Aeronautics Design Project

LIÈGE université

Aircraft Performance

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Content of the course

- Introduction to design performance
- Weight estimates
- Drag estimates
- Flight phases
  - Cruise
  - Take-off
  - Climb
  - Turning
  - Landing
- Flow diagram

Complete procedures in text books:

- D.P. Raymer, *“Aircraft Design : A conceptual Approach”*

- E. Torenbeek, *“Synthesis of Subsonic Airplane Design”*

Design for performance

Main requirement for a new aircraft = fulfilment of the mission

→ Performance calculation at the design stage

At the design stage, we choose:

- Size of the wing
- Type and size of the engines

Flight points

- Cruise → first performance analyses for an airliner
- Take off
- Landing
- Climb

Weight and drag

First step is the determination of the weight and drag

Weight → Lift

Drag → Thrust

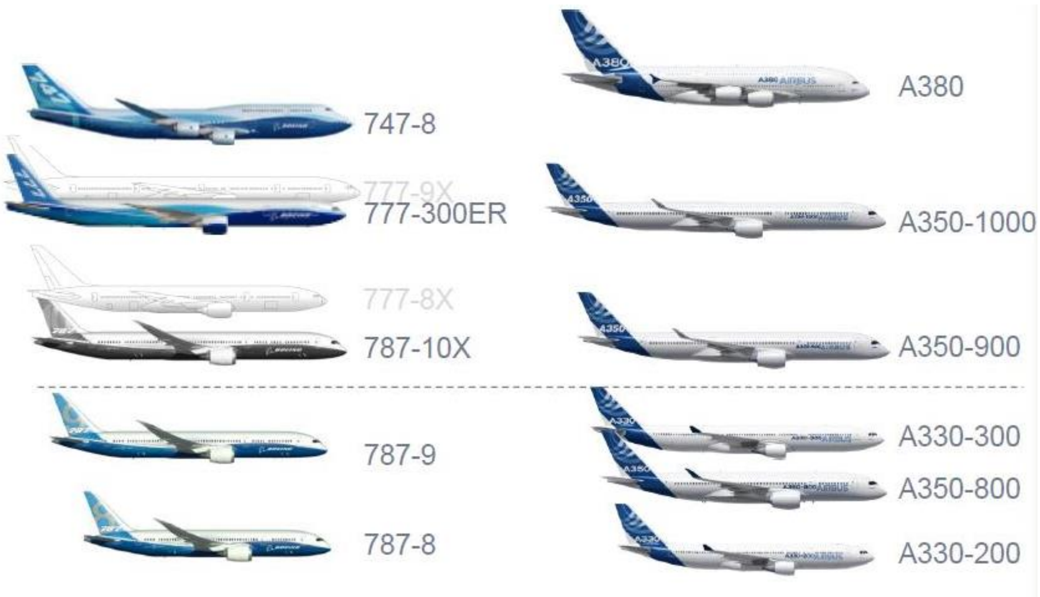
Weight and drag must be known at several points in the flight envelope (= capabilities of an aircraft in terms of structural loads and speed)

Two methodologies:

- Carry out **detailed simulations** at the conceptual design stage (very costly)
- Use previous experience: **statistical** data

Statistics

Enormous amount of data on very similar aircrafts



Source: www.jethrojeff.com

Source: www.a13x.com.au/aircraft-size-comparison/

Statistics

Enormous amount of data on very similar aircrafts

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Weight estimates

Take off weight, $\mathbf{W_{to}}$ expressed as:

$$W_{to} = W_e + W_p + W_f$$

where,

W_e = total empty weight (e.g. airframe, wings, engines)

W_p = payload weight

W_f = fuel weight

The **total empty weight** $\mathbf{W_e}$, is $W_e = W_{fix} + W_{var}$

W_{fix} = fixed empty weight (e.g. engines if pre-selected)

W_{var} = variable empty weight (e.g. fuselage, wings, avionics, equipments, …)

$$W_{to} = \frac{W_p + W_{fix}}{1 - \frac{W_{var}}{W_{to}} - \frac{W_f}{W_{to}}}$$

Weight estimates

Statistic tools are shared between:

- Light aircraft (W_{to}<5670kg) (5670kg=12500lb)
- Heavy aircraft (W_{to}>5670kg)

For light aircraft (from 100 different types):

$$\frac{W_{\text{var}}}{W_{\text{to}}} = \begin{cases} 0.45 \text{ - for normal category with fixed gear} \\ 0.47 \text{ - for normal category with retractable gear} \\ 0.50 \text{ - for utility category} \\ 0.55 \text{ - for acrobatic category} \end{cases}$$

$$\frac{W_{\text{f}}}{W_{\text{to}}} = 0.17 \frac{R}{1000} r_{uc} AR^{-0.5} + 0.35$$

with, R = aircraft's range (in km)
AR = Aspect Ratio of the main wing
r_{uc} = 1.00-1.35 is the undercarriage drag correction

Undercarriage drag correction

This drag correction is used both in the calculation of:

- the fuel weight (W_f)

- the zero-lift drag (see later)

If the landing gear is fully retractable

→ $r_{uc} = 1$

Otherwise:

$$r_{uc} = \begin{cases} 1.35 \text{ - for fixed gear without streamlined wheel fairings} \\ 1.25 \text{ - for fixed gear with streamlined wheel fairings} \\ 1.08 \text{ - main gear retracted in streamlined fairings on the fuselage} \\ 1.03 \text{ - main gear retracted in engine nacelles} \end{cases}$$

Fairings

No wheel fairing

Wheel fairing

Fairings

Fuselage fairings

Antonov 225

Engine nacelles

Bombardier Dash8

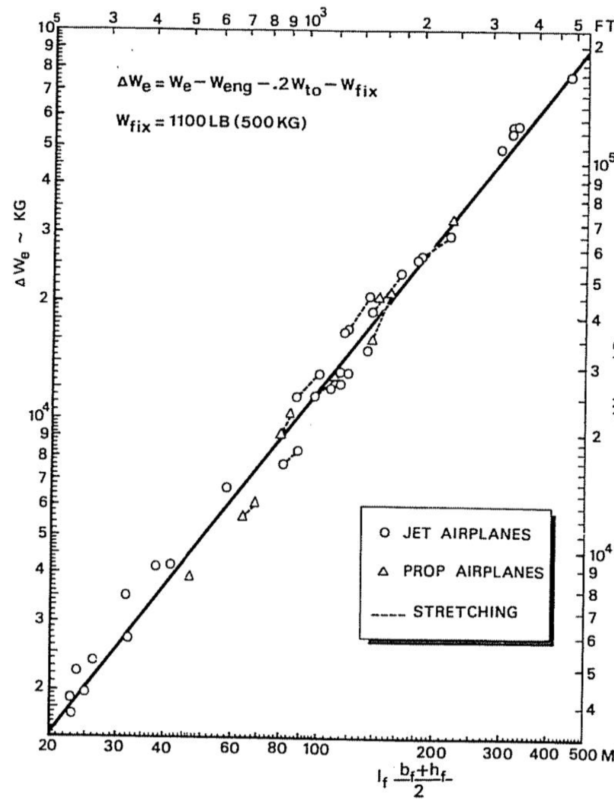
Weight guesstimates

For <u>heavy aircraft</u> (W_{to}>5670kg) :

$$\frac{W_{\text{var}}}{W_{\text{to}}} = 0.2$$

$$W_{\text{fix}} = W_{\text{eng}} + 500 + \Delta W_{\text{e}}$$

with, W_{eng} = engine weight
Δ W_e is a correction factor from the graph



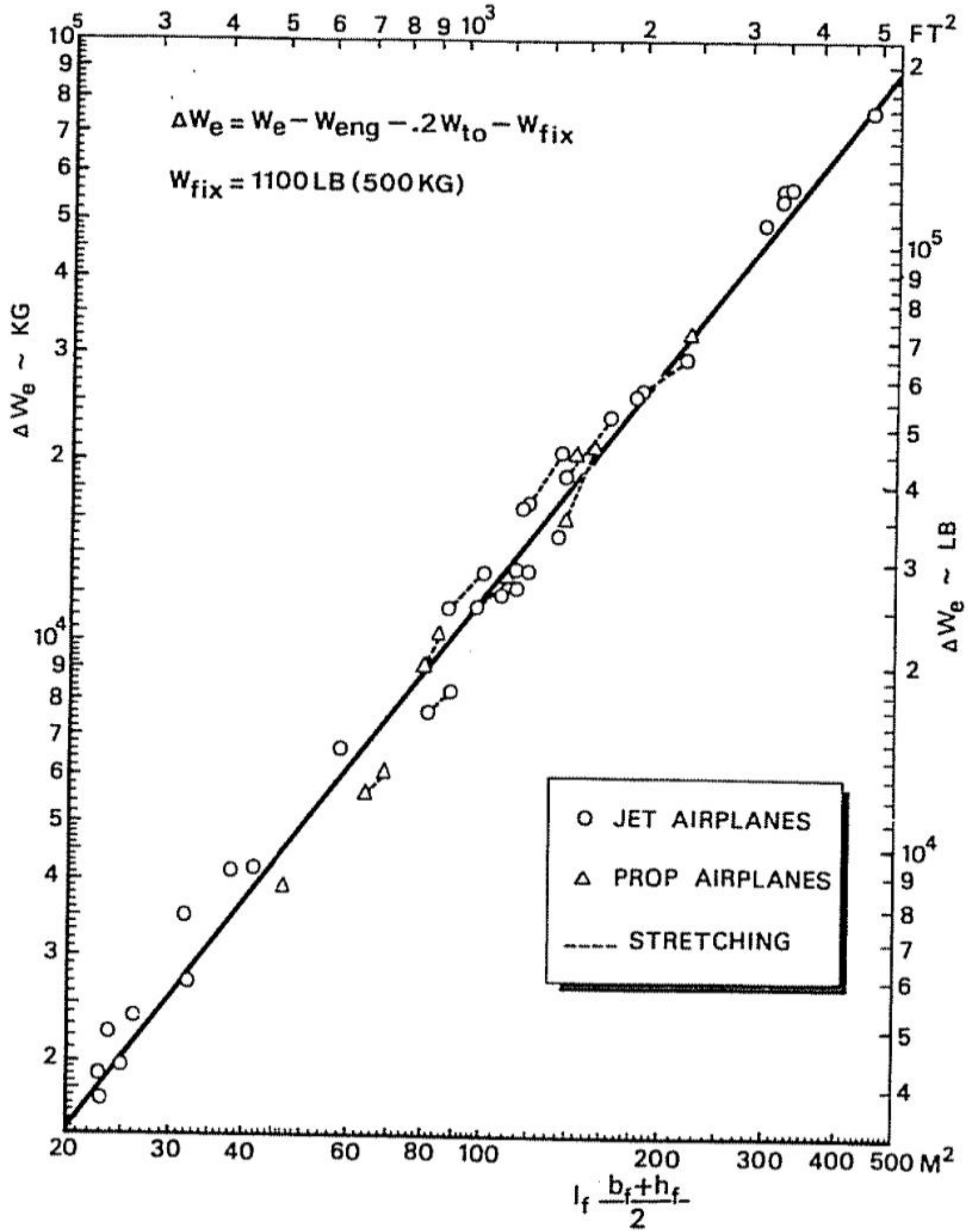
Δ W_e

l_f = fuselage length

b_f = fuselage width

h_f = fuselage height

(use metric units)



Weight estimates

For heavy aircraft (with turboprops):

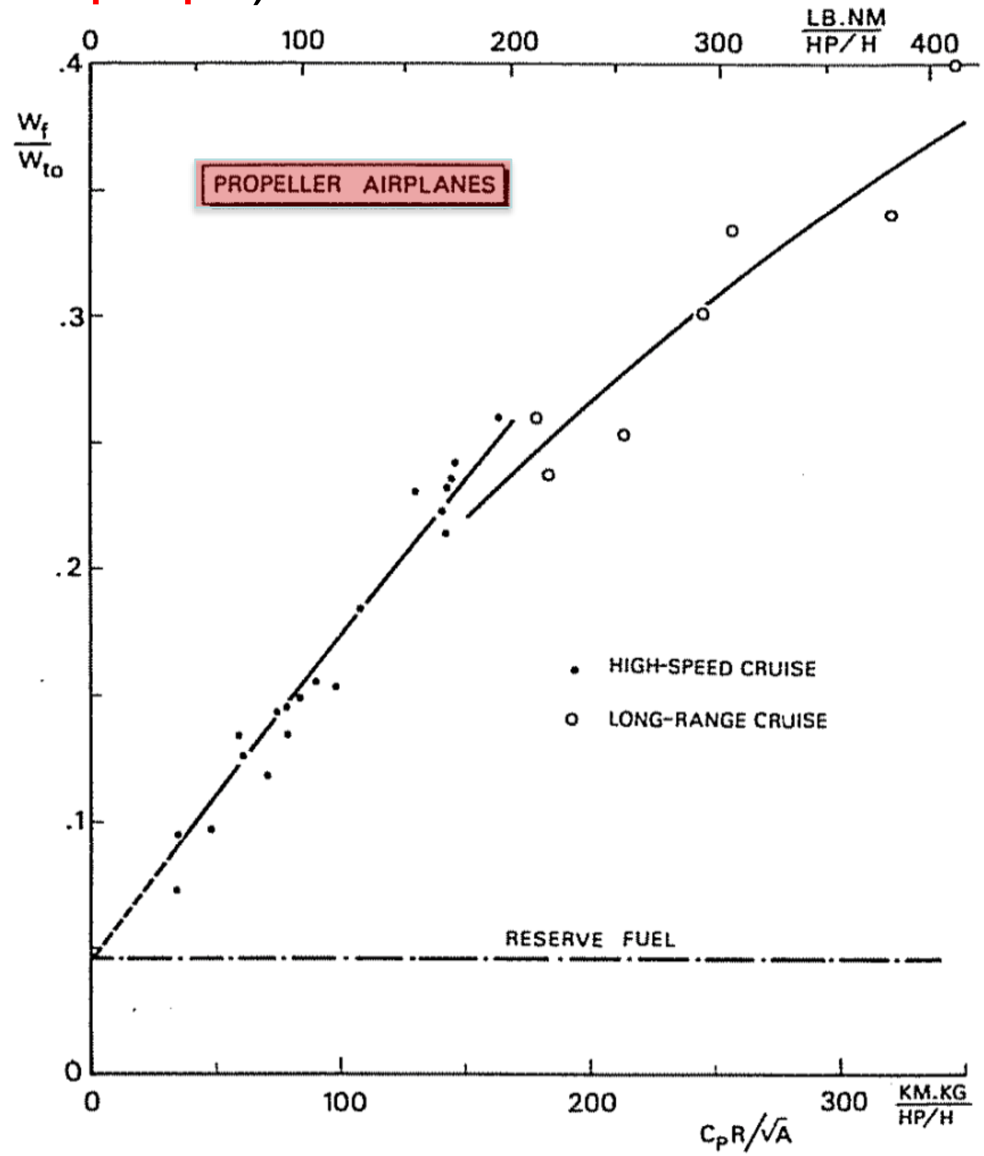
W_f (fuel weight)

with,

C_p = specific fuel consumption for propeller aircraft

R = Range
A = AR

(use metric units)

