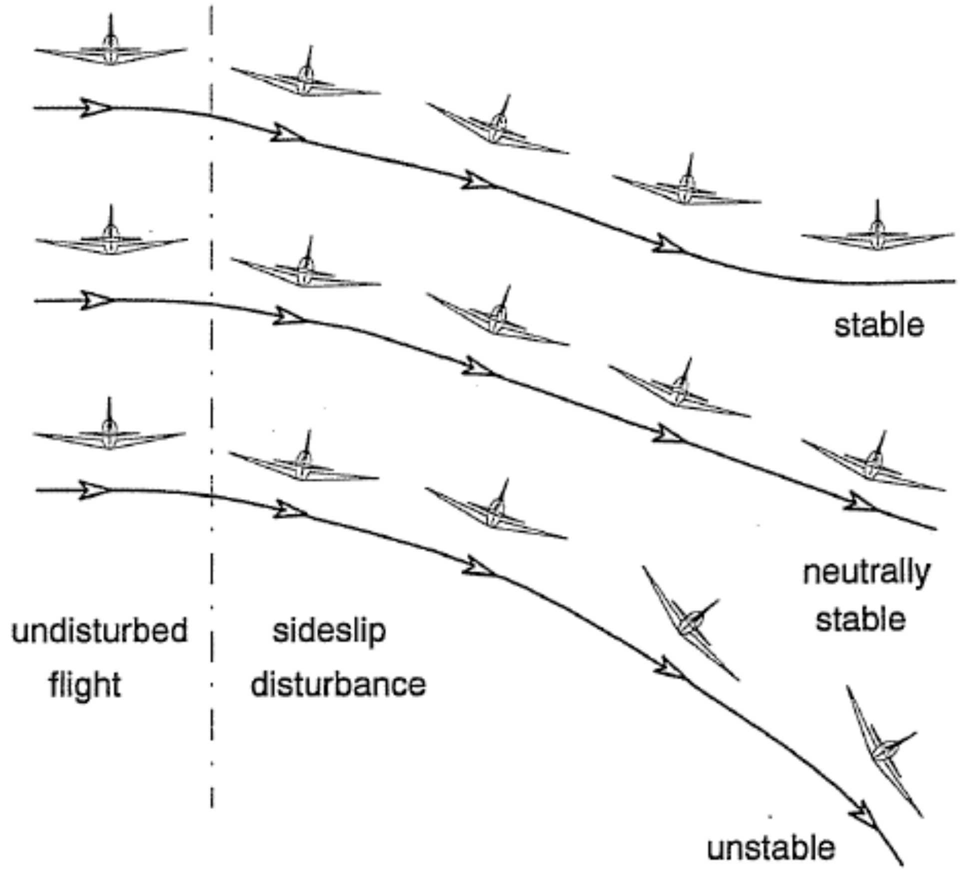


A350

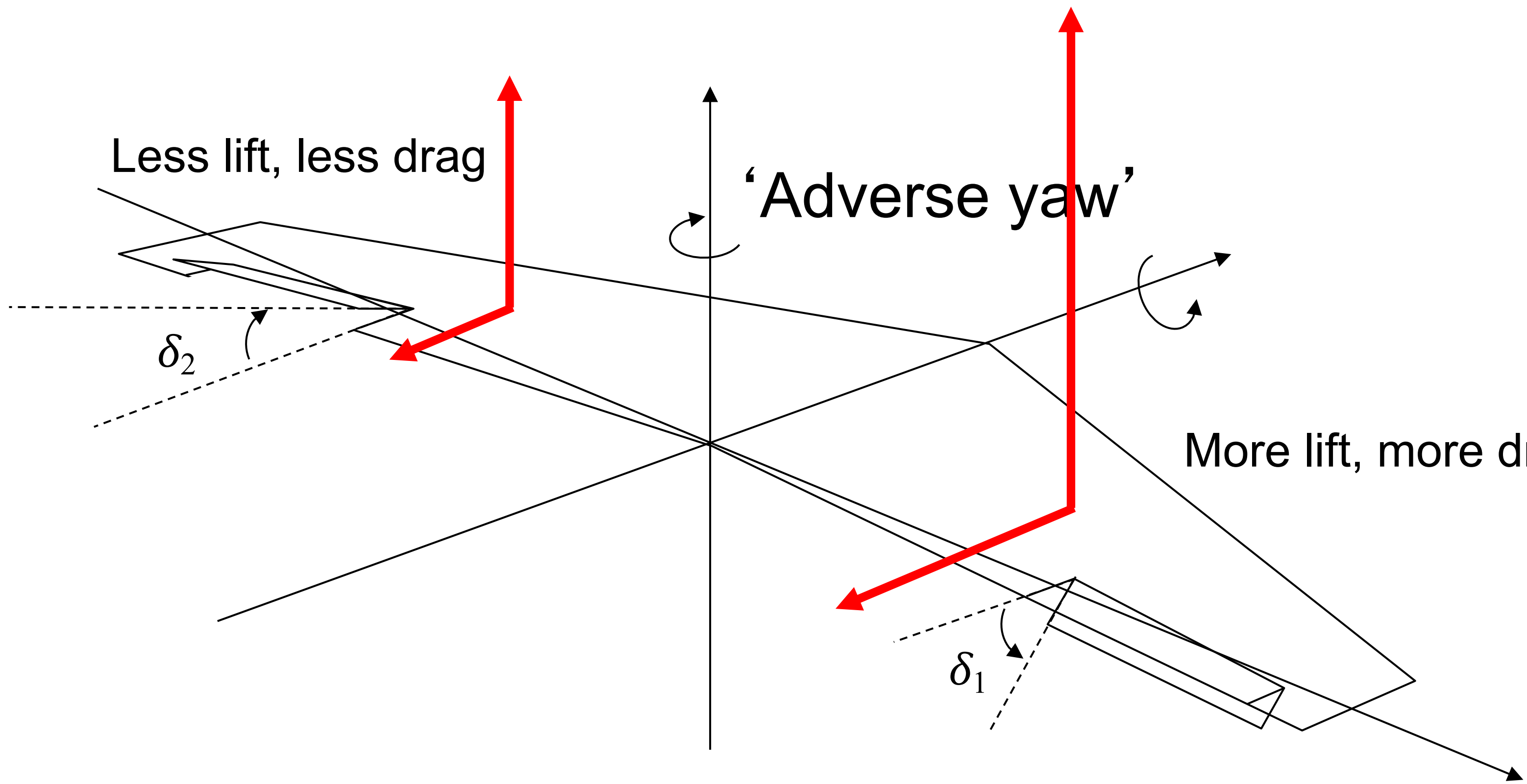
Roll stability

For a stable aircraft in roll = $\frac{dC_R}{d\varphi} < 0$





Roll control via ailerons



On the up/right wing: Increasing the lift increases drag

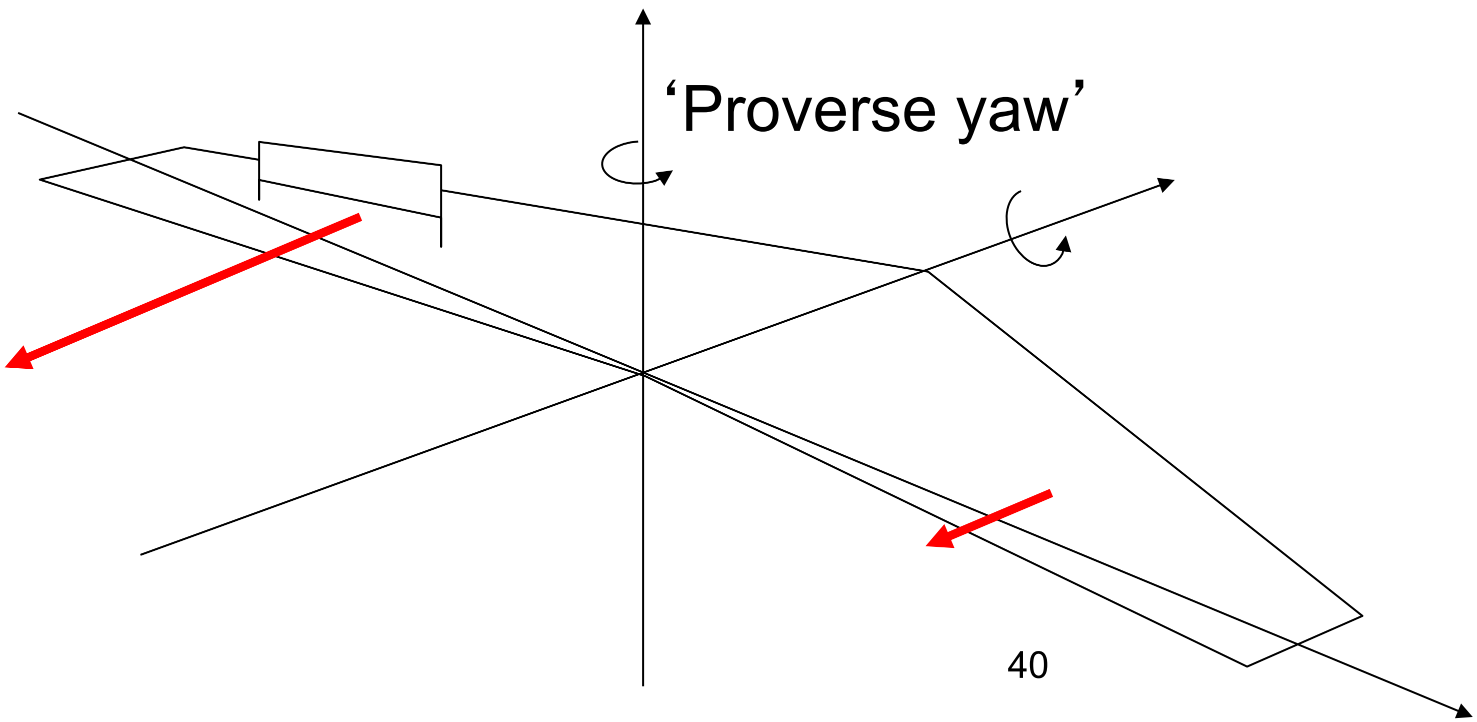
On the down/left wing: Less lift and drag decrease