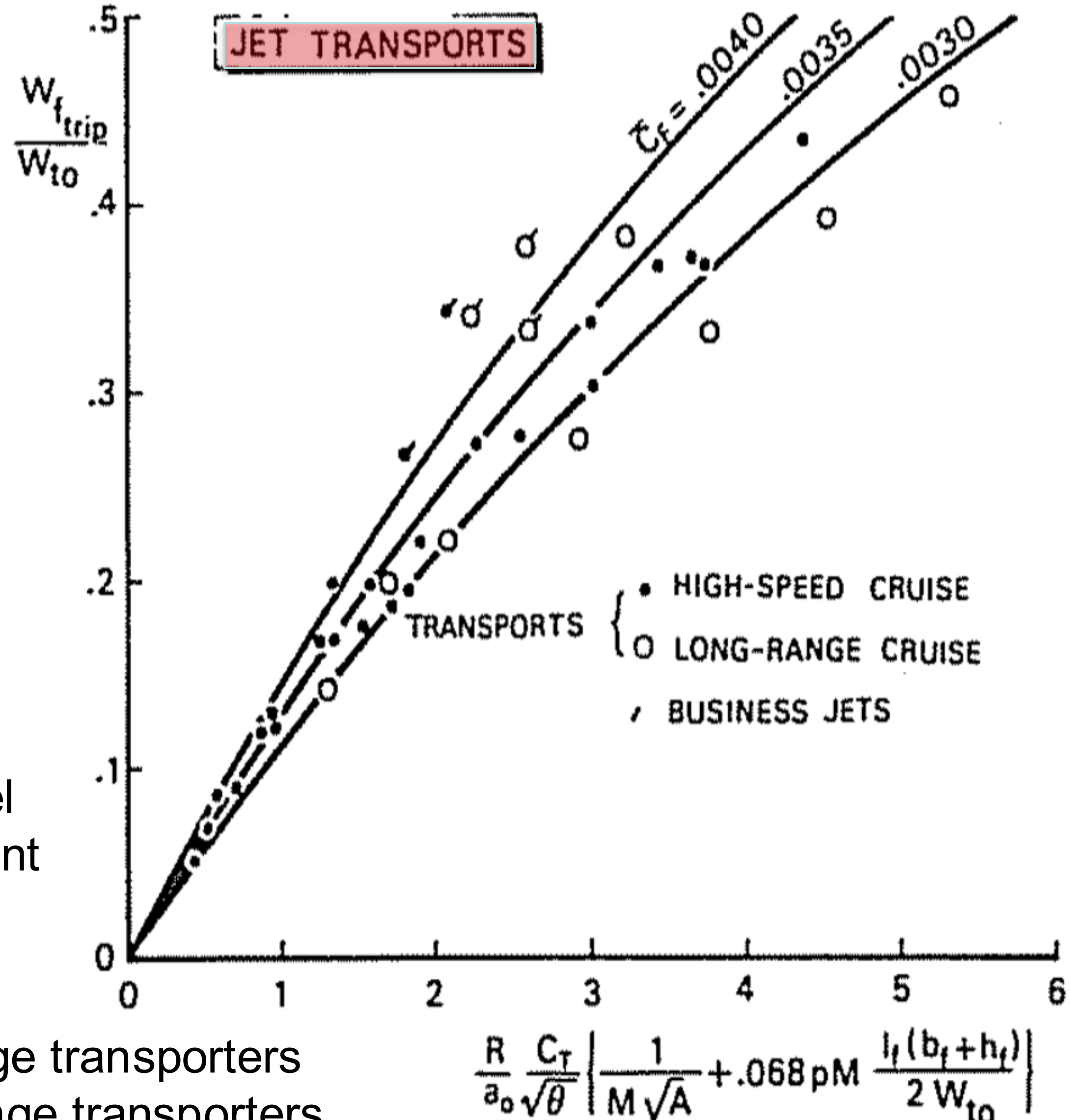


Weight estimates

For heavy jet aircraft

W_f (fuel weight)

with,



p = atm. pressure at cruise
M = Mach number at cruise
$\boldsymbol{\theta} = T/T_0$ cruise/stand. temp.
$C_T/\sqrt{\theta}$ = corrected specific fuel consumption at cruise
a_0 = speed of sound at sea level
$\overline{C_F}$ = mean skin friction coefficient based on wetted area

$$\overline{C_F} = \begin{cases} 0.003 & \text{for large, long range transporters} \\ 0.0035 & \text{for small, short range transporters} \\ 0.004 & \text{for business and executive jets} \end{cases}$$

Skin friction coefficient

- Gives an estimate of the drag force due to air friction over the full surface of the aircraft (= wetted area)

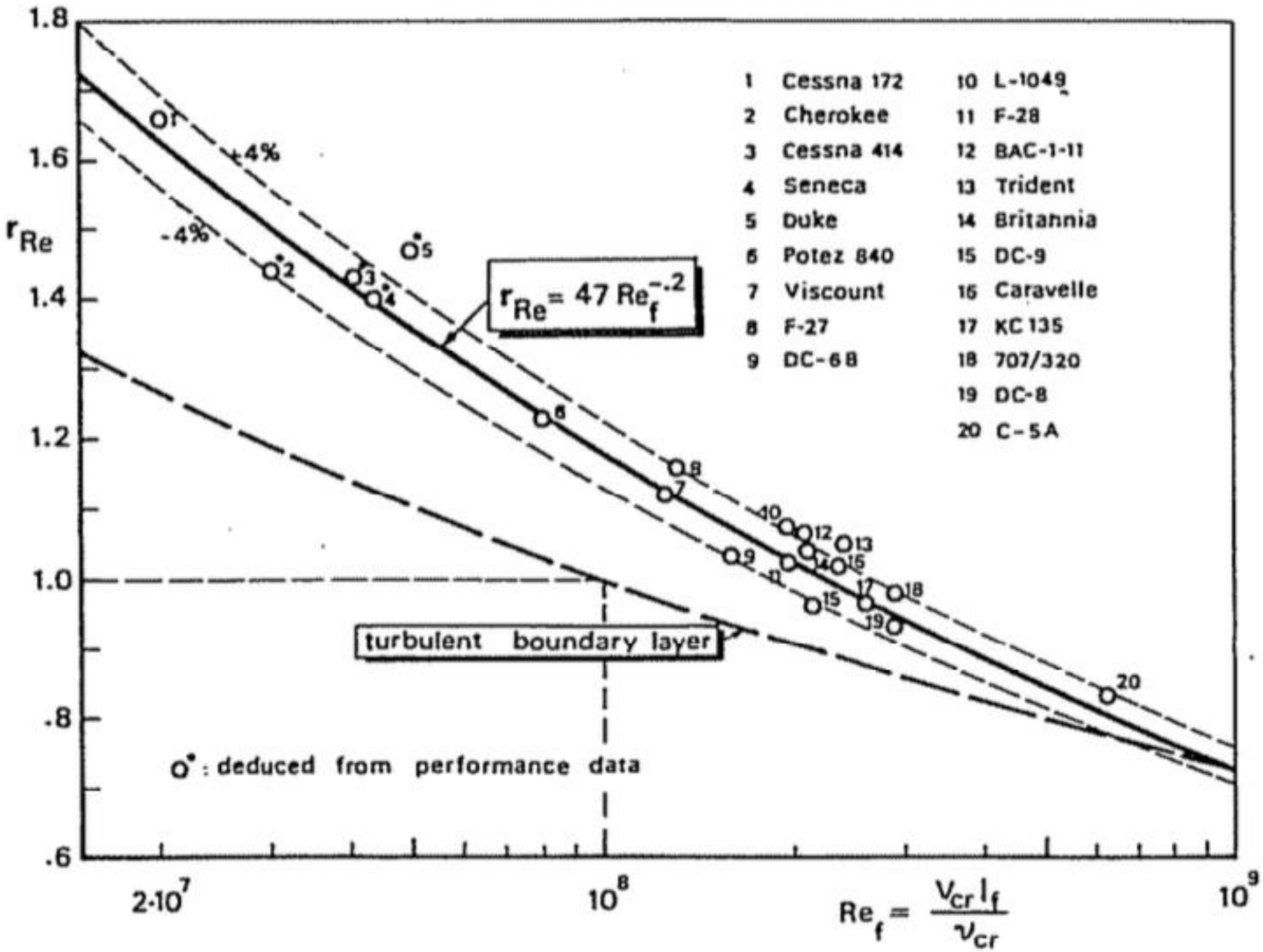
- Can be estimated by Prandtl-Schlichting theory as

$$C_F = \frac{0.455}{\left(\log_{10}\left(Re_{\mathrm{cr}}\right)\right)^{2.58}}$$

where Re_{cr} is based on the cruise conditions and the fuselage length

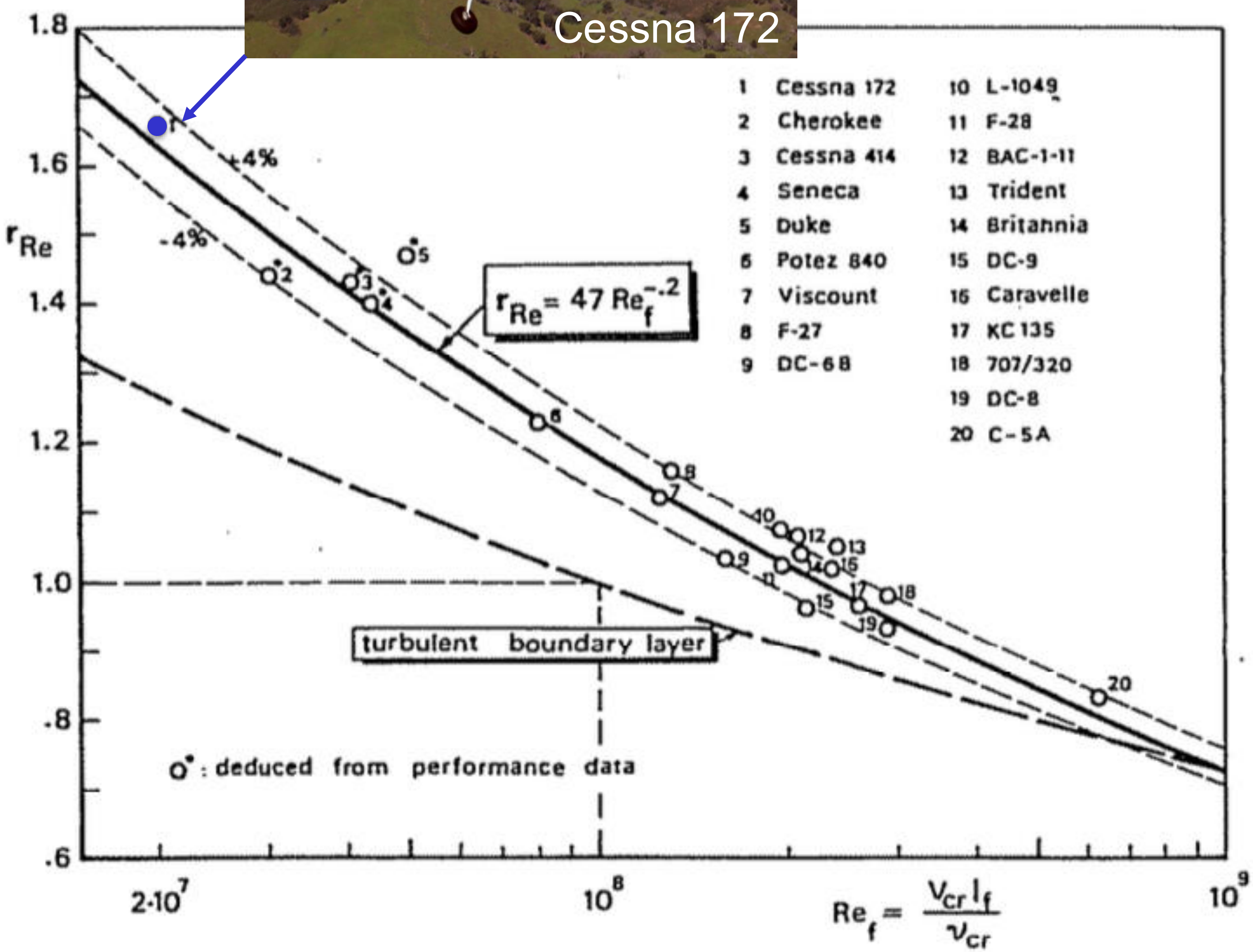
Skin friction coefficient

$$r_{Re} = C_F(\mathrm{Re})/C_F(10^8)$$



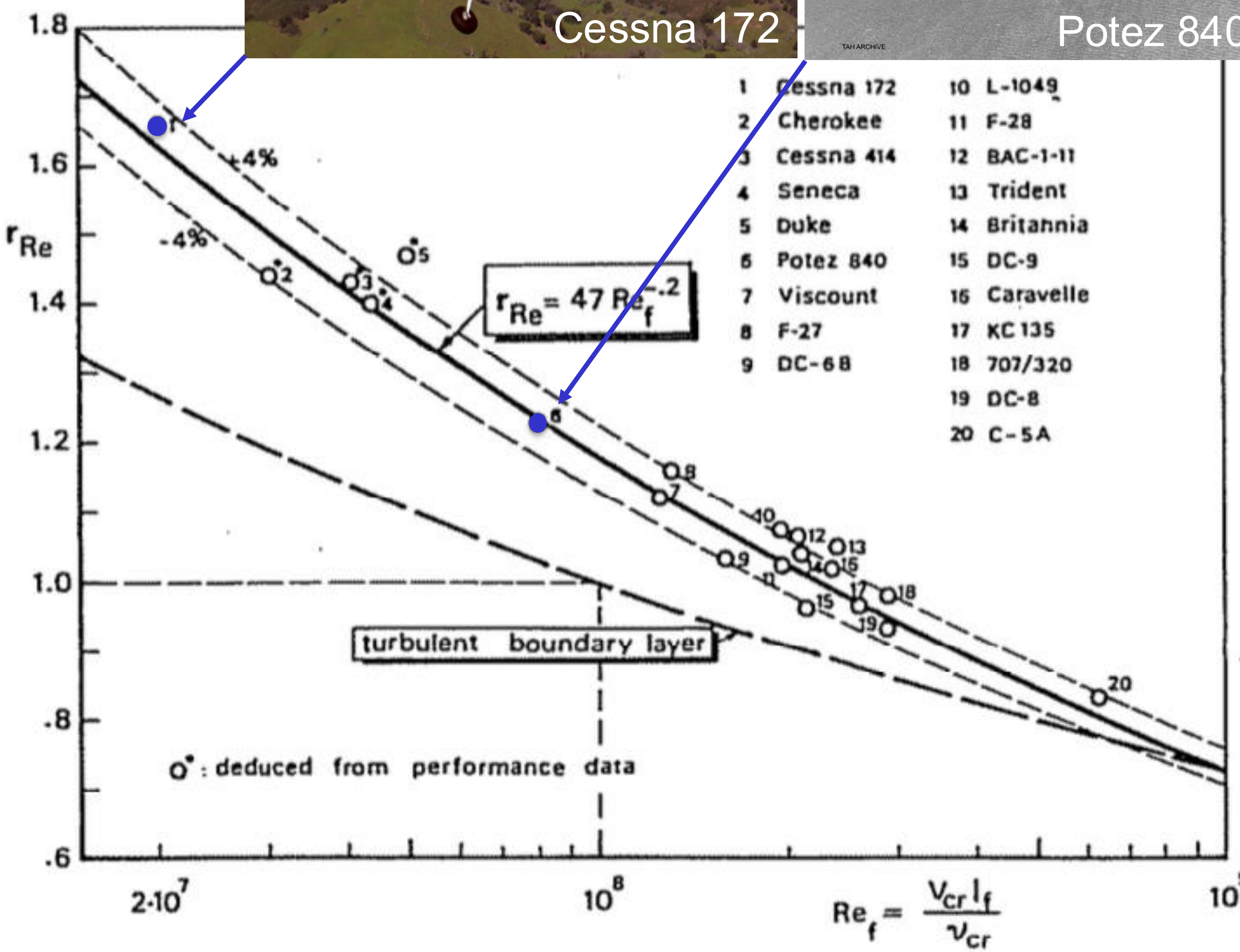
Skin frictio

$$r_{Re} = C_F(\mathrm{Re})/C_F($$



Skin frictio

$r_{Re} = C_F(\mathrm{Re})/C_F($



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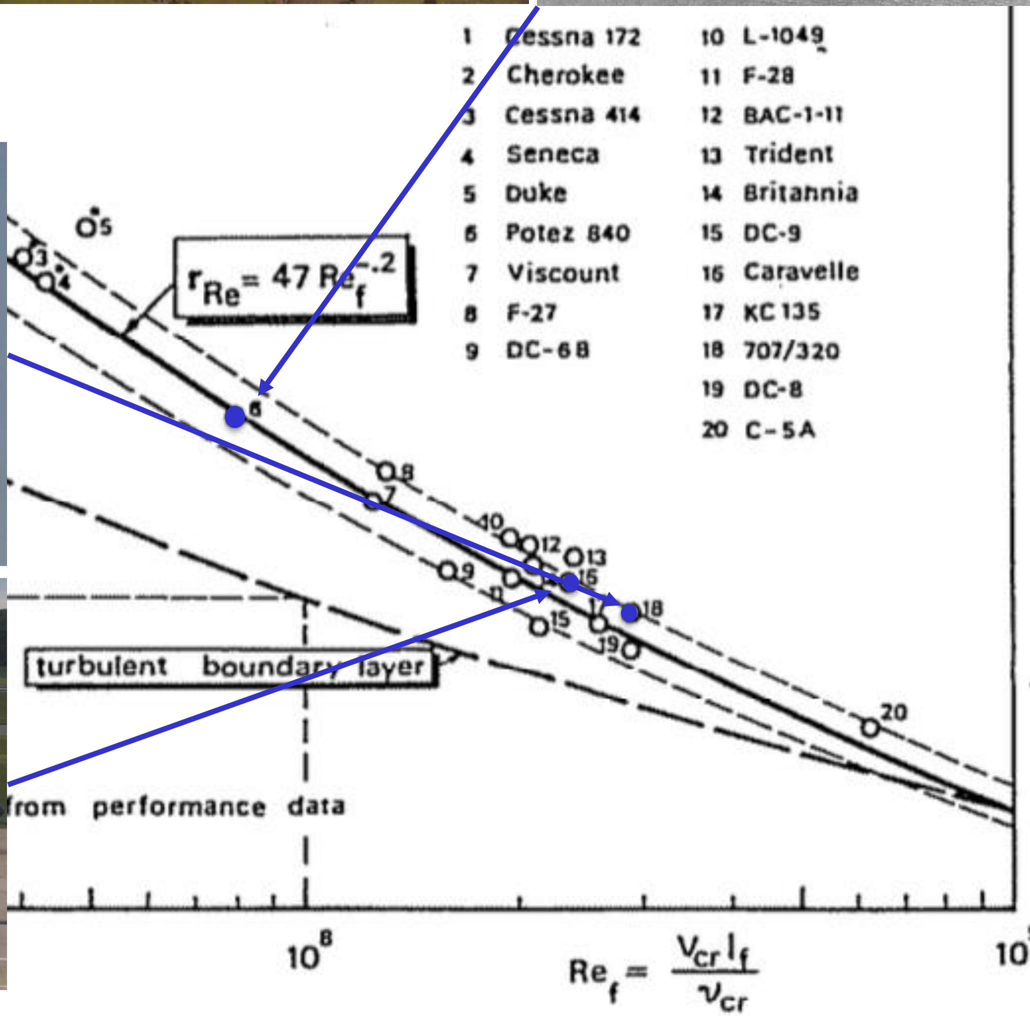