→ **Adverse yaw**

When rolling/turning left, there is a yaw moment to the right

Roll control via spoilers

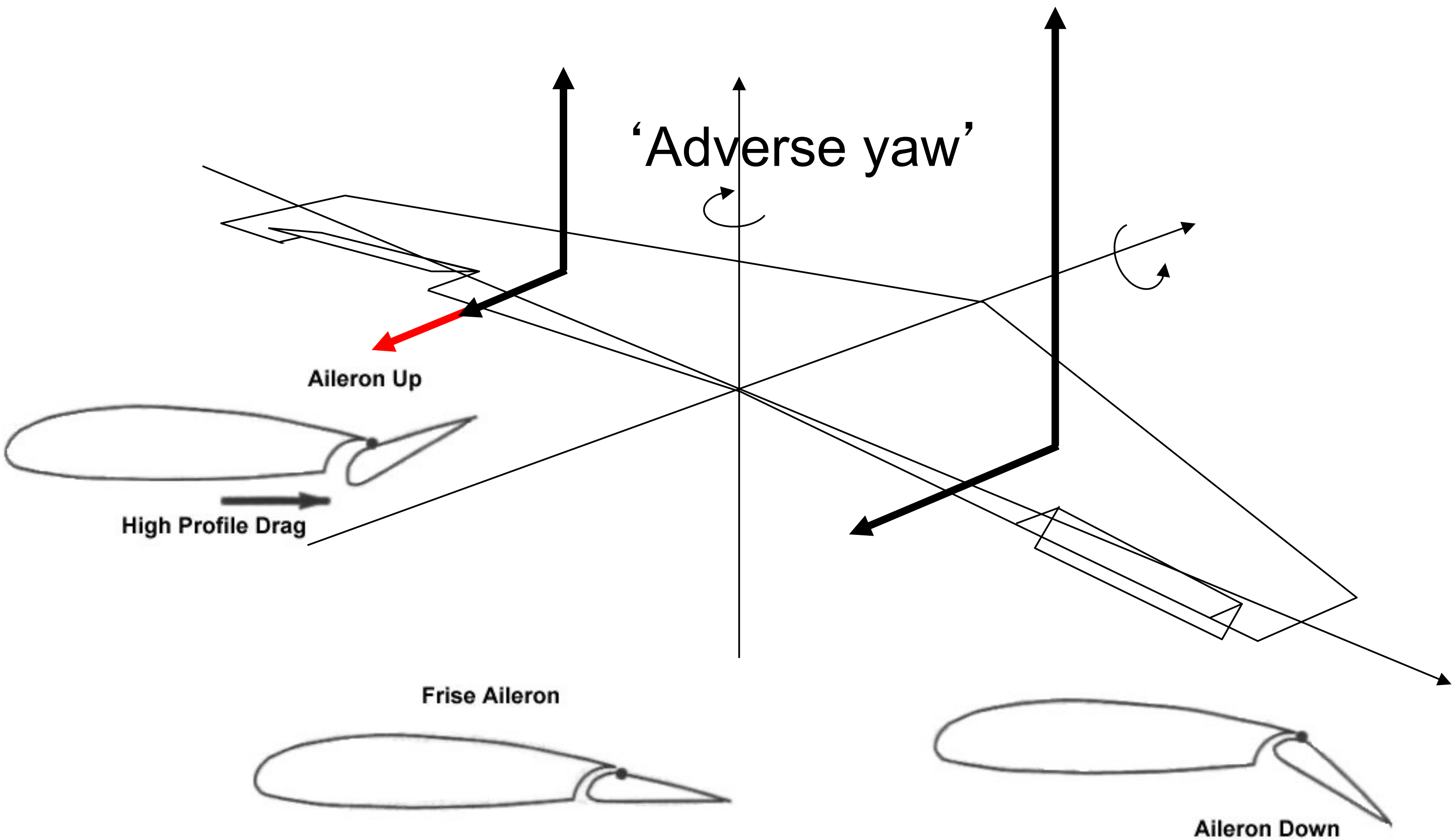
Deforming a spoiler on the wing towards which we want to turn



The spoiler decreases lift and increases drag.
→ Resulting yawing moment in the same direction as the roll (Proverse yaw)

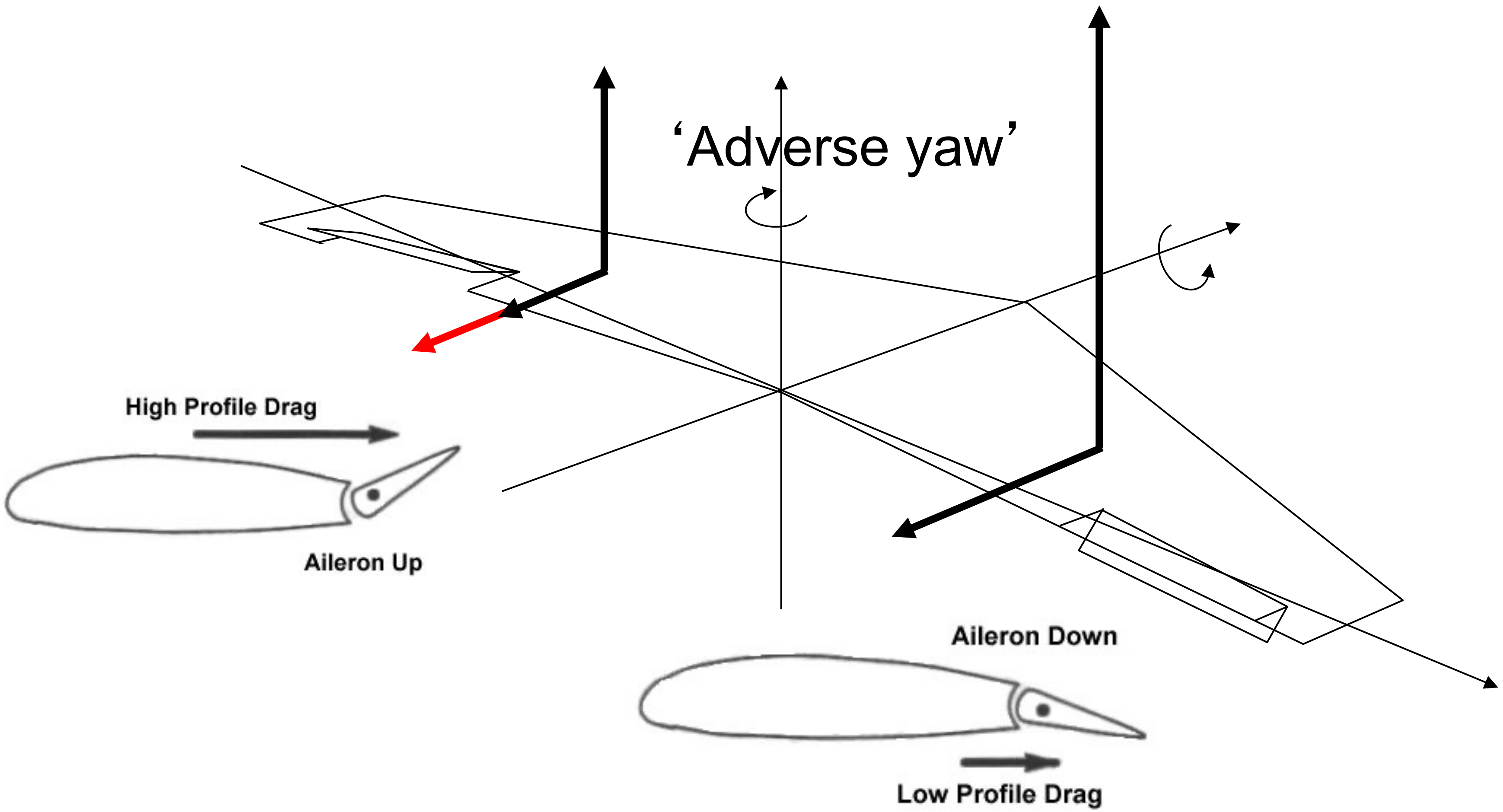
Frise ailerons

Increase (profile) drag on the down/left wing to counteract the large drag of the other wing