Skin frictio

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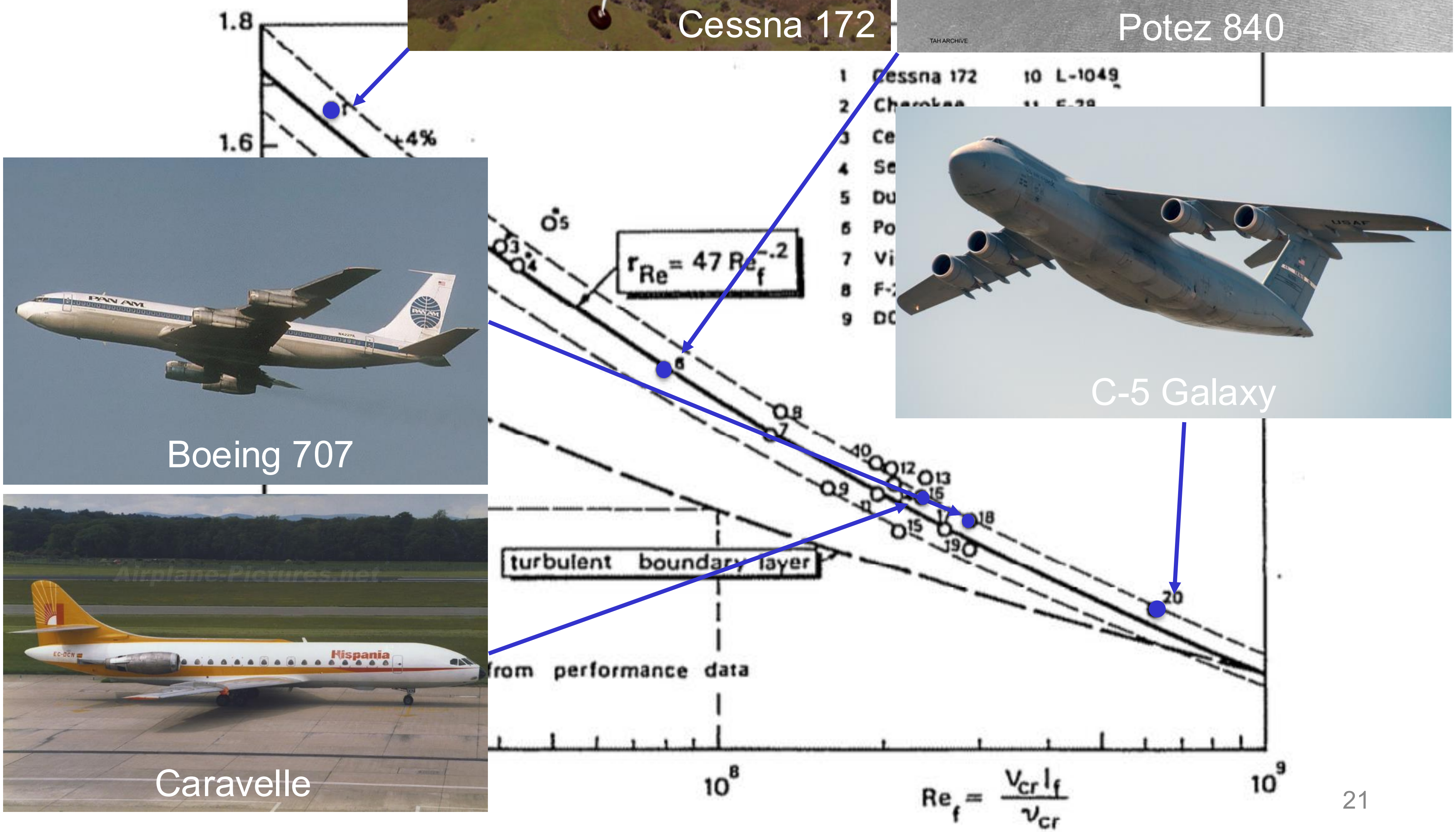


Drag calculation

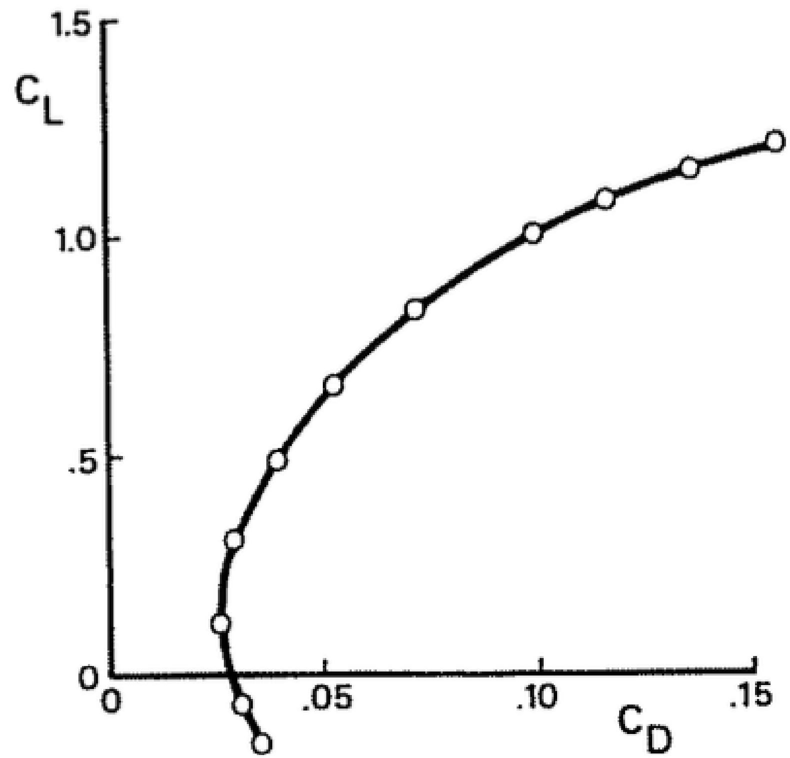
- Aircraft has several sources of drag

- It is usual to summarize them in the drag polar of the aircraft:

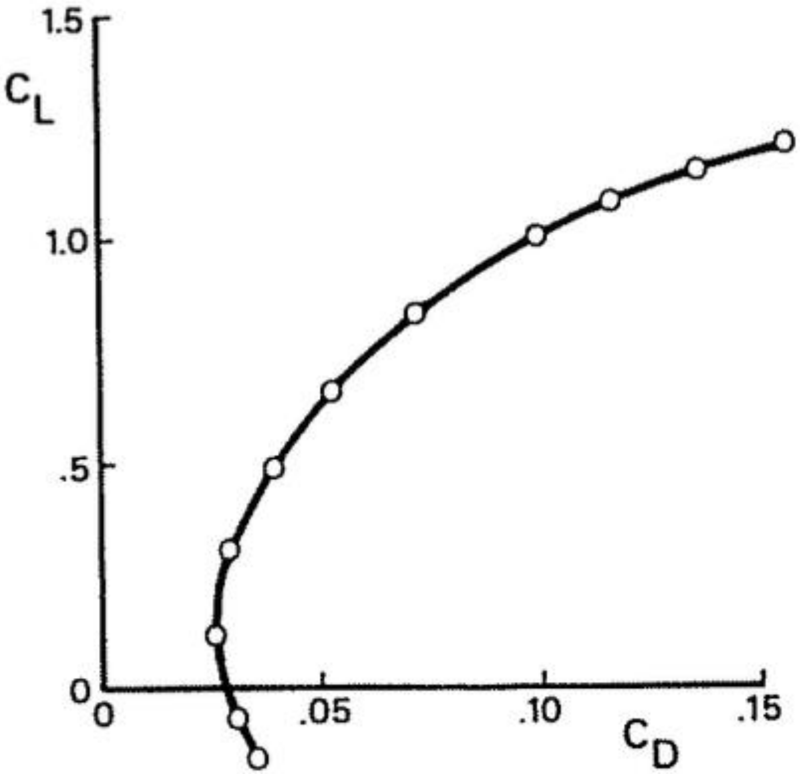
$$C_D = C_{D_0} + \frac{C_L^2}{e\pi AR}$$

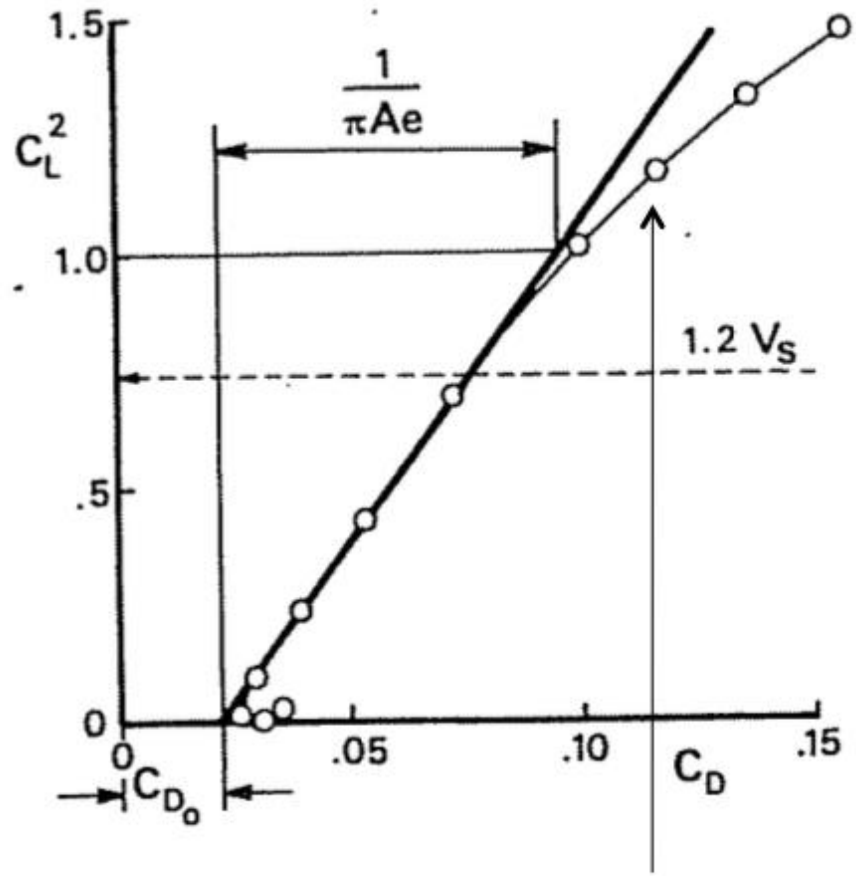
with,
C_{D0} is the parasitic drag (independent of lift)

e is the Oswald efficiency factor

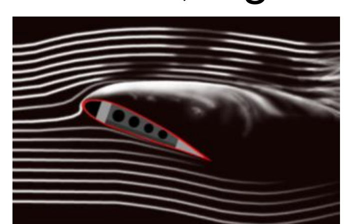


Drag polar





For high angles of attack, high lift and risk of stall

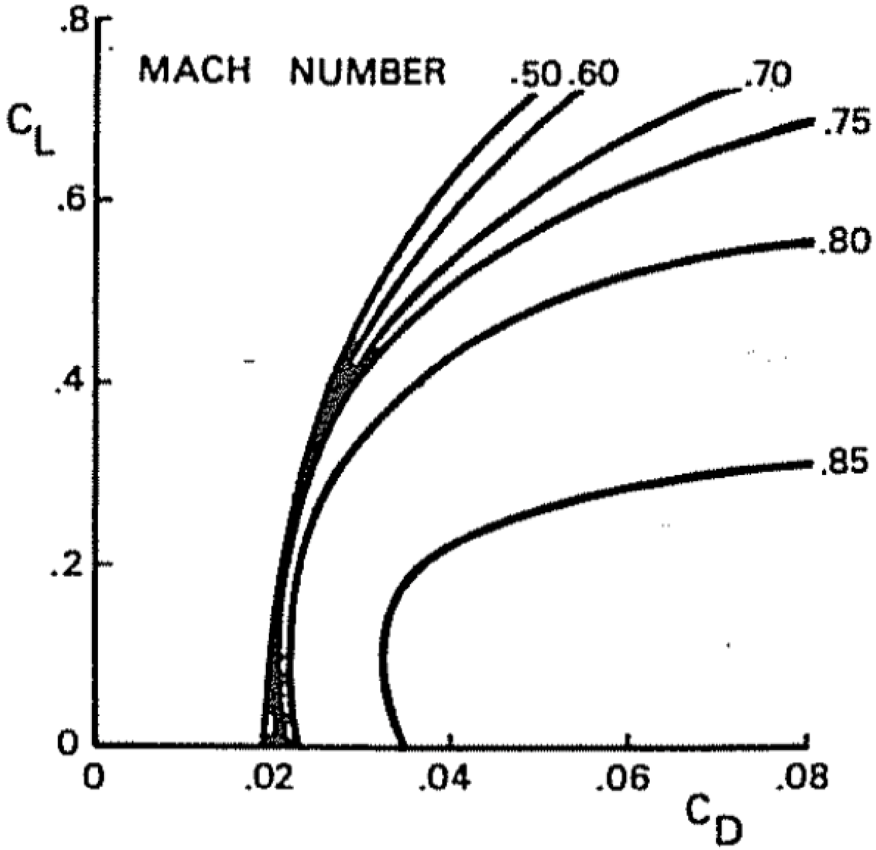


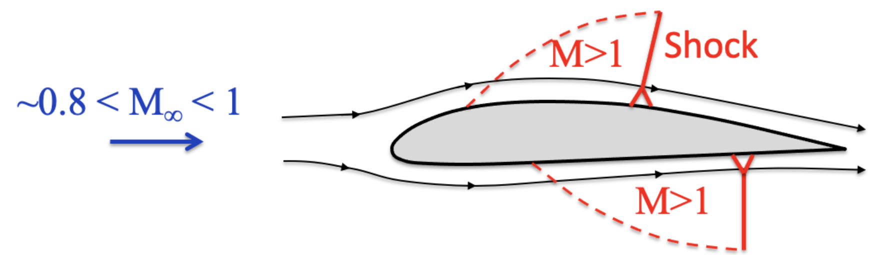
Drag figures for different aircrafts

Aircraft Type	C_{D0}	e
High-subsonic jet	0.014-0.020	0.75-0.85
Large turboprop	0.018-0.024	0.80-0.85
Twin-engine piston aircraft	0.022-0.028	0.75-0.80
Single-engine piston aircraft with fixed gear	0.020-0.030	0.75-0.80
Single-engine piston aircraft with retractable gear	0.025-0.040	0.65-0.75
Agricultural aircraft without spray system	0.060	0.65-0.75
Agricultural aircraft with spray system	0.070-0.080	0.65-0.75