Differential aileron deflection

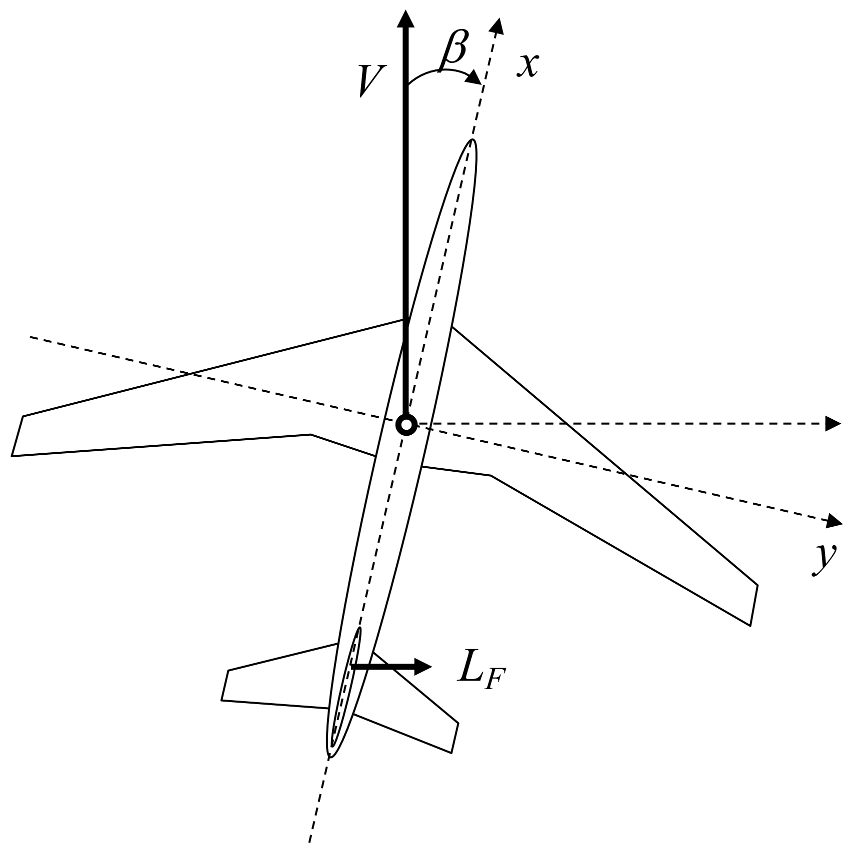
- The roll rate of the aircraft depends on the mean aileron deflection angle.
- The individual deflections δ_1 and δ_2 do not have to be equal



Yaw stability

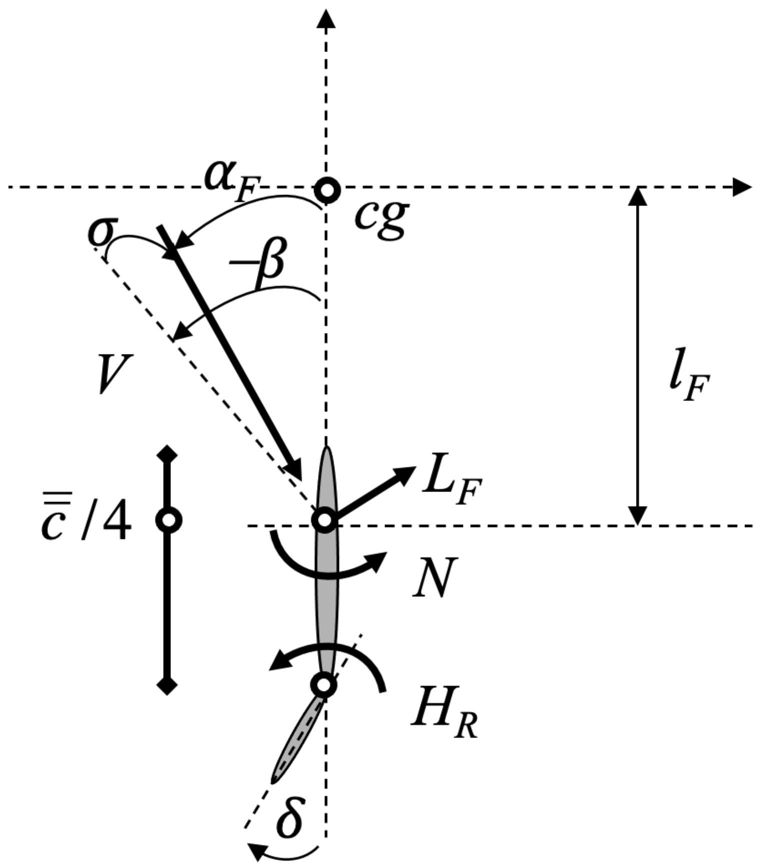
Yaw angle **β** induces a dissymmetry on the aircraft

Lift on the vertical stabilizer (fin) producing a stabilizing moment around z axis



Yaw stability

Lift coefficient of the fin : $C_{L_F} = c_1\alpha_F + c_2\delta = c_1(-\beta+\sigma) + c_2\delta$



σ = the sidewash velocity

local windspeed component induced by the effect of the fuselage, wing and possibly propellers.

Moment coefficient of the fin: $C_n = C_{L_F}\overline{V}_F$, where $\overline{V}_F = \dfrac{S_F l_F}{S\bar{\bar{c}}}$

Yaw stability

Stability condition for yaw : $\frac{dC_n}{d\beta} < 0$

which is equivalent to $\bar{V}_F \left(-c_1 + c_1 \frac{d\sigma}{d\beta} + c_2 \frac{d\delta}{d\beta}\right) < 0$

In this case, it makes no sense to differentiate the yawing moment by the lift since the two are independent

The sidewash factor $\mathrm{d}\sigma/\mathrm{d}\beta$, is very difficult to estimate

Three main contributions to the sidewash:

- **Fuselage**, which acts as a lifting body when at a yaw angle
- **Wing**, around which the flow is asymmetric. The resulting sidewash is more pronounced for low AR sweptback wings
- **Propeller**, which creates more asymmetric in its wake

Yaw control

Yaw angle must be zero during most flight conditions to minimize drag

Rudder deflection used to control yaw

Rudder power = rate of change of fin moment with rudder angle

$$= \frac{dC_n}{d\delta} = \bar{V}_F \frac{dC_{L_F}}{d\delta} = c_2 \bar{V}_F$$

This quantity must be large enough to maintain zero yaw in the most extreme flight conditions.

Take off and landing

During cruise, aircraft tend to turn towards the wind to minimize drag.

The objective is to achieve zero yaw.

At take-off and landing, this is not possible :

The aircraft must be aligned with the runway even in presence of a very strong sidewind.

→ Rudder must be able to provide enough moment to keep the aircraft aligned with the runway

Summary on control surfaces:

Elevators: contribute to pitch stability and control pitch angle

Rudder: contribute to yaw stability and control yaw angle

Ailerons: do not contribute to stability but control roll rate, not roll angle.

Elevons = ailerons that can also move up or down in unison

Flaperons = ailerons moving downwards only (like flaps)

Spoilerons = ailerons that can also move upwards only (like spoilers)

contribute to stability and control both pitch and roll

Dynamic stability

A flying aircraft has several **modes of vibration**:

- **Longitudinal modes:**
  - Short period oscillation
  - Long period oscillation (Phugoid)

- **Lateral modes**
  - Spiral mode
  - Roll subsidence
  - Dutch roll

→ These dynamic modes must be considered

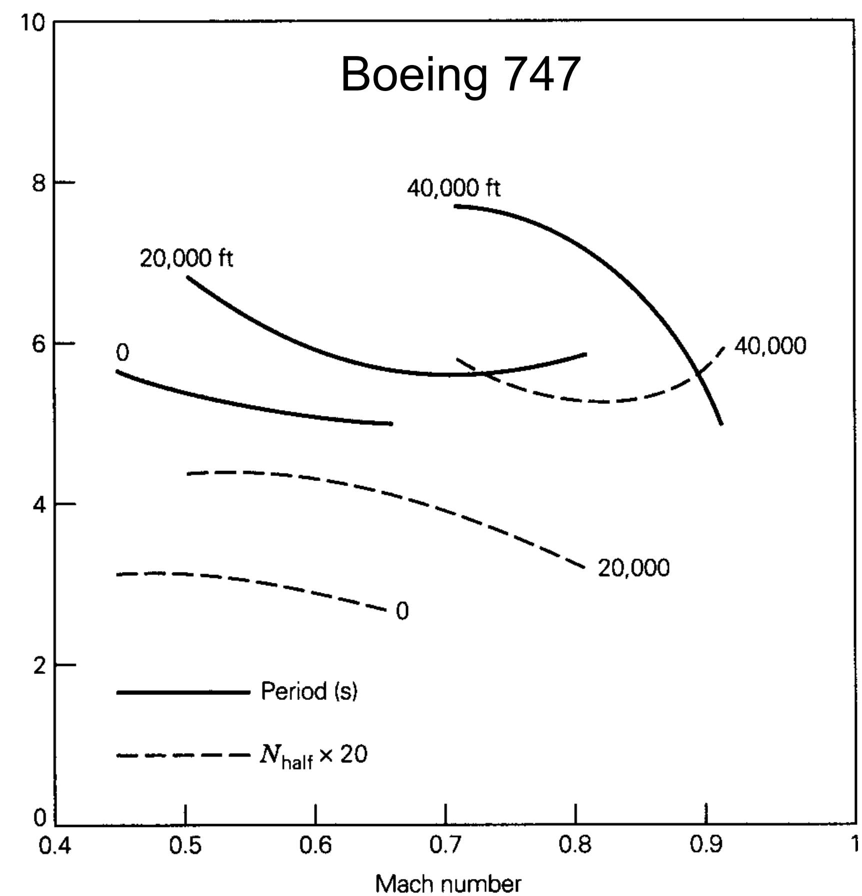
Short period oscillations

Driven by angle of attack

Take place because of abrupt input changes

Speed changes are negligible

Period of oscillation decreases with airspeed/Mach



N_{half}= number of periods until the amplitude is halved

Phugoid

= Long period oscillations in longitudinal direction

Angle of attack is constant: aircraft climbs and descends in an oscillatory manner