Compressibility drag

Compressibility effects increase drag





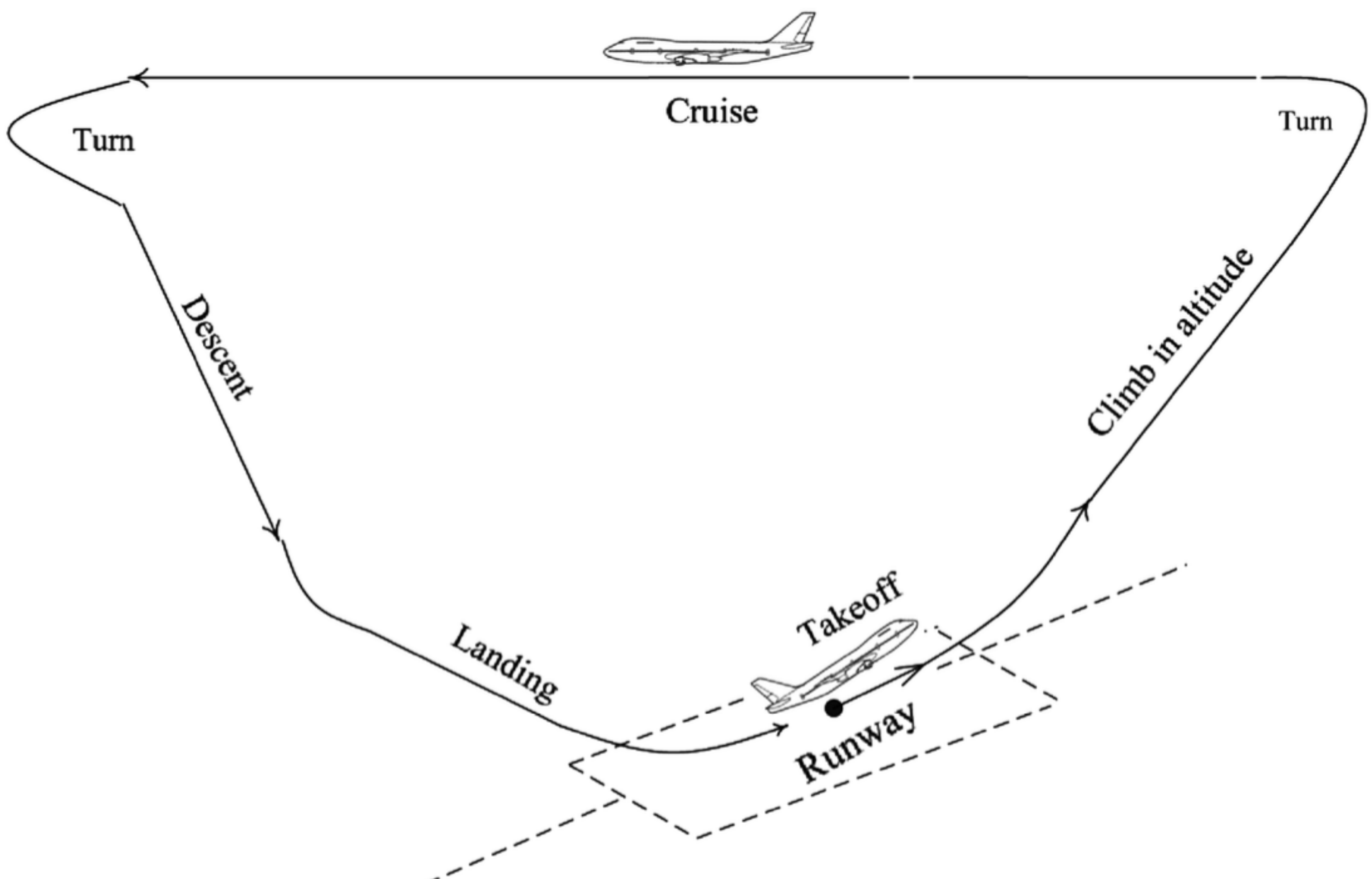
At the early design stage

→ Add ΔC_D to C_{D0}

ΔC_D = 0.0005 for long range cruise conditions

ΔC_D = 0.002 for high speed cruise conditions

Different flight phases

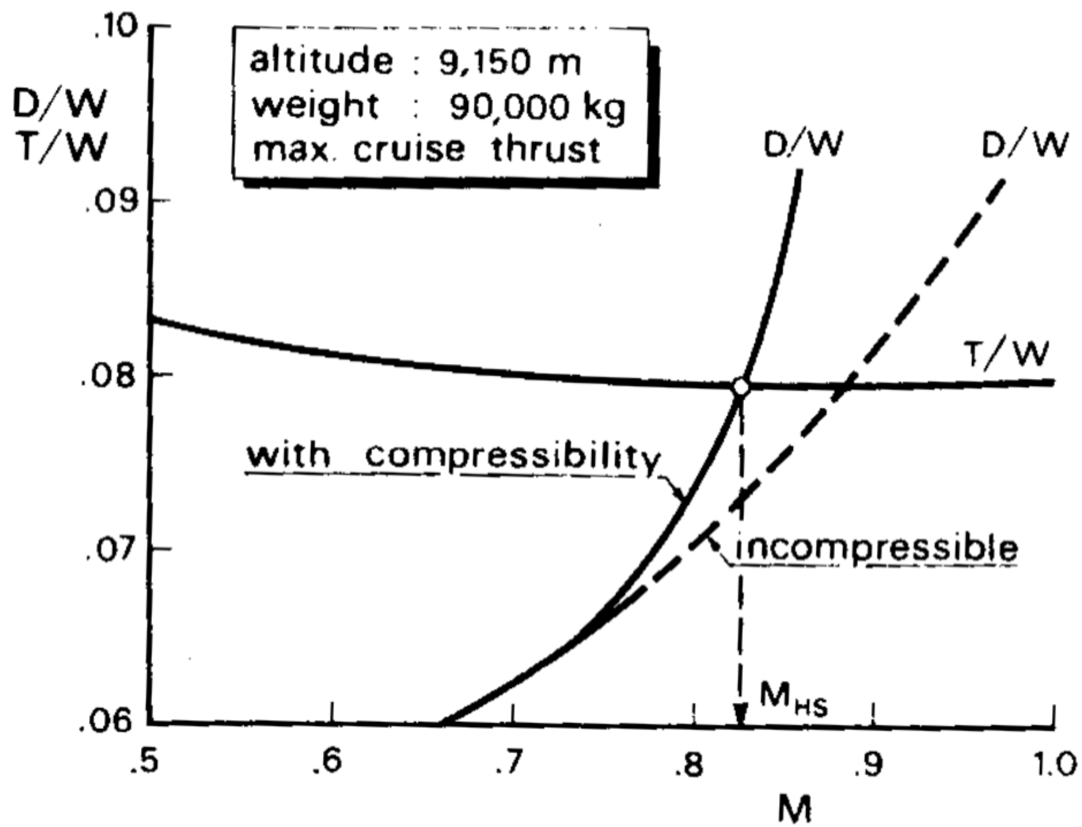


Take off → Climb → Turn → Cruise … → Landing

source: A. Ghenaiet, Journal of Aircraft vol. 44, 6

Cruise Mach

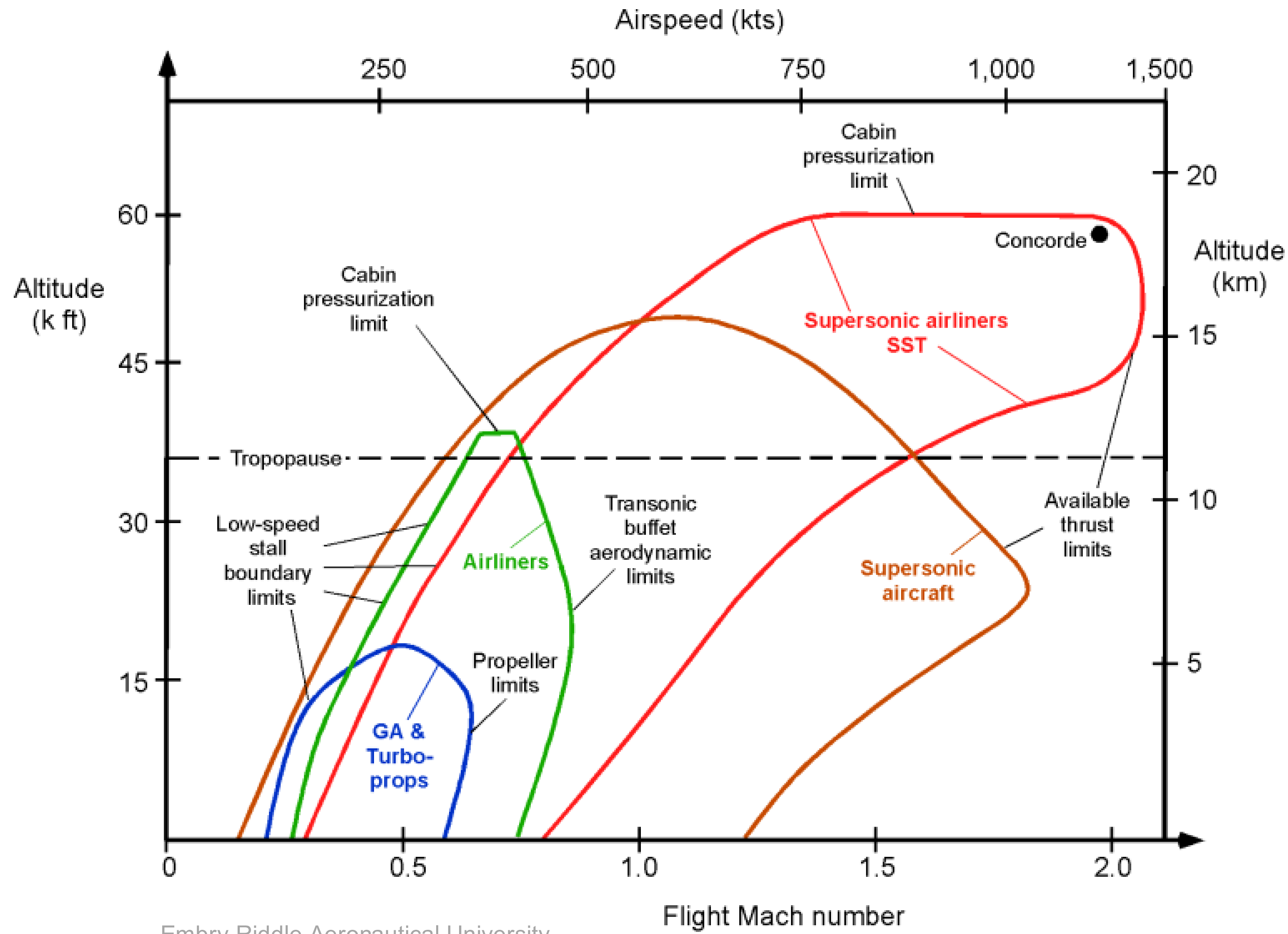
M_{HS} = Maximum Mach for High-Speed Cruise



Calculation to be repeated at several altitudes

For each altitude → a different cruise Mach

Flight envelope



Embry-Riddle Aeronautical University

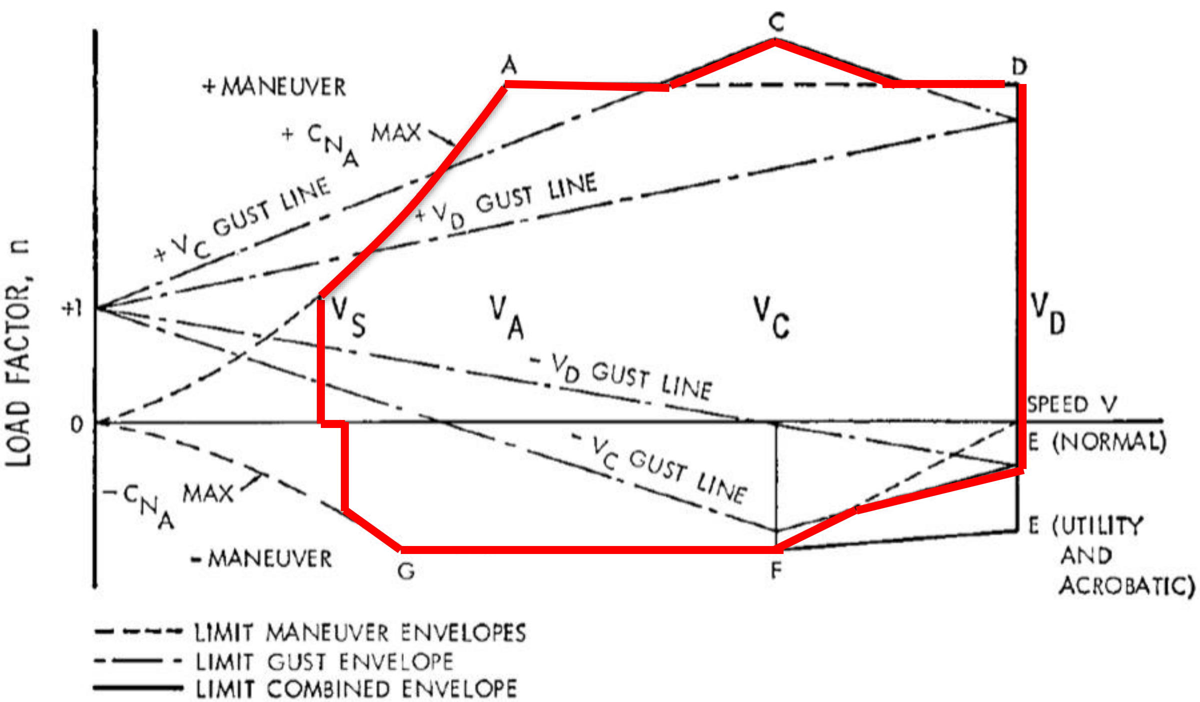
Manoeuvring envelope

V-n diagram

= load factor ($n=L/W$) on the aircraft at a given speed

Gathers information about **manoeuvre** and **gust**

Informs the pilot which flight configurations (speed/altitude) are safe



V_C = Design Cruising speed (resistance to gusts)

V_D = Design Diving speed (max speed the aircraft must resist)

V_A = Manoeuvre speed (max speed with full deflection of control surfaces)

V_S = Stall speed (min speed of the aircraft)

Cruise

At cruise, flight speed is constant

Lift (L) = Weight (W) = Vertical balance

$$\rightarrow L = W = \frac{1}{2}\rho V^2 C_L S \rightarrow C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$$

$\boldsymbol{\rho}$ = cruise air density
V = cruise speed
S = wing area

Also, **Thrust (T) = Drag (D) = Horizontal balance**

$$\rightarrow T = D = \frac{1}{2}\rho V^2 C_D S$$

where C_D is obtained from the drag polar

Cruise

Thrust to Weight ratio

$$\frac{T}{W} = \frac{D}{W} = \frac{\frac{1}{2}\rho V^2 C_D S}{W} = \frac{1}{2W}\rho V^2 S\left(C_{D0} + \frac{C_L^2}{e\pi AR}\right)$$

$$= \frac{\rho V^2 C_{D0}}{2W/S} + \frac{2W}{e\pi AR\rho V^2 S}$$

The thrust here is the installed thrust, which is 4-8% lower than the un-installed thrust.

This equation can be used to choose an engine for the cruise condition

Cruise

Minimum Thrust

The Thrust-to-Weight ratio can be minimized w.r.t. W/S

→ The minimum Thrust is required when

$$\frac{W}{S} = \frac{1}{2}\rho V^2\sqrt{d_1 e\pi AR}$$

where, d_1 = 0.008 – 0.010 for an aircraft with retractable undercarriage

→ The minimum Thrust to Weight ratio is then :

$$\left(\frac{T}{W}\right)_{min} = \frac{C_{D0} + \sqrt{d_1}}{\sqrt{d_1 e\pi AR}}$$

Cruise

Engine Thrust

The thrust of an engine at cruise can be determined from:

- Manufacturer's data

- Approximate relationship to the take off thrust:

$$\frac{T}{T_{to}} = 1 - \frac{0.454(1+\lambda)}{\sqrt{1+0.75\lambda}} M + \left(0.6 + \frac{0.13\lambda}{G}\right) M^2$$

where, λ is the bypass ratio
M is the cruise Mach number
G = 0.9 for low bypass engines
G = 1.1 for high bypass engines

Range

The range of an aircraft can be estimated from the Bréguet equation:

$$R = \frac{V}{C_T}\frac{L}{D}\ln\left(\frac{W_\mathrm{i}}{W_\mathrm{i} - W_\mathrm{f}}\right)$$

which is applicable in cruise conditions only.

with, *L/D* = cruise Lift-to-Drag ratio
V = the cruise airspeed [*m/s*]
C_T = specific fuel consumption [*1/s*]
W_i = weight of the aircraft at the beginning of cruise [*kg*]
W_f = cruise fuel weight [*kg*]

Attention to units (imperial/SI) of C_T in reference books !

Maximizing range

The range equation can also be written as

$$\frac{R}{a_0} = \frac{ML/D}{C_T/\sqrt{\theta}} \ln\left(\frac{W_i}{W_i - W_f}\right)$$

where, *M* is the cruise Mach number
a_0 is the speed of sound at sea level

The range can be maximized by maximizing *L/D* or *ML/D*

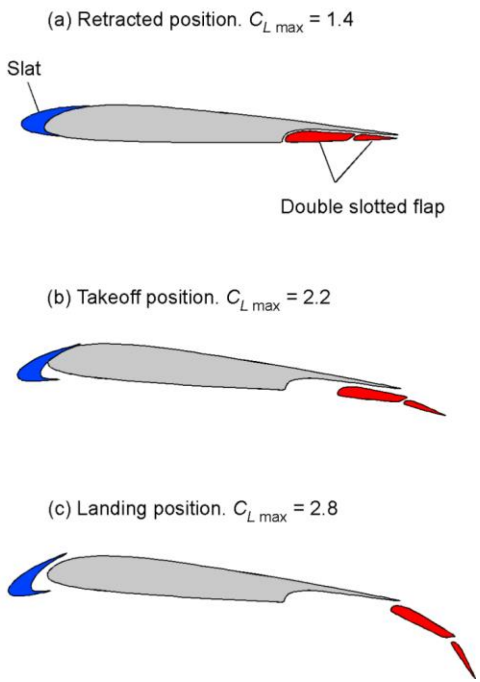
To maximize *L/D*: $C_L = \sqrt{C_{D_0} e \pi AR}$