Low damping of the motion

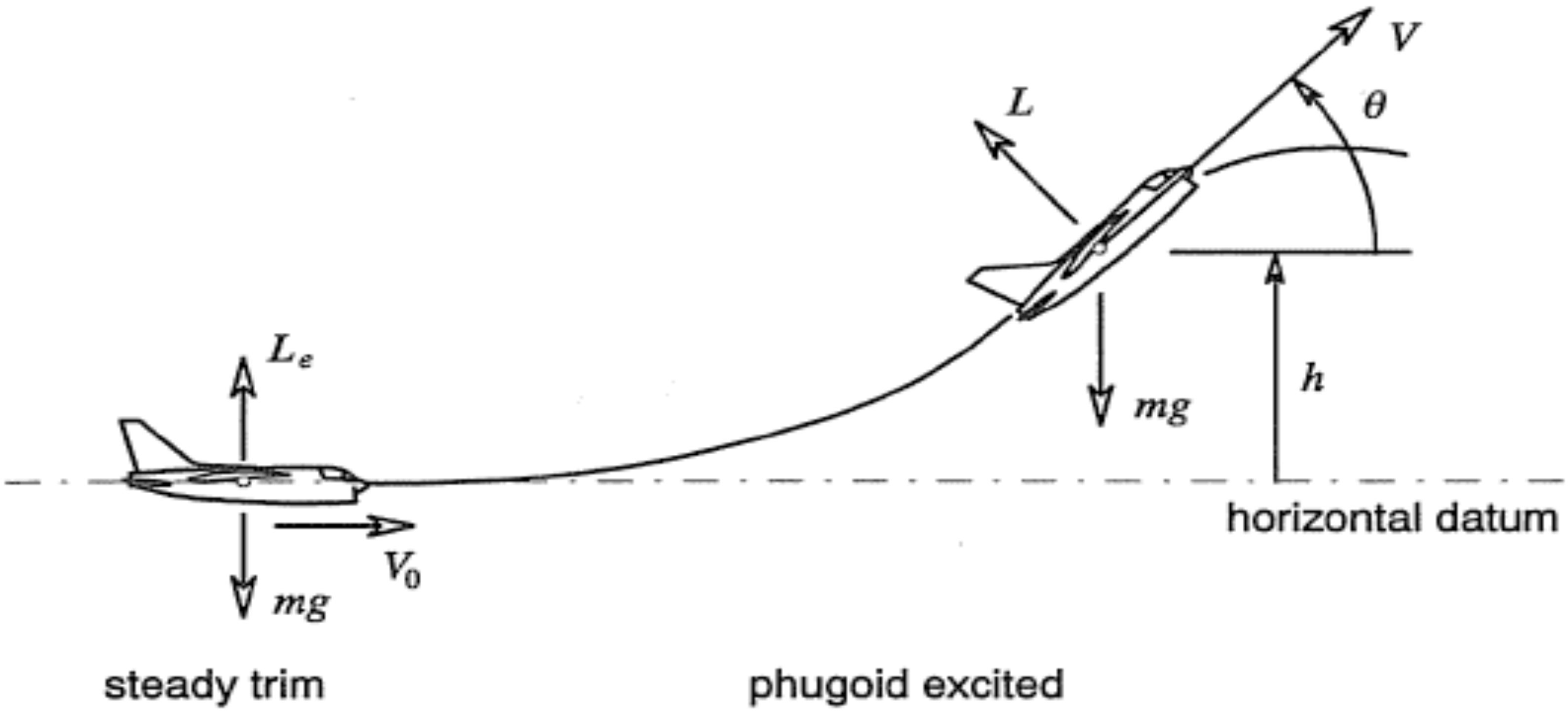
Phugoid period:

- microlight aircraft: 15-25s
- light aircraft: overs 30s
- jet aircraft: minutes

Neutralized by re-trimming the aircraft in a new flight configuration

Phugoid videos

Phugoid estimates

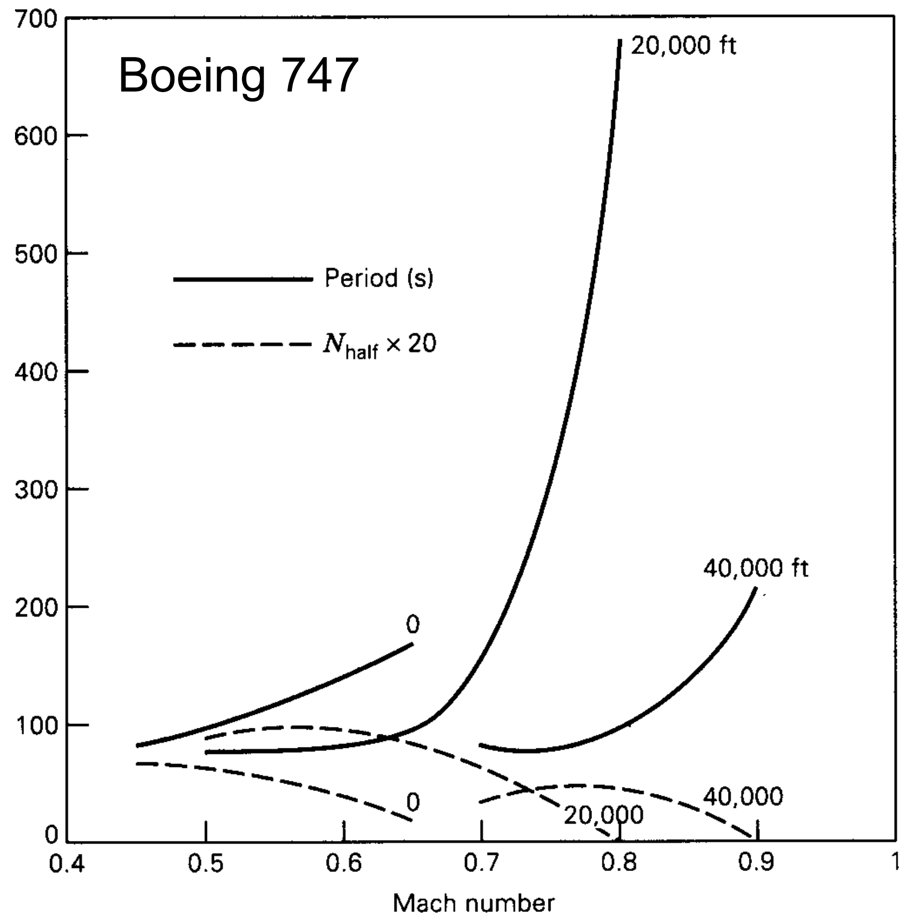


Lanchester approximations of damping and frequency:

$$\zeta_p \omega_p = \frac{g C_D}{C_L V_0}, \quad \omega_p = \frac{g\sqrt{2}}{V_0}$$

Phugoid

Phugoid period increases with airspeed
Phugoid damping slightly increase with airspeed



Spiral mode

= yaw movement with a little of roll

Spiral mode can be stable or unstable

Non-oscillatory mode with large time constant

Typical half-life of spiral mode ~ a minute

Spiral movement is usually stopped by a corrective control input

Spiral mode video

Roll subsidence

An impulse aileron → initiate aircraft rolling

In general the aircraft will stop rolling with time
(i.e. the roll rate becomes zero after sufficient time)

The aircraft will find itself at a roll angle which depends on how fast the roll rate tends to zero.

This phenomenon is called roll subsidence.

Roll subsidence video

Dutch roll

Dutch roll = Roll-Yaw coupling

Roll stability is stronger than Yaw stability because wing is bigger than the fin

Name from ice skating in Holland

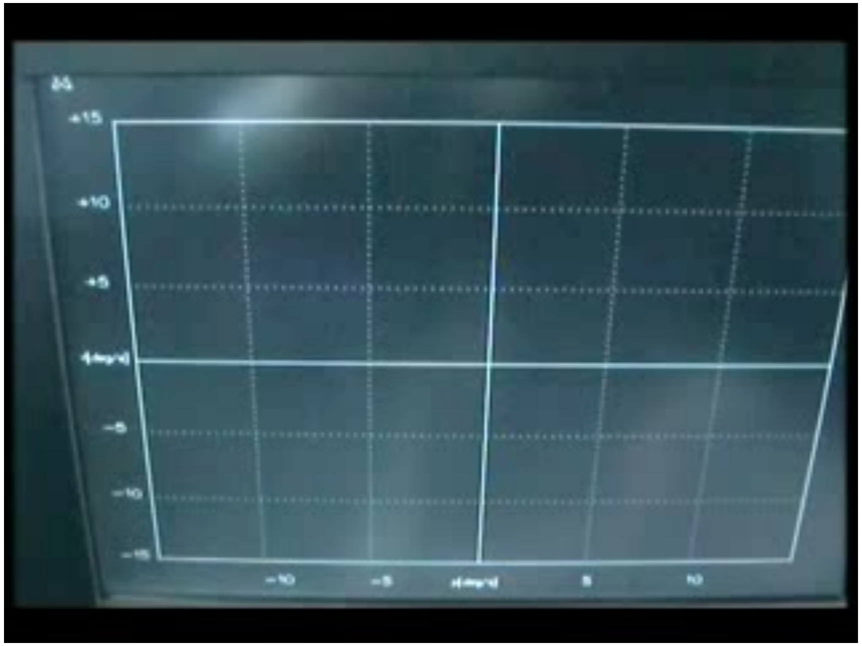
Solution :

Yaw damper = computer connected to the rudder to mitigate yaw motion

Dutch roll dangerous ? No, but : there were incidence due to improper reaction by pilots (putting rudder input too late)

Dutch roll video

Dutch roll video



Lateral modes of Boeing 747

Altitude	Mach	Spiral	Dutch Roll	
		Half-life	Period	N_{half}
0	0.45	35.7	5.98	0.87
0	0.65	34.1	4.54	0.71
20,000	0.5	76.7	7.3	1.58
20,000	0.65	64.2	5.89	1.33
20,000	0.8	67.3	4.82	1.12
40,000	0.7	-296	7.99	1.93
40,000	0.8	94.9	6.64	3.15
40,000	0.9	-89.2	6.19	1.18

Summary

Stability = ability to **keep** the aircraft in the air in the chosen flight attitude

→ Aircraft returns to the equilibrium position after a perturbation.

→ Stability must be ensured but not oversized !

Control = ability to **change** the flight direction and attitude of the aircraft

Static stability in :
- Pitch → Longitudinal
- Yaw/Roll → Lateral

Dynamic stability to avoid dangerous phenomena