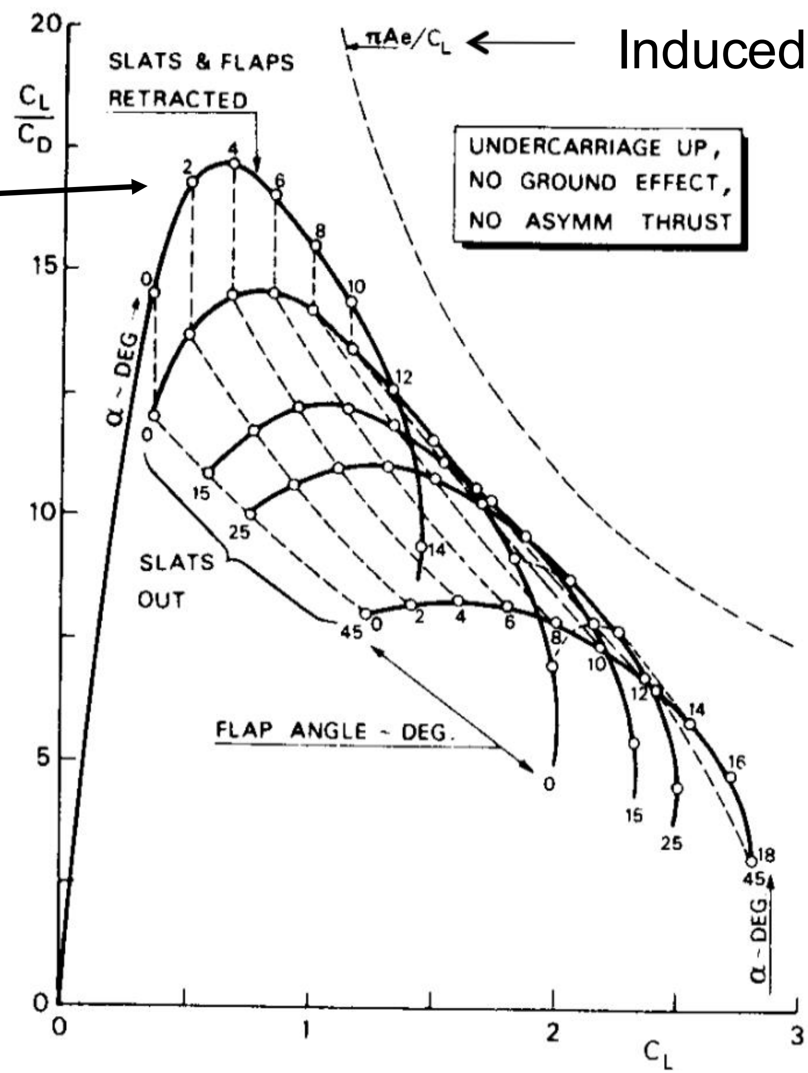
To maximize *ML/D*: $C_L = \sqrt{\frac{1}{3} C_{D_0} e \pi AR}$

Lift-to-Drag ratio

Example of Lift-to-Drag ratio variation with Lift, Angle of attack and deployment of slats/flaps

Best C_L/C_D
for the clean configuration





Induced drag only

Range design

At the early design stage, the designer must choose a **favourable combination** of:

- Speed
- Altitude
- Airplane geometry
- Engine

→ best range performance or fuel efficiency

Depending on the **objective**, most important consideration is:

Fuel efficiency → for long-haul aircraft
Engine weight → for short-haul aircraft

<u>Constraints</u>:

- Cruise fuel is not the only part of the fuel weight
- Engine thrust often determined by take off field
- Air Traffic Controls decide the allowable cruise altitudes
- An aircraft can have more than one engine
- …

Reserve fuel

ATA (Air Transport Association) regulation claims that the airliner must carry enough reserve fuel to :

- Continue flight for time equal to 10% of basic flight time at normal cruise conditions
- Execute missed approach and climb at the destination airport
- Fly to alternate airport 370km distant
- Hold at alternate airport for 30 min at 457m (1500ft) above the ground
- Descend and land at alternate airport

Approximate formula: $$W_{f_{res}} / W_{to} = 0.18 C_T / \sqrt{\theta AR}$$

$\boldsymbol{\theta}$ = T/T_0 cruise/stand. temp.

C_T = specific fuel consumption at cruise

Range for propeller aircraft

For propeller aircraft, the Bréguet range equation is

$$R = \frac{\eta_p}{C_P} \frac{L}{D} \ln\left(\frac{W_i}{W_i - W_f} \right)$$

where η_P is the propeller efficiency
C_P is the specific fuel consumption

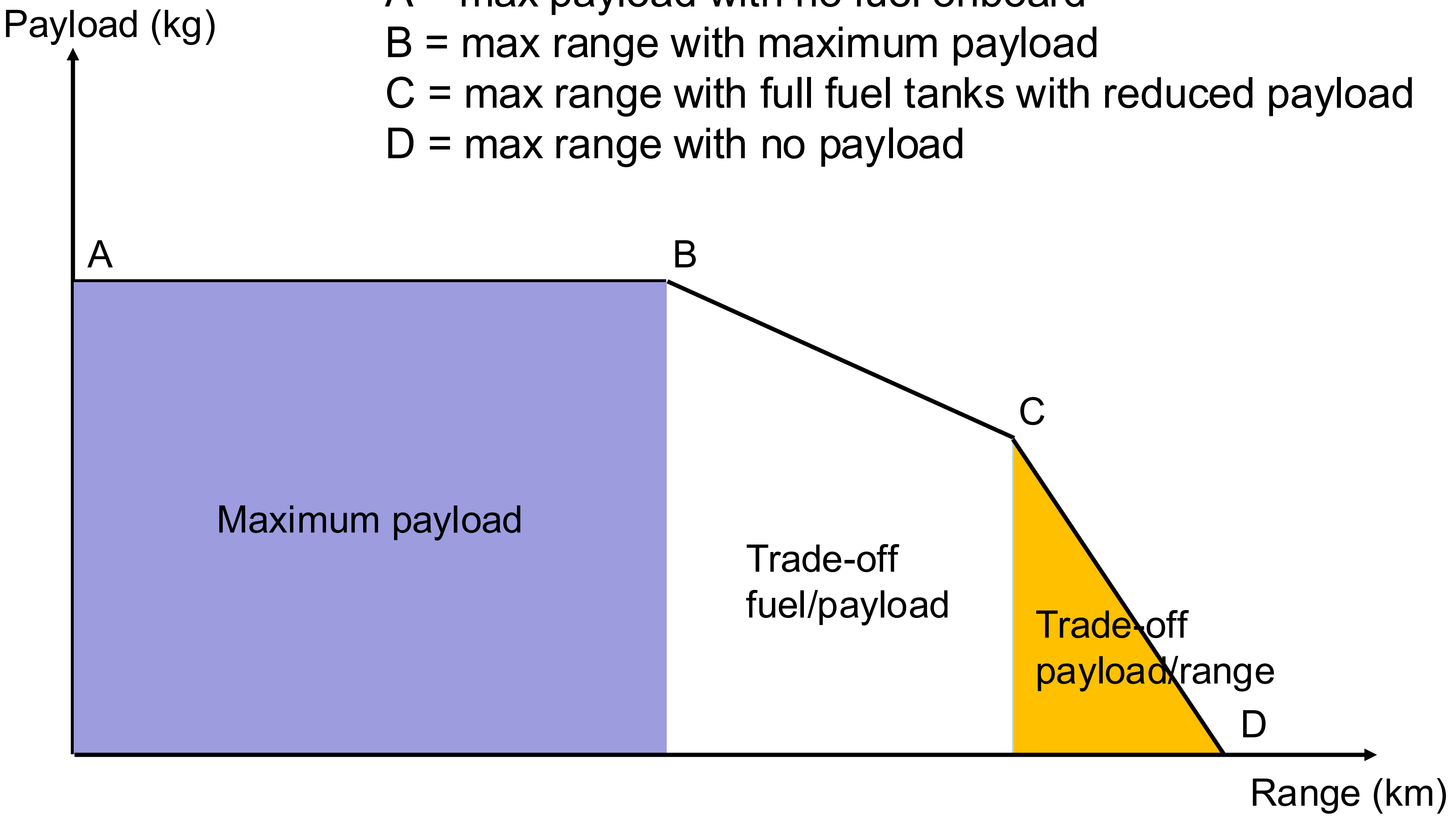
Range can be maximized by:

- Minimizing the airplane drag

- Minimizing the engine power

Payload-range diagram

A = max payload with no fuel onboard
B = max range with maximum payload
C = max range with full fuel tanks with reduced payload
D = max range with no payload

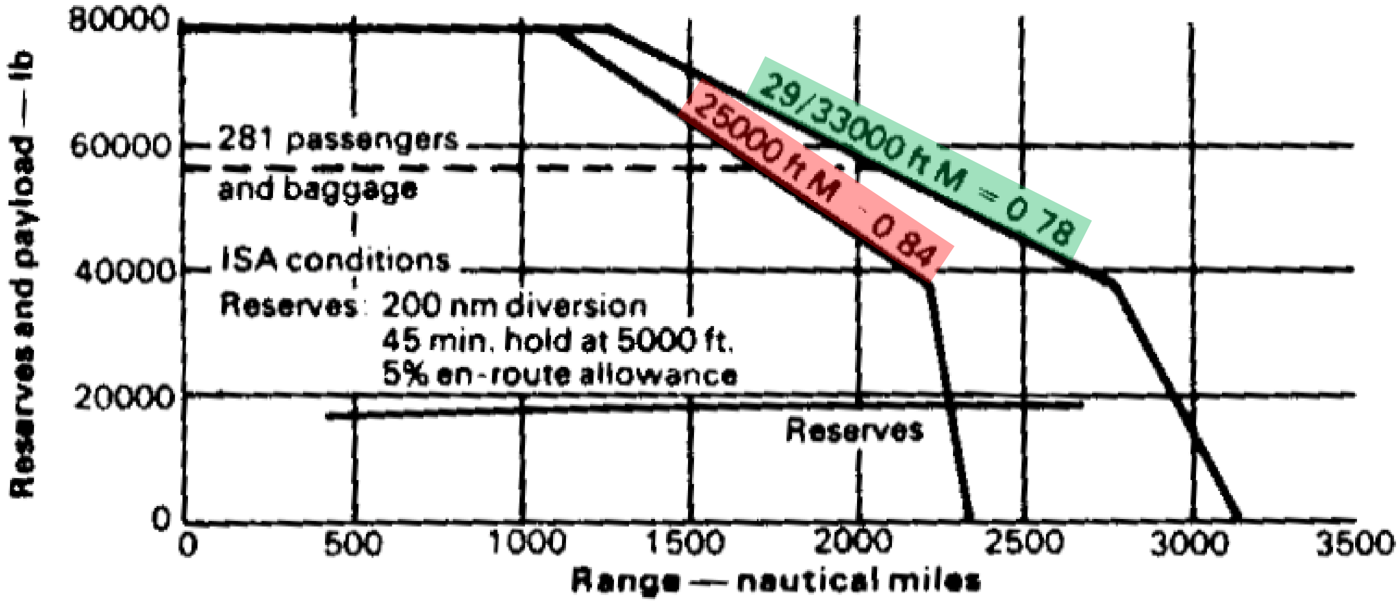


Payload-range diagram

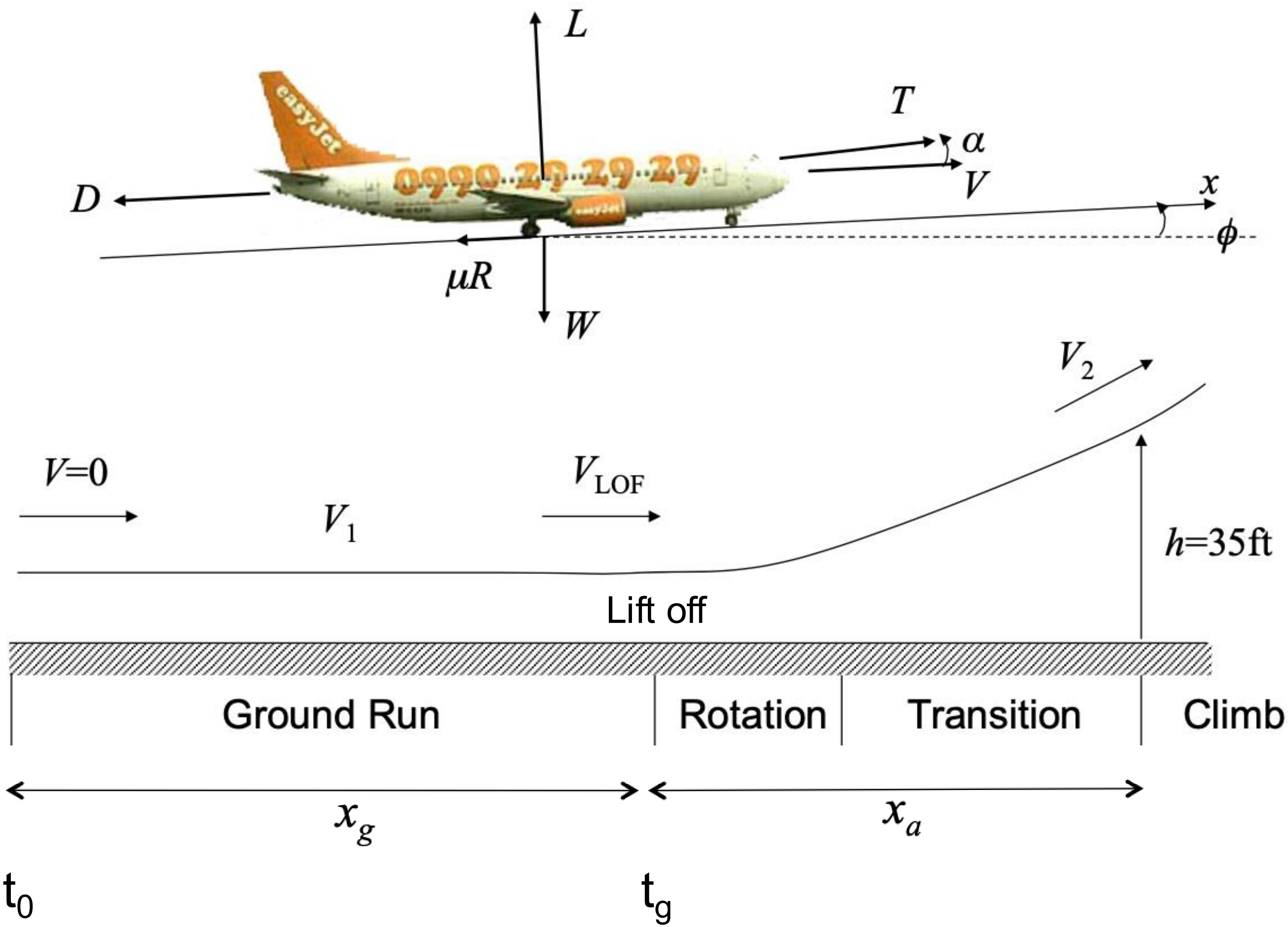
Two flight cases:

- High-Speed Cruise → Low Range
- Long-Range Cruise → Low Speed

Example : A-300B



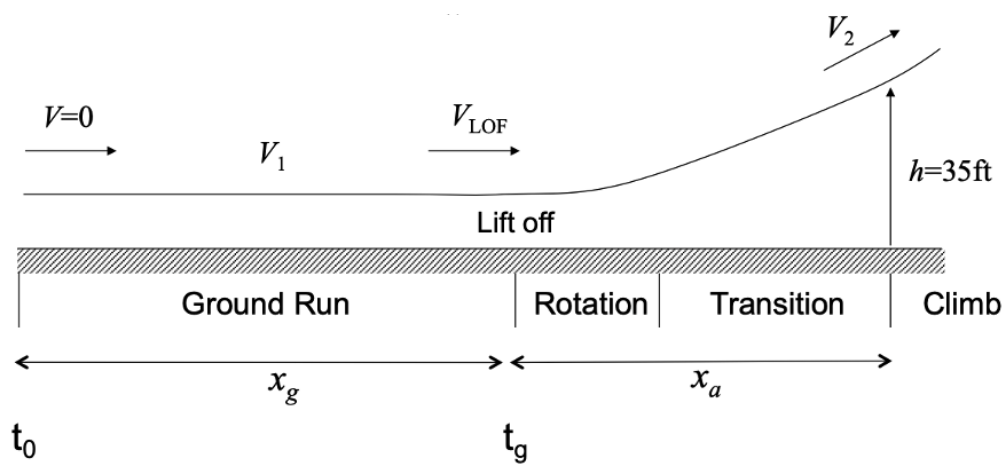
Take off



L
T
α
V
x
D
ϕ
μR
W
V2
V=0
V_LOF
V1
h=35ft
Lift off
Ground Run
Rotation
Transition
Climb
xg
xa
t0
tg

Take off

- Take off starts at time t_0, with airspeed V_0 and the runway may have an angle to horizontal of).
- Lift off occurs at time t_g, after a distance of x_g, usually at speed V_{LOF}.
- Take off is completed when the aircraft has reached sufficient height to clear an obstacle 35ft high (50ft for military aircraft)
- Finally, the climb out phase takes the aircraft to 500ft at the climb throttle setting.



Ground run

- Start at V_0 (equal to zero or not)

- Angle of attack is defined w.r.t. the thrust line

Tricycle landing gear

Low AoA

→ For sufficient speed, nose is lifted up to the optimal AoA

Tailwheel landing gear

High AoA

→ Control surfaces (when effective) are used to decrease the angle of attack
→ Decrease of drag and increase of speed

Lift off

As seen in Lecture 1 of Aerodynamics:

Vertical balance: $L = \frac{1}{2}\rho_{\infty} V_{\infty}^{2} S\ C_{L} = W$

$$V_{\text{stall}} = \sqrt{\frac{2W}{\rho_{\infty} S C_{L,\max}}}$$

In practice (for safety reasons), Lift off speed is defined as

$$V_{LOF} = k_1 V_{stall}$$

where,

- k_1 varies with the type of aircraft
- k_1 = 1.1 is an indicative value

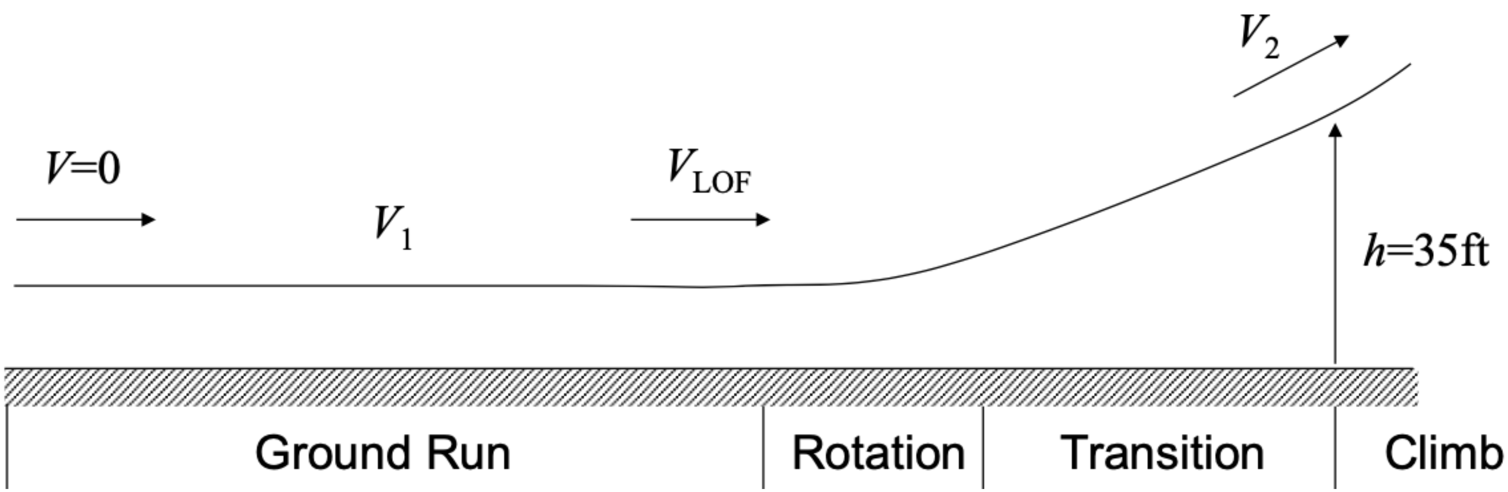
Rotation and transition

→ Aircraft **rotation** = deflection of the velocity from nearly horizontal to a few degrees upward

very short stage (few seconds)

→ **Transition** stage follows up to obstacle clearance

speed at clearance: $V_2 = k_2 V_{stall}$ with k_2=1.2

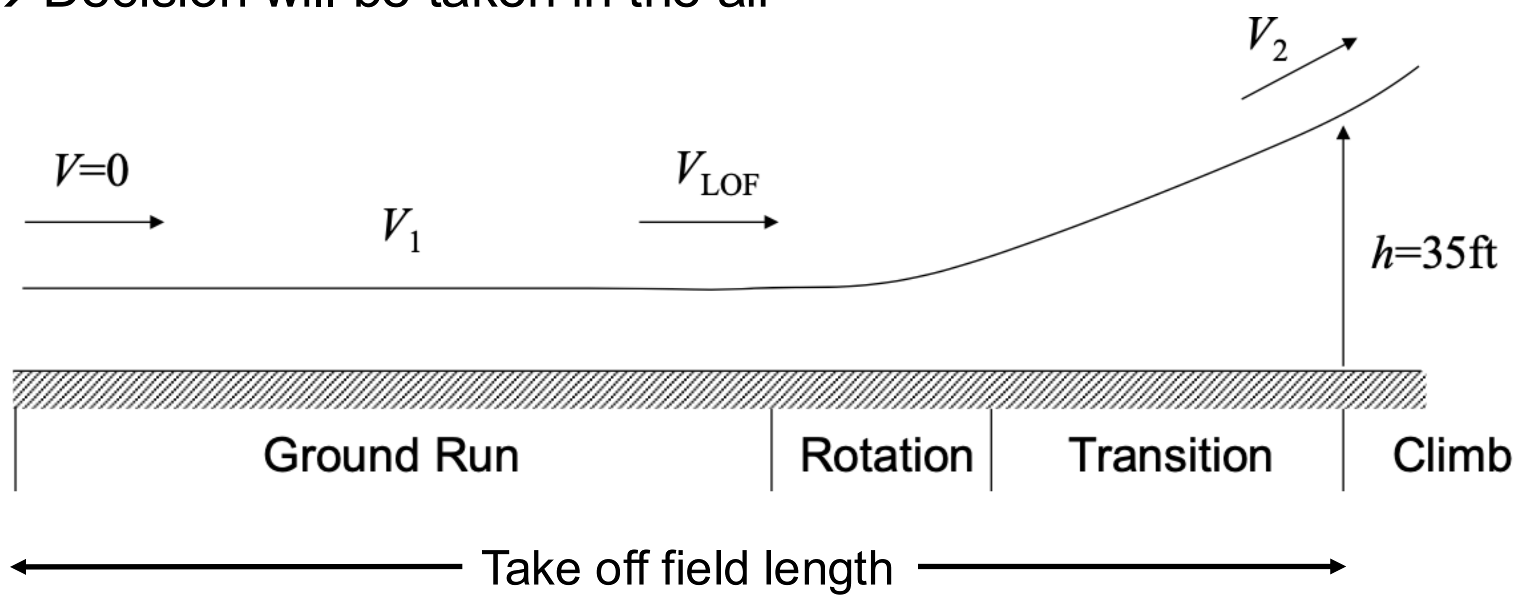


Take off details

V_1 = **Decision speed** (for multi-engine aircraft)

For $V < V_1$, if one engine fails, take off is aborted
→ The runway is long enough to stop the aircraft

For $V > V_1$, if one engine fails, take off is continued
→ Decision will be taken in the air



Take off completed at ~ 500ft and once flaps are retracted

Equations of motion

Force balance parallel to the runway:

$$F = T - \mu R - D = ma$$

Aircraft acceleration, $a = \frac{dV}{dt}$

Aircraft velocity, $\mathrm{v} = \frac{dx}{dt}$

Then, $\frac{V}{a} = \frac{dx}{dt}\frac{dt}{dV} = \frac{dx}{dV} \rightarrow \frac{dx}{dV} = \frac{V}{a}$

The displacement is obtained by integration w.r.t. V:

$$x_g = \int_0^{V_{LOF}} \frac{V}{a} dV$$

Friction force

Friction force is proportional to the vertical force:

Friction = μR with $R = W - L = W - \frac{1}{2}\rho V^2 S C_L$

(assuming the runway's inclination is small)

The total horizontal balance becomes:

$$ma = T - \mu W - \frac{1}{2}\rho V^2 S(C_D - \mu C_L)$$

Runway type	$\boldsymbol{\mu}$
Concrete, asphalt	0.02
Hard turf	0.04
Field with short grass	0.05
Field with long grass	0.1
Soft field, sand	0.1-0.3

Approximate solutions

Ground run x_g

$$x_g \approx \frac{V_{LOF}^2/2g}{\frac{\bar{T}}{W_{TO}} - \mu'}$$

where, $\bar{T}$ = Thrust at $\frac{V_{LOF}}{\sqrt{2}} \approx 0.75\frac{5+\lambda}{4+\lambda}T_{TO}$

$$C_L = \mu e \pi AR$$

$$\mu' = \mu + 0.72\frac{C_{D0}}{C_{Lmax}}$$

Increases with

- Aircraft Weight (W_{TO})
- Altitude
- Temperature
- Rolling friction
- Positive runway slope

Decreases with

- Thrust
- High lift devices

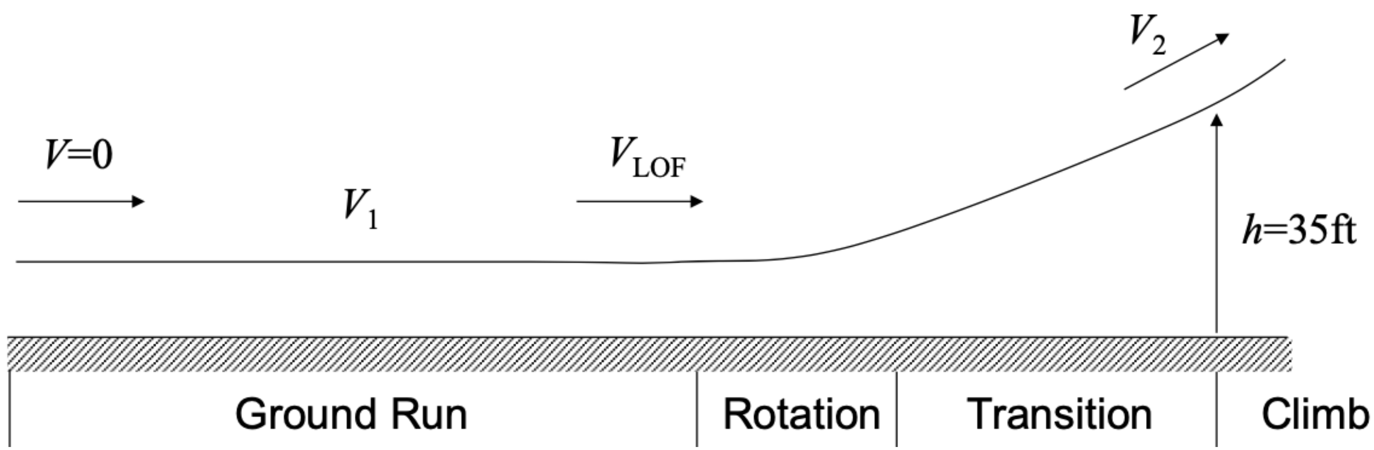
Air run x_a

$$x_a \approx \frac{V_{LOF}^2}{g\sqrt{2}} + \frac{h}{\gamma_{LOF}}$$

where, $\gamma_{LOF} = \left(\frac{T-D}{W}\right)_{LOF} \approx 0.9\frac{\bar{T}}{W_{TO}} - \frac{0.3}{\sqrt{AR}}$

Airspeed at take off $V_2 = V_{LOF}\sqrt{1 + \gamma_{LOF}\sqrt{2}}$

Climb



- Immediately follows take off
- Objective: reach the cruising altitude
- Usually performed in a vertical plane → no turning
- Short phase → aircraft's weight is assumed constant
- Small variations of speed and flight path → constant speed climb

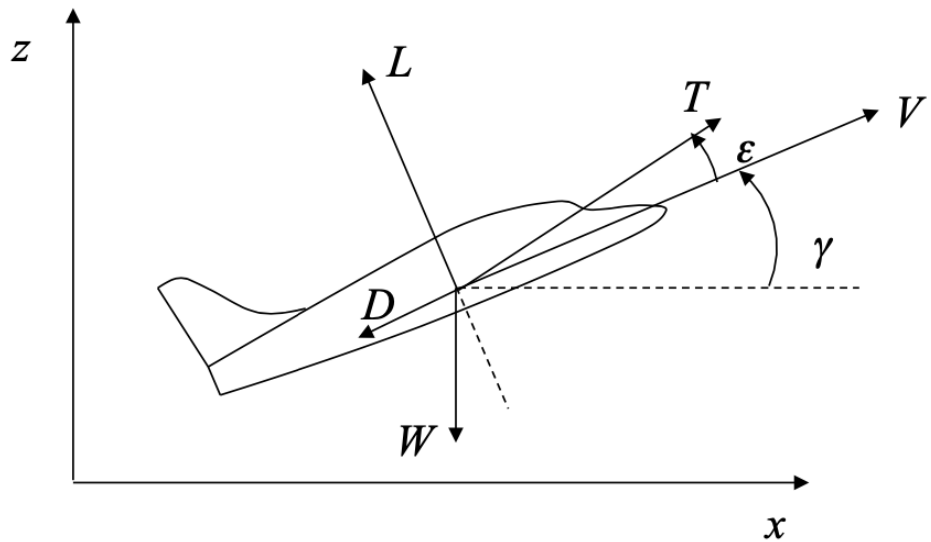
Climb

Climb performances:

- Operational requirements, e.g.
  - Rate of climb at sea level
  - Service ceiling altitude for a maximum rate of climb of 0.5m/s

- Airworthiness requirements
  - Minimum climb gradient at take off, in cruise, at landing
  - Rate of climb at a specified altitude with one engine inoperative

Climb diagram



- Thrust line is not necessarily aligned with flight path
- Constant speed V → the aircraft does not accelerate
- Thrust and lift equations: $T = \frac{1}{2}\rho S V^2 C_D + W \sin\gamma$

(assuming **ε** = 0) $W \cos\gamma = \frac{1}{2}\rho S V^2 C_L$

Thrust and Power

Thrust equation can be expressed in terms of Power

$$T = \frac{1}{2}\rho S V^2 C_D + W \sin\gamma \quad \rightarrow \quad TV = \frac{1}{2}\rho S V^3 C_D + WV \sin\gamma$$

Defining, the available power for Climb : $P_a = TV$

the power required for level flight : $P_r = \frac{1}{2}\rho S V^3 C_D$

the rate of climb $V_z = V \sin\gamma$

Then, $V_z = \frac{P_a - P_r}{W}$ and $\sin\gamma = \frac{P_a - P_r}{VW}$

Climb of a jet aircraft

For a jet aircraft :

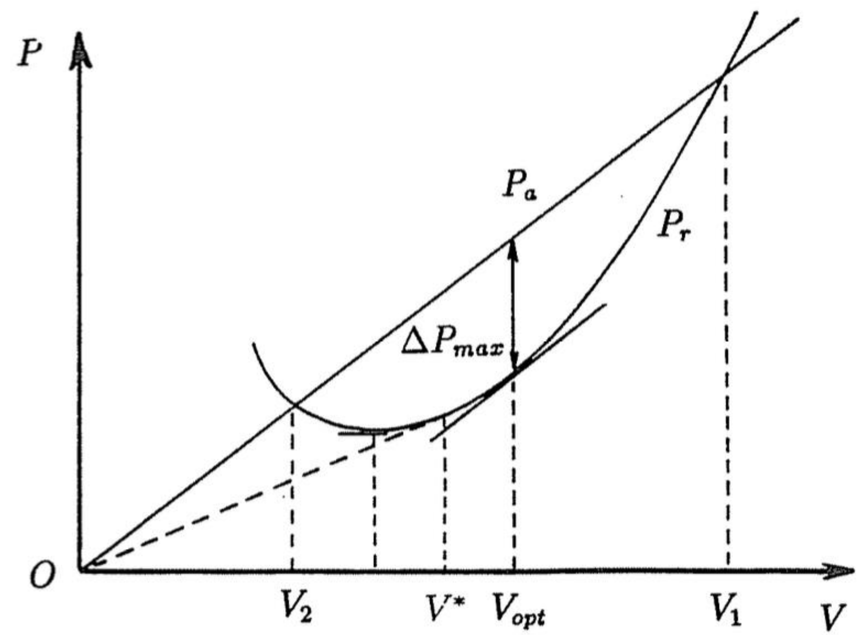
- the power available for climb varied linearly with airspeed
- the power required for level flight varies non-linearly with airspeed

→ There is an optimum airspeed to maximize the rate of climb

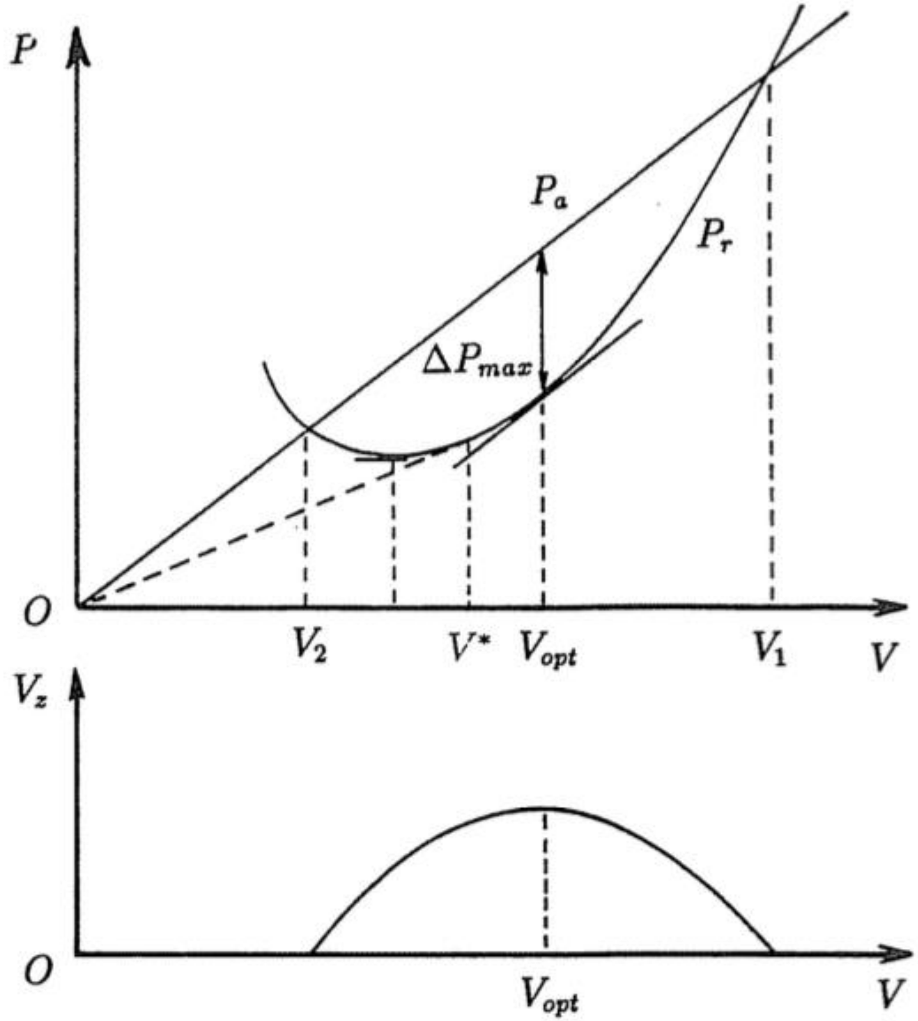
$$V_z = \frac{P_a - P_r}{W}$$

→ Maximize $P_a - P_r$

This airspeed is not necessarily the one corresponding to the minimum value of P_r



Climb of a jet aircraft



Below V_2 and above V_1 → Aircraft cannot climb

Maximum climb rate

Using the drag polar and assuming a small rate of climb

→ Equation for the rate of climb can be written as

$$V_z = \frac{P_a - P_r}{W} = \frac{1}{W}\left(TV - \frac{1}{2}\rho S V^3 C_{D0} - \frac{2kW^2}{\rho S V}\right)$$

where, $k = \dfrac{1}{e\pi AR}$

The maximum climb rate is found for $\dfrac{\partial V_z}{\partial V} = 0$

$$\rightarrow \frac{3\rho S C_D}{W} V^4 - 2\frac{T}{W} V^2 - \frac{4kW}{\rho S} = 0$$

Maximum climb rate

Solving this quartic equation

$$\frac{3\rho S C_D}{W} V^4 - 2\frac{T}{W} V^2 - \frac{4kW}{\rho S} = 0$$

one gets,

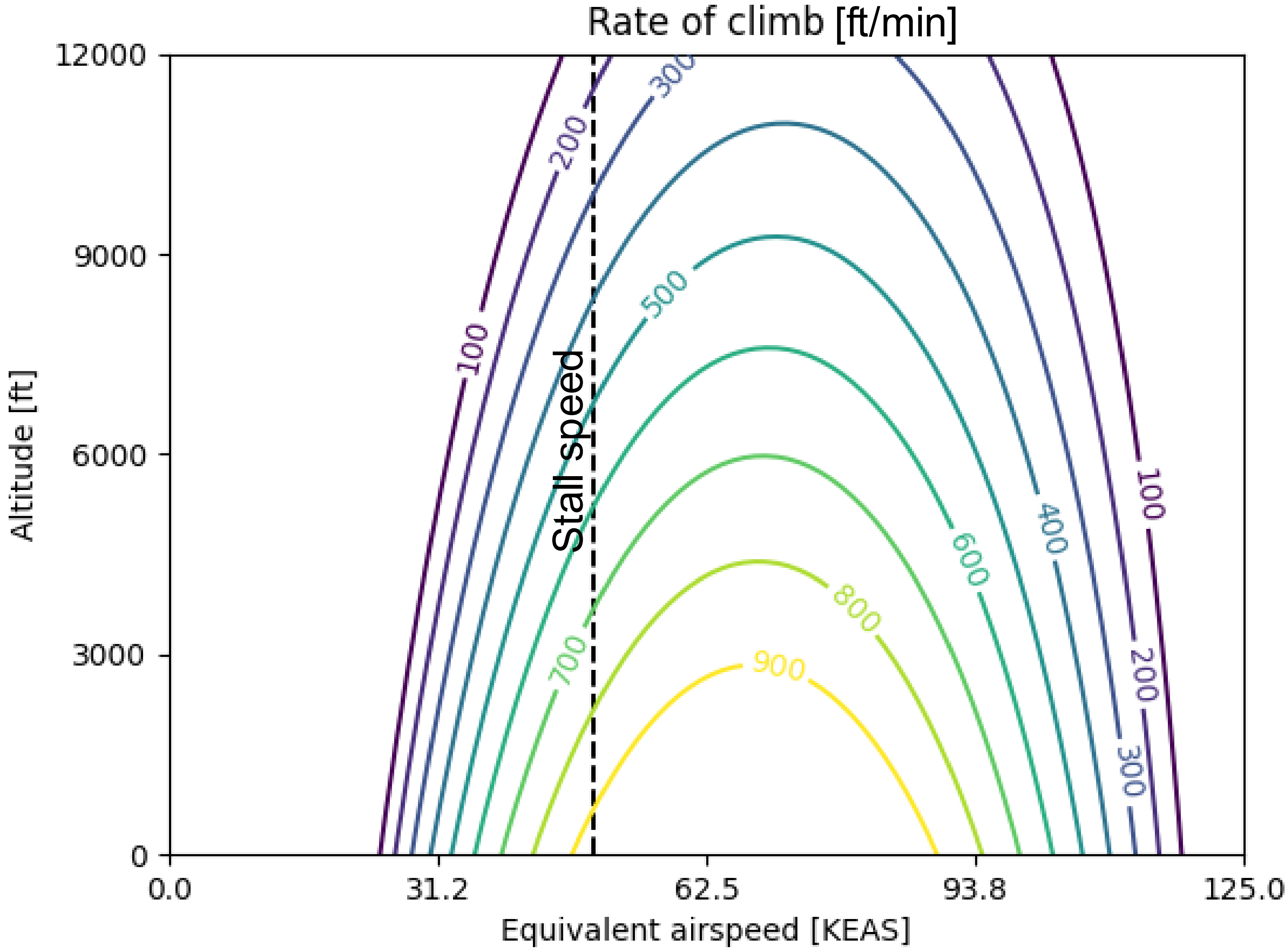
$$\rightarrow \frac{V_{zMAX}}{V} = \frac{1}{3E_{max}}\left(\tau + \sqrt{\tau^2 + 3}\right) - \frac{3}{E_{max}\left(\tau + \sqrt{\tau^2+3}\right)}$$

where, $\tau = E_{max}\, {}^{T}\!/_{W}$ and $E_{max} = \left(\frac{C_L}{C_D}\right)_{max}$

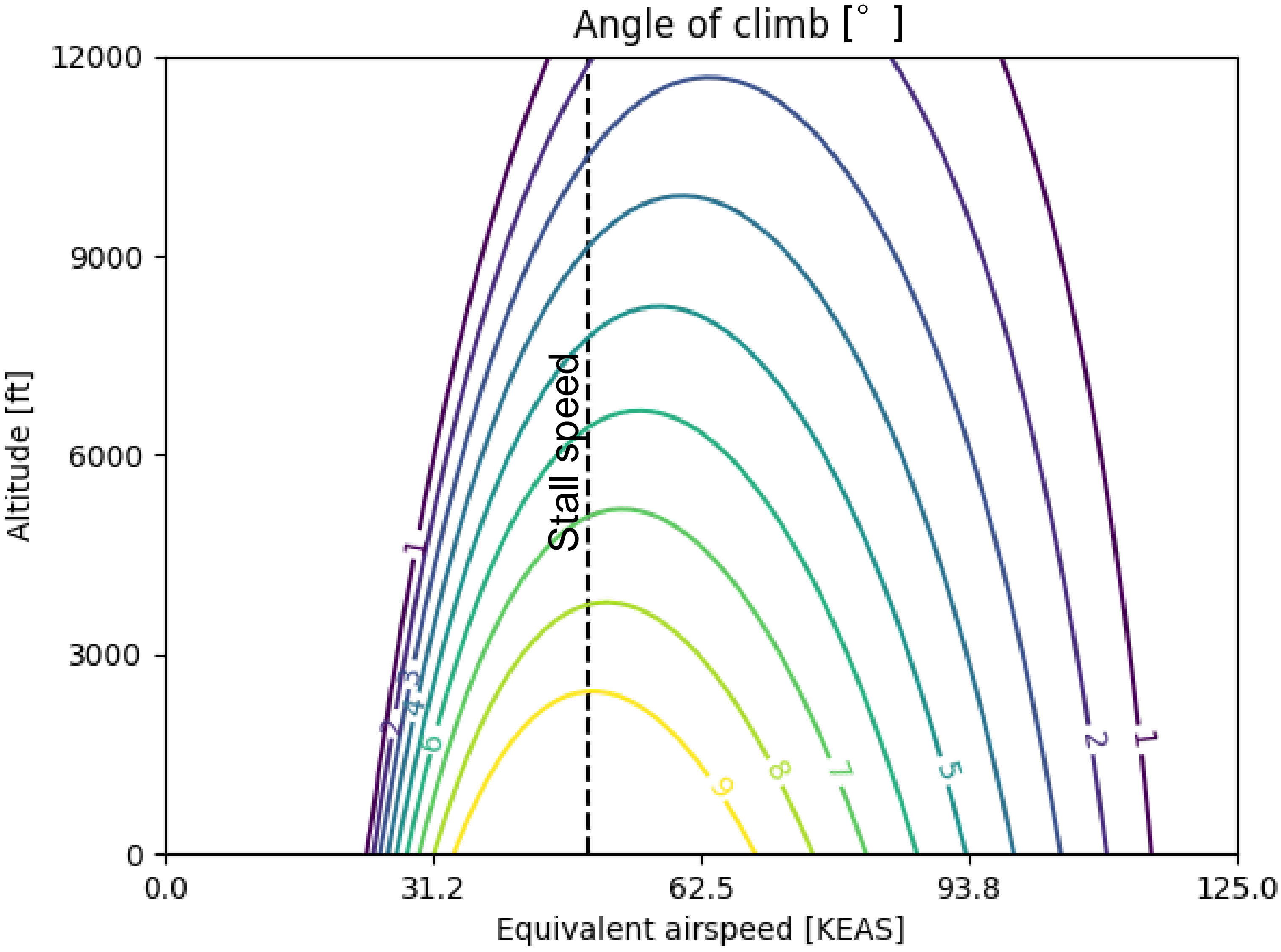
→ The maximum climb rate depends on:

- Thrust available
- Weight
- Altitude
- Wing surface

Climb rate vs. altitude



Climb gradient vs. altitude



Climb rate requirements

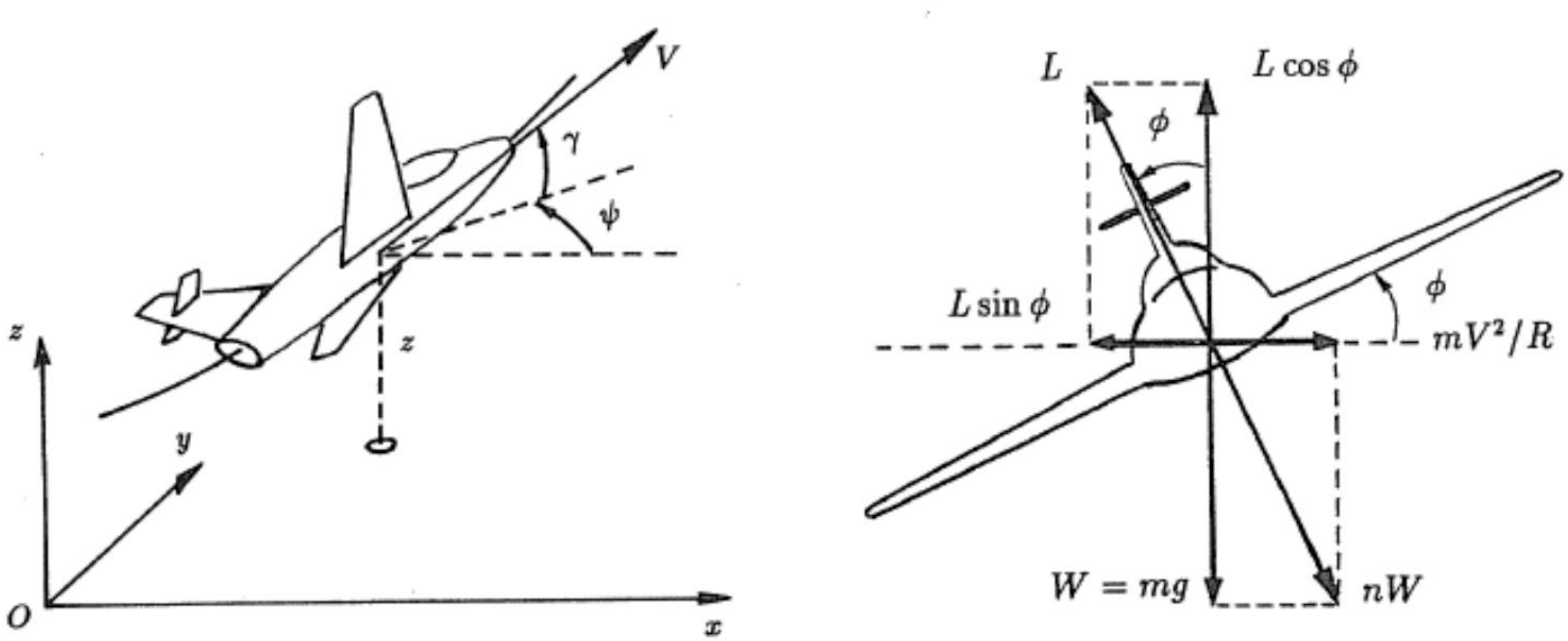
PHASE OF FLIGHT		AIRPLANE CONFIGURATION						MINIMUM CLIMB GRADIENT %		
		flap setting	u.c.	engine	thrust (power)	speed	altitude	$N_e=2$	$N_e=3$	$N_e=4$
TAKEOFF CLIMB POTENTIAL ("first segment")		t.o.	↓	one engine out	t.o.	V_{LOF}	$0 \rightarrow h_{uu}$ [1]	0	.3	.5
TAKEOFF FLIGHT PATH	"second segment"	t.o.	↑	one engine out	t.o.	V_2 [2]	$h_{uu} \rightarrow 400$ ft	2.4	2.7	3.0
TAKEOFF FLIGHT PATH	final takeoff ("third segment")	en route	↑	one engine out	max. cont.	$V \geq 1.25 V_S$	400→1,500ft	1.2	1.5	1.7
APPROACH CLIMB POTENTIAL		approach [3]	↑	one engine out	t.o.	$V \leq 1.5 V_S$	0 [1]	2.1	2.4	2.7
LANDING CLIMB POTENTIAL		landing	↓	all engines takeoff [4]		$V \leq 1.3 V_S$	0 [1]	3.2	3.2	3.2

Nomenclature:

- V_{LOF} - liftoff speed
- V_2 - takeoff safety speed
- V_R - rotation speed
- V_S - stalling speed
- u.c. - undercarriage position
- h_{uu} - height at which u.c. retraction is completed
- N_e - number of engines per a/c

1) out of ground effect
2) defined in Section 2 of Appendix K
3) flap setting such that $V_S \leq 1.10\ V_S$ for landing
4) more precisely: the engine power (thrust) available 8 seconds after throttle opening to takeoff rating
5) takeoff requirements are at actual weight, other requirements at landing (touchdown) weight

Turning



In a general turn :

- all angles (pitch, roll and yaw) are involved
- altitude changes too

In the following, let's assume turns in a horizontal plane
→ no change of altitude

Turning

- Horizontal turn → $\gamma = 0$
- Constant speed → $dV/dt = 0$

Assuming the thrust is aligned with the flight path

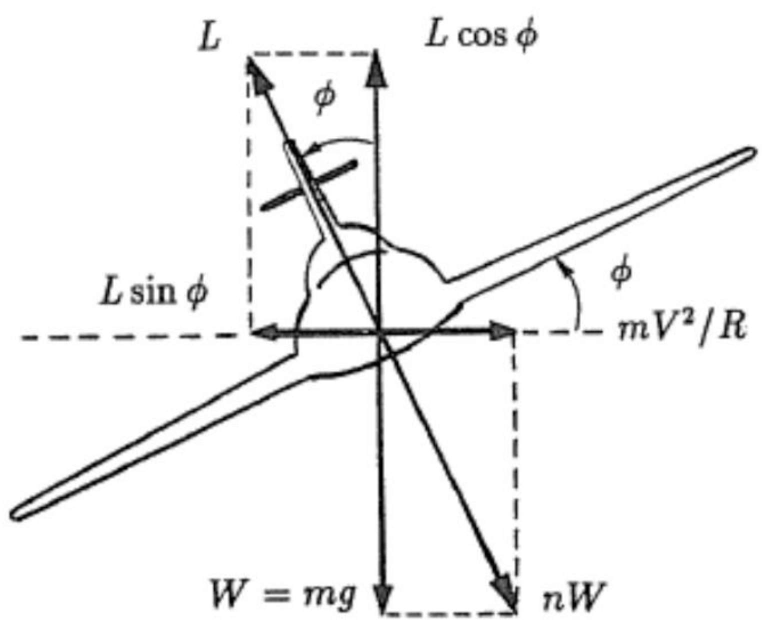
→ Equilibrium equations are simply:

$$L\cos\phi = W$$

$$L\sin\phi = \frac{mV^2}{R}$$

$$T = D$$

where, R is the radius of the circular turn



Turning

Load factor n is defined as the ratio, $n = \frac{L}{W}$

During a turn, the load factor can be so high to :
- harm the pilot
- damage the aircraft’s structure

From the balance equation in the vertical direction:

$$n = {}^{1}\!/_{\cos\phi}$$

During a turn, the lift must balance:
- the weight
- the centrifugal force

Turning radius

For a given load factor, the turning radius is expressed as

$$L\cos\phi = W$$
$$L\sin\phi = \frac{mV^2}{R} \quad \longrightarrow \quad R = \frac{V^2}{g\tan\Phi} = \frac{V^2}{g\sqrt{n^2-1}}$$

Expressing the airspeed V in terms of the load factor and lift coefficient

$$nW = \frac{1}{2}\rho S V^2 C_L \longrightarrow V^2 = \frac{2nW}{\rho S C_L}$$

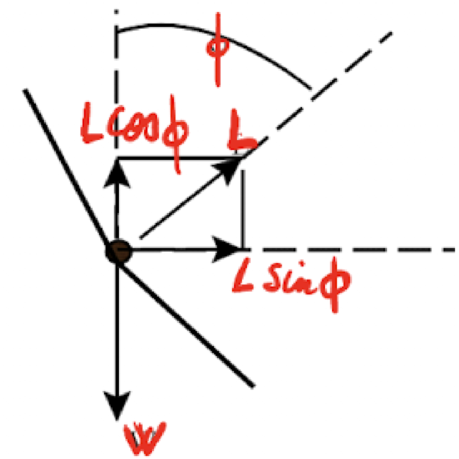
$$\longrightarrow \quad R = \frac{2W}{\rho g S C_L}\frac{n}{\sqrt{n^2-1}}$$

Low turn radius R if : large C_L, low altitude, high load factor, low wing loading (W/S)

Maximum turning rate

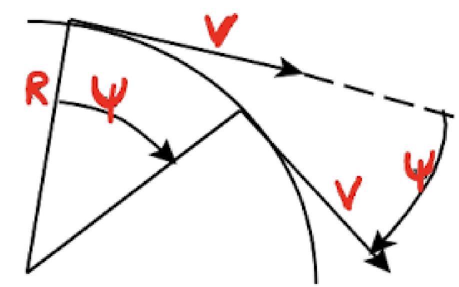
The turning rate can be expressed as

$$\frac{d\psi}{dt} = g\sqrt{\frac{\rho S C_L}{2W}\left(\frac{n^2-1}{n}\right)}$$



Rear View of Turn

Note that:
- The lift coefficient cannot exceed C_{Lmax}
- The maximum load factor is n_{max}



Top View of Turn

Then, the maximum turning rate is

$$\left(\frac{d\psi}{dt}\right)_{max} = g\sqrt{\frac{\rho S C_{Lmax}}{2W}\left(\frac{n_{max}^2-1}{n_{max}}\right)}$$

High turn rate if large C_L, low altitude, high load factor, low wing loading (W/S), i.e. same than low turn radius

Turning

Lift required for turning

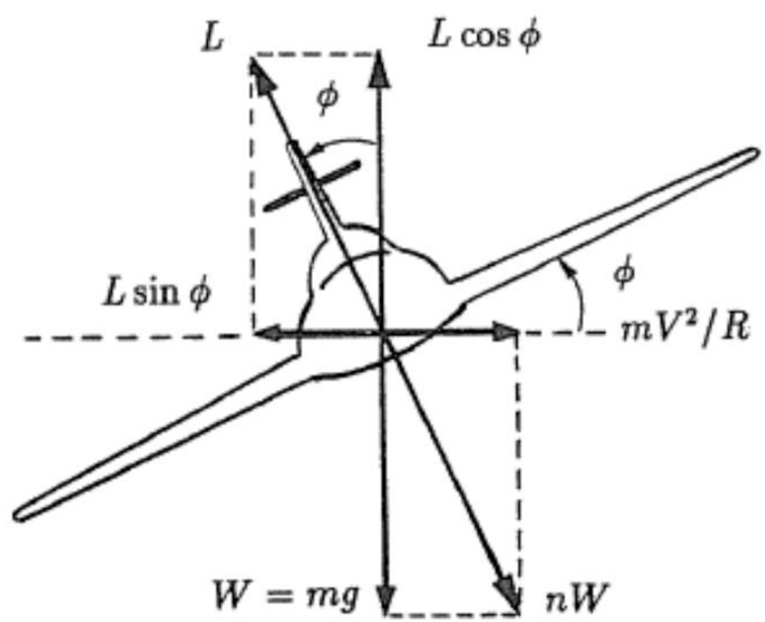
From the turn diagram:

$$nW = \sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2}$$

Furthermore, $L = nW$

Hence the required lift is simply:

$$nW = \frac{1}{2}\rho S V^2 C_L$$



Turning

Thrust required for turning

From the drag polar:

$$C_D = C_{D0} + \frac{C_L^2}{e\pi AR} = C_{D0} + \frac{1}{e\pi AR}\left(\frac{2nW}{\rho S V^2}\right)^2$$

The require thrust is then given by

$$T = \frac{1}{2}\rho S V^2 C_D = \frac{1}{2}\rho S V^2\left(C_{D0} + \frac{1}{e\pi AR}\left(\frac{2nW}{\rho S V^2}\right)^2\right)$$

Alternatively, $T = D = \frac{LD}{L} = nW\frac{C_D}{C_L}$

Assuming constant Lift/Drag ratio → Thrust proportional to load factor

Maximum load factor

Load factor cannot exceed:

- The aircraft structural limits

- The user (pilot, passenger) limits

It must be verified that the turn radius R corresponds to a load factor lower than n_{max}

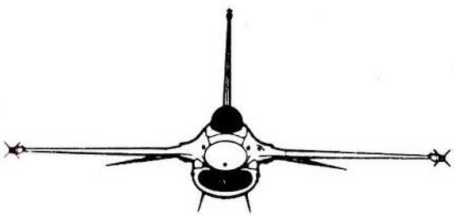
$$nW = \sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2} \quad \rightarrow \quad n_{max} = \frac{\sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2}}{W}$$

where, n_{max} is usually 2.5 for commercial transports
n_{max} can be 6 or higher for aerobatic aircraft

Turn diagram

F-16A

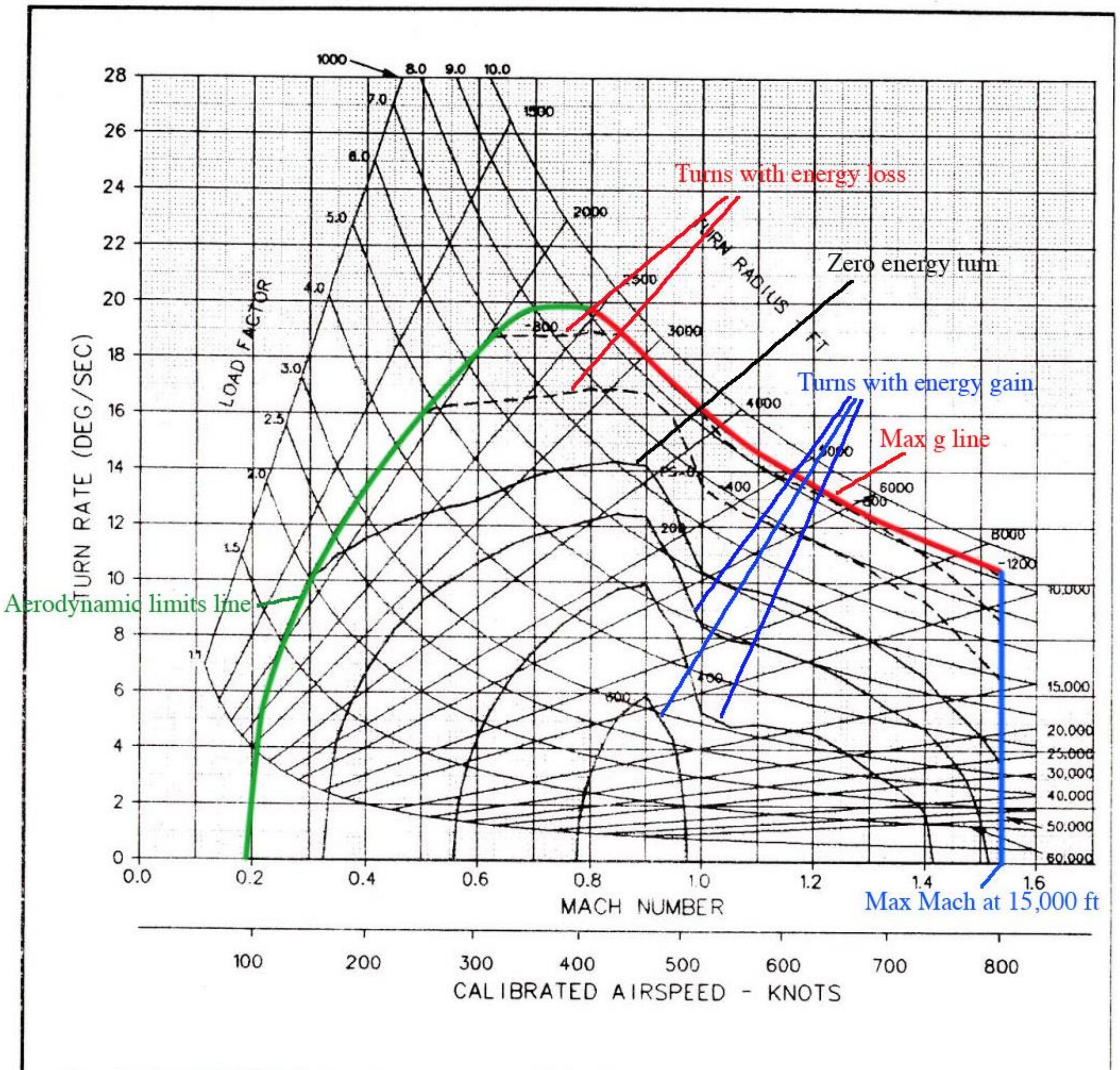
BLK 15 FMS



(2) AIM-9P3 IR Missiles
512 rds of 20 mm
50% internal Fuel = 3486 lbs (1581 kg)
Combat Weight = 20,875 lbs (9469 kg)

WING AREA	300 Sq Ft	(28 Sq M)
EMPTY WEIGHT	16131 Lbs	(7317 Kg)
INTERNAL FUEL	1073 US Gal	(4060 Liter)
	6972 Lbs	(3162 Kg)
TAKEOFF WEIGHT WITH (2) IR + GUN	24361 Lbs	(11065 Kg)
MAX EXTERNAL FUEL	1465 US Gal	(5545 Liter)
	9522 Lbs	(4318 Kg)
COMBAT WEIGHT	20875 Lbs	(9469 Kg)
MAX A/B THRUST AT SEA LEVEL (F100-PW-220NSI)	23744 Lbs	(10770 Kg)
		106 KN
MAX MIL PWR THRUST AT SEA LEVEL	14601 Lbs	(6623 Kg)
		65 KN
COMBAT T/W RATIO	1.14	
COMBAT WING LOADING	70 Lb/Sq Ft	(340 Kg/Sq M)
MAX TOGW	37500 Lbs	(17010 Kg)
MAX SUBSONIC DSGN. LOAD FACTOR	9.3 g's	

TURN PERFORMANCE AT 15000 FT (4572 m)
Utilizing Maximum Afterburner (Wet) Power



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Landing

Landing consists in **two phases**:

- Approach above a hypothetical obstacle to touch-down

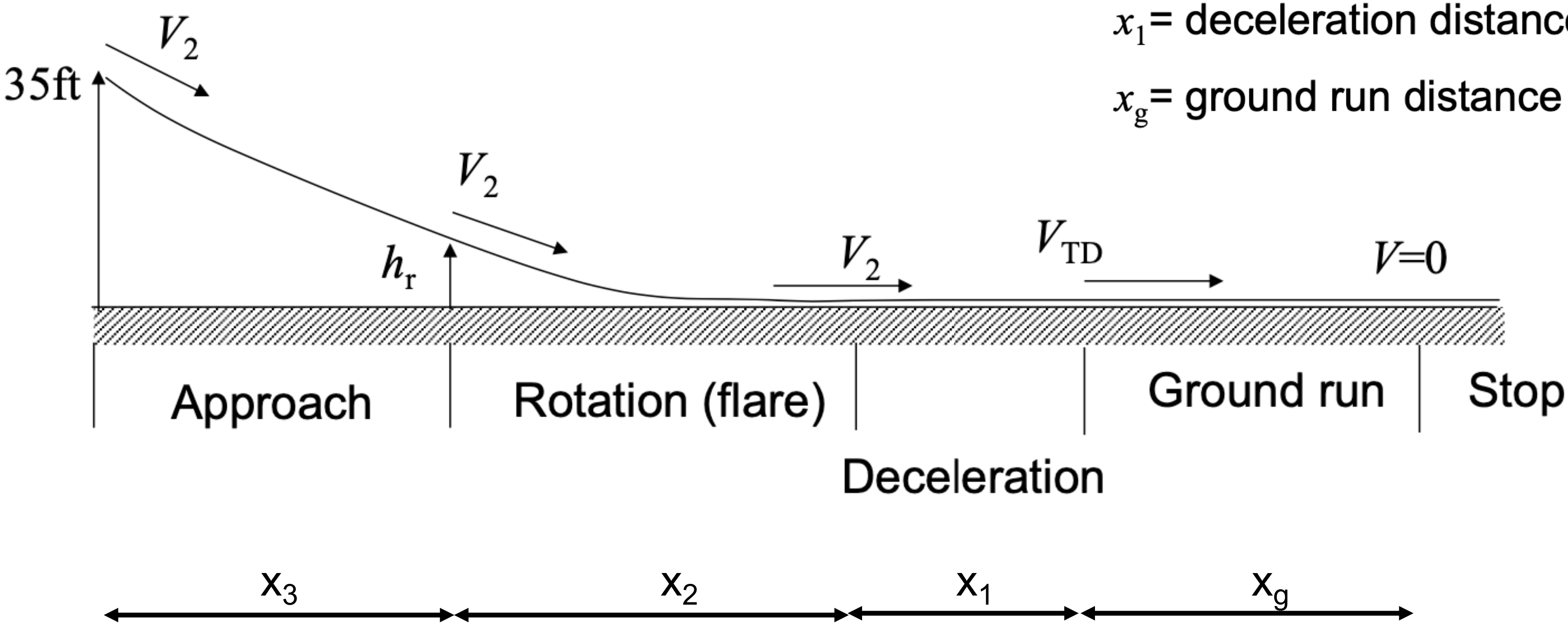
- Ground run to full stop

x_3= approach distance

x_2= rotation distance

x_1= deceleration distance

x_g= ground run distance



Landing

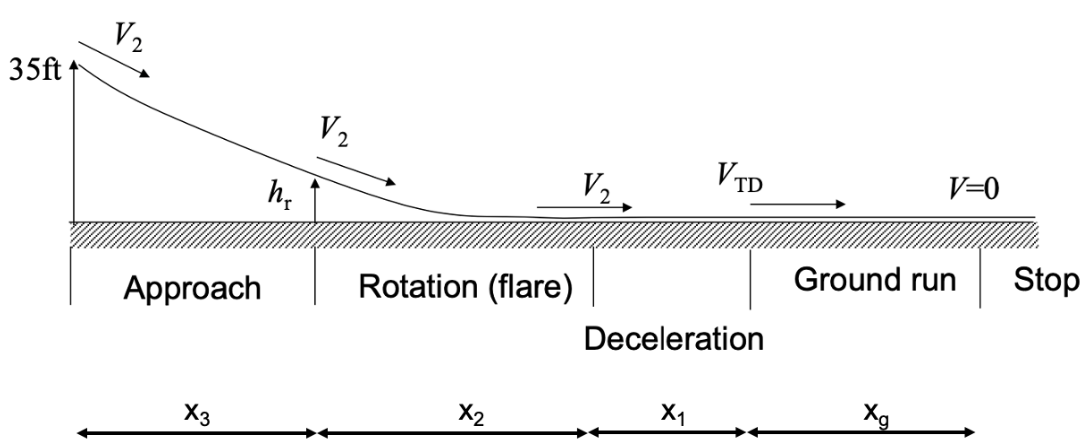
Approach

- Aircraft makes an approach along the axis of the runway
- A glide angle $\boldsymbol{\gamma}$ ranges between -2.5° and -3.5°
- The speed is $V_2 = 1.2\, V_{stall}$
- The height of the hypothetical object is h_{obj}
- The rotation height is h_r
- The approach distance is $x_3 = \frac{h_{obj} - h_r}{\tan\gamma}$
- The approach time is $t_3 = \frac{x_3}{V_2 \cos\gamma}$

Landing

Rotation

- Aircraft follows an arc with radius R
- Similarly to take off, $R = \frac{V_2^2}{g(n-1)}$
- The rotation distance is $x_2 = R \sin\gamma$
- The rotation time is $t_2 = \frac{\gamma V_2}{g(n-1)}$



Landing

Ground run

- After touch-down, aircraft speed must drop from V_{TD} to 0
- Distance of ground run can be approximated by

$$x_g = {V_{TD}^2}/{2\bar{a}}$$

where $\bar{a}$ is the mean deceleration

$$\bar{a} = \begin{cases} 0.30 - 0.35 \text{ for light aircraft with simple brakes} \\ 0.35 \text{ - } 0.45 \text{ for turboprop aircraft without reverse propeller thrust} \\ 0.40 - 0.50 \text{ for jets with spoilers, anti - skid devices, speed brakes} \\ 0.50 \text{ - } 0.60 \text{ as above, with nosewheel breaks} \end{cases}$$

Parametric design

Design for performance is an **optimization process**

<u>Objective</u>: **satisfy or exceed** all performance requirements

How: by finding the optimal combination of **parameters**:

- **Powerplant**
  - Take off thrust
  - number of engines
  - engine type
  - engine configuration

- **Wing**
  - Wing area
  - Aspect ration
  - High lift devices

Flow diagram